PYTHAGOREAN POWERS
or
A CHALLENGE TO PLATONISM

By

Colin Cheyne and Charles R. Pigden

I have tried to apprehend the Pythagorean power by which number holds sway above the flux. Bertrand Russell, *Autobiography*, vol. 1, Prologue.

The Quine/Putnam indispensability argument is regarded by many as the chief argument for the existence of platonic objects. We argue that this argument cannot establish what its proponents intend. The form of our argument is simple. Suppose indispensability to science is the only good reason for believing in the existence of platonic objects. Either the dispensability of mathematical objects to science *can* be demonstrated and, hence, there is no good reason for believing in the existence of platonic objects, or their dispensability *cannot* be demonstrated and, hence, there is no good reason for believing in the existence of mathematical objects which are genuinely platonic. Therefore, indispensability, whether true or false, does not support platonism.

Mathematical platonists claim that at least some of the objects which are the subject matter of pure mathematics (e.g. numbers, sets, groups) actually exist. Furthermore, they claim that these objects differ radically from the concrete objects (trees, cats, stars, molecules) which inhabit the material world.

We take the standard platonistic position to include the claim that platonic objects lack spatio-temporal location and causal powers. Many (perhaps most) mathematical platonists subscribe to this view.[1] But some who call themselves (or might be called) mathematical platonists eschew the standard position.[2] They maintain that mathematical objects *do* possess location and causality, although they retain some similarity to the kinds of things that Plato had in mind. We do not intend to enter into a terminological dispute as to which party of platonists truly deserves the name. Perhaps the people we shall call *non-standard* platonists are closer to the original Plato. After all, Plato himself was notoriously equivocal about the causal status of his Forms. However that may be, we take the majority (or perhaps the Anglo-American) view to be the standard view. On this

---

[1] 'As a mathematical platonist, I hold that mathematical objects are causally inert and exist independently of us and our mental lives' Resnik [19, p. 41]. See also [6, 7, 11 & 22].

[2] Notably Maddy [15].

theory, platonic objects have neither locations nor causal powers. And it is this theory that we intend to challenge.

One version of what we call non-standard platonism is particularly popular in Australia where it is associated with a sympathy for universals. The properties and relations proposed by David Armstrong are not ethereal and impotent beings confined to a transcendent non-spatial realm. Far from having *no* location, they are fully present in each of their *many* locations and contribute to the facts which are the relata of causal relations. A bonus for this robust form of realism is that mathematical entities (in particular, numbers) emerge from these universals as higher-level relations carrying with them the causal powers and the locatedness of the universals from which they emerge.[3] Lacking the 'other-worldly' aspects of standard platonism, the non-standard position might be better characterized as 'Aristotelian' or even 'Pythagorean', though some prefer the term 'Scientific Platonism'. But Australia (alas!) is not the world at large, and this Aussie Aristotelianism remains a minority opinion. Our quarrel is with standard platonism — the platonism that includes the claim that platonic objects are acausal. However, we return to the relationship between non-standard platonism and the indispensability argument towards the end of this paper.

There are well-known epistemological difficulties for standard platonism.[4] If platonic objects are so 'remote' and inert, how can we as human knowers, existing in space-time and the causal nexus, come to have knowledge of the existence and properties of such objects? Anti-platonists conclude that in the absence of a plausible account of how we acquire such knowledge, we should at least be agnostic about the existence of platonic objects.

Quine [17, ch. 1 & 18, ch. 20] and Putnam [16] argue that the methods by which we confirm scientific theories are the means by which we acquire knowledge of platonic objects. In outline, they argue as follows. Our best theories about the world postulate entities which we cannot observe (e.g. electrons) in order to make sense of our experiences. But those same theories postulate platonic objects (e.g. numbers, sets). These mathematical objects are just as indispensable to science as theoretical entities like electrons. Electron theory quantifies over numbers, just as it quantifies over electrons. So we have the same reason for thinking that numbers exist as we do electrons. Platonic knowledge is an indispensable part of scientific knowledge. Throw out the platonic bathwater and you lose the scientific baby.

Hartry Field is an anti-platonist. But he considers the Quine/Putnam

---

[3] See [1, 2, 4, 5 & 10].

[4] See Benacerraf [3].

indispensability argument to be the best available argument for platonism. Indeed, he considers there to be no other serious contenders [9, p. 8].[5] Consequently, Field has set himself the task of demonstrating that mathematical objects are not indispensable to our best current science [8]. He argues that the indispensability argument 'can be undercut if we can show that there are equally good theories and explanations that don't involve commitment to numbers and functions and the like' [9, p. 17].

There are two respects in which mathematical objects are supposed to be indispensable to science. They are indispensable when it comes to inference and they are indispensable in that our best scientific theories freely quantify over them. To prove that platonic objects are dispensable, and hence that we need not believe in them, Field has to do two things. First, he must show that platonic objects are not necessary for inference, and secondly, he must show that our best scientific theories can be nominalized, i.e. reformulated in such a way as to dispense with platonic entities.

We believe that he has completed the first part of his project.[6] He has demonstrated that if a nominalistic claim follows from a nominalized theory extended by a purely mathematical theory, then it follows from the nominalized theory alone. Thus, the mathematical theory need not be true in order that it be a useful aid to inference. But this result can only support the anti-platonist cause if our best scientific theories do not appeal to platonic objects. Hence the importance of the second part of Field's project. But this he has barely begun. All he has managed so far is a nominalized version of *Newtonian* gravitational mechanics. This theory is false and what is more has been shown to be so on empirical grounds. General Relativity Theory, which is at least unrefuted, has not been (and perhaps cannot be) nominalized. Furthermore, serious difficulties face any attempt to nominalize quantum mechanics (a current 'best' theory). These difficulties look ominous, perhaps insurmountable. Thus it may be that Field's program cannot be carried through.

Two epistemic possibilities open up before us. The first is that Field's project will succeed and our best science will be nominalized. In that case the indispensability

---

[5] In particular, Field is unimpressed by Crispin Wright's recent attempts to revive Fregean platonism. See Wright [22] and Field [9, ch. 5]. We agree with Field's admittedly contentious view, but do not defend it here.

[6] This too is a contentious claim. Shapiro [20] has argued that if consequence is given a proof-theoretic reading, Field's argument fails, but that if consequence is given a semantic reading, Field is implicitly quantifying over platonic objects. Field replies [9, ch. 4] that in [8] he did employ the semantic conception of consequence (and hence that his argument is a success) but that the same result can be obtained if consequence is interpreted as a modal concept. This seems to us a satisfactory answer.

argument collapses and with it the best case for platonism. Alternatively, Field's project will fail and the best science will resist nominalization. This is not to suppose that it will be demonstrated that nominalization is logically impossible. (Indeed, true Quineans would be shocked at the idea.) Rather, as with the search for the Loch Ness monster, the persistent failure of our best efforts to find such a nominalization will be sufficient reason to suppose that, like the Loch Ness monster, there is no such nominalization to be found. In other words, we should conclude on empirical grounds that theories which quantify over mathematical objects are better than their nominalistic rivals. In this scenario, Nature cries 'No!' to nominalized theories but 'Maybe!' to ontologically loaded ones.

Now if our best science is condemned to quantify over mathematical objects, this would tend to show that such objects exist. But by the same token, standard platonism would seem to be in trouble. Why should theories which quantify over certain objects do better than theories which do not? One explanation is readily to hand. If we are genuinely unable to leave those objects out of our best theory of what the world is like (at least, that part of the world with which we causally interact), then they must be responsible in some way for that world's being the way it is. In other words, their indispensability is explained by the fact that they are causally affecting the world, however indirectly. The indispensability argument may yet be compelling, but it would seem to be a compelling argument for the existence of entities *with* causal powers.

Why couldn't a mathematical object be a constituent of a causal fact (or event or state) and yet itself be causally inert? Perhaps it could. But either its presence would make no difference to the effects of that fact and so any mention of it could be omitted from an explanation of those effects, or its presence would make a difference to the effects of the fact in which case it would be perverse to deny it causal efficacy. For example, suppose the fact that there are three cigarette butts in the ashtray causes Sherlock to deduce that Moriarty is the murderer, and that if there had been more or fewer butts he would have deduced otherwise. The fact that there are three cigarette butts in the ashtray is clearly causal. Suppose that the number three is an indispensable constituent of that fact.[7] Could platonists then claim that the number three is an acausal constituent of the fact? On the face of it, no. It's being a constituent of the fact makes a causal difference. If the number two or the number four were in its place, the effects would differ. What more

---

[7] It isn't, of course. Frege has shown how we can say that there are three butts in the ashtray without reference to the number three. But if we want an example in which indispensability is more likely, we shall need to delve into the realms of General Relativity or quantum mechanics. If platonists believe that they can strengthen their case with such an example, we look forward to seeing it.

is needed for it to qualify as an object with causal powers?

Our challenge to platonists is for them to provide an explanation for the indispensability of objects whose presence (they claim) makes no causal difference. And it will need to be a better explanation than our suggestion that they are indispensable because their presence *does* make a causal difference.

So whether or not Field's project succeeds, standard platonism seems to be in trouble. Either the project succeeds and the indispensability argument must be abandoned, or the project fails and, although there is good reason to believe in mathematical objects, there is also good reason to believe that they are not acausal. Either way, standard platonism faces a challenge.

The challenge is most squarely directed at those platonists who believe that the indispensability argument provides our best reason for adopting standard platonism. Does this mean that other platonists ('*dispensable* standard platonists' perhaps) are off the hook? Well, it will still be a challenge (if not such a serious one) to those who believe that indispensability provides *some* reason for believing standard platonism. And some standard platonists do appear to believe this, otherwise they would not take the trouble to argue that Field's project cannot succeed.[8]

What about those who claim that the indispensabilty argument gives us *no* reason for believing in standard platonism? Presumably they will have some other reason for believing in the existence of platonic objects. Their grounds for adopting platonism will be either empirical or *a priori*. If they are empirical, then a similar argument can be run against them, and they face a similar challenge. If *a priori*, then they should not be surprised if mathematics turns out to be dispensable. On the other hand, if it proves to be indispensable, they face the challenge of providing an explanation of this fact, but one which should not appeal, however indirectly, to the causal efficacy of mathematical objects.

Many platonists believe that the success of Field's project would count against platonism. And in this they are surely correct. But what they do not realize is that the *failure* of Field's project would *also* count against platonism. For it would create a problem that standard platonists are ill-equipped to solve – how to account for the indispensability of numbers in describing the causal nexus whilst absolving those numbers from the sordid taint of causality. One option is what might be called the neo-Kantian or 'framework' solution. Numbers are needed to underwrite *any* conceivable causal order but they themselves play no part in the proceedings. They provide a sort of metaphysical

---

[8] See for example [11, 12, 13, 19 & 20].

framework for any possible physics — an indispensable, indeed, a necessary backdrop for the causal show. But though there could be no causal structure without numbers, numbers are not implicated in the causal shenanigans described by any science, whether actual or merely possible. This theory requires a lot of work if it is to be anything more than a collection of figures of speech.[9] But in so far as it can be made sense of, the framework theory is false. Field's achievements as a nominalizer have demonstrated this. For he has succeeded in nominalizing Newtonian physics. This physics describes a simpler set of worlds than the one we actually inhabit. In these unsophisticated Newtonian Edens (free from the serpent of Relativity), there are causal laws and causal histories but numbers are superfluous to requirements. They are not needed to underpin the causal goings-on, and if they exist at all, they constitute an infinity of spare parts, of underpinnings which underpin nothing. Far from constituting a necessary framework for any conceivable physics, they turn out to be unnecessary to the physics which everyone believed in until Einstein came along. According to the framework theory, numbers are necessary because they are presupposed by any conceivable causal system. By nominalizing Newtonian mechanics, Field shows that this is not so. For we can conceive of a causal order (namely that described by Newton) which can do without the framework.

The new problem that the platonists face is this: How can a set of necessary beings help explain a contingent set of facts (namely the facts accounted for by Einsteinian physics) when they would not be needed if the facts were otherwise (i.e. such as to confirm Newtonian physics)? Numbers would be like a modally capricious God who in some worlds stoops to create whilst in others he prefers to reign in splendid isolation. Such a God might exist necessarily, but his relational properties would be contingent. For in some worlds he would be causally active and in others not. So too with numbers. If they are needed to account for the goings on in some worlds and not others, this suggests that they are causally active in some worlds and not others. And if Field's project fails, this suggests that one of the worlds in which numbers are causally active is the actual one.

Given the success of Field's project so far, the ultimate failure of his enterprise would be just as damaging to standard platonism as his total triumph. On the other hand, the success or failure of Field's project would differ in their respective impacts on *non-standard*

---

[9] 'How can numbers play a necessary part in causal explanations even though they exercise no causal powers?' 'Well, they're part of the framework.' 'What is this framework?' 'Well, of course, "framework" is only a metaphor, since in the real world frameworks actually do a lot of causal work, but what I mean by "framework" is a kind of a thing which ... well, um, … what it *does* is it allows numbers to play a necessary part in causal explanations even though they exercise no causal powers.' 'Gee, thanks!' Cf. Stove [21, p. 53] on the synthetic *a priori*.

platonism and on a pure nominalism which rejects all mathematical entities. Success for Field would count against the former, but favor the latter.

Recall that by 'platonic objects', we mean entities which are acausal. For non-standard platonists (in which group we include the Aussie Aristotelians), mathematical entities are located within the causal nexus. If Field's project succeeds, then mathematical entities can be dispensed with. Non-standard platonism is in trouble, since we do not need to posit numbers to make sense of the causal flow. If the mathematical objects are *purely* mathematical (that is, if the *only* reason to believe in them is that they underwrite the truth of useful mathematical claims) then there is no reason to believe in them and they can be safely dismissed. If they 'emerge' from a system of universals which have independent claims to being (that is, if we get them as a sort of metaphysical bonus) then the system is undermined but not discredited. It ceases to be a plus for the realist metaphysic that it can explain both the truth and the utility of mathematical claims. For their utility can be satisfactorily accounted for without supposing them to be true. But the fact (if it turns out to be a fact) that mathematical entities are dispensable does not entail that they *must* be dispensed with. If there are other reasons for believing in the universals and the numbers emerge from the universals, we might still have reason to believe in the reality of numbers, even though the reasons would not be as compelling as they were before. We should perhaps conclude that numbers exist, but that they have no role in a scientific account of the world. Or better, that numbers exist, but that the utility of mathematics provides no reason for supposing that they do.

Should Field's project fail, however, there would be no threat to the non-standard position. Indeed, non-standard platonists usually see the indispensability argument as lending support to their position. In our view, they are quite right to do so.

Recently Ruth Richardson, a former finance minister in New Zealand, published a memoir under the title *Making a Difference*. Her implied boast was that she (unlike most finance ministers) was causally efficacious. The indispensability argument claims that numbers, sets, etc., *make a difference* (which is why they cannot be dispensed with). But it is difficult to see how they can do this without being causally efficacious. Hence numbers, if they are to be believable, must be like Ruth Richardson.

University of Otago

REFERENCES

1. D. M. Armstrong, *A Theory of Universals: Universals and Scientific Realism, vol. 2* (Cambridge: Cambridge University Press, 1978).

2. D. M. Armstrong, *A Combinatorial Theory of Possibility* (Cambridge: Cambridge University Press, 1989).

3. P. Benacerraf, 'Mathematical Truth', *Journal of Philosophy* 70 (1973) pp. 661-679.

4. J. Bigelow, *The Reality of Numbers* (Oxford: Clarendon Press, 1988).

5. J. Bigelow & R. Pargetter, *Science and Necessity* (Cambridge: Cambridge University Press, 1990).

6. J. R. Brown, 'π in the Sky', in [14, pp. 95-120].

7. J. P. Burgess, 'Epistemology and Nominalism', in [14, pp. 1-15].

8. H. Field, *Science Without Numbers* (Oxford: Blackwell, 1980).

9. H. Field, *Realism, Mathematics and Modality* (Oxford: Blackwell, 1989).

10. P. Forrest & D. M. Armstrong, 'The Nature of Number', *Philosophical Papers* 16 (1987) pp. 165-186.

11. B. Hale, *Abstract Objects* (Oxford: Blackwell, 1987).

12. B. Hale, 'Nominalism', in [14, pp. 121-144].

13. B. Hale & C. Wright, 'Nominalism and the Contingency of Abstract Objects', *Journal of Philosophy* 89 (1992) pp. 111-135.

14. A. D. Irvine (ed.), *Physicalism in Mathematics* (Dordrecht: Kluwer Academic Publishers, 1990).

15. P. Maddy, *Realism in Mathematics* (Oxford: Clarendon Press, 1990).

16. H. Putnam, 'Philosophy of logic', in *Mathematics, Matter and Method* 2nd edn (Cambridge: Cambridge University Press, 1979).

17. W. V. O. Quine, *From a Logical Point of View* 2nd edn (New York: Harper & Row, 1961).

18. W. V. O. Quine, *The Ways of Paradox* (New York: Random House, 1966).

19. M. D. Resnik, 'Beliefs About Mathematical Objects', in [14, pp. 41-71].

20. S. Shapiro, 'Conservativeness and Incompleteness', *Journal of Philosophy* 80 (1983) pp. 521-531.

21. D. Stove, *The Plato Cult and Other Philosophical Follies* (Oxford: Blackwell, 1991).

22. C. Wright, *Frege's Conception of Numbers as Objects* (Aberdeen: Aberdeen University Press, 1983).