Solving the Proportion Problem: A Plea for Selectivity

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Abstract. [12] argues that quantificational adverbs are unselective binders over individuals. The Lewisian analysis, however, fails to recognize the ambiguity in some quantificationally modified conditionals. That the Lewisian approach cannot predict some attested reading is known as the "proportion problem." I propose a solution based on the following ideas: (a) quantificational adverbs bind selectively; (b) a singular indefinite and its anaphoric pronoun may introduce a plural discourse referent, and (c) plural predication is elusive.

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1 The Proportion Problem

[12] argues that quantificational adverbs (hereafter Q-adverbs) such as 'always' and ''usually' are unselective binders. The Lewisian analysis consists of the following theses:

(1) a. Conditionals are analyzed as having a tripartite structure (i.e. Q-adverb: restrictor: nuclear scope)[1]
 b. Indefinites introduce free variables in the logical form.
 c. Q-adverbs range primarily over individuals.
 d. Q-adverbs are unselective binders that bind all variables in their scope.

The analysis, however, suffers from the so-called "proportion problem." ([5]) To illustrate, consider (2). Is it true given (3)?[2]

(2) If a farmer owns a donkey, he usually beats it.

[1] Lewis' analysis of 'if' is in fact three-way ambiguous: (a) 'if'-clauses in quantificationally modified conditionals are analyzed as restrictors ([12]); (b) indicative conditionals are analyzed as material implication ([13]); (c) counterfactuals are analyzed as variably strict conditionals ([11]).

[2] Underline indicates that a donkey is beaten by its owner.

(3)

Farmer	Donkey
A	$\underline{d_1, d_2, d_3, d_4, d_5}$
B	d_6
C	d_7
D	d_8

According to the Lewisian analysis, (2) is true iff the majority of the admissible value assignments that satisfy the restrictor satisfy also the nuclear scope. That is, most of the farmer-donkey pairs in which the farmer owns the donkey, the former beats the latter. (2) is true given (3). Yet there is a very natural reading that it is false. The Lewisian analysis is problematic because, if we take (2) to be ambiguous, it under-generates; if (2) is not ambiguous but simply false, the analysis is plainly wrong.

Now consider a different scenario:

(4)

Farmer	Donkey
A	$d_1,...,d_{10}$
B	$\underline{d_{11}, d_{12}},d_{13}$
C	$\underline{d_{14}, d_{15}},d_{16}$

Here again, the traditional Lewisian analysis predicts that the Q-adverb modified donkey sentence (2) is false, since merely 4 out of the 16 farmer-donkey pairs satisfy the condition specified in the nuclear scope. But this answer is lacking and shows insufficient sensitivity to the complexity of the model. To be sure, for those who think that a donkey-owing farmer is a donkey-beater just in case he beats all his donkeys, (2) is false. On the other hand, (2) is true for those that think mistreating just one donkey is enough for the bad name. For example, those who fights for animal rights or work for animal welfare and protection would not hesitate to call out a farmer who beats at least one of his donkeys. The question is, ultimately, what qualifies a donkey-owning for a donkey-beating farmer.

2 Diagnosis

Quantifying over farmer-donkey pairs, as the Lewisian analysis predicts, yields just the symmetric reading. The proportion problem shows that we need also the asymmetric reading where the farmers somehow carry more weight. However, there is no straightforward quantification over the (donkey-beating) farmers, and we do not want to quantify over just the donkeys that are mis-treated. Drawing on Heims idea that the occurrence of pronouns inserts indirect pressure on how the restrictor should be interpreted, we argue that all variables in the nuclear scope must be quantificationally bound; however, the quantification cannot be the standard unselective binding.

Meanwhile, it is unfortunate that previous studies on the proportion problem focus on consistent scenarios[3] only. Due to the oversight of non-consistent scenarios, the elusiveness of plural predication has yet to received its due attention. The puzzle about how the Q-adverb modified donkey sentence is to be evaluated with respect to (4) is essentially how relational predication works for plurals. If one thinks, as we do, that donkey sentences such as S1 is not just about farmers that own one donkey only, we need to consider not only if a relational predicate, such as 'beat, holds between a farmer and each of his donkey, but also if the predicate holds between a farmer and the collection of his donkeys as a whole.

That native speakers do oscillate their judgements regarding (4) is suggestive.[4]

It indicates a sentence like 'a farmer beats his donkeys can be made true by various types of scenarios. That there is some grey area in the truth conditions is evidence that there is a fundamental indeterminacy in plural predication. It is unfortunate that pervious studies of the proportion problem focus on scenarios where the relation described by the matrix predicate holds consistently, that is, each farmer beats either all or none of his donkeys. Due to the oversight of non-consistent scenarios (where a farmer does not beat all of his donkeys), the elusiveness of plural predication does not receive its due attention.What is puzzling about (2) when evaluated with respect to (4) is essentially how relational predication works for plurals. Once we abandon the assumption of relative uniqueness, we need to seriously consider the possibility of non-consistent relationship between a 'boss' and its many 'dependents.' If we deem it is possible that a farmer owns more than one donkey, we need to consider not only if a relational predicate (e.g. 'beat') holds between a farmer and each of his donkey, but also if it holds between a farmer and the whole collection of his donkeys.

To summarize, the proportion problem exemplifies an ambiguity in what the Q-adverbs should bind, which is connected to how the restrictor should be understood. The standard Lewisian analysis predicts only the symmetric reading, where the Q-adverbs binds unselectively; the preferred asymmetric reading, on the other hand, rests on an alternative interpretation of the restrictor according to which it is not about the farmer-donkey pairs. Therefore, to obtain the asymmetric reading, the Q-adverbs needs to bind more selectively. We maintain that the proper construal must take into account two points. First, all the variables in the consequent need to be quantificationally bound. Second, once we admit the

[3] By consistent scenarios I mean where the relation described by the matrix predicate holds consistently; e.g. a farmer beats all of his donkeys.

[4] Here is a relevant quote from [16]: "[regarding "Most farmers who own a donkey beat it."] does it mean that most farmers who own a donkey beat all of the donkeys they own, that most farmers who own a donkey beat most of the donkeys they own, or that most farmers who own a donkey beat some of the donkeys they own? I am simply not sure, and informants I have consulted have not expressed strong or consistent opinions. This does not obviate the need for an analysis, since people do have intuitions about certain situations. My own rationalization of the data is that people have firm intuitions about situations where farmers are **consistent** [my emphasis] about their donkey-beating." p.256.

need to collapse some farmer-donkey pairs, we must address plural predication. As a synthesis of these two points, the binding of the ‘donkey variable will have to be special.

3 Solution

We propose a new solution based on the following ideas:

(5) a. (a) Q-adverbs bind selectively;
 b. (b) singular indefinites and their anaphoric pronouns may introduce plural discourse referents;
 c. (c) plural predication is elusive.

Our formal analysis is couched in DRT. In standard DRT, a singular indefinite such as ‘a farmer’ introduces a discourse referent/variable, which is matched to a single individual via the embedding function. We keep the indefinite-as-variable thesis intact but argue that a singular indefinite may introduce into DRS a *plural* or *sum* discourse referent. An indefinite description can receive a ‘collective’ reading where it introduces a set-indicating variable. Such a set is maximal in the sense that its members are all the individuals satisfying the relevant conditions.[5]

Here is the construction rule: a singular indefinite such as ‘a donkey’ introduces invariably into K a sub-DRS K_1, and then there are three options:

(i) the sub-DRS remains unchanged, resulting in relative uniqueness *à* la Kadmon

(ii) the sub-DRS converges to the main DRS K

(iii) the sub-DRS triggers a “plural introduction” such that a sum discourse referent Z is added to the universe of the main DRS

$$\text{(i)}\ \begin{array}{|c|}\hline \\ \hline \begin{array}{|c|}\hline \text{y}\\ \hline \text{donkey(y)}\\ \hline\end{array}_{K_1} \\ \\ \hline\end{array}_{K} \quad \text{(ii)}\ \begin{array}{|c|}\hline \text{y}\\ \hline \text{donkey(y)}\\ \hline\end{array}_{K} \quad \text{(iii)}\ \begin{array}{|c|}\hline \text{Z} \\ \hline \text{Z}=\sum\text{y}\begin{array}{|c|}\hline \text{y}\\ \hline \text{donkey(y)}\\ \hline\end{array}_{K_1} \\ \\ \hline\end{array}_{K}$$

Solving the proportion problem calls for the asymmetric reading, which the third option facilitates. The verification conditions for various DRS conditions follows the interpretation of plurals in ([7]).

For our purpose, the only verification condition that needs to be noted is:[6]

(6) f verifies Y$=\sum$y K iff f(Y)$=\sigma a\exists g[f \subseteq g\&Dom(g) = Dom(f) \cup U(K)\&$ $g(d) = a\&g$ verifies K in M]

[5] This way, we can attribute the ambiguity between the symmetric reading and the asymmetric reading to the ambiguity in interpreting indefinites.

[6] We adopt the following from chapter 4 in [7] and [18].

Following our proposal, (2) is analyzed as:

(7)
$$\begin{array}{|c|}\hline \\ \hline \begin{array}{|c|}\hline \text{x, Z} \\ \hline \text{farmer(x)} \\ \text{Z}=\sum \text{y} \begin{array}{|c|}\hline \text{y} \\ \hline \text{donkey(y)} \\ \text{own(x,y)} \\ \hline \end{array} \\ \hline \end{array} \quad \left\langle \begin{array}{c} \textit{most} \\ x \end{array} \right\rangle \quad \begin{array}{|c|}\hline \text{u, V} \\ \hline \text{u=x} \\ \text{V=Z} \\ \text{beat(u,V)} \\ \hline \end{array} \\ \hline \end{array}$$

An embedding function *f* verifies (7) iff:

most extensions g of *f*, where Dom(g)=Dom(f)$\cup\{x, Z\}$ that verify the conditions in K_1, farmer (x) and $\text{Z}=\sum \text{y} \begin{array}{|c|}\hline \text{y} \\ \hline \text{donkey(y)} \\ \text{own(x,y)} \\ \hline \end{array}$
can be extended to h, where in this case h=g, such that h verifies K_2, i.e.,

$$\begin{array}{|c|}\hline \text{u, V} \\ \hline \text{u=x} \\ \text{V=Z} \\ \text{beat(u,V)} \\ \hline \end{array}$$

A function *f* verifies $\text{Z}=\sum \text{y} \begin{array}{|c|}\hline \text{y} \\ \hline \text{donkey(y)} \\ \text{own(x,y)} \\ \hline \end{array}$ in a model M=$< D, I, \sqcup >$ iff:

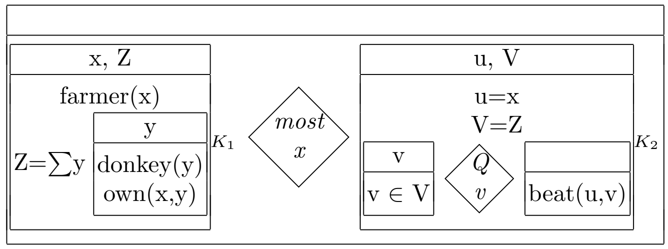
f(Z)=$\sigma a \exists g[f \subseteq g \& Dom(g) = Dom(f) \cup \{y\} \& g(y) = a \& g(y) \in I(donkey) \&$
$< g(x), g(y) > \in I(own)$

Given the verification conditions so sketched, it should be clear that the *primary quantification* involved in (7) is the quantification over x, or the farmers. However, the verification condition of DRS K_2 is tricky. This is due to the problem of plural predication. We may address the issue using two approaches. The first is to be minimalistic, the second explicit. Suppose a function maps u to an individual A, and his three donkeys d_1, d_2 and d_3. We then state:

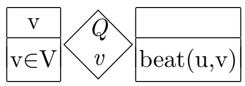
(8) beat(u,V) is true iff A beats d_1, d_2 and d_3.

(8) acknowledge the underlying looseness in plural predication and declare that it is not the job of the semanticists to decide when a beating relation holds between a man and his donkeys. That is the job for people who study the metaphysics of beating. This is the minimalistic approach.

Alternatively, we may expand (7) like this:

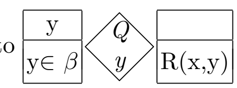
(9) 

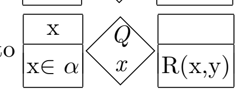
The crucial difference between (7) and (9) is the transition from 'beat(u,V)' to the duplex condition

(10) 

More generally, we postulate a construction rule for "plural elimination":

(11) Let x and y stand for singular discoure referents and α and β stand for plural discourse referents:

a. A condition R(x,β) is transformed to 

b. A condition R(α,y) is transformed to 

The representation delineated in (9) has the advantage of making the *second* quantification explicit, confirming the idea that all pronouns occurring in the nuclear scope must be quantified over. The pronoun 'he', represented by u, is quantified by 'most'; the pronoun 'it', represented by V, is quantified indirectly via the secondary quantification Q over v. The second quantifier Q is left unspecified in (9). This underspecification is intended to reflect the flexibility with respect to the secondary quantification over the donkeys. While we prefer this second, more explicit solution, we remain neutral whether the first approach is feasible.

Below we illustrate how our proposal would deal with a number of different scenarios. Take (12):

(12)

Farmer	Donkey
A	d_1,...,d_{10}
B	$\underline{d_{11}}$, d_{12},d_{13}
C	$\underline{d_{14}}$, d_{15},d_{16}

If someone judges the donkey sentence 'If a farmer owns a donkey, he usually beats it' to be true with respect to (12), we can infer that for her, the secondary quantifier Q is equivalent to the existential quantification '$\exists$.' In contrast, if someone judges the sentence to be false, then we know that for her, Q has a quantificational force stronger than '$\exists$.'

Consider again (2), which is repeated here as (13):

(13)

Farmer	Donkey
A	d_1,...,d_{10}
B	$\underline{d_{11}, d_{12}}, d_{13}$
C	$\underline{d_{14}, d_{15}}, d_{16}$

With respect to (13), if one judges the adverbially modified donkey conditional to be false, it means that for her, Q is not existential. On the other hand, if she judges the sentence to be true, then Q might be '$\exists$' or something stronger, such as 'most.' Note that reelative to the same scenario, different speakers may have divergent criterion regarding Q, and one speaker can have varying specifications of Q relative to different scenarios. Furthermore, depending on the relational predicate in question ('beat' in the current example), one may have a particular preference for some Q.

4 Final Remarks

The analysis we advance here have four central theses: (i) every pronoun that appears in the nuclear scope of a conditional must be quantified over; (ii) a singular indefinite and its anaphoric pronoun may introduce a plural discourse referent; (iii) when a conditional contains two pronouns, the corresponding discourse referents may receive distinct quantifications, and finally (iv) plural predication is elusive.

Recall Heim's observation that the presence of a pronoun in the nuclear scope exerts pressure on how the restrictor should be constructed. Our first point that all the pronouns in the nuclear scope must receive some quantification is much like an extended argument stemming from that idea.

Our second point addresses the challenge presented by conditionals with two pronouns. We argue that singular indefinites and their anaphoric pronouns may receive a collective reading and introduce into DRS a sum discourse referent. This effectively echoes Neale's (1990) claim that singular donkey pronouns are semantically numberless.[7] What we argue is that for the sake of obtaining the proper asymmetric reading, we adopt the "plural introduction" strategy so that a singular indefinite and its anaphoric pronoun can be understood as possibly plural.

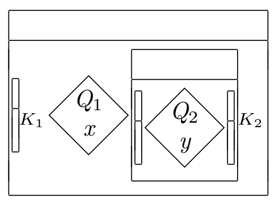
In the foregoing discussion, we only interpret 'a donkey' collectively, but it is possible that 'a farmer' receives a pluralization treatment. If we consider

[7] See [14], Chapter 6, especially section 6.3. However, due to the difficulties [9] raises to salience, I remain neutral about applying the number neutrality account across the board.

scenarios where the co-ownership of donkeys are relevant, this is what we will need. In principle, we can have either 'a farmer,' 'a donkey,' or both to be read as introducing a plural individual. However, if we confine ourselves to scenarios where each farmer owns at most one donkey and any donkey is owned by at most one farmer, it does not matter which indefinite description receives the collective reading or if they both do.

Regarding the third point, we believe the split of quantification should be welcomed. Given a uniform treatment of conditionals as triggering duplex conditions on a par with sentences with quantified noun phrases, we now have a general scheme for their DRS representation:

(14) For any conditional 'Q-adverb, if ϕ, ψ,' let K_1 represent the restrictor and K_2 represent the nuclear scope. The DRS for the conditional is:

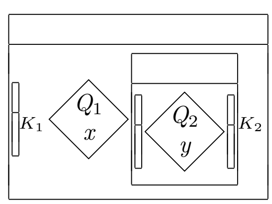


where Q_1 corresponds to the quantificational force of the Q-adverb in use.

The secondary quantification Q_2 is optional in two senses. First, when there is only one donkey pronoun in the nuclear scope, nothing will trigger the secondary quantification. Secondly, even if there are two donkey pronouns, we may choose to be parsimonious with respect to the representation and leave the secondary quantification to interpretation (and metaphysics).

Complex DRSs triggered by quantified phrases also undergo minor changes. $K_1(Q_1x)K_2$ now becomes:

(15)



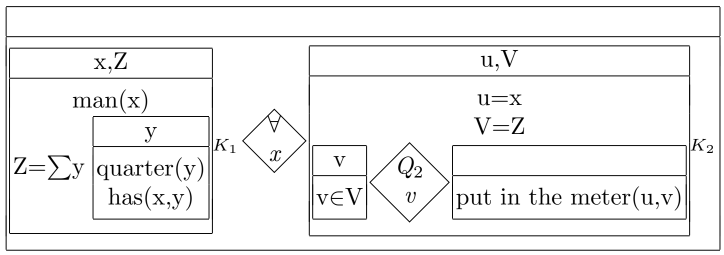
where the secondary quantification Q_2 is needed only when there is a donkey pronoun. When there is no donkey pronoun in the quantified sentence, e.g., 'Most farmers who own a donkey are rich," nothing will trigger the secondary quantification.

Besides contributing to the desired asymmetric reading to handle the proportion problem, the separation of quantification naturally lends itself as an explanation of the difference between the weak and strong readings.[8]

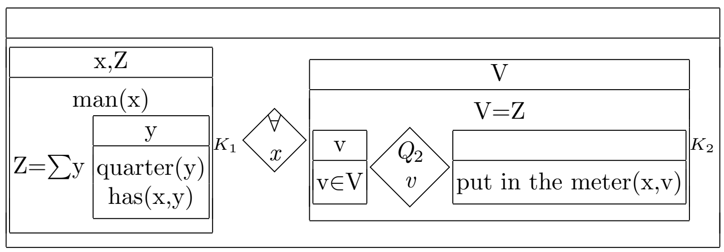
(16) a. If a man has a quarter, he puts it in the meter.
b. Every man who has a quarter puts it in the meter.

The respective representation for (16a) and (16b) are:

(17) a.



b.



The strong reading results from taking the secondary quantification Q_2 to be universal, and the weak reading results from taking Q_2 to be existential.

In short, while the proportion problem demonstrates the need to separate the quantification so that the (non-universal) Q-adverb binds more selectively, the demand for splitting the quantification is already present when we are charged with accounting for the weak reading. It is desirable to have a unified machinery that provides the required division.

Finally, regarding the fourth point, that plural predication is loose is clearly exemplified in the following:

(18) a. At the end of the press conference, the reporters asked the president questions.[9]
b. At the end of the press conference, the reporters asked the presidents questions.
c. At the end of the press conference, the reporters asked the president a question about gun control regulations.

[8] For more discussions on the weak and strong readings, see [15], [1], [8], [10] and [3], among others.
[9] This is from [17], which he attributes to [2].

The truth of (18a) does not depend on every single reporters at the press conference asked the president a question. One or more reporters might have asked one or more questions, but there might be one or more reporters who did not asked any. Similarly, the truth of (18b) does not require that every reporters asked a question and/or that every president was asked a question. Perhaps bearing more directly on my proposal is (18c). It is not transparent the singular indefinite 'a question' entails that there was only one question asked about the new stimulus package. Our intuition is that several reporters could have each asked such a question, but other speakers might have a different judgement.

Returning to the initial Lewisian account of adverbs of quantification and conditionals, we have come to realize that in order to handle the proportion problem, besides the assumption of unselective binding, other modifications are necessary:

(19) a. Conditionals are analyzed as having a tripartite structure.

b. Indefinites introduce free variables in the logical form. ⇒ **singular** indefinites may introduce **plural** variables.

c. Q-adverbs range primarily over individuals.

d. Q-adverbs are unselective binders that bind all variables in their scope. ⇒ Q-adverbs bind **selectively** one of the variables in their scope.[10]

The separation of quantification together with the elusiveness of plural predication suggests that there may be an inherent indeterminacy of donkey sentences. The indeterminacy is subject to various constraints such as world knowledge, the predicate in question, the Q-adverb in use, and the logical properties (monotonic features) of the determiner, to name but a few. After all, "it may be sometimes be futile if not wrong to suppose that donkey sentences must have a definite reading."[11] Nevertheless, the representation helps to elucidate, given a particular reading (i.e., a determinate truth or falsity with respect to a scenario), what the discourse content, or structure of information, must and might be.

[10] A different formulation is this: Q-adverbs **may** bind multiple variables, but need not do so; when the primary and secondary quantification coincide in their quantificational force, we have what appears as unselective binding from one single quantifier.
[11] [4], p.151.

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