The Nomic Likelihood Account of Laws

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Abstract

An adequate account of laws should satisfy at least five desiderata: it should provide a unified account of laws and chances, it should yield plausible relations between laws and chances, it should vindicate numerical chance assignments, it should accommodate dynamical and non-dynamical chances, and it should accommodate a plausible range of nomic possibilities. No extant account of laws satisfies these desiderata. This paper presents a non-Humean account of laws, the *Nomic Likelihood Account*, that does.

1 Introduction

This paper defends a new account of laws, the *Nomic Likelihood Account*. The motivation for this account comes from the desire for an account that satisfies five desiderata, desiderata I take to be necessary conditions on an adequate account of laws. Roughly, these desiderata are (1) providing a unified account of laws and chances, (2) entailing plausible relations between laws and chances, (3) explaining why chance events deserve the numerical values values we assign them, (4) accommodating both dynamical and non-dynamical chances, and (5) accommodating a plausible range of nomic possibilities.

The Nomic Likelihood Account satisfies all of these desiderata. In broad strokes, the nomic likelihood account proceeds as follows. First, it posits a single fundamental nomic relation – the "nomic likelihood" relation – which satisfies certain constraints. Then it characterizes laws and chances in terms of this relation. So on this account, laws and chances end up being things that encode facts about the web of nomic likelihood relations.

I'll present the Nomic Likelihood Account in a largely theory-neutral manner. The main assumption I'll make, following Lewis (1983), is that there's a special subset of properties, the *perfectly natural* or *fundamental* properties, that fix all qualitative truths. Thus to describe what the world is like, it suffices to describe what there is and what fundamental properties those things have. And to provide an adequate account of some important feature of the world, one must ultimately be able to spell it out in the language of fundamental properties.[1]

Here is a road map for the rest of this paper. In section 2, I spell out the desiderata on an adequate account of laws sketched above. After presenting and motivating these desiderata (section 2.1), I suggest that none of the extant accounts of laws satisfy these desiderata, and show how several popular accounts fail to do so (section 2.2). In section 3, I offer an intuitive sketch of the Nomic Likelihood Account. In section 4, I present the nomic likelihood relation and the constraints I take this relation to satisfy. In section 5, I present a representation and uniqueness theorem showing that the pattern of instantiations of the nomic likelihood relation can be uniquely represented by things that look a lot like laws and chances (section 5.1). This theorem has some unique features that are of independent interest – it can distinguish between nomically forbidden events and chance 0 events that aren't nomically forbidden (e.g., an infinite number of fair coin tosses landing heads), and it doesn't employ the kind of "richness" assumptions that such theorems typically require. Using these results, I propose an account of laws and chances (section 5.2), describe some features of laws and chances that follow from this account (section 5.3), and apply the account to a toy example (section 5.4). In section 6, I show how the Nomic Likelihood Account satisfies the desiderata described above. In section 7, I consider some worries for the Nomic Likelihood Account. I conclude in section 8. Appendices A, B, and C, contain proofs of the main results.

2 Desiderata For An Adequate Account of Laws

2.1 The Desiderata

I'll now present five desiderata that I think must be satisfied by any adequate account of laws. While I'll briefly motivate these desiderata, I won't engage in an extended defense of them here. Those who are inclined to contest some of these desiderata can understand my case for the Nomic Likelihood Account as taking conditional form: *if* one takes these to be desiderata for an adequate account of laws, then we have reason to accept something like the Nomic Likelihood Account.

Desideratum 1. An adequate account should provide a unified (and appropriately discriminating) account of laws and chances.

An adequate account of laws should provide a unified account of laws and chances. It should allow for both probabilistic and non-probabilistic laws, and it should recognize non-probabilistic laws as a limiting case of probabilistic laws. That is, it should

[1] Some have argued that instead of taking the distinction between fundamental and non-fundamental properties to be primitive, one should take something like a grounding relation to be primitive, and characterize the fundamental properties in terms of this grounding relation (e.g., see Schaffer (2009)). I take what I say here to be largely compatible with such an approach.

recognize that nomic requirements/forbiddings and chances are of a kind, differing only on where they lie on the spectrum of nomic likelihood, with nomic requirements at one end, nomic forbiddings at the other, and non-trivial chances in-between. Moreover, it should do this without conflating being nomically required/forbidden with having a chance of 1/0. After all, there are events that have a chance of 0 that aren't nomically forbidden (e.g., infinitely many fair coin tosses landing heads), and events that have a chance of 1 that aren't nomically required (e.g., infinitely many fair coin tosses not all landing heads).[2]

Desideratum 2. An adequate account should yield plausible connections between laws and chances, laws and other laws, and chances and other chances.

An adequate account of laws should yield plausible relations between laws and chances, laws and other laws, and chances and other chances. For example, it should entail that nomically required events have a chance of 1. It should entail that something can't be nomically forbidden and nomically required at the same time. And it should say something about how the dynamical chances at one time are related to the dynamical chances at another.

Desideratum 3. An adequate account should describe what, at the fundamental level, makes it the case that chance events deserve the numerical values they're assigned.

An adequate account of laws should provide a satisfactory explanation for why chance events deserve the numerical values we assign them. That is, it should provide an account of the metaphysical structure underlying chances that explains why these numerical assignments are a "good fit" with the underlying metaphysical reality.

To get a feel for what this desideratum requires, let's consider an unsatisfactory attempt to meet this demand. Suppose one tried to satisfy this desideratum by stipulating that, as a primitive fact, the world has a nomic disposition of 0.6 strength to bring about one state of affairs given some other state of affairs. What, at the fundamental level, does this posit amount to?

At first glance, this would seem to amount to positing a fundamental "nomic disposition" relation between one state of affairs, another state of affairs, and the number 0.6. But it's implausible to think that, at the fundamental level, the chance facts boil down to relations to numbers of this kind. After all, the choice to assign chances values between 0 and 1 is purely conventional; we could assign chances using values between 0 and 2, or 0 and 0.5, just as well.[3] A more plausible story would provide

[2]I speak loosely here of chance *events*, but it will be more convenient to follow Lewis (1980) and take the objects of chance to be propositions. That said, little of importance hangs on this; see section 7 for a discussion of some of the ways in which one can modify the account defended here to fit one's particular ontological sensibilities.

[3]For further worries regarding such appeals to fundamental relations to numbers, see section 4 of Eddon (2013a) and Eddon (2013b).

some non-numerical relations whose structure justifies these numerical assignments. But this would, of course, require saying more than simply stipulating the existence of a nomic disposition of a certain numerical strength.

Desideratum 4. An adequate account should be able to accommodate both dynamical and non-dynamical chances (like those of statistical mechanics).[4]

An adequate account of laws should be able to accommodate both dynamical chances – such as those of the GRW interpretation of quantum mechanics – and non-dynamical chances – such as those of statistical mechanics.[5] Since statistical mechanical chances are macrostate-relative and compatible with determinism, it follows that an adequate account of laws should be able to make sense of macrostate-relative chances and non-trivial chances at deterministic worlds.[6]

Desideratum 5. An adequate account should be able to accommodate plausible nomic possibilities.

An adequate account of laws should be able to make sense of a plausible range of nomic possibilities. For example, it should be able to make sense of laws concerning particular locations, times, or objects, like the Smith's garden case discussed by Tooley (1977). It should be able to make sense of uninstantiated laws, such as worlds where $F = ma$ is a law but there are no massive objects. It should be able to make sense of world in which there is only one chance event – a coin toss, say – with a chance of 0.6 of landing heads and a chance of 0.4 of landing tails. And it should be able to distinguish such a world from an otherwise identical world in which the chance of heads is 0.7 and the chance of tails is 0.3.

While this is a desideratum that many accounts of laws and chances fail to fully satisfy (see section 2.2), it's most notably violated by Humean accounts – accounts on which the laws and chances supervene on the distribution of local qualities. For example, such accounts cannot make sense of uninstantiated laws, nor can they distinguish between worlds which differ only with respect to their chance assignments.

[4] *Dynamical chances*, or transition chances, are chances of the world evolving from some state S at one time into another state S' at another. *Non-dynamical chances* are chances that can't be thought of in this way; chances of the initial conditions being a certain way are a standard example (though see Demarest (2016) for a discussion of how to reinterpret such chances dynamically).

[5] The claim that an adequate account of laws should be able to accommodate non-dynamical chances is somewhat contentious, but it's been defended by a number of people, including Loewer (2001), Meacham (2005), Winsberg (2008), Frigg & Hoefer (2010), Strevens (2011), Emery (2013), Handfield & Wilson (2014), and Elliott (2018).

[6] Some have suggested understanding non-dynamical chances, such as those of statistical mechanics, as measures of rational indifference. If one adopted this stance, then one could dispense with this fourth desideratum, since one would only need an account of laws to accommodate dynamical chances. But there are well-known reasons for being skeptical of this understanding of statistical mechanical chances. For some of these reasons, see Strevens (1998), Albert (2000), Loewer (2001), North (2010), and Meacham (forthcoming); for a survey of this debate, see Meacham (2010).

Humeans take this to be a bullet worth biting in order to avoid positing fundamental nomic properties or powers. As such, Humeans won't take desideratum 5 to be a *requirement* on an adequate account of laws, even though they might concede that failing to accommodate plausible nomic possibilities is a mark against their view. The debate between Humeans and non-Humeans is a long one, and I won't attempt to settle it here. Instead, I'll simply side with the non-Humeans, and assume that desideratum 5 is a requirement on an adequate account of laws.

2.2 Other Accounts

To my knowledge, no existing account of laws satisfies the five desiderata described above. Due to space constraints, I won't try to provide an exhaustive discussion of the existing accounts and why they fall short. Instead, I'll just briefly discuss seven prominent accounts, and flag the desiderata that each fails to satisfy.

1. Carroll's (1994) primitivist account fails to satisfy desiderata 2 and 3. Carroll's account takes what the laws and chances are to be primitive. But simply stating that such-and-such laws and chances hold doesn't suffice to tell us what relations can hold between laws/chances and other laws/chances (desideratum 2). For example, it doesn't tell us anything about how the dynamical chances at one time should be related to the dynamical chances at another.

Likewise, simply stating that it's a primitive fact that a certain event has a chance of 0.6 doesn't provide a plausible story for what, at the fundamental level, makes this event deserve this numerical assignment (desideratum 3). At first glance, the claim that it's a fundamental fact that a certain event has a chance of 0.6 seems to be asserting that some kind of fundamental relation holds between that event and a number. But as we saw in section 2.1, this story is deeply implausible. Alternatively, one might understand claims about numerical chance assignments as concise ways of describing some more fundamental non-numerical structure that underlies these numerical assignments. But such a story requires a description of what this more fundamental non-numerical structure is, and Carroll's account doesn't provide us with these details.[7]

2. Lewis's (1994) best system account of laws fails to satisfy desiderata 4 and 5. Lewis's account requires all chances to be dynamical chances, and so fails to satisfy desideratum 4.[8] And as a Humean account – an account which takes the laws to supervene on the distribution of local qualities – it fails to satisfy desideratum 5, since it's unable to accommodate a plausible range of nomic possibilities. For (as we saw in section 2.1) there are plausible nomic possibilities – such as pairs of worlds that are

[7]Maudlin's (2007) primitivist account doesn't satisfy desiderata 2 and 3 for similar reasons. Maudlin's account also fails to satisfy desiderata 4 since it takes all chances to be dynamical chances. But Maudlin takes this to be a feature, not a bug.

[8]Though there are variants of Lewis's proposal that allow for such chances; e.g., see Loewer (2001), Winsberg (2008), and Frigg & Hoefer (2010).

identical with respect to the distribution of local qualities but different with respect to the chances – that Humean accounts cannot recognize.

3. Armstrong's (1983) universalist account fails to satisfy desiderata 2, 3 and 5. On one natural reading of Armstrong's account, it takes the nomic facts to be entailed by infinitely many fundamental necessitation relations – each intuitively corresponding to a different chance value – which hold between pairs of fundamental properties (universals) F and G.[9] Armstrong's account fails to satisfy desideratum 3 because it doesn't provide these necessitation relations with any structure that would justify one numerical assignment over any other. For example, nothing about the account tells us whether the necessitation relation N_a is stronger than the necessitation relation N_b, or whether N_a is closer in strength to N_b than N_c, or whether N_a is twice as strong as N_b. In a similar vein Armstrong's account fails to satisfy desideratum 2, since it doesn't say enough about these necessitation relations to determine what, for example, the relation between dynamical chances at different times is. Finally, Armstrong's account rules out plausible nomic possibilities (desideratum 5), since it rules out the possibility of worlds with uninstantiated laws or chances (such as a world where Newton's gravitational force law holds but there are no masses).[10]

4-5. Swoyer's (1982) necessitarian account and Lange's (2009) counterfactual account both fail to satisfy desiderata 1, 2 and 3. While these accounts differ in a number of ways, they are similar in that they both don't take chances to be part of the laws. Instead, they take chances to be just another quantitative property like mass or charge, and their accounts say little more about what chances are like. As a result, these accounts fail to satisfy the first three desiderata: they fail to provide a unified account of laws and chances (desideratum 1), they fail to yield plausible relations between laws/chances and other laws/chances (desideratum 2), and they fail to explain what, at the fundamental level, makes chance events deserve the numerical values they're assigned (desideratum 3).[11]

The preceding discussion suggests that most extant accounts of laws have particular trouble satisfying desiderata 2 and 3. This is likely because these accounts have largely focused on non-probabilistic laws, with probabilistic laws being something of a sideshow. So I'll conclude by assessing two accounts of chances that do better with respect to desiderata 2 and 3. Since these accounts are only intended as accounts of

[9]For a discussion of this and other ways of understanding Armstrong's account of probabilistic laws, see Jacobs & Hartman (2017). That said, for the purposes of this paper, figuring out the most plausible reading of Armstrong isn't important, since Armstrong's account will fail to satisfy desiderata 2, 3 and 5 on all of these readings.

[10]Tooley's (1987) universalist account fails to satisfy the same desiderata, though Tooley's account fails to satisfy desideratum 5 for a different reason (namely, it's unable to make sense of laws regarding particular locations, like Smith's garden; see Carroll (1994), Appendix A, footnote 6). Tooley's account also takes all chances to be dynamical chances, so it also fails to satisfy desideratum 4.

[11]Lange's (2009) account does say some things about the relationship between laws and other laws, and laws and chances (cf. section 3.7 of Lange (2009)), but says little about the relationship between different chance distributions.

chance, they won't provide a unified account of laws and chances (desideratum 1), nor say everything we'd like about how laws/chances bear on other laws/chances (desideratum 2). But it's worth seeing how they fare.

6. Suppes's (1973) propensity account of chances fails to satisfy desiderata 1, 2 and 3, though it does better with respect to desideratum 3 than the other accounts we've considered. Suppes takes an "at least as probable than" relation as primitive, imposes certain constraints on this relation, and then uses these constraints to provide representation theorems for various kinds of probabilistic phenomena, such as radioactive decay and coin tosses.[12] These representation theorems show, roughly, that one can assign numerical values to chance events that will line up with the "at least as probable than" relation and satisfy the probability axioms.

Suppes's account fails to satisfy desiderata 1 and 2 for the reasons given above – since it only provides an account of chances, not laws and chances, it doesn't provide a unified account of laws and chances, or the relationships between them. Moreover, Suppes's account doesn't provide a unified account of chances. For Suppes takes different probabilistic phenomena to impose different kinds of constraints, and goes on to provide different representation theorems for these different phenomena. Thus Suppes's account of chances is highly heterogeneous.[13]

Suppes's account does better with respect to desideratum 3, making substantial progress with respect to explaining what, at the fundamental level, makes chance events deserve the numerical values they're assigned. Unfortunately, it still falls short of providing a satisfactory justification. For while Suppes's approach yields a representation theorem, it doesn't yield the uniqueness theorem required to show that these numerical representations are unique. Thus this account doesn't justify our assigning the particular numerical values that we do.

7. Konek's (2014) propensity account of chances fails to satisfy desiderata 1, 2 and 5. Konek's account employs a primitive "comparative propensity ordering" that satisfies certain constraints, and then uses these constraints to provide a representation and uniqueness theorem. Thus we finally have an account which fully satisfies desideratum 3 – an account that explains what, at the fundamental level, makes chance events deserve the numerical values we assign them.

But Konek's account fails to satisfy desiderata 1 and 2 for reasons we've already seen – since it's not an account of laws and chances, just chances, it doesn't provide a unified account of laws and chances, or describe the relations that hold between them. Moreover, Konek's account also doesn't yield all of the relations between chances that one would like. For example, it doesn't say anything about how dynamical chances at different times are related.[14]

[12]See Suppes (1987).

[13]This is something Suppes takes to be a merit of his account. For he takes the expectation that there will be some unified account in the offing to be wrong-headed. Like much of the contemporary literature, I'm inclined to disagree.

[14]Of course, it would be unfair to raise any of this as a criticism of Konek. Konek's goal is simply

Finally, Konek's account fails to recognize some plausible nomic possibilities (desideratum 5). It seems possible for there to be a world with only one chance event – a coin toss – with a chance of 0.6 of landing heads (cf. section 2.1). And this possibility seems distinct from an otherwise identical world where the chance of heads is 0.7. But on Konek's account neither of these worlds are possible – the comparative propensity ordering facts that line up with these numbers will be too weak to yield a precise numerical chance assignment, so Konek's account will take such worlds to have imprecise chances. And since the comparative ordering facts that line up with these numbers will be the same in both worlds, Konek's account can't recognize these possibilities as distinct.

3 The Nomic Likelihood Account (I): The Intuitive Picture

Let's start by sketching the intuitive picture behind the Nomic Likelihood Account.

It's natural to think that laws and chances are of a kind. Deterministic laws tell us that if one state of affairs obtains, then another state of affairs is nomically required to obtain. Chances tell us that if one state of affairs obtains, then another state of affairs has a certain nomic likelihood of obtaining. And nomic requirements and nomic likelihoods seem to be instances of the same kind of thing. Nomic requirements are just what you get when you turn the nomic likelihood "all way up".

Now, the nomic likelihood of one state of affairs given another is a quantitative feature of the world. You can have different degrees of nomic likelihood. And these degrees can be characterized in precise, numerical ways — one state of affairs can be twice as likely as another, for example. So what undergirds these quantitative features of the world? What's the metaphysical structure underlying nomic likelihoods?

The view I propose takes its cue from a popular account of quantitative properties like mass.[15] Consider an object that has a certain amount of mass. What undergirds the fact that it has *that* quantity of mass? According to one popular account, it's the mass relations that hold between the object and all other massive objects. For example, this object might be more massive than some objects, and less massive than others. And it's this web of mass relations that fixes the particular amount of mass the object has. What it is for an object to have a particular amount of mass is just for it to bear the right relations of this kind to everything else.

to show that proponents of propensity accounts of chances can provide a principled story for why they expect propensities to satisfy the probability axioms. And just as it would be unfair to criticize Konek for presenting a view which doesn't provide an account of laws (since Konek wasn't trying to provide an account of laws), it would be unfair to criticize Konek for failing to yield relations between dynamical chances at different times (since Konek wasn't trying to provide a comprehensive account of chances).

[15]For a survey of different accounts of quantitative properties, see Eddon (2013b).

The Nomic Likelihood Account adopts a similar approach to nomic likelihood. In the case of mass, what bears a quantity of mass is an object.[16] In the case of nomic likelihood, what bears a quantity of nomic likelihood is a pair of states of affairs — given this state of affairs, there's such-and-such likelihood of this other state of affairs coming about. Or, if we factor in the fact that these likelihoods can vary from world to world, what bears a quantity of nomic likelihood is a triple — a pair of states of affairs and a world.

Now consider a triple that has a certain nomic likelihood – at this world, given this state of affairs, there's such-and-such likelihood of this other state of affairs coming about. What undergirds the fact that this triple has *that* nomic likelihood? According to the Nomic Likelihood Account, it's the relations that hold between that triple and all other triples that have nomic likelihoods. For example, this triple might be more nomically likely than some triples, and less nomically likely than others. And it's this web of nomic likelihood relations that fixes the particular amount of nomic likelihood this triple has. What it is for a triple to have a particular nomic likelihood is just for it to bear the right relations to other triples.[17]

Of course, a satisfying account has to do more than just gesture at certain relations. Return to the case of mass. A satisfying account of quantities of mass has to do more than gesture at some mass relations. It has to tell us what these relations are, what these relations are like, and how these relations vindicate taking masses to be quantitative, i.e., vindicate assigning numerical values to these quantities in the way that we do. And this is what accounts of quantitative properties like mass do. They propose certain fundamental mass relations, present some "axioms" that describe how these relations behave, and provide a representation and uniqueness theorem showing that these relations vindicate our using numbers to represent the amount of mass things have in the way that we do.

Providing a satisfying account of nomic likelihood requires doing something similar. We need to spell out what the fundamental relations are, what these relations are like, and how these relations vindicate assigning numerical values to chances in the way that we do. This is what I'll do in the next two sections. I'll spell out the fundamental nomic likelihood relation, present some "axioms" describing how this relation behaves, and provide a representation and uniqueness theorem showing that these relations vindicate our using numbers to represent amounts of nomic likelihood in the way that we do. And with an account of nomic likelihood in hand, it's straight-

[16]Assuming we're taking objects to be world-bound. If we don't, then since an object's mass can vary from world to world, we might take the bearer of mass to be an object and world pair.

[17]For those familiar with the literature on quantitative properties, the account of nomic likelihood described here is analogous to the version of the first-order relations account of quantitative properties discussed by Eddon (2013a) that allows these relations to hold between individuals in different possible worlds. An alternative way of developing the Nomic Likelihood Account is sketched in section 7 in the discussion of the third worry. This alternative "two layer" account of nomic likelihood is analogous to the second-order relations account of quantitative properties defended by Mundy (1987) and Eddon (2013a).

forward to provide an account of laws and chances.

While proponents of the Nomic Likelihood Account can remain neutral about many metaphysical debates, it's hard to sketch an intuitive picture of the view in a theory-neutral manner. So I've made some assumptions in this section while presenting the picture; for example, I've appealed to things like Chisholm-style states of affairs. But these aren't assumptions that the Nomic Likelihood Account is wedded to; we'll return to discuss some alternative approaches in section 7.[18]

4 The Nomic Likelihood Account (II): The Posit

In this section I'll present the key posit of the Nomic Likelihood Account, the nomic likelihood relation. In section 4.1 I'll introduce the nomic likelihood relation. In section 4.2 I'll introduce some helpful terminology. In section 4.3 I'll describe the constraints (i.e., axioms) that I take the nomic likelihood relation to satisfy.

Two comments before we get started. First, in section 3 I talked about nomic likelihoods in terms of states of affairs. As it turns out, it will be formally more convenient to characterize nomic likelihoods in terms of propositions instead of states of affairs. But this is purely for convenience – we could formulate everything in terms of states of affairs instead, albeit in a slightly clunkier way.[19] In what follows I'll assume that a proposition can be identified with the set of possible worlds at which it's true.[20] I'll take Ω to be the set of all possible worlds, i.e., the trivially true proposition that some possibility obtains, and I'll take $\varnothing$ to be the empty set, i.e., the trivially false proposition that no possibility obtains.

Second, it's worth saying something about the representation and uniqueness theorem this approach employs in order to help the reader understand the motivation for some of the axioms. The measurement theory literature contains a number of representation and uniqueness theorems which take an ordering relation that satisfies certain constraints, and show that there's a unique numerical representation that lines up with that relation. Given this, working out the axioms of the nomic likelihood relation and providing a representation and uniqueness theorem for it seems like a straightforward task. All that's required to complete this project, it seems, is to take one of these formal results and change its interpretation.

Unfortunately, none of the results in the literature can do the work required, for two reasons. First, none of the results in the literature I'm aware of can distinguish between having a probability of 1 and being required to be true. Or, given the interpretation we're interested in, can distinguish between having a chance of 1 and being nomically required. So while these results provide us with something to identify

[18]For example, we can replace the role of states of affairs with properties or propositions (as I do in section 4), or replace the role of worlds with states of affairs or properties.

[19]For discussion of some different ways of characterizing the nomic likelihood relation, see section 7.

[20]I'll use the term "set" here loosely to cover both sets and classes.

chances with, they don't provide us with something to identify nomic requirements with. (Recall that we can't just take nomic requirements to be the things that have a probability of 1, for there are things which have a probability of 1 that aren't nomically required – e.g., an infinite number of fair coin tosses not all landing heads.) In order to satisfy the first desideratum of section 2.1, we need an account that can make such distinctions.

Second, all of the theorems in the literature I know of require strong "richness" assumptions in order to derive their result.[21] These richness assumptions impose strong constraints on the probability function, such as, e.g., that for every value in the unit interval, there's something that has that probability. This rules out plausible nomic possibilities like there being a world with only a single chance event, e.g., a coin toss, which has a chance of 0.6 of heads and a chance of 0.4 of tails. In order to satisfy the fifth desideratum of section 2.1, we need an account that can recognize such possibilities.

The framework I'll present will allow us to distinguish between having a chance of 1 and being nomically required. It does so by introducing, in addition to the unique largest and smallest nomic likelihoods, held by Ω and $\emptyset$, unique next largest and next smallest likelihoods. Likewise, the framework I'll present doesn't need to posit the kind of richness axioms the existing theorems require. This is because it introduces cross-world relations that effectively allow us to "import" richness from other worlds. Of course, these changes require replacing many of the standard axioms that the results in the literature employ, and showing that we can still derive everything we want from their replacements.

4.1 The Nomic Likelihood Relation

Here is the fundamental posit of the Nomic Likelihood Account:

The Nomic Likelihood Relation: There exists a fundamental six-place *nomic likelihood* relation, $\succeq (C, A, w, C', A', w')$ ("C given A at w is at least as nomically likely as C' given A' at w'"), that satisfies the 12 nomic axioms (cf. section 4.3), where w, w' are worlds, and A, A', C, C' are propositions that supervene on the fundamental properties and relations other than $\succeq$.

The last clause ensures that the propositions the nomic likelihood relation holds of aren't themselves about nomic facts. I take this constraint to be independently plausible, and it ensures that we won't run into self-reference paradoxes. Now let's turn to the 12 nomic axioms that the nomic likelihood relation is required to satisfy.

[21] For some discussions of worries regarding these richness axioms in the context of standard theories of quantitative properties, see Melia (1998), Eddon (2013a), Eddon (2013b), and Perry (2015).

4.2 Terminology

Let me start by introducing some terminology.

Let $\boldsymbol{C}_{A,w}$ be an ordered triple consisting of a pair of propositions $A, C \subseteq \Omega$ and a world $w \in \Omega$. I'll call A and C the *antecedent* and *consequent* propositions of the triple, respectively. When expressing such triples, everything that's bolded should be understood as describing the consequent proposition of the triple. E.g., $(\boldsymbol{C} \cap \boldsymbol{C'})_{A,w}$ is a triple whose consequent proposition is $C \cap C'$, whose antecedent proposition is A, and whose world is w. When talking about triples which share the same indices, I'll leave the indices implicit.

At the risk of abusing notation, I'll often express the nomic likelihood relation in terms of these triples. Thus I'll use "$\boldsymbol{C}_{A,w} \succeq \boldsymbol{C'}_{A',w'}$" as shorthand for "$\succeq (C, A, w, C', A', w')$". Using this notation, we can define the "more nomically likely than" relation $\succ$ as follows: $\boldsymbol{C} \succ \boldsymbol{C'}$ *iff* $\boldsymbol{C} \succeq \boldsymbol{C'}$ and $\boldsymbol{C'} \not\succeq \boldsymbol{C}$. Likewise, we can define the "nomically on a par" relation $\sim$ as follows: $\boldsymbol{C} \sim \boldsymbol{C'}$ *iff* $\boldsymbol{C} \succeq \boldsymbol{C'}$ and $\boldsymbol{C'} \succeq \boldsymbol{C}$.

Let NS (for "nomic space") be the set of all triples $\boldsymbol{C}_{A,w}$ such that C, A, and w are either the first three or last three arguments of some instantiation of $\succeq$. Intuitively, NS is the set of all triples that have nomic likelihoods.

Let the *(A,w)-cluster* be the subset of NS containing all the triples with A and w as their second and third members. Intuitively, A and w pick out a situation, and the (A,w)-cluster identifies the consequent propositions that nomic likelihoods are assigned to in that situation. For example, if A and w pick out a chance distribution, the (A,w)-cluster will consist of the triples whose consequent propositions are assigned chances by this distribution. Note that clusters can be "gappy", in the sense that for some propositions C, the (A,w)-cluster won't contain $\boldsymbol{C}_{A,w}$. This is because, holding A and w fixed, there can be nomic constraints on some consequent propositions but not others. For example, A and w might pick out a chance distribution which assigns chances to propositions about the behavior of particles, but not to propositions about the behavior of incorporeal spirits. Likewise, note that clusters can be empty. For example, if w is a lawless world, then the (A,w)-cluster will be empty, since no triples of the form $\boldsymbol{C}_{A,w}$ are assigned nomic likelihoods.

With this notation in hand, let's turn to the 12 nomic axioms.

4.3 The Nomic Axioms

1. We haven't imposed any constraints on which consequent propositions C are assigned nomic likelihoods in an (A,w)-cluster. For example, as it stands, it could be the case that C is assigned a nomic likelihood but $\overline{C}$ is not; or that C and C' are assigned nomic likelihoods but $C \cup C'$ is not. The first axiom ensures that the consequent propositions that are assigned nomic likelihoods are closed under natural operations like negation and disjunction. E.g., it ensures that if given certain meteorological conditions A at world w there's some nomic likelihood of it raining (C), then there's also some nomic likelihood of it not raining ($\overline{C}$); and if there's some nomic likelihood of

it raining (C) and some nomic likelihood of it snowing (C'), then there's also some nomic likelihood of it raining or snowing ($C \cup C'$).

Axiom 1 (σ-algebra):

1. If $\boldsymbol{C}$ is in NS, then $\overline{\boldsymbol{C}}$ is in NS.
2. If $\boldsymbol{C_1}, \boldsymbol{C_2}, \ldots$ are in NS, then $\bigcup_{i=1}^{\infty} \boldsymbol{C_i}$ is in NS.

Formally, this axiom ensures that for every non-empty (A,w)-cluster, the consequent propositions in that cluster form a σ-algebra.

2. Nothing we've said so far requires all triples with nomic likelihoods to be comparable, or requires comparisons between triples to be transitive. For all we've said, it could be the case that it raining (given meteorological conditions A at w) is more nomically likely than it snowing (given A' at w'), and it snowing (given A' at w') is more nomically likely than it being sunny (given A'' at w''), but it raining (given A at w) is neither more nomically likely than, less nomically likely than, or on a par with, it being sunny (given A'' at w''). The second axiom rules this out, by ensuring that all triples with nomic likelihoods are comparable, and that these comparisons are transitive.

Axiom 2 (Weak Order):

1. $\succeq$ is *connected*: for all $\boldsymbol{C}_{A,w}$, $\boldsymbol{C'}_{A',w'}$ in NS, either $\boldsymbol{C}_{A,w} \succeq \boldsymbol{C'}_{A',w'}$ or $\boldsymbol{C'}_{A',w'} \succeq \boldsymbol{C}_{A,w}$.
2. $\succeq$ is *transitive*: for all $\boldsymbol{C}_{A,w}$, $\boldsymbol{C'}_{A',w'}$, $\boldsymbol{C''}_{A'',w''}$ in NS, if $\boldsymbol{C}_{A,w} \succeq \boldsymbol{C'}_{A',w'}$ and $\boldsymbol{C'}_{A',w'} \succeq \boldsymbol{C''}_{A'',w''}$, then $\boldsymbol{C}_{A,w} \succeq \boldsymbol{C''}_{A'',w''}$.

Formally, this axiom ensures that the nomic likelihood relation provides a weak ordering of NS.

3. The previous axioms haven't imposed any constraints on how the nomic likelihoods assigned to members of different (A,w)-clusters line up with each other. For example, as it stands, it could be that all the triples in one cluster are more nomically likely than all the triples in another. The third axiom ensures that triples whose consequent propositions are trivially true (Ω) or trivially false ($\varnothing$) have the same nomic likelihoods in all (A,w)-clusters. Intuitively, this ensures that the "ceiling" and "floor" of nomic likelihoods is the same at all clusters.

Axiom 3 (Cross-algebra Comparisons):

1. If $\boldsymbol{\Omega}_{A,w}$ and $\boldsymbol{\Omega}_{A',w'}$ are in NS: $\boldsymbol{\Omega}_{A,w} \sim \boldsymbol{\Omega}_{A',w'}$.
2. If $\varnothing_{A,w}$ and $\varnothing_{A',w'}$ are in NS: $\varnothing_{A,w} \sim \varnothing_{A',w'}$.

4. So far, nothing we've said requires there to actually *be* any triples with nomic likelihoods. For all we've said, it could be the case that all (A,w)-clusters are empty. And even if we assume there are non-empty clusters, nothing we've said requires them to be fine-grained. E.g., it could be the case that every triple which has a nomic

likelihood is on a par with (say) one of three triples, entailing that there are effectively only three degrees of nomic likelihood. And even if we assume there is a cluster with a rich range of nomic likelihoods, nothing we've said requires these nomic likelihoods to be fine-grained enough to distinguish between consequent propositions that are nomically required and ones which are "just" overwhelmingly likely (e.g., that at least one of infinitely many fair coin tosses lands heads). The fourth axiom imposes "richness" requirements that ensure there's an appropriately fine-grained range of nomic likelihoods.

Axiom 4 (Rich Algebra): There exists a particular cluster, call it "R" (for "rich"), with the following features:

1. There is a pair of triples in R, call them "$\varnothing$**+**" and "$\boldsymbol{\Omega}$**-**", such that:
   (a) $\varnothing \prec \varnothing\textbf{+} \prec \boldsymbol{\Omega}\textbf{-} \prec \boldsymbol{\Omega}$.
   (b) For all $\boldsymbol{C}$ such that $\boldsymbol{C} \not\sim \varnothing$, $\boldsymbol{C} \succeq \varnothing\textbf{+}$.
   (c) For all $\boldsymbol{C}$ such that $\boldsymbol{C} \not\sim \boldsymbol{\Omega}$, $\boldsymbol{C} \preceq \boldsymbol{\Omega}\textbf{-}$.
2. There are no $\boldsymbol{C} \succ \varnothing\textbf{+}$ in R such that, for any $\boldsymbol{C'}$ in R such that $C' \subset C$, either:
   (a) $\boldsymbol{C'} \sim \boldsymbol{C}$.
   (b) $\boldsymbol{C'} \sim \varnothing$.
   (c) $\boldsymbol{C'} \sim \varnothing\textbf{+}$.
   (d) $\boldsymbol{C'} \sim \boldsymbol{\Omega}\textbf{-}$ and $\boldsymbol{C} \sim \boldsymbol{\Omega}$.
3. For any $\boldsymbol{C}_{A,w}$ and $\boldsymbol{C'}_{A,w}$ in NS such that $C \cap C' = \varnothing$, there's some $\boldsymbol{C''_R}$ and $\boldsymbol{C'''_R}$ in R such that $\boldsymbol{C}_{A,w} \sim \boldsymbol{C''_R}$, $\boldsymbol{C'}_{A,w} \sim \boldsymbol{C'''_R}$, and $C'' \cap C''' = \varnothing$.

This is an important axiom, so it's worth talking through what it says in a bit more detail. This axiom posits the existence of a "rich" cluster, R. The three clauses of this axiom ensure that R is rich in three different ways. (This axiom is compatible with there being multiple clusters which satisfy these clauses. But "R" is a name for a particular one of them.)

The first clause entails that in this rich cluster there's (i) a "next highest" rank of nomic likelihood, which sits below $\boldsymbol{\Omega}$ but above every other rank, and (ii) a "next lowest" rank of nomic likelihood, which sits above $\varnothing$ but below every other rank. I use the names "$\boldsymbol{\Omega}$**-**" and "$\varnothing$**+**" for some particular triples in R that have these ranks. (This clause is compatible with there being multiple triples in R which have these ranks. But "$\boldsymbol{\Omega}$**-**" and "$\varnothing$**+**" are names for a particular pair of them.)

It's worth emphasizing that "$\boldsymbol{\Omega}$**-**" and "$\varnothing$**+**" are names for two particular *triples* in R, not names for the consequent propositions of some triples whose indices have been left implicit. (E.g., I'm not using "$\boldsymbol{\Omega}$**-**" as shorthand for "$\boldsymbol{\Omega}\textbf{-}_R$"; "$\Omega$-" is not the name of a proposition.) Thus $\boldsymbol{\Omega}$**-** and $\varnothing$**+** will never be expressed with indices; the second and third elements of these triples are fixed.

In what follows, it will be convenient to have a name for triples $\boldsymbol{C}$ whose rank is such that $\varnothing\textbf{+} \prec \boldsymbol{C} \prec \boldsymbol{\Omega}\textbf{-}$. I'll say that such triples have a *middling* rank.

The second clause is the analog of the standard "atomless" assumption.[22] Roughly, it ensures that in this rich cluster, any triple $\boldsymbol{C}$ of at least middling rank can be always be decomposed into smaller triples of middling rank.

The third clause ensures that every degree of nomic likelihood is instantiated in R. That is, it entails that R is rich enough to be such that every triple in NS is nomically on a par with some triple in R.[23]

5. Intuitively, nomic likelihoods should satisfy something like a qualitative notion of additivity. For example, given meteorological conditions A at world w, if it raining (C) is more nomically likely than it snowing (C'), then it raining or being sunny ($C \cup C''$) should be more nomically likely than it snowing or being sunny ($C' \cup C''$).[24] The fifth axiom ensures that nomic likelihoods will satisfy this kind of additivity requirement.

Axiom 5 (Restricted Cross-algebra Additivity): Suppose that $(\boldsymbol{C} \cap \boldsymbol{C'})_{A,w} \sim (\boldsymbol{C''} \cap \boldsymbol{C'''})_{A',w'} \sim \varnothing_{A,w}$, that $\boldsymbol{C}_{A,w} \sim \boldsymbol{C''}_{A',w'}$, and that none of the following three conditions hold: (i) $\boldsymbol{C}_{A,w} \sim \boldsymbol{\Omega}$**-**, $\boldsymbol{C'}_{A,w} \sim \boldsymbol{C'''}_{A',w'} \sim \varnothing$**+**, (ii) $\boldsymbol{C}_{A,w} \sim \varnothing$**+**, $\boldsymbol{C'}_{A,w} \sim \boldsymbol{C'''}_{A',w'} \sim \boldsymbol{\Omega}$**-**, (iii) $\boldsymbol{\Omega}_{A,w} \succ \boldsymbol{C}_{A,w} \succ \varnothing_{A,w}$, $\boldsymbol{C'}_{A,w} \sim \varnothing_{A,w}$, $\boldsymbol{C'''}_{A',w'} \sim \varnothing$**+**. Then $\boldsymbol{C'}_{A,w} \succeq \boldsymbol{C'''}_{A',w'}$ *iff* $(\boldsymbol{C} \cup \boldsymbol{C'})_{A,w} \succeq (\boldsymbol{C''} \cup \boldsymbol{C'''})_{A',w'}$.

It's worth flagging two ways in which this qualitative additivity axiom differs from typical qualitative additivity axioms. First, typical qualitative additivity axioms don't include conditions (i)-(iii). But the introduction of $\boldsymbol{\Omega}$**-** and $\varnothing$**+** requires the additivity claim to be restricted to cases where none of conditions (i)-(iii) hold.[25] Second, typical qualitative additivity axioms effectively only apply within a single cluster. But in order to "import" richness facts from other clusters, we need the additivity claim to

[22] An *atom* is a triple $\boldsymbol{C} \succ \varnothing$ such that any $\boldsymbol{C'}$ that $\boldsymbol{C}$ contains is either on a par with $\boldsymbol{C}$ or $\varnothing$. So, intuitively, an atom is a triple with some nomic likelihood which can't be decomposed into anything that's strictly less nomically likely, but still at least somewhat nomically likely. The standard atomless assumption is just the assumption that there are no atoms: there are no $\boldsymbol{C} \succ \varnothing$ such that, for any $\boldsymbol{C'}$ such that $C' \subset C$, either (a) $\boldsymbol{C'} \sim \boldsymbol{C}$, or (b) $\boldsymbol{C'} \sim \varnothing$. Introducing $\varnothing$**+** and $\boldsymbol{\Omega}$**-** requires modifying the standard atomless assumption. This modified assumption (the second clause of Axiom 4) entails that if we remove all triples on a par with $\varnothing$**+** and $\boldsymbol{\Omega}$**-**, then this rich cluster will be atomless.

[23] Though this is not all it entails; it also entails that for any two disjoint triples in any cluster, there are two disjoint triples in R that have those same ranks.

[24] Where I'm assuming here that raining, snowing, and being sunny are mutually exclusive.

[25] We need to add these restrictions because if any of (i)-(iii) obtain, we can construct counterexamples to the additivity claim (that $\boldsymbol{C'}_{A,w} \succeq \boldsymbol{C'''}_{A',w'}$ *iff* $(\boldsymbol{C} \cup \boldsymbol{C'})_{A,w} \succeq (\boldsymbol{C''} \cup \boldsymbol{C'''})_{A',w'}$). For an intuitive example within a single algebra, let $\boldsymbol{C} = \boldsymbol{C''}$ be the proposition that at least two of infinitely many coin tosses landed tails, let $\boldsymbol{C'}$ be the proposition that none of infinitely many coin tosses landed tails, and let $\boldsymbol{C'''}$ be the proposition that no more than one of infinitely many coin tosses landed tails. This is an instance of (ii): $\boldsymbol{C} \sim \boldsymbol{\Omega}$**-**, and $\boldsymbol{C'} \sim \boldsymbol{C'''} \sim \varnothing$**+**. Now note that the rest of the conditions this axiom imposes (other than (i)-(iii)) are satisfied: $\boldsymbol{C} \cap \boldsymbol{C'} \sim \boldsymbol{C''} \cap \boldsymbol{C'''} \sim \varnothing$, and $\boldsymbol{C} \sim \boldsymbol{C''}$. But while $\boldsymbol{C'} \succeq \boldsymbol{C'''}$ is true, $\boldsymbol{C} \cup \boldsymbol{C'} \sim \boldsymbol{\Omega}$**-** $\succeq \boldsymbol{C''} \cup \boldsymbol{C'''} \sim \boldsymbol{\Omega}$ is false. Thus without the restriction ruling out cases of type (ii), axiom 5 would be false. And we can construct similar counterexamples if we omit conditions (i) or (iii).

apply to triples belonging to different clusters.[26]

6. The sixth axiom plays an important role in establishing the representation and uniqueness theorem, but it's a bit harder to get an intuitive grip on than the other axioms. Consider a sequence of triples from some cluster that's "expanding", in the sense that the consequent proposition of each triple in the sequence is entailed by the consequent propositions of all the earlier members of the sequence. And suppose some other triple $\boldsymbol{C}$ is more nomically likely than any triple in this sequence. Then it's natural to think that $\boldsymbol{C}$ should also be more nomically likely than a triple whose consequent proposition is the disjunction of all of the consequent propositions in this sequence. This is what the sixth axiom requires.

Axiom 6 (Continuity): If for all i, $\boldsymbol{C} \succeq \boldsymbol{C_i}$ and $C_i \subseteq C_{i+1}$, then $\boldsymbol{C} \succeq \bigcup_{i=1}^{\infty} \boldsymbol{C_i}$.

Formally, this axiom ensures that the $\succeq$ relation is monotonically continuous.

7. So far we've said little about how the nomic likelihoods of triples on a par with $\varnothing$**+** and $\boldsymbol{\Omega}$**-** behave. For example, given conditions A at world w, suppose the nomic likelihood of a certain coin landing heads (C) is middling, the nomic likelihood of an independent sequence of infinitely many coins all landing heads (C') is on a par with $\varnothing$**+**, and the nomic likelihood of at least one coin in this infinite sequence landing tails ($\overline{C'}$) is on a par with $\boldsymbol{\Omega}$**-**. How does the nomic likelihood of the coin landing heads (C) compare to that of the coin landing heads *or* the infinite sequence of coins all landing heads ($C \cup C'$)? Likewise, how does the nomic likelihood of the coin landing heads (C) compare to that of the coin landing heads *and* at least one of an independent infinite sequence of coins landing tails ($C \cap \overline{C'}$)? The seventh axiom settles the answer to these questions, holding in both cases that the likelihoods are the same.

In particular, the seventh axiom entails that adding things on a par with $\varnothing$**+** can only result in a change of likelihood in extremal cases, when it's added to something on a par with $\varnothing$ or $\boldsymbol{\Omega}$**-**. Likewise, it entails that intersecting things on a par with $\boldsymbol{\Omega}$**-** can only result in a change of likelihood in extremal cases, when it's intersecting something on a par with $\varnothing$**+** or $\boldsymbol{\Omega}$.

Axiom 7 ($\varnothing$+/Ω- Differences):

1. If $\varnothing\mathbf{+} \preceq \boldsymbol{C} \prec \boldsymbol{\Omega}\mathbf{-}$, and $\boldsymbol{C'} \sim \varnothing\mathbf{+}$, then $\boldsymbol{C} \sim \boldsymbol{C} \cup \boldsymbol{C'}$.
2. If $\varnothing\mathbf{+} \prec \boldsymbol{C} \preceq \boldsymbol{\Omega}\mathbf{-}$, and $\boldsymbol{C'} \sim \boldsymbol{\Omega}\mathbf{-}$, then $\boldsymbol{C} \cap \boldsymbol{C'} \sim \boldsymbol{C}$.

8. We haven't yet imposed any requirements tying nomic likelihood to truth. For all we've said, it could be the case that if meteorological conditions A hold at world w then it's maximally likely that it will rain (C), and meteorological conditions A do hold at w, and yet it doesn't rain at w. The eighth axiom ensures that nomic likelihood is tied to truth in the way we'd expect.

[26] A third and more subtle way in which it differs from typical qualitative additivity axioms is that it doesn't require C and C' (and C'' and C''') to actually be disjoint. Instead, it merely requires the triples corresponding to these intersections to be on a par with $\varnothing$.

Axiom 8 (Ω Instantiation): If $\boldsymbol{C}_{A,w} \sim \boldsymbol{\Omega}_{A,w}$, and $w \in A$, then $w \in C$.

9. Nothing we've said so far has imposed conditions tying the fact that *if A obtained at w then C would have a certain likelihood* to the possibility of A obtaining. Consider the set of worlds L_w containing all the worlds that assign the same nomic likelihoods as world w. (I.e., if $w' \in L_w$, then for all $\boldsymbol{C''}_{A'',w''}$ in NS, $\boldsymbol{C}_{A,w} \succeq \boldsymbol{C''}_{A'',w''}$ *iff* $\boldsymbol{C}_{A,w'} \succeq \boldsymbol{C''}_{A'',w''}$.) And suppose that given meteorological conditions A at a world in L_w, there's a certain nomic likelihood of rain (C). As it stands, this could be true even though there's *no* world in L_w at which conditions A hold. One might take this to be implausible. If there's a certain likelihood of rain given certain meteorological conditions at w, then there should be some nomically similar world where those meteorological conditions obtain. The ninth axiom ensures that this is the case.

Axiom 9 (Antecedent Instantiation): If $\boldsymbol{C}_{A,w}$ is in NS, then there exists a $w' \in A$ such that for all $\boldsymbol{C''}_{A'',w''}$ in NS, $\boldsymbol{C}_{A,w} \succeq \boldsymbol{C''}_{A'',w''}$ *iff* $\boldsymbol{C}_{A,w'} \succeq \boldsymbol{C''}_{A'',w''}$.

10. Nothing we've said so far has imposed conditions tying the fact that *if A obtained at w then C would have a middling likelihood* to the possibility of C obtaining. Suppose that given meteorological conditions A at a world in L_w, there's a middling nomic likelihood of rain (C). As things stand, it could be the case that it rains at every world in L_w where A obtains, even though it only has a middling likelihood of doing so. Likewise, it could be the case that it doesn't rain at any world in L_w where A obtains, even though it has a middling nomic likelihood of doing so. Both scenarios are implausible: if there's a middling likelihood of rain, then there should be some A-worlds in L_w where it rains, and some where it does not. The tenth axiom ensures that this is the case.

Axiom 10 (Chancy Instantiation): If $\varnothing_{A,w} \prec \boldsymbol{C}_{A,w} \prec \boldsymbol{\Omega}_{A,w}$, then there exists a w' and w'' such that:

1. For all $\boldsymbol{C'''}_{A''',w'''}$ in NS, $\boldsymbol{C}_{A,w} \succeq \boldsymbol{C'''}_{A''',w'''}$ *iff* $\boldsymbol{C}_{w',A} \succeq \boldsymbol{C'''}_{A''',w'''}$ *iff* $\boldsymbol{C}_{A,w''} \succeq \boldsymbol{C'''}_{A''',w'''}$.
2. $w' \in A$ and $w'' \in A$.
3. $w' \in C$ and $w'' \notin C$.

11. The previous axioms haven't imposed any constraints on what triples there are in different (A,w)-clusters indexed to the same world. Suppose that given meteorological conditions A at w, there's a middling likelihood of it raining the next day (C) and a middling likelihood of it raining the day after that (C'). And consider the nomic likelihoods that might obtain at w given those meteorological conditions and that it rains the first day ($A \cap C$). For all we've said so far, it could be that given $A \cap C$ at w there's a maximal likelihood assigned to it raining the first day (C), but no likelihood at all – whether high or low – assigned to it raining the second day (C'). That is, it could be that the ($A \cap C$,w)-cluster is simply silent about the likelihood of it raining the second day. This is odd. If the (A,w)-cluster assigns a nomic likelihood to C', it

seems the $(A \cap C, w)$-cluster should as well. The eleventh axiom ensures this, by requiring clusters at the same world to have consequent propositions that line up with each other.

Axiom 11 (Same Algebra): Suppose that $A \supset A'$, that $\boldsymbol{A'}_{A,w} \succ \boldsymbol{\emptyset+}$, and that the *$(A',w)$*-cluster is not empty. Then $\boldsymbol{C}_{A',w}$ is in NS *iff* $\boldsymbol{C}_{A,w}$ is in NS.[27]

12. Axiom 11 ensures that clusters at the same world have consequent propositions that line up with each other. But while axiom 11 ensures that these clusters will assign nomic likelihoods to the appropriate propositions, we haven't yet said anything about what the magnitudes of these nomic likelihoods should be. Suppose that given meteorological conditions A at w, there's a middling likelihood of it raining the next day (C), a middling likelihood of it raining the day after that (C'), and a smaller but still middling likelihood of it raining both days ($C \cap C'$). Given those meteorological conditions and that it rains the next day ($A \cap C$), what should the likelihood of it raining both days be? For all we've said so far, it could be anything, including on a par with the trivially true proposition Ω or the trivially false proposition $\emptyset$. This is implausible: the likelihood of it raining both days should be middling. The twelfth axiom ensures this, by requiring the nomic likelihoods assigned by same-world clusters to line up in the way you'd expect.

Formulating the twelfth axiom precisely requires a little stage-setting. Let an *n-equipartition* P of a cluster be a set of n triples $\boldsymbol{P_i}$ which are all nomically on a par with each other, and whose consequent propositions are mutually exclusive and exhaustive.[28] Let $f : \mathbb{N} \times NS \to \mathbb{N}$ be a function such that: $f(n, \boldsymbol{C}_{A,w}) = x$ *iff* for any n-equipartition P of the rich cluster R, and any $\boldsymbol{C}_{A,w}$ in NS:

$$f(n, \boldsymbol{C}_{A,w}) = n \text{ if } \boldsymbol{C}_{A,w} \sim (\cup_{\boldsymbol{i=1}}^{\boldsymbol{i=n}} \boldsymbol{P_i})_{A,w}.$$
$$f(n, \boldsymbol{C}_{A,w}) = m \text{ if } n > m > 0 \text{ and } (\cup_{\boldsymbol{i=1}}^{\boldsymbol{i=m+1}} \boldsymbol{P_i})_{A,w} \succ \boldsymbol{C}_{A,w} \succeq (\cup_{\boldsymbol{i=1}}^{\boldsymbol{i=m}} \boldsymbol{P_i})_{A,w}.$$
$$f(n, \boldsymbol{C}_{A,w}) = 0 \text{ if } \boldsymbol{P_1}_{A,w} \succ \boldsymbol{C}_{A,w}.$$

Intuitively, f takes a natural number n and a triple $\boldsymbol{C}$, and spits out a natural number x indicating that the nomic likelihood of $\boldsymbol{C}$ is at least $\frac{x}{n}$ that of Ω, but less than $\frac{x+1}{n}$ that of Ω. Thus if $f(n, \boldsymbol{C}) = 0$, we know the nomic likelihood of $\boldsymbol{C}$ is less than $\frac{1}{n}$ of Ω; if $f(n, \boldsymbol{C}) = 1$, we know the nomic likelihood of $\boldsymbol{C}$ is at least $\frac{1}{n}$ but less than $\frac{2}{n}$ of Ω; and

[27] To see why the $\boldsymbol{A'}_{A,w} \succ \boldsymbol{\emptyset+}$ clause is required, consider the chance of a dart landing on various points in the 0 to 1 cm interval, with uniform probability. Let A be the proposition that a dart landed in the 0 to 1 cm interval, A' be the proposition that the dart landed on some rational number in the 0 to 1 cm interval, and C be the proposition that the dart landed on the 1/2 point in the 0 to 1 cm interval. Then $A \supset A'$, and the (A',w)-cluster is not empty, but while $\boldsymbol{C}_{A,w}$ is plausibly in NS (since given A, C has a chance of 0), $\boldsymbol{C}_{A',w}$ is plausibly not in NS (since given A' and the uniform probability assumption, no well-defined chance can be assigned to C without violating countable additivity).

[28] That is, an *n-equipartition* P is a set of n triples $\boldsymbol{P_i}$ such that (i) $\forall i, j$, $\boldsymbol{P_i} \sim \boldsymbol{P_j}$, (ii) $\forall i, j$, $P_i \cap P_j = \emptyset$, and (iii) $\forall i$, $\cup_{i=1}^{i=n} P_i = \Omega$.

so on; and if $f(n, \boldsymbol{C}) = n$, we know the nomic likelihood of $\boldsymbol{C}$ is at least $\frac{n}{n}$ of Ω, i.e., is exactly that of Ω.

Axiom 12 (Algebra Coordination): Suppose that $\boldsymbol{A}_{A,w}$, $\boldsymbol{A}_{A',w}$, $(\boldsymbol{C} \cap \boldsymbol{A})_{A,w}$, and $(\boldsymbol{C} \cap \boldsymbol{A})_{A',w}$ are in NS. If $A \subseteq A'$, and if it's not the case that there's some m such that for all $n > m$, $f(n, \boldsymbol{A}_{A,w}) = 0$ or $f(n, \boldsymbol{A}_{A',w}) = 0$, then:

$$\lim_{n\to\infty} \frac{f(n, (\boldsymbol{C} \cap \boldsymbol{A})_{A,w})}{f(n, \boldsymbol{A}_{A,w})} = \lim_{n\to\infty} \frac{f(n, (\boldsymbol{C} \cap \boldsymbol{A})_{A',w})}{f(n, \boldsymbol{A}_{A',w})}.$$

This axiom ensures that if $A \subseteq A'$, the (A,w)-cluster and the (A',w)-cluster agree on the proportion of $\boldsymbol{A}$'s nomic likelihood that contributes to $\boldsymbol{C}$'s likelihood.

Some key lemmas that follow from the axioms are described in appendix A.1. The proofs of these lemmas are given in appendix A.2.

5 The Nomic Likelihood Account (III): The Account

In this section I finish developing the Nomic Likelihood Account. In section 5.1 I'll present a representation and uniqueness theorem regarding the nomic likelihood relation. In section 5.2, using these results, I'll present the Nomic Likelihood Account of laws and chances. In section 5.3 I'll present some consequences of this account regarding laws and chances. And in section 5.4 I'll present a toy example of some complete laws given the Nomic Likelihood Account.

Before we proceed, it's worth sketching the role that the representation and uniqueness theorem plays in this account. It's helpful to start with an analogy. In the decision theory literature, people have offered representation and uniqueness theorems showing that if a subject's preferences satisfy certain conditions, then there's a (more or less) unique pair of functions that line up with these preferences in the way you'd expect rational credences and utilities to line up with them. One popular account of credences and utilities identifies them with the functions picked out by these theorems.[29] On this account, credences and utilities are just things that encode facts about a subject's preferences. And if we adopt this account, the theorem provides a straightforward explanation for why credences and utilities deserve the numerical values we assign them – because these are the only numerical assignments that line up with preferences in the right way.

Similarly, the representation and uniqueness theorem described in section 5.1 shows that if the nomic likelihood relation satisfies certain conditions, then there's a unique function and pair of relations that line up with these nomic likelihood relations in the way you'd expect chances and nomic requirements/forbiddings to line up with them. The Nomic Likelihood Account identifies chances and nomic requirements/forbiddings

[29]For classic presentations, see Savage (1954) and Jeffrey (1965). For criticisms of these accounts, see Eriksson & Hájek (2007) and Meacham & Weisberg (2011).

with the function and relations picked out by the theorem. On this account, chances and nomic requirements/forbiddings are just things that encode facts about the web of nomic likelihood relations. And if we adopt this account, the theorem provides a straightforward explanation for why chances deserve the numerical values we assign them – because these are the only numerical assignments that line up with the nomic likelihood relations in the right way.

5.1 The Representation and Uniqueness Theorem

We can partition the space of worlds such that two worlds w' and w'' are in the same cell of the partition *iff*, for all C', A', and all $\boldsymbol{C}_{A,w}$ in NS: $\boldsymbol{C}_{A,w} \succeq \boldsymbol{C'}_{A',w'}$ *iff* $\boldsymbol{C}_{A,w} \succeq \boldsymbol{C'}_{A',w''}$. Intuitively, two worlds are in the same cell of this partition *iff* the same nomic facts hold at both worlds. I'll call this the *nomic partition*. I'll use L, L', L'', etc., to denote different cells of this partition, and L_w to denote the cell w is in.

The following representation and uniqueness theorem is shown in appendix B:[30]

The Representation and Uniqueness Theorem: If $\succeq$ satisfies the nomic likelihood axioms, then there's a unique function $ch_{A,L}(C)$ (that takes three propositions C, A, and L as arguments, and spits out a real number between 0 and 1), and a unique pair of three-place relations $NR(\boldsymbol{C}_{A,w})$ and $NF(\boldsymbol{C}_{A,w})$ (that hold between a pair of propositions C and A and a world w),[31] such that:

1. $ch_{A,L}(C) \geq ch_{A',L'}(C')$ *iff* for any $w \in L$ and $w' \in L'$, either:
   (a) $\boldsymbol{C}_{A,w} \succeq \boldsymbol{C'}_{A',w'}$.
   (b) $\boldsymbol{C}_{A,w} \not\succeq \boldsymbol{C'}_{A',w'}$, and $\boldsymbol{C}_{A,w} \sim \boldsymbol{\Omega}\text{-}$ and $\boldsymbol{C'}_{A',w'} \sim \boldsymbol{\Omega}_{A',w'}$.
   (c) $\boldsymbol{C}_{A,w} \not\succeq \boldsymbol{C'}_{A',w'}$, and $\boldsymbol{C}_{A,w} \sim \boldsymbol{\varnothing}_{A,w}$ and $\boldsymbol{C'}_{A',w'} \sim \boldsymbol{\varnothing+}$.
2. $NR(\boldsymbol{C}_{A,w})$ *iff* $\boldsymbol{C}_{A,w} \sim \boldsymbol{\Omega}_{A,w}$.
3. $NF(\boldsymbol{C}_{A,w})$ *iff* $\boldsymbol{C}_{A,w} \sim \boldsymbol{\varnothing}_{A,w}$.

Furthermore, the function $ch_{A,L}(\cdot)$ will be a countably additive probability function.[32]

[30] As it turns out, only the first seven nomic axioms are required to obtain this result. The last five nomic axioms only come into play when deriving the lemmas regarding laws and chances given in section 5.3.

[31] One would typically express these relations as $NR(C, A, w)$ and $NF(C, A, w)$. But in what follows it will be more convenient (if a slight abuse of notation) to express these relations in terms of triples.

[32] That is, for all A and L, $ch_{A,L}(\cdot)$ will be such that:

1. For all C such that $\boldsymbol{C}_{A,w} \in NS$, $ch_{A,L_w}(C) \geq 0$.
2. For any $\boldsymbol{\Omega}_{A,w} \in NS$, $ch_{A,L_w}(\Omega) = 1$.
3. For any sequence $C_1, ..., C_i, ...$ such that for all C_i, $\boldsymbol{C_i}_{A,w} \in NS$, and for all $i \neq j$, $C_i \cap C_j = \varnothing$,

$$ch_{A,L_w}\left(\bigcup_{\boldsymbol{i=1}}^{\boldsymbol{\infty}} \boldsymbol{C_i}\right)_{w,A} = \sum_{i=1}^{\infty} ch_{A,L_w}(C_i).$$

This theorem shows that the nomic likelihood relation can be uniquely represented by a countably additive probability function *ch* which assigns numbers that line up with the nomic likelihood relation, and a pair of relations NR (nomically required) and NF (nomically forbidden) that hold between the members of a triple $\boldsymbol{C}_{A,w}$ *iff* it's maximally or minimally nomically likely, respectively.

5.2 The Account of Laws and Chances

Given the representation and uniqueness theorem, we can provide an account of laws, chances, and nomic requirements and forbiddings, as follows.

Complete Laws of Nature: A world w has *complete laws of nature* L *iff* $L = L_w$.[33]

It will be convenient to follow Lewis (1979) and identify properties with the set of possible individuals that instantiate them. Since the property $\mathcal{L}$ of being a world with laws L_w picks out the same set of worlds as the proposition L that laws L_w obtain, it follows that $\mathcal{L} = L$. Thus we can refer to the laws as both properties and propositions, since they're both.

The Nomic Likelihood Account then identifies chances, nomic requirements and nomic forbiddings with the *ch* function and NR and NF relations provided by the representation and uniqueness theorem:

Chances: The *chance* of C given complete laws L and antecedent A is x *iff* $ch_{A,L}(C) = x$.

Nomic Requirements: If A holds at w then C is *nomically required* to hold at w *iff* $NR(\boldsymbol{C}_{A,w})$.

Nomic Forbiddings: If A holds at w then C is *nomically forbidden* from holding at w *iff* $NF(\boldsymbol{C}_{A,w})$.

5.3 Some Lemmas Regarding Laws and Chances

The second desideratum discussed in section 2.1 was that an adequate account should yield plausible connections among laws and chances. We can now show some of the ways in which the Nomic Likelihood Account satisfies this desideratum by describing some further lemmas that follow from the nomic axioms described in section 4.3,

[33] Given this account of complete laws, how do we determine whether a given proposition (e.g., a statement of Newton's gravitational force law) is a law? Presumably a necessary condition is that it should be entailed by the complete laws. One might take this to be a sufficient condition as well, or one might add various other requirements – that it express a regularity, be appropriately general, etc. From the perspective of the Nomic Likelihood Account, this is merely a terminological matter – what really matters, metaphysically speaking, are the complete laws. (In a similar vein, there won't be an interesting distinction to draw between "fundamental laws" and "derived laws" on the Nomic Likelihood Account (Johansson (2005), Frisch (2014)), since the only plausible candidate for a "fundamental" law would be the complete laws.)

and the account of laws and chances offered in section 5.2. (The numbering of these lemmas starts at 10 because they follow the 9 lemmas given in appendix A.1. The derivations of these lemmas are given in appendix C.)

10. If (given A at w) there's some likelihood of C, and A entails C, then it seems C should be nomically required. E.g., suppose there's some nomic likelihood of rain (C) given that it's raining hard (A) at w. Then, given that it's raining hard at w, it should be nomically required that it rains. This is what the tenth lemma shows.

Lemma 10: If $\boldsymbol{C}_{A,w}$ is in NS, and A entails C, then $NR(\boldsymbol{C}_{A,w})$.

11. It seems like nomic requirements should be closed under entailment. For example, if (given A at w) it's nomically required that it be rainy (C), and nomically required that it be windy (C'), then it should be nomically required that it be rainy and windy ($C \cap C'$). This is what the eleventh lemma says.

Lemma 11: For all $\boldsymbol{C}$ in NS: If $C_1, ..., C_n$, entail C, and $NR(\boldsymbol{C_1}), ..., NR(\boldsymbol{C_n})$, then $NR(\boldsymbol{C})$.

12. It seems nomic requirements and nomic forbiddings should be linked: if C is nomically required, then $\overline{C}$ should be nomically forbidden, and vice versa. For example, if (given A at w) it's nomically required that it rain (C), then it should be nomically forbidden that it not rain ($\overline{C}$), and vice versa. This is what the twelfth lemma states.

Lemma 12: $NR(\boldsymbol{C})$ *iff* $NF(\overline{\boldsymbol{C}})$.

13. It seems like nomic requirements and forbiddings should be tied to the truth. For example, if (given A at w) rain is nomically required, and A obtains, then it should rain. Likewise, if (given A at w) rain is nomically forbidden, and A obtains, then it shouldn't rain. This is what the thirteenth lemma asserts.

Lemma 13:

1. If $NR(\boldsymbol{C}_{A,w})$ and $w \in A$, then $w \in C$.
2. If $NF(\boldsymbol{C}_{A,w})$ and $w \in A$, then $w \in \overline{C}$.

14. It seems nomic likelihoods should be tied to chances. For example, if (given A at w) the nomic likelihood of rain is on a par with **Ω-** or **Ω**, then the chance of rain (given A and the laws that hold at w) should be 1. Likewise, if the nomic likelihood of rain is on a par with ∅**+** or ∅, then the chance of rain should be 0. And if the nomic likelihood of rain is middling, then the chance of rain should be greater than 0 but smaller than 1. This is what the fourteenth lemma says.

Lemma 14:

1. If $\boldsymbol{C}_{A,w} \succeq \boldsymbol{\Omega\text{-}}$, then $ch_{A,L_w}(C) = 1$.
2. If $\boldsymbol{C}_{A,w} \preceq \emptyset\boldsymbol{+}$, then $ch_{A,L_w}(C) = 0$.
3. If $\boldsymbol{\Omega\text{-}} \succ \boldsymbol{C}_{A,w} \succ \emptyset\boldsymbol{+}$, then $ch_{A,L_w}(C) \in (0, 1)$.

15. It seems related chance distributions should assign chances to the same propositions. For example, suppose A and L yield a well-defined chance distribution over a sequence of fair coin tosses. And suppose the conjunction of A and the first fair coin toss landing heads (call this conjunction A') and L also yield a well-defined chance distribution. It would be strange if $ch_{A',L}$ assigned chances to coin tosses that $ch_{A,L}$ did not assign chances to, or vice versa. Rather, it seems $ch_{A,L}$ and $ch_{A',L}$ should assign chances to the same propositions. This is what the fifteenth lemma says.

Lemma 15: If $A \supset A'$, $ch_{A,L}(A') > 0$, and $ch_{A',L}(\Omega)$ is well-defined, then for all C, $ch_{A',L}(C)$ is well-defined *iff* $ch_{A,L}(C)$ is well-defined.

16. It seems related chance distributions should have related chance assignments. For example, suppose A and L yield a well-defined chance distribution over a sequence of independent coin tosses, and this distribution assigns a chance of 1/2 to the first coin landing heads (C), and chance of 1/4 to the first two coin tosses landing heads ($C \cap C'$). And suppose the conjunction of A and the first fair coin toss landing heads – i.e., $A \cap C$ – and L also yield a well-defined chance distribution. What chance should $ch_{A \cap C,L}$ assign to the first two coin tosses landing heads? Given the chances $ch_{A,L}$ assigns, it seems the right answer is 1/2. This is what the sixteenth lemma entails.

Lemma 16: If $A \supseteq A'$, and $ch_{A,L}(C \mid A')$ and $ch_{A',L}(C)$ are well-defined, then $ch_{A',L}(C) = ch_{A,L}(C \mid A')$.

5.4 A Toy Example

It can be helpful to see a concrete example of some complete laws L_w on the Nomic Likelihood Account. But it's hard to do so concisely for realistic physical theories. So I'll instead present a toy example corresponding to a pair of cases discussed in section 2.1: a pair of worlds in which there's only one chance event, a coin toss, where the chance of heads is 0.6 in one world, and 0.7 in the other.

Let A be a proposition describing the state of a world at t consisting of a certain coin toss set-up, and let C be a proposition stating that the outcome of this coin toss was heads. Let w be a world such that there are only four triples indexed to w in NS: $\emptyset_{A,w}$, $\boldsymbol{C}_{A,w}$, $\overline{\boldsymbol{C}}_{A,w}$, and $\boldsymbol{\Omega}_{A,w}$. Let $\boldsymbol{C}_{A,w}$ be on a par with the triples in the rich cluster that are assigned a value of 0.6 by the representation and uniqueness theorem.

The complete laws of w, L_w, will consist of the set of worlds in w's cell of the nomic partition. And these laws describe a world in which there's almost nothing of nomic interest going on: there's only a single non-trivial chance event – a coin toss – which has a chance of 0.6 of landing heads.

We can also consider a world w' such that the only triples indexed to w' in NS are: $\emptyset_{A,w'}$, $\boldsymbol{C}_{A,w'}$, $\overline{\boldsymbol{C}}_{A,w'}$, and $\boldsymbol{\Omega}_{A,w'}$. And in this case, $\boldsymbol{C}_{A,w'}$ is on a par with the triples in the rich cluster that are assigned a value of 0.7. The complete laws $L_{w'}$ will consist of the set of worlds with the same nomic facts as w', and these laws describe a world in which there's only a single non-trivial chance event – a coin toss – which has a chance of 0.7 of landing heads.

6 The Nomic Likelihood Account and the Desiderata

Now let's turn to see how the Nomic Likelihood Account fares with respect to the five desiderata given in section 2.1.

Desideratum 1. An adequate account should provide a unified (and appropriately discriminating) account of laws and chances.

The Nomic Likelihood Account provides a unified account of laws and chances, characterizing both in terms of the nomic likelihood relation (cf. section 5.2). Probabilistic and non-probabilistic laws are treated similarly, with the laws that impose nomic requirements just being stronger versions of the laws that impose chances. And the Nomic Likelihood Account is appropriately discriminating, distinguishing between propositions that are nomically required and propositions that have a chance of 1 but aren't nomically required.

Desideratum 2. An adequate account should yield plausible connections between laws and chances, laws and other laws, and chances and other chances.

The Nomic Likelihood Account yields the kinds of relations between laws and chances that one would expect (cf. section 5.3). For example, it entails that nomically required propositions are not nomically forbidden, and vice versa; it entails that nomic requirements are closed under entailment;[34] it entails that nomically required propositions will have a chance of 1, and nomically forbidden propositions a chance of 0; it entails that chance distributions at the same world will be related by conditionalization; and so on.

Desideratum 3. An adequate account should describe what, at the fundamental level, makes it the case that chance events deserve the numerical values they're assigned.

The Nomic Likelihood Account provides a satisfactory explanation for why chance events deserve the numerical values we assign them. At the fundamental level we have various instantiations of the nomic likelihood relation which satisfy certain constraints (cf. sections 4.1 and 4.3). And we have a representation and uniqueness theorem that shows that there is exactly one way of assigning numbers in the $[0, 1]$-interval to propositions so that these assignments line up with these nomic likelihood relations (cf. sections 5.1 and 5.2). Since the Nomic Likelihood Account identifies chances with

[34] Assuming that the entailed propositions bear any likelihood relations at all.

these assignments, it provides an explanation for why chance events deserve the numerical values we assign them.

Desideratum 4. An adequate account should be able to accommodate both dynamical and non-dynamical chances (like those of statistical mechanics).

The Nomic Likelihood Account itself doesn't appeal to a distinction between "dynamical" and "non-dynamical" chances. But we can distinguish between different kinds of chances, and see what the Nomic Likelihood Account entails about them.

Here is one way to draw such a distinction. Let's say that a world w has *non-trivial chances iff* there are middling likelihood triples indexed to w. Call these chances *dynamical iff* all of the middling likelihood triples indexed to w have an antecedent proposition H describing a complete history up to some time.[35] Call these chances *non-dynamical iff* they're not dynamical.[36]

Given this characterization of dynamical chances, the Nomic Likelihood Account will entail that dynamical chances will have the features they're expected to have. For example, the Nomic Likelihood Account will entail that worlds with dynamical chances can't have deterministic laws. If w has deterministic laws, then every likelihood-having triple indexed to w that has a complete history H as its antecedent proposition will either be nomically required or nomically forbidden (depending on whether H and L_w entail the triple's consequent proposition or its negation). Since none of these triples have a middling likelihood, it follows that w can't have dynamical chances.[37]

Likewise, the Nomic Likelihood Account will entail that at worlds with dynamical chances, propositions about the past can only be assigned a chance of 0 or 1. Let w be a world with dynamical chances, H a history up to t, and C some proposition about what the world is like prior to t such that $C_{H,w}$ has some likelihood. By construction

[35]Or at relativistic worlds, propositions describing a complete history up to some Cauchy slice.

[36]Although this is one way to draw the distinction between "dynamical" and "non-dynamical" chances, it is not the only way. A different (and to my mind, equally reasonable) way to draw the distinction is to call these chances *dynamical iff* all of the middling likelihood triples indexed to w have an antecedent proposition S which includes a description of the complete state of the world at some time. This alternative characterization of "dynamical" chances won't yield the result that propositions about the past can only be assigned a dynamical chance of 0 or 1. Those who hold that dynamical chances should only be able to assign propositions about the past a chance of 0 or 1 (like Lewis (1980)) will take this to be a reason to favor the characterization of dynamical chances given in the text. Those who want to permit the possibility of future-to-past dynamical chances, or even temporally symmetric dynamical chances (like Meacham (2005)) will take this to be a reason to favor the alternative characterization just described. In any case, on the Nomic Likelihood Account, this is merely a terminological matter. Nothing of substance hangs on our choice about which chances to call "dynamical".

[37]On some ways of characterizing determinism, such as Lewis's (1983), a complete history and deterministic laws will only fix the truth of every *qualitative* proposition, not every proposition *simpliciter*. Given this understanding of determinism, the Nomic Likelihood Account will only entail that deterministic laws are incompatible with dynamical chances that assign middling likelihoods to qualitative propositions.

H will entail either C or $\overline{C}$, from which it follows (by lemmas 10 and 12) that $\boldsymbol{C}_{H,w}$ is either nomically required or nomically forbidden. Thus (by lemma 14) the chance of $\boldsymbol{C}_{H,w}$ is either 0 or 1.

By contrast, the Nomic Likelihood Account will allow worlds with deterministic laws to have non-dynamical chances, and so can accommodate classical mechanical worlds with statistical mechanical chances. For example, let the laws of w be those of classical statistical mechanics,[38] let A be the claim that the world at t consists of a small isolated system containing uniform lukewarm water, and let C be the claim that the world five minutes after t consists of a small isolated system containing an ice cube in hot water. L_w and A don't entail whether C is true or not – the laws and the fact that the world consists of uniform lukewarm water doesn't entail that there will be an ice cube in five minutes, nor does it entail that there won't be. So $C_{A,w}$ can have a middling likelihood even though the laws at w are deterministic.

Likewise, the Nomic Likelihood Account doesn't require non-dynamical chances to assign propositions about the past a chance of 0 or 1. Consider a variant of the example from above, where w has classical statistical mechanical laws, A asserts that the world at t consists of lukewarm water, and C asserts that the world 5 minutes *before* t consists of an ice cube in hot water. A is compatible with both the truth and falsity of C – the world consisting of lukewarm water at t is compatible with both there being an ice cube five minutes ago and there not being such an ice cube. So $C_{A,w}$ can have a middling likelihood, even though C is a proposition about the past.[39]

Desideratum 5. An adequate account should be able to accommodate plausible nomic possibilities.

The Nomic Likelihood Account can accommodate a wide range of plausible nomic possibilities. For example, since the only kind of consequent proposition the account can't assign nomic likelihoods to are propositions concerning nomic facts (section 4.1), the account allows nomic likelihoods to be assigned to propositions about particular locations, times, and objects. Thus the account allows for laws about particular locations, times, and objects, like the case of Smith's garden discussed by Tooley (1977).

[38]Following Albert (2000), we can take these to be the conjunction of Newton's laws of motion, the Past Hypothesis, and the Statistical Postulate.

[39]I'm evaluating the claim that "all propositions about the past get a chance of 0 or 1" by taking the antecedent proposition to pick out a particular time – the earliest time which the proposition says something about – and taking consequent propositions to be "about the past" if they say things about times earlier than that. This way of understanding when propositions are about the past yields the result that non-dynamical chances can assign chances other than 0 or 1 to propositions about the past. A different (and to my mind, equally reasonable) approach would be to maintain that the antecedent propositions of non-dynamical chances (like those of statistical mechanics) aren't naturally time-indexed. And claims about whether "all propositions about the past get a chance of 0 or 1" simply don't make sense in the context of non-dynamical chances, since there's no good way of picking out a "now" time that we can use to determine whether a proposition is about the past.

Likewise, the account can assign nomic likelihoods to triples even if both their consequent and antecedent propositions are false (section 4.3, axiom 8). Thus it can allow for worlds with uninstantiated laws, like a world where $F = ma$ is a law but there are no massive objects. And as we saw in section 5.4, the account can can make sense of a world w with a single chance event, a coin toss, where the chance of heads is 0.6, and an otherwise identical world w' where the chance of heads is 0.7.

7 Worries

Let's turn to assess some worries one might raise for the Nomic Likelihood Account.

1. The Ontological Worry: The nomic likelihood relation is a fundamental relation defined over propositions and worlds. Characterizing the laws in terms of such a relation commits one to having propositions and possible worlds in one's ontology.

Reply: First, note that the Nomic Likelihood Account doesn't require one to understand propositions and worlds in a metaphysically heavyweight way. For example, one might identify propositions with sets of worlds, and adopt a metaphysically lightweight understanding of worlds themselves, like the one advocated by Stalnaker (2011).

Second, although I've characterized the nomic likelihood relation as taking propositions and worlds as relata, one could characterize the relation in other ways to avoid these commitments. If one doesn't like propositions, one could replace the appeal to propositions with an appeal to properties, i.e., the property of being a world at which the relevant proposition is true. Or one could replace the appeal to propositions with an appeal to Chisholm-style states of affairs.[40]

Likewise, if one doesn't like worlds, one could replace the appeal to worlds with an appeal to propositions, i.e., the maximally specific propositions describing that possibility. (On this approach, of course, one would not identify propositions with sets of worlds.) Or one could replace the appeal to worlds with an appeal to very detailed properties or states of affairs. These alternative characterizations of the nomic likelihood relation would require only superficial modifications to the details presented in sections 4 and 5.[41]

[40]See Chisholm (1976).

[41]A related complaint is that the account described in sections 3-5 commits one to taking propositions and worlds to be more fundamental than (say) chance events and lawful states of the world at a time. But, one might argue, the latter should be more fundamental than the former – for example, it's natural to take chance events to be more fundamental than the propositions describing them. (I owe an anonymous referee for raising this concern.)

So far I've followed Lewis (1983) in taking the fundamental/non-fundamental distinction to only apply to properties, not to things like propositions and events. So properly articulating this concern would require spelling out a broader account of the fundamental/non-fundamental distinction. But, putting that aside, I

2. The Explanatory Worry: The Nomic Likelihood Account allows for pairs of worlds that, nomic facts aside, are qualitatively identical, and yet which differ with respect to their laws. (For example, the pair of worlds discussed in section 5.4.) But it's hard to see how such an account could explain why these worlds differ with respect to their laws, other than simply stipulating that different nomic likelihood relations hold of them. And that seems little better than being a primitivist about laws.[42]

Reply: The Nomic Likelihood Account is, indeed, similar to primitivist accounts of laws in these respects.[43] But I don't take this to be a problem for the Nomic Likelhood Account. The complaint I raised in section 2.2 about primitivist accounts like Carroll's (1994) wasn't that they took nomic facts to be primitive, or that they couldn't explain why certain laws obtained without appealing to nomic facts. After all, pretty much any non-Humean account is going to have to appeal to some kind of brute modal or nomic facts. Rather, the complaint was that accounts like Carroll's don't provide the kind of detailed framework needed to satisfy desiderata 2 and 3 – to yield plausible connections among laws and chances, and to show why chance events deserve the numerical values we assign them. And this is a demerit the Nomic Likelihood Account does not share.

3. The Duplication/Intrinsicality Worry: Following David Lewis (1983), let's say two worlds are *duplicates*$_{DL}$ *iff* there is a bijection between their parts that preserves their fundamental properties and the fundamental relations holding between them. Since the nomic likelihood relation holds between a world and things that aren't a part of that world (i.e., another world and several propositions), it won't play a role in our assessment of whether worlds are duplicates$_{DL}$. Indeed, one world can be a duplicate$_{DL}$ of another even if one bears various nomic likelihood relations and the other bears no nomic likelihood relations at all. And since the laws of a world are determined by its nomic likelihood relations, it follows that duplicate$_{DL}$ worlds needn't have the same laws. This is implausible.

Likewise, following David Lewis (1983), let's say that a property is *intrinsic*$_{DL}$ *iff* it never divides duplicates – any two things that are duplicates either both have

take this concern to be in the same vein as the ontology worry described in the text, and to be amenable to the same kind of reply. Just as one can adjust the account to fit one's ontological sensibilities by changing the relata of the nomic likelihood relation, one can also adjust the account to fit one's sensibilities regarding what's ontologically fundamental by changing these relata. For example, if we replace the appeal to propositions with an appeal to states of affairs, and we take events to be a kind of state of affairs (Chisholm (1990)), then we can avoid any suggestion that chance events are less fundamental than propositions.

[42]I'd like to thank an anonymous referee for bringing this worry to my attention.

[43]Indeed, the two-layerversion of the Nomic Likelihood Account discussed in worry 3 – which posits both fundamental first-order "complete law" properties of worlds, and a fundamental second order nomic likelihood relation that holds of these properties and propositions – might naturally be classified as a form of primitivism (cf. footnote 50).

this property or both fail to have this property.[44] It follows that the laws of a world aren't intrinsic$_{DL}$ properties of that world. This is implausible.

Reply: To begin, it's worth noting that an analogous worry arises for a popular measurement theoretic account of quantitative properties like mass and charge.[45] This account posits some fundamental relations over objects corresponding to each quantitative property – e.g., in the case of mass, a mass ordering and a mass concatenation relation – and then use those relations to characterize the quantitative structure of that property. Now, note that the up quark and the charm quark are identical in every way except for their mass. Since on this account these differences of mass are the result of the different mass relations they stand in, it follows that the up quark and the charm quark will be duplicates$_{DL}$. Indeed, given a similar account of other quantitative properties, it will follow that all fundamental particles are duplicates$_{DL}$. This seems implausible. Likewise, it will follow that all of the derivative monadic quantitative properties – e.g., having $\frac{2.2\ MeV}{c^2}$-mass – will not be intrinsic$_{DL}$. Again, this seems implausible.

There are three ways for the proponent of the Nomic Likelihood Account to reply to the worries raised above. These replies mirror the options available to the proponents of the popular measurement theoretic account of quantitative properties just described. They can (1) challenge the characterizations of duplication and intrinsicality given above, (2) modify the posits the theory makes, or (3) bite the bullet. I won't discuss the third reply,[46] but let's look at each of the first two replies more carefully.

(1) Let's start by distinguishing between two kinds of relations. First, there are relations that only hold between things located at the same possible world; call these *connecting* relations. Spatiotemporal relations are connecting relations – you can't be five feet from something located at a different possible world. Second, there are relations that can hold between things that are located at different possible worlds; call these *non-connecting* relations. The more-mass-than relation is a non-connecting relation – we can make sense of something at another possible world having more mass than me.[47]

Intuitively, qualitative duplicates are perfectly alike "in and of themselves". That is, duplicates must share their monadic fundamental properties. By contrast, dupli-

[44]See Shumener (forthcoming) for some arguments for why we should take laws to be intrinsic. Of course, there are various worries one might raise regarding whether Lewis's account of intrinsic properties is fine-grained enough; e.g. see Eddon (2011). But those worries are orthogonal to the worries being raised here.

[45]For some early and influential accounts along these lines see Krantz et al. (1971) and Field (1980). For a survey of this literature, see Eddon (2013b).

[46]For a defense of this third reply, see Dasgupta (2013).

[47]Lewis's (1986) conception of possible worlds relies on a distinction of this kind, taking possible worlds to be fusions of possible individuals that are related by some chain of connecting relations. Lewis (1986) also endorsed a particular conception of connecting relations, taking them to consist of all and only those fundamental relations that are 'spatiotemporal or analogously spatiotemporal' (p. 76).

cates need not be alike in how they are connected to other things – two copies of a book may differ in their spatiotemporal relations to me and still be duplicates. That is, duplicates can differ with respect to their fundamental connecting relations. But these two truisms leave open the question of whether duplicates should be alike with respect to their non-connecting relations. One thought is that duplicates must also be alike with respect to their fundamental non-connecting relations. So in order for two objects to be duplicates, they must not only share their monadic fundamental properties, they must also stand in the same kinds of fundamental non-connecting relations – e.g., they must bear the more-mass-than relation to the same things.

This suggests an alternative to Lewis's account of duplication. Let's say that a pair of objects a and b are *interchangeable* with respect to a relation R *iff* $R(..., a, ...) \leftrightarrow R(..., b, ...)$. So two objects are interchangeable with respect to a relation *iff* whenever that relation holds between the first object and certain other things, it also holds between the second object and those same other things. Now let's say that two things a and b are *duplicates*$_O$ *iff* (i) one can form a bijection between a's parts and b's parts that preserves the fundamental properties and fundamental relations between them (i.e., they're *duplicates*$_{DL}$), and (ii) a and b are interchangeable with respect to all fundamental non-connecting relations. One might propose that our ordinary notion of duplication is duplication$_O$.[48]

We saw above that given a popular measurement theoretic account of mass, the up quark and the charm quark will be duplicates$_{DL}$. But they won't be duplicates$_O$. The mass ordering and mass concatenation relations are paradigmatic instances of non-connecting relations, and the up and charm quarks aren't interchangeable with respect to them. So this alternate account of duplication avoids the unpleasant result that the up and charm quarks are duplicates, in the ordinary sense.

Likewise, on the Nomic Likelihood Account, two otherwise identical worlds with different laws will be duplicates$_{DL}$. But they won't be duplicates$_O$. The nomic likelihood relation is a non-connecting relation, and these two worlds won't be interchangeable with respect to it. So given this alternate account of duplication, the proponent of the Nomic Likelihood Account can maintain that worlds with different laws aren't duplicates, in the ordinary sense.

Turning to intrinsicality, let's say that a property is *intrinsic*$_O$ *iff* it doesn't divide duplicates$_O$. One might propose that our ordinary notion of intrinsicality is intrinsic$_O$.[49] If this is correct, then proponents of this popular measurement theoretic account of mass can maintain that monadic quantitative properties (like having $\frac{2.2\ MeV}{c^2}$-mass) *are* intrinsic in the ordinary sense.

Likewise, proponents of the Nomic Likelihood Account can maintain that the property of being a world where the laws are L is intrinsic in the ordinary sense.

[48]Or, if one takes our ordinary notion of duplication to be vague, that duplication$_O$ is one of the possible disambiguations of our ordinary notion of duplication.

[49]Or more or less our ordinary notion. For some remaining issues that crop up, see Eddon (2011), and the references therein.

(2) Those who would prefer to keep Lewis's characterizations of duplication and intrinsicality can respond to this objection in a different way.

As we saw above, according to a popular measurement theoretic account of quantitative properties, things that differ solely with respect to their quantitative properties (e.g., the up and charm quarks) will be duplicates$_{DL}$, and the derivative monadic quantitative properties (e.g., having $\frac{2.2\ MeV}{c^2}$-mass) will not not be intrinsic$_{DL}$. Mundy (1987) and Eddon (2013a) have argued that we should avoid these difficulties by modifying the account. In particular, instead of just positing one layer of fundamental quantitative properties – fundamental quantitative relations that hold between objects – we should posit two layers of fundamental quantitative properties – fundamental monadic quantitative properties instantiated by objects, and fundamental second-order quantitative relations that hold between these monadic properties. Thus, for example, instead of positing fundamental mass-concatenation and mass-ordering relations over objects, we can posit fundamental monadic mass-properties (e.g., having $\frac{2.2\ MeV}{c^2}$-mass) that hold of objects, and fundamental mass-concatenation and mass-ordering relations over these monadic mass properties. If we do this, then the up and charm quarks won't be duplicates$_{DL}$, and monadic quantitative properties (like having $\frac{2.2\ MeV}{c^2}$-mass) will be intrinsic$_{DL}$.

We can avoid the analogous worries for the Nomic Likelihood Account presented in sections 3-5 by modifying it in a similar fashion. Namely, instead of positing one layer of fundamental nomic likelihood properties – fundamental nomic likelihood relations over worlds and propositions – we can posit two layers of fundamental nomic likelihood properties – fundamental monadic nomic properties instantiated by worlds, and fundamental second-order nomic likelihood relations that hold between these monadic properties and propositions. In this two-layerpicture, the monadic properties will intuitively line up with the complete laws instantiated by that world, L. And the nomic likelihood relation will replace the appeal to worlds with an appeal to these complete laws, where $\succeq(C, A, L, C', A', L')$ holds when C given A if the laws are L is at least as nomically likely as C' given A' if the laws are L'.[50] If we adopt this two-layerversion of the Nomic Likelihood Account, then otherwise identical worlds with different laws won't be duplicates$_{DL}$, and the property of having complete laws L will be intrinsic$_{DL}$, as desired.

4. The Holism Worry: Grant that the laws and chances are intrinsic features of the world (cf. worry 3). On the Nomic Likelihood Account, the laws and chances will still be holistic features of the world. This contrasts with a local picture on which, for example, the chance of a coin toss is determined by local features of the coin toss set-up. On this local picture, a local duplicate of this coin toss set-up

[50]Since this two-layer version of the Nomic Likelihood Account posits a range of fundamental complete law properties in addition to the fundamental nomic likelihood relation, this account might be classified as a form of primitivism about laws. I have no objection to this classification, since I don't think there's anything inherently problematic about primitivist accounts. What matters is not whether an account is primitivist, but whether it satisfies the desiderata an adequate account of laws should satisfy.

in another world would have the same chance of landing heads. On the Nomic Likelihood Account, this needn't be the case.[51]

Reply: Let's first get clear on what the distinction between holistic and local pictures of laws and chances amounts to. At a first pass, we can take the disagreement to be about whether there are local regions (regions smaller than a world) such that any duplicate of these regions, in any world, will have the same operative laws and chances. On local pictures there are regions like this: since the laws and chances are local features of regions, and a duplicate of such a region will share its local features, any duplicate of such a region will be governed by the same laws and chances. On holistic pictures, like the one provided by the Nomic Likelihood Account, there aren't regions like this: since the laws and chances are determined at the world level, and vary from world to world, duplicates of local regions in different worlds generally won't be governed by the same laws and chances.

I don't have any strong intuitions about whether the holistic or the local picture is correct.[52] So I'm inclined to take this to be a case of spoils to the victor – we should adopt the picture suggested by the account of laws and chances that we independently find most plausible. But I grant that if one is strongly attracted to a local picture of laws and chances, then one has a reason to dislike the Nomic Likelihood Account.

5. The Wrong Grain Worry: The nomic likelihood relation is fine-grained in some respects – for example, it allows us to distinguish between chance 1 propositions that are nomically required and chance 1 propositions that are not. But in other respects it still seems too coarse-grained to capture all of the nomic likelihood facts. For example, suppose a point-like dart is thrown at a one meter interval, with the probability of it hitting any point determined by a bell-curve centered around the 0.5-meter point. The nomic likelihood relation will treat the dart landing on the 0.5-meter point and the dart landing on the 0.9-meter point as on a par ($\sim \emptyset$**+**). But surely the dart landing on the 0.5-meter point is nomically more likely than the dart landing on the 0.9-meter point.

Reply: It's true that given the version of the Nomic Likelihood Account developed here, the fundamental nomic likelihood relation won't be sensitive to such facts. But the proponent of this account can explain (and partially vindicate) these intuitions regarding more fine-grained nomic likelihood facts.[53]

[51] I'd like to thank an anonymous referee for encouraging me to address this worry.

[52] Some (like Armstrong (1983) and Maudlin (2007)) want to allow for worlds where the laws and chances differ in different epochs. It's natural to think that this might be a consideration which tells between a local and holistic picture of laws. But both pictures can make sense of such possibilities. Holistic laws can accommodate such worlds by having laws that assert that regions in different spatiotemporal locations behave differently. And local laws can accommodate such worlds by positing different local laws in different spatiotemporal locations.

[53] A different response to this objection would be to develop a variant of the Nomic Likelihood Account whose axioms provide a representation and uniqueness theorem that yields non-standard probability as-

For example, it's true that this account will take the dart landing on the 0.9-meter point and the dart landing on the 0.5-meter point to be on a par ($\sim \emptyset\mathbf{+}$). But if we consider arbitrarily small neighborhoods surrounding these points (i.e., all points within $\pm\epsilon$ meters), then the nomic likelihood of landing in the neighborhood of the 0.5-meter point will be greater than that of landing in the neighborhood of the 0.9-meter point. And we can use this fact to explain the intuition that the dart landing on the 0.5-meter point is more likely than it landing on the 0.9-meter point.

Likewise, if the probability measure representing the chances is absolutely continuous with respect to some other salient (σ-finite) measure, then it follows from the Radon-Nikodym theorem that one can define a probability *density* with respect to that salient measure.[54] In the dart case described above, the salient measure is length, and we can define the probability density at each point of the one meter interval of the dart landing there. The probability density of the dart landing on the 0.5-meter point will be larger than the probability density of the dart landing on the 0.9-meter point. And we can use this fact to explain our intuition that the former is more nomically likely.[55]

8 Conclusion

I've suggested (section 2.1) that an adequate account of laws should satisfy five desiderata: it should (1) provide a unified account of laws and chances, (2) yield plausible relations between laws and chances, (3) explain why we're justified in assigning numerical values to chance events in the way that we do, (4) allow for both dynamical and non-dynamical chances, and (5) allow for an appropriately expansive range of nomic possibilities. I've argued (section 2.2) that no extant account of laws satisfies these desiderata.

In this paper I've developed an account of laws, the Nomic Likelihood Account (sections 3-5), that satisfies all five desiderata (section 6). On this account, the fundamental nomic property is a nomic likelihood relation. And laws and chances are

signments (e.g., hyperreal valued-probabilities). Although this is an interesting avenue for future research, there are some *prima facie* reasons to be skeptical that chances are this fine-grained; see Pruss (2018) and Easwaren & Towsner (2018).

[54]Billingsley (1995).

[55]One might be tempted to construct a second and more fine-grained nomic likelihood relation in light of such facts, and take this to be the "real" nomic-likelihood relation. I think this would be a mistake. For these densities will only be defined with respect to a second measure; so at best they're providing us with something like comparisons of nomic likelihood *with respect to such-and-such a measure*, not comparative nomic likelihoods *simpliciter*.

A similar obstacle prevents us from skipping over having to posit the $\mathbf{\Omega-}$ and $\emptyset\mathbf{+}$ likelihoods, and simply distinguishing between chance 0 events that are nomically forbidden and those that are not by appealing to whether they have non-zero densities. For again, these densities will only be defined relative to some further measure.

things that encode facts about the web of nomic likelihood relations. As I've noted, there are various challenges one might raise for this account (section 7). But I think this is ultimately the most attractive account of laws and chances on offer.[56]

A Some Lemmas Regarding Nomic Likelihood

A.1 Some Key Lemmas

Lemma 1: For all $\boldsymbol{C}$ in NS, $\boldsymbol{\Omega} \succeq \boldsymbol{C} \succeq \emptyset$.

Lemma 2: For all $\boldsymbol{C}$, $\boldsymbol{C'}$ in NS, if $C \subseteq C'$, then $\boldsymbol{C} \preceq \boldsymbol{C'}$.

Lemma 3: For all $\boldsymbol{C'}$ in NS:

1. If $\boldsymbol{C} \sim \emptyset$, then $\boldsymbol{C} \cap \boldsymbol{C'} \sim \emptyset$.
2. If $\boldsymbol{C} \sim \boldsymbol{\Omega}$, then $\boldsymbol{C} \cup \boldsymbol{C'} \sim \boldsymbol{\Omega}$.

Lemma 4: For all $\boldsymbol{C'}$ in NS:

1. If $\boldsymbol{C} \sim \emptyset$, then $\boldsymbol{C} \cup \boldsymbol{C'} \sim \boldsymbol{C'}$.
2. If $\boldsymbol{C} \sim \boldsymbol{\Omega}$, then $\boldsymbol{C} \cap \boldsymbol{C'} \sim \boldsymbol{C'}$.

Lemma 5: $\boldsymbol{C} \sim \emptyset$ *iff* $\overline{\boldsymbol{C}} \sim \boldsymbol{\Omega}$.

Lemma 6: $\boldsymbol{C} \sim \boldsymbol{\emptyset+}$ *iff* $\overline{\boldsymbol{C}} \sim \boldsymbol{\Omega\text{-}}$.

Lemma 7: If $\boldsymbol{\emptyset+} \prec \boldsymbol{C} \prec \boldsymbol{\Omega\text{-}}$, then $\boldsymbol{\emptyset+} \prec \overline{\boldsymbol{C}} \prec \boldsymbol{\Omega\text{-}}$.

Lemma 8: If $\boldsymbol{C} \succeq \boldsymbol{C'}$, then $\overline{\boldsymbol{C'}} \succeq \overline{\boldsymbol{C}}$.

Lemma 9: For all $\boldsymbol{C}$ in NS:

1. If $\boldsymbol{C} \cap \boldsymbol{C'} \sim \boldsymbol{C} \cap \boldsymbol{C''} \sim \emptyset$, and none of the following three conditions hold: (i) $\boldsymbol{C} \sim \boldsymbol{\Omega\text{-}}$, $\boldsymbol{C'} \sim \boldsymbol{C''} \sim \boldsymbol{\emptyset+}$, (ii) $\boldsymbol{C} \sim \boldsymbol{\emptyset+}$, $\boldsymbol{C'} \sim \boldsymbol{C''} \sim \boldsymbol{\Omega\text{-}}$, or (iii) $\boldsymbol{\Omega} \succ \boldsymbol{C} \succ \emptyset$, $\boldsymbol{C'} \sim \emptyset$, $\boldsymbol{C''} \sim \boldsymbol{\emptyset+}$, then $\boldsymbol{C'} \succeq \boldsymbol{C''}$ *iff* $\boldsymbol{C} \cup \boldsymbol{C'} \succeq \boldsymbol{C} \cup \boldsymbol{C''}$.
2. If $\boldsymbol{C} \cap \boldsymbol{C'} \sim \boldsymbol{C} \cap \boldsymbol{C''} \sim \emptyset$, and none of the following four conditions hold: (i) $\boldsymbol{C} \sim \boldsymbol{\Omega\text{-}}$, $\boldsymbol{C'} \sim \boldsymbol{C''} \sim \boldsymbol{\emptyset+}$, (ii) $\boldsymbol{C} \sim \boldsymbol{\emptyset+}$, $\boldsymbol{C'} \sim \boldsymbol{C''} \sim \boldsymbol{\Omega\text{-}}$, (iii) $\boldsymbol{\Omega} \succ \boldsymbol{C} \succ \emptyset$, $\boldsymbol{C'} \sim \emptyset$, $\boldsymbol{C''} \sim \boldsymbol{\emptyset+}$, or (iv) $\boldsymbol{\Omega} \succ \boldsymbol{C} \succ \emptyset$, $\boldsymbol{C'} \sim \boldsymbol{\emptyset+}$, $\boldsymbol{C''} \sim \emptyset$, then $\boldsymbol{C'} \succ \boldsymbol{C''}$ *iff* $\boldsymbol{C} \cup \boldsymbol{C'} \succ \boldsymbol{C} \cup \boldsymbol{C''}$.

[56]For helpful comments and discussion, I'd like to thank Maya Eddon, Jenn McDonald, Alejandro Perez-Carballo, two anonymous referees, and participants of the Fall 2019 UMass Brown Bag group, the 2019 Rutgers Conference on the Philosophy of Probability, the 2021 Canadian Society for the History and Philosophy of Science conference, the 2021 Formal Philosophy conference, and the 2021 Society for the Metaphysics of Science conference.

A.2 Proofs

While the lemmas in section A.1 are ordered thematically, the proofs are presented in order of dependence (with later lemmas depending on earlier ones, but not vice versa). Most of these proofs implicitly appeal to axioms like 1 and 2 to discharge the existence assumptions of the other axioms they employ; to avoid needless clutter, I'll leave such appeals implicit.

• *Proof of Lemma 9:* (1) The first part of the lemma is a special case of axiom 5, where all of the relevant propositions belong to the same cluster, and $C = C''$. (Note that while axiom 5 imposes the condition that $\boldsymbol{C}_{A,w} \sim \boldsymbol{C}''_{A',w'}$, which entails that $\boldsymbol{C}_{A,w}$ is in NS, lemma 5 doesn't have such a clause since $C = C''$. Thus lemma 5 needs to explicitly add the existence assumption "For all $\mathbf{C}$ in NS".)

(2) The second part of the lemma follows from the first and the assumption that it's also not the case that (iv) $\boldsymbol{\Omega} \succ \boldsymbol{C} \succ \varnothing$, $\boldsymbol{C}' \sim \varnothing$+, $\boldsymbol{C}'' \sim \varnothing$. To see this, suppose that the relevant triples are on a par with the emptyset, and none of conditions (i)-(iv) hold.

First, let's establish that if $\boldsymbol{C}' \succ \boldsymbol{C}''$, then $\boldsymbol{C}' \cup \boldsymbol{C} \succ \boldsymbol{C} \cup \boldsymbol{C}''$. If $\boldsymbol{C}' \succ \boldsymbol{C}''$, then $\boldsymbol{C}' \succeq \boldsymbol{C}''$, and since none of (i)-(iii) hold, the first part of the lemma entails that $\boldsymbol{C}' \cup \boldsymbol{C} \succeq \boldsymbol{C} \cup \boldsymbol{C}''$. Furthermore, $\boldsymbol{C}' \succ \boldsymbol{C}''$ entails that $\boldsymbol{C}' \not\preceq \boldsymbol{C}''$, and since none of (i), (ii) or (iv) hold, the first part of the lemma entails that $\boldsymbol{C}' \cup \boldsymbol{C} \not\preceq \boldsymbol{C} \cup \boldsymbol{C}''$. (The conditions change because C' and C'' switch places. Conditions (i) and (ii) treat C' and C'' symmetrically, but condition (iii) does not; condition (iv) is what you get when you swap C' and C'' in condition (iii).) So if $\boldsymbol{C}' \succ \boldsymbol{C}''$, then $\boldsymbol{C}' \cup \boldsymbol{C} \succ \boldsymbol{C} \cup \boldsymbol{C}''$.

Second, let's establish that if $\boldsymbol{C}' \cup \boldsymbol{C} \succ \boldsymbol{C} \cup \boldsymbol{C}''$, then $\boldsymbol{C}' \succ \boldsymbol{C}''$. If $\boldsymbol{C}' \cup \boldsymbol{C} \succ \boldsymbol{C} \cup \boldsymbol{C}''$, then $\boldsymbol{C}' \cup \boldsymbol{C} \succeq \boldsymbol{C} \cup \boldsymbol{C}''$, and since none of (i)-(iii) hold, the first part of the lemma entails that $\boldsymbol{C}' \succeq \boldsymbol{C}''$. Furthermore, $\boldsymbol{C}' \cup \boldsymbol{C} \succ \boldsymbol{C} \cup \boldsymbol{C}''$ entails that $\boldsymbol{C}' \cup \boldsymbol{C} \not\preceq \boldsymbol{C} \cup \boldsymbol{C}''$, and since none of (i), (ii) or (iv) hold, the first part of the lemma entails that $\boldsymbol{C}' \not\preceq \boldsymbol{C}''$. So if $\boldsymbol{C}' \cup \boldsymbol{C} \succ \boldsymbol{C} \cup \boldsymbol{C}''$, then $\boldsymbol{C}' \succ \boldsymbol{C}''$.

• *Proof of Lemma 2:* Suppose that $A \supseteq B$. Let $C = B$, $C' = A - B$ and $C'' = \varnothing$. Note that $\boldsymbol{C} \cap \boldsymbol{C}' \sim \boldsymbol{C} \cap \boldsymbol{C}'' \sim \varnothing$. Note also that none of conditions (i)-(iii) of lemma 9 obtain (since in all of them $\boldsymbol{C}'' \not\sim \varnothing$). Thus by lemma 9, $\boldsymbol{C}' \succeq \boldsymbol{C}''$ *iff* $\boldsymbol{C} \cup \boldsymbol{C}' \succeq \boldsymbol{C} \cup \boldsymbol{C}''$, i.e., $(\boldsymbol{A} - \boldsymbol{B}) \succeq \varnothing$ *iff* $\boldsymbol{B} \cup (\boldsymbol{A} - \boldsymbol{B}) \succeq \boldsymbol{B} \cup \varnothing$ *iff* $\boldsymbol{A} \succeq \boldsymbol{B}$. Since the left-hand side is true, the right-hand side must be true as well.

• *Proof of Lemma 1:* (1) Since $\varnothing$ is a subset of every proposition, lemma 2 entails that $\varnothing \preceq$ every proposition. (2) Likewise, since every proposition is a subset of Ω, lemma 2 entails that $\boldsymbol{\Omega} \succeq$ every proposition.

• *Proof of Lemma 3:* (1) Since $C \cap C'$ is a subset of C, lemma 2 entails that $\boldsymbol{C} \cap \boldsymbol{C}'$ has to be $\preceq$ to $\boldsymbol{C}$. Since nothing is ranked lower than $\varnothing$ (lemma 1), $\boldsymbol{C} \cap \boldsymbol{C}'$ is on a par with $\varnothing$. (2) Likewise, since $C \cup C'$ is a superset of C, lemma 2 entails that $\boldsymbol{C} \cup \boldsymbol{C}'$ has to be $\succeq$ to $\boldsymbol{C}$. Since nothing is ranked higher than $\boldsymbol{\Omega}$ (lemma 1), $\boldsymbol{C} \cup \boldsymbol{C}'$ is on a par with $\boldsymbol{\Omega}$.

• *Proof of Lemma 4, part (1):* Let $\boldsymbol{C} \sim \varnothing$, let C' be an arbitrary proposition, and let $C'' = \varnothing$. Since $\boldsymbol{C} \sim \varnothing$, lemma 3 entails that $\boldsymbol{C} \cap \boldsymbol{C}' \sim \varnothing$; likewise since $\boldsymbol{C}'' \sim \varnothing$, it follows that $\boldsymbol{C}' \cap \boldsymbol{C}'' \sim \varnothing$. Given this and the fact that none of conditions (i)-(iii) of lemma 9 hold (since in each of (i)-(iii), $\boldsymbol{C''} \not\sim \varnothing$), lemma 9 entails that $\boldsymbol{C''} \succeq \boldsymbol{C}$ *iff* $\boldsymbol{C''} \cup \boldsymbol{C'} \succeq \boldsymbol{C} \cup \boldsymbol{C'}$. Since $\boldsymbol{C''} \sim \boldsymbol{C}$, we know the left hand side is true, so $\boldsymbol{C''} \cup \boldsymbol{C'} \succeq \boldsymbol{C} \cup \boldsymbol{C'}$ must be true. Since $C'' = \varnothing$, $C'' \cup C' = C'$, so $\boldsymbol{C'} \succeq \boldsymbol{C} \cup \boldsymbol{C'}$ must be true. And since C' is a subset of $C \cup C'$, lemma 2 entails that $\boldsymbol{C'} \preceq \boldsymbol{C} \cup \boldsymbol{C'}$. Thus $\boldsymbol{C'} \sim \boldsymbol{C} \cup \boldsymbol{C'}$.

• *Proof of Lemma 5:* First, let's establish that if $\boldsymbol{C} \sim \varnothing$, then $\overline{\boldsymbol{C}} \sim \boldsymbol{\Omega}$. If $\boldsymbol{C} \sim \varnothing$, then it follows from part (1) of lemma 4 that $\boldsymbol{C} \cup \overline{\boldsymbol{C}} \sim \overline{\boldsymbol{C}}$. Since $\boldsymbol{C} \cup \overline{\boldsymbol{C}} \sim \boldsymbol{\Omega}$, it follows that $\overline{\boldsymbol{C}} \sim \boldsymbol{\Omega}$. Second, let's establish that if $\boldsymbol{C} \sim \boldsymbol{\Omega}$, then $\overline{\boldsymbol{C}} \succ \varnothing$. Suppose for *reductio* that $\boldsymbol{C} \sim \boldsymbol{\Omega}$, but that $\overline{\boldsymbol{C}} \not\sim \varnothing$, and thus (given lemma 1) that $\overline{\boldsymbol{C}} \succ \varnothing$. Note that $C \cap \overline{C} = C \cap \varnothing = \varnothing$. And, letting $C = C$, $C' = \overline{C}$, and $C'' = \varnothing$, note that none of conditions (i)-(iv) of lemma 9 hold. Thus lemma 9 entails that $\overline{\boldsymbol{C}} \succ \varnothing$ *iff* $\overline{\boldsymbol{C}} \cup \boldsymbol{C} \succ \varnothing \cup \boldsymbol{C}$. Since $\overline{\boldsymbol{C}} \succ \varnothing$ is true by supposition, the left-hand side of this bijection must be true. But since $\varnothing \cup \boldsymbol{C} \sim \boldsymbol{\Omega}$ (by lemma 3), the right-hand side of this bijection must be false. By *reductio*, $\overline{\boldsymbol{C}} \sim \varnothing$.

• *Proof of Lemma 6:* First, let's establish that if $\boldsymbol{C} \sim \varnothing\text{+}$, then $\overline{\boldsymbol{C}} \sim \boldsymbol{\Omega}\text{-}$. Suppose $\boldsymbol{C} \sim \varnothing\text{+}$. Note that either (i) $\overline{\boldsymbol{C}} \sim \varnothing$, (ii) $\overline{\boldsymbol{C}} \sim \varnothing\text{+}$, (iii) $\varnothing\text{+} \prec \overline{\boldsymbol{C}} \prec \boldsymbol{\Omega}\text{-}$, (iv) $\overline{\boldsymbol{C}} \sim \boldsymbol{\Omega}\text{-}$, or (v) $\overline{\boldsymbol{C}} \sim \boldsymbol{\Omega}$. (i),(v): If $\overline{\boldsymbol{C}} \sim \varnothing/\boldsymbol{\Omega}$, then by lemma 5 it follows that $\boldsymbol{C} \sim \boldsymbol{\Omega}/\varnothing$, contra supposition. (ii),(iii): If $\overline{\boldsymbol{C}} \sim \varnothing\text{+}$ or $\varnothing\text{+} \prec \overline{\boldsymbol{C}} \prec \boldsymbol{\Omega}\text{-}$, then (since $\boldsymbol{C} \sim \varnothing\text{+}$) the first part of axiom 7 entails that $\boldsymbol{C} \cup \overline{\boldsymbol{C}} \sim \overline{\boldsymbol{C}} \prec \boldsymbol{\Omega}$, which is impossible since $\boldsymbol{C} \cup \overline{\boldsymbol{C}} \sim \boldsymbol{\Omega}$. Thus the only remaining option is (iv): $\overline{\boldsymbol{C}} \sim \boldsymbol{\Omega}\text{-}$.

Second, let's establish that if $\boldsymbol{C} \sim \boldsymbol{\Omega}\text{-}$, then $\overline{\boldsymbol{C}} \sim \varnothing\text{+}$. Suppose $\boldsymbol{C} \sim \boldsymbol{\Omega}\text{-}$. Note that either (i) $\overline{\boldsymbol{C}} \sim \varnothing$, (ii) $\overline{\boldsymbol{C}} \sim \varnothing\text{+}$, (iii) $\varnothing\text{+} \prec \overline{\boldsymbol{C}} \prec \boldsymbol{\Omega}\text{-}$, (iv) $\overline{\boldsymbol{C}} \sim \boldsymbol{\Omega}\text{-}$, or (v) $\overline{\boldsymbol{C}} \sim \boldsymbol{\Omega}$. (i),(v): If $\overline{\boldsymbol{C}} \sim \varnothing/\boldsymbol{\Omega}$, then by lemma 5 it follows that $\boldsymbol{C} \sim \boldsymbol{\Omega}/\varnothing$, contra supposition. (iii),(iv): If $\varnothing\text{+} \prec \overline{\boldsymbol{C}} \prec \boldsymbol{\Omega}\text{-}$ or $\overline{\boldsymbol{C}} \sim \boldsymbol{\Omega}\text{-}$, then axiom 7 entails that $\boldsymbol{C} \cap \overline{\boldsymbol{C}} \sim \overline{\boldsymbol{C}} \succ \varnothing$, which is impossible since $\boldsymbol{C} \cap \overline{\boldsymbol{C}} \sim \varnothing$. Thus the only remaining option is (ii): $\overline{\boldsymbol{C}} \sim \varnothing\text{+}$.

• *Proof of Lemma 7:* For *reductio* suppose otherwise – that $\varnothing\text{+} \prec \boldsymbol{C} \prec \boldsymbol{\Omega}\text{-}$, but not $\varnothing\text{+} \prec \overline{\boldsymbol{C}} \prec \boldsymbol{\Omega}\text{-}$. For this to be the case, $\overline{\boldsymbol{C}}$ must be on a par with either $\varnothing$, $\varnothing\text{+}$, $\boldsymbol{\Omega}\text{-}$, $\boldsymbol{\Omega}$. If $\overline{\boldsymbol{C}} \sim \varnothing/\boldsymbol{\Omega}$, then by lemma 5 it follows that $\boldsymbol{C} \sim \boldsymbol{\Omega}/\varnothing$, contra supposition. If $\overline{\boldsymbol{C}} \sim \varnothing\text{+}/\boldsymbol{\Omega}\text{-}$, then by lemma 6 it follows that $\boldsymbol{C} \sim \boldsymbol{\Omega}\text{-}/\varnothing\text{+}$, contra supposition. By *reductio*, $\varnothing\text{+} \prec \overline{\boldsymbol{C}} \prec \boldsymbol{\Omega}\text{-}$.

• *Proof of Lemma 8:* Suppose for that $\boldsymbol{C} \succeq \boldsymbol{C'}$.

Note that $(C \cap C') \cap (C - C') = (C \cap C') \cap (C' - C) = \varnothing$. If it's not the case that either (i) $(\boldsymbol{C} \cap \boldsymbol{C'}) \sim \boldsymbol{\Omega}\text{-}$, $(\boldsymbol{C} - \boldsymbol{C'}) \sim (\boldsymbol{C'} - \boldsymbol{C}) \sim \varnothing\text{+}$, (ii) $(\boldsymbol{C} \cap \boldsymbol{C'}) \sim \varnothing\text{+}$, $(\boldsymbol{C} - \boldsymbol{C'}) \sim (\boldsymbol{C'} - \boldsymbol{C}) \sim \boldsymbol{\Omega}\text{-}$, or (iii) $\varnothing\text{+} \prec (\boldsymbol{C} \cap \boldsymbol{C'}) \prec \boldsymbol{\Omega}\text{-}$, $(\boldsymbol{C} - \boldsymbol{C'}) \sim \varnothing$, $(\boldsymbol{C'} - \boldsymbol{C}) \sim \varnothing\text{+}$, then by lemma 9 it follows that $(\boldsymbol{C} - \boldsymbol{C'}) \succeq (\boldsymbol{C'} - \boldsymbol{C})$ *iff* $(\boldsymbol{C} \cap \boldsymbol{C'}) \cup (\boldsymbol{C} - \boldsymbol{C'}) = \boldsymbol{C} \succeq (\boldsymbol{C} \cap \boldsymbol{C'}) \cup (\boldsymbol{C'} - \boldsymbol{C}) = \boldsymbol{C'}$. Since the right hand side is true by supposition, the left hand side must be true too.

Note also that $(\overline{C} \cap \overline{C'}) \cap (C - C') = (\overline{C} \cap \overline{C'}) \cap (C' - C) = \varnothing$. If it's not the case that either (i*) $(\overline{\boldsymbol{C}} \cap \overline{\boldsymbol{C'}}) \sim \boldsymbol{\Omega}\text{-}$, $(\boldsymbol{C} - \boldsymbol{C'}) \sim (\boldsymbol{C'} - \boldsymbol{C}) \sim \varnothing\text{+}$, (ii*) $(\overline{\boldsymbol{C}} \cap \overline{\boldsymbol{C'}}) \sim \varnothing\text{+}$, $(\boldsymbol{C} - \boldsymbol{C'}) \sim (\boldsymbol{C'} - \boldsymbol{C}) \sim \boldsymbol{\Omega}\text{-}$, (iii*) $\varnothing\text{+} \prec (\overline{\boldsymbol{C}} \cap \overline{\boldsymbol{C'}}) \prec \boldsymbol{\Omega}\text{-}$, $(\boldsymbol{C} - \boldsymbol{C'}) \sim \varnothing$, $(\boldsymbol{C'} - \boldsymbol{C}) \sim \varnothing\text{+}$, or (iv*) $\varnothing\text{+} \prec (\overline{\boldsymbol{C}} \cap \overline{\boldsymbol{C'}}) \prec \boldsymbol{\Omega}\text{-}$, $(\boldsymbol{C} - \boldsymbol{C'}) \sim \varnothing\text{+}$, $(\boldsymbol{C'} - \boldsymbol{C}) \sim \varnothing$, then by lemma 9 it follows that $(\boldsymbol{C'} - \boldsymbol{C}) \succ (\boldsymbol{C} - \boldsymbol{C'})$ *iff* $(\overline{\boldsymbol{C}} \cap \overline{\boldsymbol{C'}}) \cup (\boldsymbol{C'} - \boldsymbol{C}) = \overline{\boldsymbol{C}} \succ (\overline{\boldsymbol{C}} \cap \overline{\boldsymbol{C'}}) \cup (\boldsymbol{C} - \boldsymbol{C'}) = \overline{\boldsymbol{C'}}$. Note that we derived the falsity of the left hand side above (we derived that $(C - C') \succeq (C' - C)$ is true, which entails that $(C' - C) \succ (C - C')$ is false). Thus the right hand side must be false too. Thus $\overline{C} \not\succ \overline{C'}$, or (equivalently) $\overline{C} \preceq \overline{C'}$. So if $C \succeq C'$, then $\overline{C} \preceq \overline{C'}$.

We've only shown this result, though, in cases where none of (i)-(iii), (i*)-(iv*) obtain. To establish the result in full generality, we need to show that in each of these cases lemma 8 will still hold. So suppose $C \succeq C'$:

(i) Suppose $(C \cap C') \sim$ **Ω-**, $(C - C') \sim (C' - C) \sim \emptyset$**+**. By lemma 6, $\overline{C \cap C'} \sim \emptyset$**+**. And since both $\overline{C}$ and $\overline{C'}$ are subsets of $\overline{C \cap C'}$, it follows from lemma 2 that both $\overline{C}$ and $\overline{C'}$ can't be more nomically likely than ∅**+**. Since $\overline{C}$ has $(C' - C)$ as a subset, and $\overline{C'}$ has $(C - C')$ as a subset, lemma 2 entails that both $\overline{C}$ and $\overline{C'}$ can't be less nomically likely than ∅**+**. Thus both $\overline{C}$ and $\overline{C'}$ must be on a par with ∅**+**, and thus $\overline{C} \preceq \overline{C'}$, as desired.

(ii) Suppose $(C \cap C') \sim \emptyset$**+**, $(C - C') \sim (C' - C) \sim$ **Ω-**. We can ignore this possibility, since the latter two assignments are impossible. (If both $(C - C')$ and $(C' - C)$ were a par with **Ω-**, then by axiom 7 $(C - C') \cap (C' - C) = \emptyset$ must be on a par with **Ω-**, which is impossible.)

(iii) Suppose ∅**+** $\prec (C \cap C') \prec$ **Ω-**, $(C - C') \sim \emptyset$, $(C' - C) \sim \emptyset$**+**. Note that by the first part of lemma 4 and lemma 6, $(C \cap C') \cup (C - C') \cup (C' - C) \sim (C \cap C')$, which we know is of middling rank. Note also that $(C \cap C') \cup (C - C') \cup (C' - C) = (C \cup C')$, and by lemma 7 the triple corresponding to its negation $\overline{(C \cup C')} = (\overline{C} \cap \overline{C'})$ must also be middling. Since $\overline{C} = (C' - C) \cup (\overline{C} \cap \overline{C'})$, it follows from axiom 7 that $\overline{C} \sim (\overline{C} \cap \overline{C'})$. Likewise, since $\overline{C'} = (C - C') \cup (\overline{C} \cap \overline{C'})$, it follows from the first part of lemma 4 that $\overline{C'} \sim (\overline{C} \cap \overline{C'})$. Thus $\overline{C} \sim \overline{C'}$, and so $\overline{C} \preceq \overline{C'}$, as desired.

(i*) Suppose $(\overline{C} \cap \overline{C'}) \sim$ **Ω-**, $(C - C') \sim (C' - C) \sim \emptyset$**+**. Since $(\overline{C} \cap \overline{C'})$ is a subset of both $\overline{C}$ and $\overline{C'}$, it follows from lemma 2 that both $\overline{C}$ and $\overline{C'}$ must be at least as nomically likely as **Ω-**. Note also that neither $\overline{C}$ nor $\overline{C'}$ can be on a par with **Ω**. (Suppose for *reductio* that $\overline{C} \sim \overline{C'} \sim$ **Ω**. Then C and C' would be on a par with ∅ (by lemma 5). But C and C' are supersets of $(C - C')$ and $(C' - C)$, and $(C - C') \sim (C' - C) \sim \emptyset$**+**, so by lemma 2 C and C' must be at least as nomically likely ∅**+**. But that's impossible if they're on a par with ∅.) Thus both $\overline{C}$ and $\overline{C'}$ must be on a par with **Ω-**, and thus $\overline{C} \preceq \overline{C'}$, as desired.

(ii*) Suppose $(\overline{C} \cap \overline{C'}) \sim \emptyset$**+**, $(C - C') \sim (C' - C) \sim$ **Ω-**. We can ignore this possibility, since the latter two assignments are impossible (see (ii), above).

(iii*) Suppose ∅**+** $\prec (\overline{C} \cap \overline{C'}) \prec$ **Ω-**, $(C - C') \sim \emptyset$, $(C' - C) \sim \emptyset$**+**. Since $\overline{C} = (C' - C) \cup (\overline{C} \cap \overline{C'})$, it follows from axiom 7 that $\overline{C} \sim (\overline{C} \cap \overline{C'})$. Likewise, since $\overline{C'} = (C - C') \cup (\overline{C} \cap \overline{C'})$, it follows from the first part of lemma 4 that $\overline{C'} \sim (\overline{C} \cap \overline{C'})$. Thus $\overline{C} \sim \overline{C'}$, which entails $\overline{C} \preceq \overline{C'}$, as desired.

(iv*) Suppose ∅**+** $\prec (\overline{C} \cap \overline{C'}) \prec$ **Ω-**, $(C - C') \sim \emptyset$**+**, $(C' - C) \sim \emptyset$. By swapping C and C' throughout, the reasoning offered for (iii*) above shows that $\overline{C} \preceq \overline{C'}$ here too.

• *Proof of Lemma 4, part (2):* Let $\boldsymbol{C'}$ be an arbitrary proposition. $\boldsymbol{C} \sim \boldsymbol{\Omega}$ *iff* $\overline{\boldsymbol{C}} \sim \varnothing$ (by lemma 5). It follows from part (1) of lemma 4 that $\overline{\boldsymbol{C}} \cup \overline{\boldsymbol{C'}} \sim \overline{\boldsymbol{C'}}$, which is logically equivalent to $\overline{\boldsymbol{C} \cap \boldsymbol{C'}} \sim \overline{\boldsymbol{C'}}$. It follows from lemma 8 that $\boldsymbol{C} \cap \boldsymbol{C'} \sim \boldsymbol{C'}$.

B The Representation and Uniqueness Theorem

This representation and uniqueness theorem can be broken down into three steps. First, I'll show that given the nomic likelihood relation, we can define a relation $\succeq_k$ that, restricting our attention to the R-algebra posited by axiom 4, satisfies some axioms (which I'll call the "k-axioms"). As Villegas (1964) and Krantz et al. (1971) show, if a relation over an algebra satisfies the k-axioms, then there exists a unique order-preserving function from this algebra to the real interval $[0, 1]$, and it will be a countably additive probability function. Second, I'll show that given such a countably additive probabilistic representation, we can assign a countably additive probabilistic representation to all of the proposition in NS, and that this representation is also unique. Third, I'll show that there's a unique way of assigning NR and NF relations over the propositions in NS. Together, these steps establish the theorem.

• **Step I(a).** Given a nomic likelihood relation that satisfies the axioms given in section 4.3, we can define a coarser relation $\succeq_k$ that, restricting our attention to the R-algebra, satisfies the following K-axioms required for a countably additive probabilistic representation of these relations.

Define $\succeq_k$ in terms of $\succeq$ as follows: $\boldsymbol{C}_{A,w} \succeq_k \boldsymbol{C'}_{A',w'}$ *iff* either (i) $\boldsymbol{C}_{A,w} \succeq_k \boldsymbol{C'}_{A',w'}$, or (ii) $\boldsymbol{C}_{A,w} \sim \boldsymbol{\Omega}$**-** and $\boldsymbol{C'}_{A',w'} \sim \boldsymbol{\Omega}_{A',w'}$, or (iii) $\boldsymbol{C}_{A,w} \sim \varnothing_{A,w}$ and $\boldsymbol{C'}_{A',w'} \sim \varnothing$**+**. Intuitively, $\succeq_k$ is a coarser version of $\succeq$, which is blind to the difference between $\varnothing$ and $\varnothing$**+**, and $\boldsymbol{\Omega}$ and $\boldsymbol{\Omega}$**-**.

K-Axiom 1.

1. If $\boldsymbol{C}$ is in R, then $\overline{\boldsymbol{C}}$ is in R.
2. If $\boldsymbol{C_1}, \boldsymbol{C_2}, ...$ are in R, then $\bigcup_{i=1}^{\infty} \boldsymbol{C_i}$ is in R.

Proof: Axiom 1 entails that this holds for any cluster in NS, so it holds for R.

K-Axiom 2. $\succeq_k$ is a weak order over R. That is:

1. $\succeq_k$ is *connected*: for all $\boldsymbol{C}$ and $\boldsymbol{C'}$ in R, either $\boldsymbol{C} \succeq_k \boldsymbol{C'}$ or $\boldsymbol{C'} \succeq_k \boldsymbol{C}$.
2. $\succeq_k$ is *transitive*: for all $\boldsymbol{C}$, $\boldsymbol{C'}$ and $\boldsymbol{C''}$ in R, if $\boldsymbol{C} \succeq_k \boldsymbol{C'}$ and $\boldsymbol{C'} \succeq_k \boldsymbol{C''}$, then $\boldsymbol{C} \succeq_k \boldsymbol{C''}$.

Proof: (1) If $\succeq$ is connected, then it's trivially the case that $\succeq_k$ will be connected as well.

(2) If $\succeq$ is transitive, then $\succeq_k$ will be transitive as well. To see this, suppose for *reductio* that $\succeq$ is transitive, but $\succeq_k$ is not – there's some $\boldsymbol{C}$, $\boldsymbol{C'}$, and $\boldsymbol{C''}$ such that

$\boldsymbol{C} \succeq_k \boldsymbol{C'}$, $\boldsymbol{C'} \succeq_k \boldsymbol{C''}$, but $\boldsymbol{C} \not\succeq_k \boldsymbol{C''}$. Either (i) $\boldsymbol{C} \succeq \boldsymbol{C'}$ and $\boldsymbol{C'} \succeq \boldsymbol{C''}$, (ii) $\boldsymbol{C} \not\succeq \boldsymbol{C'}$ and $\boldsymbol{C'} \succeq \boldsymbol{C''}$, (iii) $\boldsymbol{C} \succeq \boldsymbol{C'}$ and $\boldsymbol{C'} \not\succeq \boldsymbol{C''}$, or (iv) $\boldsymbol{C} \not\succeq \boldsymbol{C'}$ and $\boldsymbol{C'} \not\succeq \boldsymbol{C''}$.

(i) Suppose $\boldsymbol{C} \succeq \boldsymbol{C'}$ and $\boldsymbol{C'} \succeq \boldsymbol{C''}$. Then since $\succeq$ is transitive, $\boldsymbol{C} \succeq \boldsymbol{C''}$, which entails that $\boldsymbol{C} \succeq_k \boldsymbol{C''}$, contra our supposition.

(ii) Suppose $\boldsymbol{C} \not\succeq \boldsymbol{C'}$ and $\boldsymbol{C'} \succeq \boldsymbol{C''}$. Then since $\boldsymbol{C} \succeq_k \boldsymbol{C'}$ but $\boldsymbol{C} \not\succeq \boldsymbol{C'}$, it follows that either (a) $\boldsymbol{C} \sim \boldsymbol{\Omega}$**-** and $\boldsymbol{C'} \sim \boldsymbol{\Omega}$, or (b) $\boldsymbol{C} \sim \varnothing$ and $\boldsymbol{C'} \sim \varnothing$**+**. (a) If $\boldsymbol{C} \sim \boldsymbol{\Omega}$**-** and $\boldsymbol{C'} \sim \boldsymbol{\Omega}$, then since anything on a par with $\boldsymbol{\Omega}$**-** will be $\succeq_k$ to everything, it follows that $\boldsymbol{C} \succeq_k \boldsymbol{C''}$, contra our supposition. (b) If $\boldsymbol{C} \sim \varnothing$ and $\boldsymbol{C'} \sim \varnothing$**+**, then since $\boldsymbol{C'} \succeq \boldsymbol{C''}$, either $\boldsymbol{C''} \sim \varnothing$ or $\boldsymbol{C''} \sim \varnothing$**+**. Either way, since anything on a par with $\varnothing$ or $\varnothing$**+** will be $\preceq_k$ everything, it follows that $\boldsymbol{C} \succeq_k \boldsymbol{C''}$, contra our supposition.

(iii) Suppose $\boldsymbol{C} \succeq \boldsymbol{C'}$ and $\boldsymbol{C'} \not\succeq \boldsymbol{C''}$. Then since $\boldsymbol{C'} \succeq_k \boldsymbol{C''}$ but $\boldsymbol{C'} \not\succeq \boldsymbol{C''}$, it follows that either (a) $\boldsymbol{C'} \sim \boldsymbol{\Omega}$**-** and $\boldsymbol{C''} \sim \boldsymbol{\Omega}$, or (b) $\boldsymbol{C'} \sim \varnothing$ and $\boldsymbol{C''} \sim \varnothing$**+**. (a) If $\boldsymbol{C'} \sim \boldsymbol{\Omega}$**-** and $\boldsymbol{C''} \sim \boldsymbol{\Omega}$, then since $\boldsymbol{C} \succeq \boldsymbol{C'}$, either $\boldsymbol{C} \sim \boldsymbol{\Omega}$ or $\boldsymbol{C} \sim \boldsymbol{\Omega}$**-**. Either way, since anything on a par with $\boldsymbol{\Omega}$**-** will be $\succeq_k$ to everything, it follows that $\boldsymbol{C} \succeq_k \boldsymbol{C''}$, contra our supposition. (b) If $\boldsymbol{C'} \sim \varnothing$ and $\boldsymbol{C''} \sim \varnothing$**+**, then since anything on a par with $\varnothing$**+** will be $\preceq_k$ everything, it follows that $\boldsymbol{C} \succeq_k \boldsymbol{C''}$, contra our supposition.

(iv) Suppose $\boldsymbol{C} \not\succeq \boldsymbol{C'}$ and $\boldsymbol{C'} \not\succeq \boldsymbol{C''}$. Then since $\boldsymbol{C} \succeq_k \boldsymbol{C'}$ and $\boldsymbol{C'} \succeq_k \boldsymbol{C''}$, it follows that either (a) $\boldsymbol{C} \sim \boldsymbol{\Omega}$**-** and $\boldsymbol{C'} \sim \boldsymbol{\Omega}$, or (b) $\boldsymbol{C} \sim \varnothing$ and $\boldsymbol{C'} \sim \varnothing$**+**, and either ($\alpha$) $\boldsymbol{C'} \sim \boldsymbol{\Omega}$**-** and $\boldsymbol{C''} \sim \boldsymbol{\Omega}$, or ($\beta$) $\boldsymbol{C'} \sim \varnothing$ and $\boldsymbol{C''} \sim \varnothing$**+**. But neither (a) nor (b) is compatible with either (α) or (β), so this is impossible.

K-Axiom 3.

1. $\boldsymbol{\Omega_R} \succ_k \varnothing_{\boldsymbol{R}}$.
2. For all $\boldsymbol{A_R}$, $\boldsymbol{A_R} \succeq_k \varnothing_{\boldsymbol{R}}$.

Proof: (1) It follows from the first part of axiom 4 that $\boldsymbol{\Omega_R} \succ \varnothing_{\boldsymbol{R}}$. This entails that $\boldsymbol{\Omega_R} \succeq \varnothing_{\boldsymbol{R}}$, and thus that $\boldsymbol{\Omega_R} \succeq_k \varnothing_{\boldsymbol{R}}$. This also entails that $\varnothing_{\boldsymbol{R}} \not\succeq \boldsymbol{\Omega_R}$; which combined with the fact that $\varnothing_{\boldsymbol{R}} \not\sim \boldsymbol{\Omega}$**-** and $\boldsymbol{\Omega_R} \not\sim \varnothing$**+** entails that $\varnothing_{\boldsymbol{R}} \not\succeq_k \boldsymbol{\Omega_R}$. Thus $\boldsymbol{\Omega_R} \succ_k \varnothing_{\boldsymbol{R}}$.

(2) It follows from lemma 1 that for all $\boldsymbol{A_R}$, $\boldsymbol{A_R} \succeq \varnothing_{\boldsymbol{R}}$, which entails that for all $\boldsymbol{A_R}$, $\boldsymbol{A_R} \succeq_k \varnothing_{\boldsymbol{R}}$.

K-Axiom 4. For all C, C', C'' in NS, if $C \cap C' = C \cap C'' = \varnothing$, then $\boldsymbol{C'} \succeq_k \boldsymbol{C''}$ *iff* $\boldsymbol{C \cup C'} \succeq_k \boldsymbol{C \cup C''}$.

Proof: As a preliminary, consider the following conditions: (i) $\boldsymbol{C} \sim \boldsymbol{\Omega}$**-**, $\boldsymbol{C'} \sim \boldsymbol{C''} \sim \varnothing$**+**, (ii) $\boldsymbol{C} \sim \varnothing$**+**, $\boldsymbol{C'} \sim \boldsymbol{C''} \sim \boldsymbol{\Omega}$**-**, (iii) $\boldsymbol{\Omega} \succ \boldsymbol{C} \succ \varnothing$, $\boldsymbol{C'} \sim \varnothing$, $\boldsymbol{C''} \sim \varnothing$**+**. Note that if any of these conditions hold, then $\boldsymbol{C'} \succeq_k \boldsymbol{C''}$ *iff* $\boldsymbol{C \cup C'} \succeq_k \boldsymbol{C \cup C''}$. (Call this biconditional "kiff".) If condition (i) holds, then the left hand side of kiff is true (since $\boldsymbol{C'} \sim \boldsymbol{C''}$). And by lemma 2, both $\boldsymbol{C \cup C'} \succeq \boldsymbol{\Omega}$**-** and $\boldsymbol{C \cup C''} \succeq \boldsymbol{\Omega}$**-**. Since $\boldsymbol{\Omega}$**-** is $\succeq_k$ everything, it follows that the right hand side of kiff is true too. For precisely the same reasons, if condition (ii) holds then both the right and left hand sides of kiff are true. If condition (iii) holds, then since everything $\succeq_k \varnothing$**+**, the left hand side of kiff is true. And since $\boldsymbol{C}$ is middling, $\boldsymbol{C \cup C'} \sim \boldsymbol{C} \sim \boldsymbol{C \cup C''}$ (by axiom 7 and the first part of lemma 4), and thus the right hand side of kiff is true too.

Now, suppose that $C \cap C' = C \cap C'' = \emptyset$. To establish K-axiom 4, we need to show that if this is the case, kiff will be true. We just saw that if any of conditions (i)-(iii) hold, kiff will be true. So we just have to show that if none of conditions (i)-(iii) hold, kiff will also be true. By lemma 9, if conditions (i)-(iii) don't obtain then $\boldsymbol{C'} \succeq \boldsymbol{C''}$ *iff* $\boldsymbol{C} \cup \boldsymbol{C'} \succeq \boldsymbol{C} \cup \boldsymbol{C''}$. (Call this biconditional "niff".) Now, either both sides of niff are true, or both are false. We can establish K-axiom 4 if we can show that either way kiff will be true.

If both sides of niff are true, then since $\succeq$ entails $\succeq_k$, it trivially follows that $\boldsymbol{C'} \succeq_k \boldsymbol{C''}$ *iff* $\boldsymbol{C} \cup \boldsymbol{C'} \succeq_k \boldsymbol{C} \cup \boldsymbol{C''}$.

What if both sides of this niff are false? For the left hand side of niff to be false, one of the following three possibilities must obtain: (a) $\boldsymbol{C'} \not\succeq_k \boldsymbol{C''}$, (b) $\boldsymbol{C'} \succeq_k \boldsymbol{C''}$ and $\boldsymbol{C'} \sim \boldsymbol{\Omega}$**-**, $\boldsymbol{C''} \sim \boldsymbol{\Omega}$, or (c) $\boldsymbol{C'} \succeq_k \boldsymbol{C''}$ and $\boldsymbol{C'} \sim \emptyset$, $\boldsymbol{C''} \sim \emptyset$**+**. For the right hand side of niff to be false, one of the following three possibilities must obtain: (a*) $\boldsymbol{C} \cup \boldsymbol{C'} \not\succeq_k \boldsymbol{C} \cup \boldsymbol{C''}$, (b*) $\boldsymbol{C} \cup \boldsymbol{C'} \succeq_k \boldsymbol{C} \cup \boldsymbol{C''}$ and $\boldsymbol{C} \cup \boldsymbol{C'} \sim \boldsymbol{\Omega}$**-**, $\boldsymbol{C} \cup \boldsymbol{C''} \sim \boldsymbol{\Omega}$, or (c*) $\boldsymbol{C} \cup \boldsymbol{C'} \succeq_k \boldsymbol{C} \cup \boldsymbol{C''}$ and $\boldsymbol{C} \cup \boldsymbol{C'} \sim \emptyset$, $\boldsymbol{C} \cup \boldsymbol{C''} \sim \emptyset$**+**. So both sides of niff being false presents us with nine possibilities, and we need to show that kiff will be true given each one.

(a&a*): Suppose $\boldsymbol{C'} \not\succeq_k \boldsymbol{C''}$, and $\boldsymbol{C} \cup \boldsymbol{C'} \not\succeq_k \boldsymbol{C} \cup \boldsymbol{C''}$. This entails that both sides of kiff are false, and thus that kiff holds.

(a&b*): Suppose that $\boldsymbol{C'} \not\succeq_k \boldsymbol{C''}$, $\boldsymbol{C} \cup \boldsymbol{C'} \succeq_k \boldsymbol{C} \cup \boldsymbol{C''}$, $\boldsymbol{C} \cup \boldsymbol{C'} \sim \boldsymbol{\Omega}$**-**, and $\boldsymbol{C} \cup \boldsymbol{C''} \sim \boldsymbol{\Omega}$. There are five possibilities to consider: (α) $\boldsymbol{C} \sim \boldsymbol{\Omega}$, ($\beta$) $\boldsymbol{C} \sim \boldsymbol{\Omega}$**-**, ($\gamma$) $\boldsymbol{C}$ is middling, (δ) $\boldsymbol{C} \sim \emptyset$**+**, ($\epsilon$) $\boldsymbol{C} \sim \emptyset$.

(α) Suppose $\boldsymbol{C} \sim \boldsymbol{\Omega}$. Then it follows from lemma 3 that $\boldsymbol{C} \cup \boldsymbol{C'} \sim \boldsymbol{\Omega}$, contra supposition. So this is impossible.

(β) Suppose $\boldsymbol{C} \sim \boldsymbol{\Omega}$**-**. C is disjoint with C' and C'', and thus C' and C'' are subsets of $\overline{C}$. It follows from lemma 6 that $\overline{\boldsymbol{C}} \sim \emptyset$**+**, and thus from lemma 2 that $\boldsymbol{C'}$ and $\boldsymbol{C''}$ are on a par with either $\emptyset$ or $\emptyset$**+**. Either way, $\boldsymbol{C'} \succeq_k \boldsymbol{C''}$, contra supposition. So this is impossible.

(γ) Suppose $\boldsymbol{C}$ is middling. It follows that $\boldsymbol{C''}$ must also be middling. (For if $\boldsymbol{C''} \sim \emptyset$ then by lemma 4 $\boldsymbol{C} \cup \boldsymbol{C''}$ wouldn't be on a par with $\boldsymbol{\Omega}$**-**; if $\boldsymbol{C''} \sim \emptyset$**+** then by axiom 7 $\boldsymbol{C} \cup \boldsymbol{C''}$ wouldn't be on a par with $\boldsymbol{\Omega}$**-**; if $\boldsymbol{C''}$ were on a par with $\boldsymbol{\Omega}$ or $\boldsymbol{\Omega}$**-** then C and C'' couldn't be disjoint (since if C and C'' are disjoint then $C'' \subseteq \overline{C}$ and so $\boldsymbol{C''} \preceq \overline{\boldsymbol{C}}$ (by lemma 2), and since $\overline{\boldsymbol{C}}$ must be middling (by lemma 7) it follows that $\boldsymbol{C''} \preceq \boldsymbol{\Omega}$**-**).) For similar reasons, $\boldsymbol{C'}$ must also be middling.

Now, note that the fact that $\boldsymbol{C} \cup \boldsymbol{C''} \sim \boldsymbol{\Omega}$ entails that any triple associated with a set of worlds outside of $S1$, $S2$ and $S3$ in figure 1 will be a par with $\emptyset$ (by lemma 5). Since $\boldsymbol{C'}$ must be middling, and $\boldsymbol{C'} - \boldsymbol{S2} \sim \emptyset$, it follows from lemma 4 that $\boldsymbol{S2}$ must also be middling.

Since $\boldsymbol{C} \cup \boldsymbol{C'} \sim \boldsymbol{\Omega}$**-**, it follows (from lemma 6) that $\overline{\boldsymbol{C} \cup \boldsymbol{C'}} \sim \emptyset$**+**, and since $S1$ is a subset of $\overline{C \cup C'}$, it follows (from lemma 2) that $\boldsymbol{S1} \preceq \emptyset$**+**. But that, and the fact that $\boldsymbol{S2}$ is middling, entails that $\boldsymbol{S1} \cup \boldsymbol{S2}$ must be on a par with $\boldsymbol{S2}$ (by lemma 4 and axiom 7), and thus that $\boldsymbol{C'} \sim \boldsymbol{C''}$. This entails that $\boldsymbol{C'} \sim_k \boldsymbol{C''}$, which contradicts the supposition

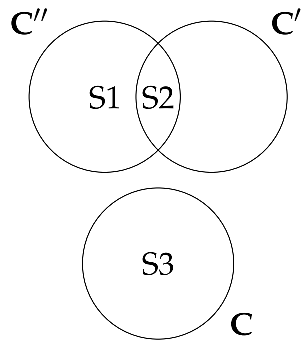

Figure 1

that $\boldsymbol{C'} \not\succeq_k \boldsymbol{C''}$. So this is impossible.

(δ) Suppose $\boldsymbol{C} \sim \emptyset$**+**. It follows from this, and the fact that $\boldsymbol{C} \cup \boldsymbol{C'} \sim \boldsymbol{\Omega}$**-** and $\boldsymbol{C} \cup \boldsymbol{C''} \sim \boldsymbol{\Omega}$, that $\boldsymbol{C'}$ and $\boldsymbol{C''}$ are on a par with either $\boldsymbol{\Omega}$ or $\boldsymbol{\Omega}$**-**. (If not, then it follows by lemma 4 and axiom 7 that $\boldsymbol{C} \cup \boldsymbol{C'}$ and $\boldsymbol{C} \cup \boldsymbol{C''}$ would be $\preceq \boldsymbol{\Omega}$**-**, contra supposition.) Either way, $\boldsymbol{C'} \succeq_k \boldsymbol{C''}$, contra supposition. So this is impossible.

(ϵ) Suppose $\boldsymbol{C} \sim \emptyset$. It follows from this, lemma 4, and the fact that $\boldsymbol{C} \cup \boldsymbol{C'} \sim \boldsymbol{\Omega}$**-** and $\boldsymbol{C} \cup \boldsymbol{C''} \sim \boldsymbol{\Omega}$, that $\boldsymbol{C'}$ and $\boldsymbol{C''}$ are on a par with either $\boldsymbol{\Omega}$ or $\boldsymbol{\Omega}$**-**. Either way, $\boldsymbol{C'} \succeq_k \boldsymbol{C''}$, contra supposition. So this is impossible.

(a&c*): Suppose $\boldsymbol{C'} \not\succeq_k \boldsymbol{C''}$, $\boldsymbol{C} \cup \boldsymbol{C'} \succeq_k \boldsymbol{C} \cup \boldsymbol{C''}$, $\boldsymbol{C} \cup \boldsymbol{C'} \sim \emptyset$ and $\boldsymbol{C} \cup \boldsymbol{C''} \sim \emptyset$**+**. It follows (by lemma 4) that $\boldsymbol{C} \sim \boldsymbol{C'} \sim \emptyset$, and thus that $\boldsymbol{C''} \sim \emptyset$**+**. But that entails that $\boldsymbol{C'} \succeq_k \boldsymbol{C''}$ is true, contra supposition. So this is impossible.

(b&(a*)–(c*)): Suppose $\boldsymbol{C'} \succeq_k \boldsymbol{C''}$, $\boldsymbol{C'} \sim \boldsymbol{\Omega}$**-**, and $\boldsymbol{C''} \sim \boldsymbol{\Omega}$. Since $C'' \cap C = \emptyset$, it follows that $\boldsymbol{C} \sim \emptyset$. So (by lemma 4) $\boldsymbol{C} \cup \boldsymbol{C'} \sim \boldsymbol{C'}$ and $\boldsymbol{C} \cup \boldsymbol{C''} \sim \boldsymbol{C''}$. It follows that the left hand side of kiff is true *iff* the right hand side of kiff is true, and thus that kiff holds.

(c&(a*)–(c*)): Suppose $\boldsymbol{C'} \succeq_k \boldsymbol{C''}$ and $\boldsymbol{C'} \sim \emptyset$, $\boldsymbol{C''} \sim \emptyset$**+**. $\boldsymbol{C}$ must either (α) be on a par with $\boldsymbol{\Omega}$, (β) be on a par with $\emptyset$, or (γ) be between those two. (α) If $\boldsymbol{C} \sim \boldsymbol{\Omega}$, then the fact that C and C'' are disjoint entails (by lemmas 5 and 1) that $\boldsymbol{C''} \sim \emptyset$, contra the supposition that $\boldsymbol{C''} \sim \emptyset$**+**. So this is impossible. (β) If $\boldsymbol{C} \sim \emptyset$, then (by lemma 4) $\boldsymbol{C} \cup \boldsymbol{C'} \sim \boldsymbol{C'}$ and $\boldsymbol{C} \cup \boldsymbol{C''} \sim \boldsymbol{C''}$, and so the the left hand side of kiff is true *iff* the right hand side of kiff is true. So kiff holds. (γ) If $\boldsymbol{C}$ is between the two, then since $\boldsymbol{C'} \sim \emptyset$ and $\boldsymbol{C''} \sim \emptyset$**+**, condition (iii) holds, and thus (as we saw above) kiff holds.

K-Axiom 5. There's no $\boldsymbol{C}$ in R such that (i) $\boldsymbol{C} \succ_k \emptyset$, and (ii) for any $\boldsymbol{C'}$ in R such that C' is a proper subset of C, either:

(a) $\boldsymbol{C'} \sim_k \boldsymbol{C}$,

(b) $\boldsymbol{C'} \sim_k \emptyset$.

Proof: Suppose otherwise for *reductio* – that there is a $\boldsymbol{C}$ in R such that (i) $\boldsymbol{C} \succ_k \emptyset$, and (ii) for any $\boldsymbol{C'}$ in R such that C' is a proper subset of C, either: (a) $\boldsymbol{C'} \sim_k \boldsymbol{C}$ or (b)

$\boldsymbol{C'} \sim_k \emptyset$. First, note that $\boldsymbol{C'} \sim_k \boldsymbol{C}$ entails that one of the following five things must be true: (α) $\boldsymbol{C'} \sim \boldsymbol{C}$, ($\beta$) $\boldsymbol{C'} \sim \boldsymbol{\Omega}$ and $\boldsymbol{C} \sim \boldsymbol{\Omega}$**-**, ($\gamma$) $\boldsymbol{C'} \sim \boldsymbol{\Omega}$**-** and $\boldsymbol{C} \sim \boldsymbol{\Omega}$, ($\delta$) $\boldsymbol{C'} \sim \emptyset$ and $\boldsymbol{C} \sim \emptyset$**+**, or ($\epsilon$) $\boldsymbol{C'} \sim \emptyset$**+** and $\boldsymbol{C} \sim \emptyset$. Second, note that $\boldsymbol{C'} \sim_k \emptyset$ entails that either (ζ) $\boldsymbol{C'} \sim \emptyset$ or (η) $\boldsymbol{C'} \sim \emptyset$**+** must be true. Third, note that if $\boldsymbol{C} \succ_k \emptyset$, then $\boldsymbol{C} \succ \emptyset$**+**. Since C' is a subset of C, lemma 2 entails that $\boldsymbol{C'} \not\succ \boldsymbol{C}$, which rules out ($\beta$) and ($\epsilon$). And ($\zeta$) makes ($\delta$) redundant. So, putting this together, it follows that there is a $\boldsymbol{C}$ in R such that (i) $\boldsymbol{C} \succ \emptyset$**+**, and (ii) for any $\boldsymbol{C'}$ in R such that C' is a proper subset of C, either: (α) $\boldsymbol{C'} \sim \boldsymbol{C}$, ($\gamma$) $\boldsymbol{C'} \sim \boldsymbol{\Omega}$**-** and $\boldsymbol{C} \sim \boldsymbol{\Omega}$, ($\zeta$) $\boldsymbol{C'} \sim \emptyset$, or ($\eta$) $\boldsymbol{C'} \sim \emptyset$**+**. But this is precisely what part 2 of axiom 4 denies. *Reductio.*

K-Axiom 6. Suppose $\boldsymbol{C}$, $\boldsymbol{C_1}$, $\boldsymbol{C_2}$, ..., and $\bigcup_{i=1}^{\infty} \boldsymbol{C_i}$ are in R. If for all i, $\boldsymbol{C} \succeq_k \boldsymbol{C_i}$ and $C_i \subseteq C_{i+1}$, then $\boldsymbol{C} \succeq_k \bigcup_{i=1}^{\infty} \boldsymbol{C_i}$.

Proof: Suppose that $\boldsymbol{C}$, $\boldsymbol{C_1}$, $\boldsymbol{C_2}$, ..., and $\bigcup_{i=1}^{\infty} \boldsymbol{C_i}$ are in R, and for all i, $\boldsymbol{C} \succeq_k \boldsymbol{C_i}$ and $C_i \subseteq C_{i+1}$. For every i, the fact that $\boldsymbol{C} \succeq_k \boldsymbol{C_i}$ entails that either $\boldsymbol{C} \succeq \boldsymbol{C_i}$, $\boldsymbol{C} \sim \boldsymbol{\Omega}$**-** and $\boldsymbol{C_i} \sim \boldsymbol{\Omega}$, or $\boldsymbol{C} \sim \emptyset$ and $\boldsymbol{C_i} \sim \emptyset$**+**. Thus there are three (somewhat overlapping) possibilities: (i) for every i, $\boldsymbol{C} \succeq \boldsymbol{C_i}$, (ii) for some i, $\boldsymbol{C} \sim \boldsymbol{\Omega}$**-** and $\boldsymbol{C_i} \sim \boldsymbol{\Omega}$, or (iii) for some i, $\boldsymbol{C} \sim \emptyset$ and $\boldsymbol{C_i} \sim \emptyset$**+**.

(i) If for all i, $\boldsymbol{C} \succeq \boldsymbol{C_i}$, then axiom 6 entails that $\boldsymbol{C} \succeq \bigcup_{i=1}^{\infty} \boldsymbol{C_i}$, which entails that $\boldsymbol{C} \succeq_k \bigcup_{i=1}^{\infty} \boldsymbol{C_i}$.

(ii) If for some i, $\boldsymbol{C} \sim \boldsymbol{\Omega}$**-** and $\boldsymbol{C_i} \sim \boldsymbol{\Omega}$, then since $C_i \subseteq \bigcup_{j=1}^{\infty} C_j$, it follows from lemma 2 that $\bigcup_{i=1}^{\infty} \boldsymbol{C_i} \sim \boldsymbol{\Omega}$. Since Ω- $\succeq_k \boldsymbol{\Omega}$, it follows that $\boldsymbol{C} \succeq_k \bigcup_{i=1}^{\infty} \boldsymbol{C_i}$.

(iii) Finally, suppose that $\boldsymbol{C} \sim \emptyset$. Since $\boldsymbol{C} \sim \emptyset$ and for all i, $\boldsymbol{C} \succeq_k \boldsymbol{C_i}$, it follows that for all i, $\emptyset$**+** $\succeq \boldsymbol{C_i}$. It follows from axiom 6 that $\emptyset$**+** $\succeq \bigcup_{i=1}^{\infty} \boldsymbol{C_i}$. And it follows from the fact that $C_i \subseteq \bigcup_{i=1}^{\infty} C_i$ and lemma 2 that $\bigcup_{i=1}^{\infty} \boldsymbol{C_i} \succeq \emptyset$**+**, and thus $\bigcup_{i=1}^{\infty} \boldsymbol{C_i} \sim \emptyset$**+**. Since $\emptyset \succeq_k \emptyset$**+**, it follows that $\boldsymbol{C} \succeq_k \bigcup_{i=1}^{\infty} \boldsymbol{C_i}$.

• **Step I(b).** Consider the condition that $ch_{A,L}(C) \geq ch_{A',L'}(C')$ *iff* for any $w \in L$ and $w' \in L'$, either:

1. $\boldsymbol{C}_{A,w} \succeq \boldsymbol{C'}_{A',w'}$,
2. $\boldsymbol{C}_{A,w} \sim \boldsymbol{\Omega}$**-** and $\boldsymbol{C'}_{A',w'} \sim \boldsymbol{\Omega}_{A',w'}$,
3. $\boldsymbol{C}_{A,w} \sim \emptyset_{A,w}$ and $\boldsymbol{C'}_{A',w'} \sim \emptyset$**+**.

Note that this is equivalent to the condition that $ch_{A,L}(C) \geq ch_{A',L'}(C')$ *iff* for any $w \in L$ and $w' \in L'$, $\boldsymbol{C}_{A,w} \succeq_k \boldsymbol{C'}_{A',w'}$. Let's say that a *ch* which satisfies this condition is *order-preserving* with respect to $\succeq_k$, and *order-encoding* with respect to $\succeq$.

We've established that the nomic likelihood relation over R satisfies k-axioms 1-6. It follows from a result by Villegas (1964) that if this relation satisfies k-axioms 1-6, then there is a unique order-preserving function p from the algebra the relation is defined over (R) to the unit interval $[0,1]$, and p is a countably additive probability function. (See Krantz et al. (1971), p216.)

Now, strictly speaking p is a function which takes one argument (a triple in R), whereas *ch* is a function of three arguments, each corresponding to an element of that triple. But we can uniquely pair each p with a *ch* function such that for all C, A, and w,

$ch_{A,L_w}(C) = p(\boldsymbol{C}_{A,w})$.[57] Since there's a unique order-preserving (with respect to $\succeq_k$) function p from R to [0,1] that's a countably additive probability function, it follows that there's a unique order-preserving (with respect to $\succeq_k$) and order-encoding (with respect to $\succeq$) function ch from triples of propositions corresponding to the elements of R to $[0,1]$, and it's a countably additive probability function.

In what follows I'll speak loosely of ch as assigning values to triples like $\boldsymbol{C}_{A,w}$, and the like, even though this is only strictly true for p, not ch.

• **Step II.** Now we'll extend the result from R to any (A,w)-algebra in NS. Given a probability function over R, we'll show that there's a unique countably additive assignment to every triple in NS that is order-preserving with respect to $\succeq_k$ and order-encoding with respect to $\succeq$. First, we'll establish that there's a unique assignment that is order-preserving with respect to $\succeq_k$/order-encoding with respect to $\succeq$. Second, we'll establish that this assignment is a countably additive probability function.

1. Recall that in order for ch to be order-preserving with respect to $\succeq_k$/order-encoding with respect to $\succeq$, it must be the case that $ch_{A,L}(C) \geq ch_{A',L'}(C')$ *iff* for any $w \in L$ and $w' \in L'$, $\boldsymbol{C}_{A,w} \succeq_k \boldsymbol{C'}_{A',w'}$. This entails that in order for ch to be order-preserving with respect to $\succeq_k$/order-encoding with respect to $\succeq$, it must be the case that $ch_{A,L}(C) = ch_{A',L'}(C')$ *iff* for any $w \in L$ and $w' \in L'$, $\boldsymbol{C}_{A,w} \sim_k \boldsymbol{C'}_{A',w'}$.

It follows from part 3 of axiom 4 that every triple in NS is $\sim$ with a triple in R. That entails that every triple in NS is $\sim_k$ with a triple in R. Thus in order for ch to be order-preserving with respect to $\succeq_k$/order-encoding with respect to $\succeq$, it must be the case that ch assigns to each triple in NS the same value it assigns to the triple(s) in R they're on a par$_k$ with. Since part 3 of axiom 4 entails that there will be such a ch, and this uniquely identifies what ch must be, and it follows that there is a unique ch over all NS that is order-preserving with respect to $\succeq_k$/order-encoding with respect to $\succeq$.

2. Now let's establish that this ch is a countably additive probability function.

The first probability axiom requires that every assignment in NS be positive. Since every assignment in R is positive (since ch is probabilistic over R), and every assignment in NS is equal to some assignment in R, it follows that every assignment over NS is positive.

The second probability axiom requires every $\boldsymbol{\Omega}_{A,w}$ in NS to be assigned 1. $\boldsymbol{\Omega}_R$ is assigned 1, and by axiom 3 every $\boldsymbol{\Omega}_{A,w} \sim \boldsymbol{\Omega}_R$ and thus every $\boldsymbol{\Omega}_{A,w} \sim_k \boldsymbol{\Omega}_R$. Since these are assigned the same values, it follows that every $\boldsymbol{\Omega}_{A,w}$ in NS is assigned 1.

Let's establish that the third probability axiom is satisfied in two steps, first (a) showing that ch is finitely additive, and then (b) showing that ch is countably additive.

(a) Let's start by showing that ch will be finitely additive. So we want to show that for any arbitrary (A,w)-cluster containing $\boldsymbol{C}$ and $\boldsymbol{C'}$ (where C and C' are disjoint), it will be the case that $ch_{A,L_w}(C) + ch_{A,L_w}(C') = ch_{A,L_w}(C \cup C')$.

57 Of course, this identification requires it to be the case that for all w, w' in the same L, $p(\boldsymbol{C}_{A,w}) = p(\boldsymbol{C}_{A,w'})$. To see that this is the case, recall that if w and w' are in the same L, it follows from the definition of L that $\boldsymbol{C}_{A,w} \sim \boldsymbol{C}_{A,w'}$, which entails that $\boldsymbol{C}_{A,w} \sim_k \boldsymbol{C}_{A,w'}$. And since ps assignments line up with $\succeq_k$, it follows that $p(\boldsymbol{C}_{A,w}) = p(\boldsymbol{C}_{A,w'})$.

By part 3 of axiom 4, the rich algebra R contains some $\boldsymbol{C}^{\star}_{\boldsymbol{R}}$ and $\boldsymbol{C}'^{\star}_{\boldsymbol{R}}$ (where $C^{\star}$ and $C'^{\star}$ are disjoint) such that $\boldsymbol{C}_{A,w} \sim \boldsymbol{C}^{\star}_{R}$ and $\boldsymbol{C}'_{A,w} \sim \boldsymbol{C}'^{\star}_{R}$. Assume that none of the following conditions obtain: (i) $\boldsymbol{\Omega}_{A,w} \succ \boldsymbol{C}_{A,w} \succ \varnothing_{A,w}$, $\boldsymbol{C}'_{A,w} \sim \varnothing_{A,w}$, $\boldsymbol{C}'^{\star}_{R} \sim \varnothing$**+**, (ii) $\boldsymbol{C}_{A,w} \sim \boldsymbol{\Omega}$**-**, $\boldsymbol{C}'_{A,w} \sim \boldsymbol{C}'^{\star}_{R} \sim \varnothing$**+**, (iii) $\boldsymbol{C}_{A,w} \sim \varnothing$**+**, $\boldsymbol{C}'_{A,w} \sim \boldsymbol{C}'^{\star}_{R} \sim \boldsymbol{\Omega}$**-**. (In a moment we'll return to consider cases where one of these conditions does obtain.) Then it follows from axiom 5 that $\boldsymbol{C}_{A,w} \succeq \boldsymbol{C}^{\star}_{R}$ *iff* $(\boldsymbol{C} \cup \boldsymbol{C}')_{A,w} \succeq (\boldsymbol{C}^{\star} \cup \boldsymbol{C}'^{\star})_{R}$ and $\boldsymbol{C}^{\star}_{R} \succeq \boldsymbol{C}_{A,w}$ *iff* $(\boldsymbol{C}^{\star} \cup \boldsymbol{C}'^{\star})_{R} \succeq (\boldsymbol{C} \cup \boldsymbol{C}')_{A,w}$.

Since the left hand side of both biconditionals are true, it follows that $(\boldsymbol{C} \cup \boldsymbol{C}')_{A,w} \sim (\boldsymbol{C}^{\star} \cup \boldsymbol{C}'^{\star})_{R}$. We know from above that $ch_{A,L}(C) = ch_{A',L'}(C')$ *iff* for any $w \in L$ and $w' \in L'$, $\boldsymbol{C}_{A,w} \sim_k \boldsymbol{C}'_{A',w'}$. Letting $R = A', w'$, it follows that $ch_{A,L_w}(C \cup C') = ch_{A',L'_w}(C^{\star} \cup C'^{\star})$. Likewise, it follows that $ch_{A,L_w}(C) = ch_{A',L'_w}(C^{\star})$ and $ch_{A,L_w}(C') = ch_{A',L'_w}(C'^{\star})$. We've established that ch is finitely additive over $R = A', w'$, so $ch_{A',L'_w}(C^{\star}) + ch_{A',L'_w}(C'^{\star}) = ch_{A',L'_w}(C^{\star} \cup C'^{\star})$. Thus it follows that $ch_{A,L_w}(C) + ch_{A,L_w}(C') = ch_{A,L_w}(C \cup C')$.

To derive this result, we assumed that none of the conditions (i)-(iii) obtained. Now let's relax that assumption, and show that it will still be the case that $ch_{A,L_w}(C) + ch_{A,L_w}(C') = ch_{A,L_w}(C \cup C')$.

(i) Suppose that $\boldsymbol{\Omega}_{A,w} \succ \boldsymbol{C}_{A,w} \succ \varnothing_{A,w}$, $\boldsymbol{C}'_{A,w} \sim \varnothing_{A,w}$, $\boldsymbol{C}'^{\star}_{R} \sim \varnothing$**+**. But by stipulation, $\boldsymbol{C}'_{A,w} \sim \boldsymbol{C}'^{\star}_{R}$, so it's impossible for this condition to obtain.

(ii) Suppose that $\boldsymbol{C}_{A,w} \sim \boldsymbol{\Omega}$**-**, $\boldsymbol{C}'_{A,w} \sim \boldsymbol{C}'^{\star}_{R} \sim \varnothing$**+**.

We know $\varnothing_R$ is assigned 0, and by axiom 3 every $\varnothing_{A,w} \sim \varnothing_R$ and thus every $\varnothing_{A,w} \sim_k \varnothing_R$. Recall that $ch_{A,L}(C) = ch_{A',L'}(C')$ *iff* for any $w \in L$ and $w' \in L'$, $\boldsymbol{C}_{A,w} \sim_k \boldsymbol{C}'_{A',w'}$. It follows that every $\varnothing_{A,w}$ in NS is assigned 0.

Now, since $\boldsymbol{C}'_{A,w} \sim_k \varnothing\boldsymbol{+}_{A,w} \sim_k \varnothing_{A,w}$, it follows that $ch_{A,L_w}(C') = ch_{A,L_w}(\varnothing) = 0$. Likewise, since $\boldsymbol{C}_{A,w} \sim_k \boldsymbol{\Omega}\text{-} \sim_k \boldsymbol{\Omega}_{A,w}$, it follows that $ch_{A,L_w}(C) = ch_{A,L_w}(\Omega) = 1$.

It follows from lemma 2 and the fact that ch is order preserving/encoding, that if $C \subseteq C'$, then $ch_{A,L_w}(C) \leq ch_{A,L_w}(C')$. Since $C \subseteq C \cup C'$, it follows that $ch_{A,L_w}(C) \leq ch_{A,L_w}(C \cup C')$. And since $\subseteq \Omega$, it follows that $ch_{A,L_w}(C \cup C') \leq ch_{A,L_w}(\Omega)$. Since $ch_{A,L_w}(C) = ch_{A,L_w}(\Omega) = 1$, it follows that $ch_{A,L_w}(C \cup C') = 1$. Thus if condition (ii) obtains, $ch_{A,L_w}(C) + ch_{A,L_w}(C') = 1 + 0 = ch_{A,L_w}(C \cup C') = 1$. So additivity is not violated.

(iii) Suppose that $\boldsymbol{C}_{A,w} \sim \varnothing$**+**, $\boldsymbol{C}'_{A,w} \sim \boldsymbol{C}'^{\star}_{R} \sim \boldsymbol{\Omega}$**-**. By switching C and C' in the argument for condition (ii), we get the result that if condition (iii) obtains, additivity is still not violated.

(b) Now let's establish that ch is countably additive.

It follows from a result by Villegas (1964) (see also Fishburn (1986), p342) that if (α) a (A,w)-cluster is a σ-algebra, and (β) ch is a *finitely* additive probability measure that is order-preserving with respect to $\succeq_k$, and (γ) $\succeq_k$ is monotonically continuous, then ch is countably additive. Earlier, we used the core axioms to derive K-axiom 6, which states that $\succeq_k$ is monotonically continuous over R. Note that nothing about the derivation depended on the (A,w)-cluster in question being R – one can use precisely the same derivation to establish that $\succeq_k$ is monotonically continuous over any cluster in NS. So we can conclude that $\succeq_k$ is monotonically continuous in general.

Since (α) it follows from axiom 1 that every (A,w)-cluster in NS is a σ-algebra, (β) we've established above that ch is order-preserving with respect to $\succeq_k$ and is a finitely additive probability function over every (A,w)-cluster in NS, and (γ) we've established that $\succeq_k$ is monotonically continuous, it follows from Villegas's result that ch is always countably additive.

• **Step III.** We've established that ch is a unique countably additive probability function over the (A,w)-clusters in NS. To conclude the theorem, we just need to show that there's a unique nomic requirement function NR and nomic forbidding function NF such that $NR(\boldsymbol{C}_{A,w})$ *iff* $\boldsymbol{C}_{A,w} \sim \boldsymbol{\Omega}_{A,w}$, and $NF(\boldsymbol{C}_{A,w})$ *iff* $\boldsymbol{C}_{A,w} \sim \emptyset_{A,w}$. But that's trivially done, since we can use those biconditionals to define NR and NF. Thus the representation and uniqueness theorem holds.

C Some Lemmas Regarding Laws and Chances

• *Proof of Lemma 10:* Suppose for *reductio* that A logically entails C, and $\boldsymbol{C}_{A,w}$ is in NS, but it's not the case that $NR(\boldsymbol{C}_{A,w})$. It follows (from the representation and uniqueness theorem) that $\boldsymbol{C}_{A,w} \not\sim \boldsymbol{\Omega}_{A,w}$.

It will also follow that $\boldsymbol{C}_{A,w} \not\sim \emptyset_{A,w}$. To see this, suppose otherwise: that $\boldsymbol{C}_{A,w} \sim \emptyset_{A,w}$, and thus $\overline{\boldsymbol{C}}_{A,w} \sim \boldsymbol{\Omega}_{A,w}$. Either $w \in A$ or $w \notin A$. If $w \in A$, then (by axiom 8) $w \in \overline{C}$, which is impossible since A entails C. If $w \notin A$, then (by axiom 9) there exists a $w' \in A$ such that $\overline{\boldsymbol{C}}_{A,w'} \sim \boldsymbol{\Omega}_{A,w'}$. But then (by axiom 8) $w' \in \overline{C}$, which is impossible since A entails C. *Reductio.*

Together, these results entail (by lemma 1) that $\emptyset_{A,w} \prec \boldsymbol{C}_{A,w} \prec \boldsymbol{\Omega}_{A,w}$. It follows from axiom 10 that there's some $w'' \in A$ with the same laws as w such that $w'' \notin C$. But this is impossible, since A entails C. By *reductio*, it must be the case that $NR(\boldsymbol{C}_{A,w})$.

• *Proof of Lemma 11:* Suppose $\boldsymbol{C}$ is in NS. If $NR(\boldsymbol{C_1})$, ..., $NR(\boldsymbol{C_n})$, then it follows (from the representation and uniqueness theorem) that $\boldsymbol{C_1} \sim ... \sim \boldsymbol{C_n} \sim \boldsymbol{\Omega}$. It follows from lemma 4 that $\cap_{\boldsymbol{i=1}}^{\boldsymbol{n}}\boldsymbol{C_i}$ is also on a par with $\boldsymbol{\Omega}$. Now, if $C_1, ..., C_n$ logically entail C, then C is a superset of their intersection, $\cap_{i=1}^{n}C_i$. By lemma 2, it follows that $\boldsymbol{C}$ must be at least as likely as $\cap_{\boldsymbol{i=1}}^{\boldsymbol{n}}\boldsymbol{C_i}$, and thus (by lemma 1) that $\boldsymbol{C}$ must be on a par with $\boldsymbol{\Omega}$. It follows from the representation and uniqueness theorem that $NR(\boldsymbol{C})$.

• *Proof of Lemma 12:* Suppose $NR(\boldsymbol{C})$. It follows from the representation and uniqueness theorem that $\boldsymbol{C} \sim \boldsymbol{\Omega}$. Thus $\overline{\boldsymbol{C}} \sim \emptyset$ (by lemma 5), and (by the representation and uniqueness theorem) $NF(\overline{\boldsymbol{C}})$. Thus $NR(\boldsymbol{C})$ entails $NF(\overline{\boldsymbol{C}})$.

Likewise, suppose $NF(\overline{\boldsymbol{C}})$. It follows from the representation and uniqueness theorem that $\overline{\boldsymbol{C}} \sim \emptyset$. Thus $\boldsymbol{C} \sim \boldsymbol{\Omega}$ (by lemma 5), and (by the representation and uniqueness theorem) $NR(\boldsymbol{C})$. Thus $NF(\overline{\boldsymbol{C}})$ entails $NR(\boldsymbol{C})$.

• *Proof of Lemma 13:* (1) Suppose $NR(\boldsymbol{C}_{A,w})$ and $w \in A$. By the representation and uniqueness theorem, $\boldsymbol{C}_{A,w} \sim \boldsymbol{\Omega}_{A,w}$. By axiom 8, it follows that $w \in C$. (2) Suppose $NF(\boldsymbol{C}_{A,w})$ and $w \in A$. By the representation and uniqueness theorem, $\boldsymbol{C}_{A,w} \sim \emptyset_{A,w}$.

It follows from lemma 5 that $\overline{\boldsymbol{C}}_{A,w} \sim \boldsymbol{\Omega}_{A,w}$, and thus by axiom 8 that $w \in \overline{C}$.

• *Proof of Lemma 14:* (1) Since the representation theorem assigns chances using $\succeq_k$, a relation that fails to distinguish between $\boldsymbol{\Omega}$ and **$\boldsymbol{\Omega}$-**, it follows that the same chance will be assigned to propositions on a par with $\boldsymbol{\Omega}$ and **$\boldsymbol{\Omega}$-**. It follows from the representation and uniqueness theorem that the chance of propositions on a par with $\boldsymbol{\Omega}$ is 1; thus the chance assigned to propositions on a par with **$\boldsymbol{\Omega}$-** will also be 1.

(2) The representation and uniqueness theorem entails that ch is additive, and that for any $\boldsymbol{\Omega}_{A,w} \in NS$, $ch_{A,L_w}(\Omega) = 1$. It follows that $ch_{A,L_w}(\Omega) + ch_{A,L_w}(\emptyset) = ch_{A,L_w}(\Omega \cup \emptyset) = ch_{A,L_w}(\Omega) = 1$, and thus that $ch_{A,L_w}(\emptyset) = 0$. Since $\succeq_k$ fails to distinguish between $\emptyset$ and **$\emptyset$+**, it follows that the same chance will be assigned to propositions on a par with $\emptyset$ and **$\emptyset$+**. Thus the chance assigned to propositions on a par with **$\emptyset$+** will also be 0.

(3) It follows from the representation and uniqueness theorem that ch is probabilistic, so $ch_{A,L_w}(C) \in [0,1]$. It also follows from the representation and uniqueness theorem that $\boldsymbol{C} \succeq_k \boldsymbol{C'}$ *iff* the chance of C is greater than the chance of C'. Thus if **$\boldsymbol{\Omega}$-** $\succ \boldsymbol{C}_{A,w} \succ$ **$\emptyset$+** then it can't be the case that $ch_{A,L_w}(C) = 0$, since then $ch_{A,L_w}(C) = ch_{A,L_w}(\emptyset)$ even though $\boldsymbol{C}_{A,w} \succ \emptyset_{A,w}$. Likewise, it can't be the case that $ch_{A,L_w}(C) = 1$, since then $ch_{A,L_w}(C) = ch_{A,L_w}(\Omega)$ even though $\boldsymbol{C}_{A,w} \prec \boldsymbol{\Omega}_{A,w}$. Thus if **$\boldsymbol{\Omega}$-** $\succ \boldsymbol{C}_{A,w} \succ$ **$\emptyset$+**, then $ch_{A,L_w}(C) = 0 \in (0,1)$.

• *Proof of Lemma 15:* Suppose, for *reductio*, that $A \supset A'$, $ch_{A,L}(A') > 0$, and $ch_{A',L}(\Omega)$ is well-defined, but either (i) for some C, $ch_{A',L}(C)$ is well-defined but $ch_{A,L}(C)$ is not, or (ii) for some C, $ch_{A,L}(C)$ is well-defined but $ch_{A',L}(C)$ is not.

(i): Since $ch_{A',L}(C)$ is well-defined, it follows from the representation and uniqueness theorem that, for some world $w \in L$, $\boldsymbol{C}_{A',w}$ is in NS. And since $ch_{A,L}(A') \neq 0$, it follows from lemma 14 that $\boldsymbol{A'}_{A,w} \not\preceq$ **$\emptyset$+**, or (equivalently) $\boldsymbol{A'}_{A,w} \succ$ **$\emptyset$+**. Given this and the fact that $A \supset A'$, it follows from axiom 11 that $\boldsymbol{C}_{A,w}$ is in NS. It follows from this and axiom 2 that $\boldsymbol{C}_{A',w} \succeq \boldsymbol{C}_{A,w}$ or $\boldsymbol{C}_{A,w} \succeq \boldsymbol{C}_{A',w}$. Thus it follows from the representation and uniqueness theorem that either $ch_{A',L}(C) \geq ch_{A,L}(C)$ or $ch_{A,L}(C) \geq ch_{A',L}(C)$. Either way, $ch_{A,L}(C)$ must be well-defined, contra supposition. *Reductio*.

(ii): Since $ch_{A,L}(C)$ is well-defined, it follows from the representation and uniqueness theorem that, for some world $w \in L$, $\boldsymbol{C}_{A,w}$ is in NS. And since $ch_{A,L}(A') \neq 0$, it follows from lemma 14 that $\boldsymbol{A'}_{A,w} \not\preceq$ **$\emptyset$+**, or (equivalently) $\boldsymbol{A'}_{A,w} \succ$ **$\emptyset$+**. Given this and the fact that $A \supset A'$, it follows from axiom 11 that $\boldsymbol{C}_{A',w}$ is in NS. It follows from this and axiom 2 that $\boldsymbol{C}_{A',w} \succeq \boldsymbol{C}_{A,w}$ or $\boldsymbol{C}_{A,w} \succeq \boldsymbol{C}_{A',w}$. Thus it follows from the representation and uniqueness theorem that either $ch_{A',L}(C) \geq ch_{A,L}(C)$ or $ch_{A,L}(C) \geq ch_{A',L}(C)$. Either way, $ch_{A',L}(C)$ must be well-defined, contra supposition. *Reductio*.

• *Proof of Lemma 16:* First, let's establish two preliminary results, lemmas 17 and 18.

Lemma 17: For every natural number n, there exists a n-equipartition of R.

• *Proof of Lemma 17:* Call a n-equipartition with respect to $\succeq_k$ (instead of $\succeq$) a *n-equipartition$_k$*. It follows from a result of Villegas (1964) that if $\succeq$ over an algebra satisfies K-axioms 1-6, then that algebra satisfies the "fineness" and "tightness" conditions (see Krantz et al. (1971), p216 for details, though these details don't matter for our purposes). It follows from a result by Savage (1954) that if an algebra satisfies these two conditions (in addition to the other K-axioms), then for any natural n, there exists a n-equipartition of that algebra (see Krantz et al. (1971) p206-207). Thus from the results shown above it follows that for every natural number n, there exists a n-equipartition$_k$ of R.

Now, the members of an n-equipartition can't be on a par with $\boldsymbol{\Omega}$**-** (if $n = 1$ then the set would fail to be exhaustive, while if $n > 1$ then the set couldn't be mutually exclusive, given lemmas 2 and 6). Likewise, the members of an n-equipartition can't be on a par with $\emptyset$**+** (since the set wouldn't be exhaustive – by axiom 7, for all n, $\cup_{i=1}^{n}\boldsymbol{P_i} \sim \emptyset\textbf{+} \prec \boldsymbol{\Omega}$). Note that if $\boldsymbol{A}, \boldsymbol{B} \not\sim \boldsymbol{\Omega}$**-** and $\boldsymbol{A}, \boldsymbol{B} \not\sim \emptyset$**+**, then $\boldsymbol{A} \succeq_k \boldsymbol{B}$ *iff* $\boldsymbol{A} \succeq \boldsymbol{B}$. Thus any n-equipartition$_k$ of R is also a n-equipartition of R.

Lemma 18: If $\boldsymbol{C}_{A,w}$ is in NS, and we know the values of $f(n, \boldsymbol{C}_{A,w})$ for all n, then we can identify the unique real number r such that $ch_{A,L_w}(C) = r$.

• *Proof of Lemma 18:* Suppose $\boldsymbol{C}_{A,w}$ is in NS. The representation and uniqueness theorem entails (i) that ch is additive, (ii) that $ch_{A,L_w}(\Omega) = 1$, and (iii) that $\boldsymbol{C}_{A,w} \sim \boldsymbol{C'}_{A',w'}$ which entails $ch_{A,L_w}(C) = ch_{A',L'_{w'}}(C')$. It follows from this that the chance of each member of an n-equipartition is $\frac{1}{n}$, and the chance of the union of m members of the n-equipartition is $\frac{m}{n}$.

It follows from the above that if $n = 10$, $f(n, \boldsymbol{C}_{A,w})$ yields the first 2 values of the decimal expansion of $ch_{A,L_w}(C)$. (I.e., "1.0" if $x = 10$, and "0.x" if $x < 10$). More generally, note that if $n = 10^l$, $f(n, \boldsymbol{C}_{A,w})$ yields the first $l + 1$ values of the decimal expansion of $ch_{A,L_w}(C)$. (I.e., "1.0...0" if $x = 10^l$, and "0.x" if $x < 10^l$).

It follows from this that if we know the values of $f(n, \boldsymbol{C}_{A,w})$ for all n, then we can identify the unique real number $r = ch_{A,L_w}(C)$. For by looking at arbitrarily fine-grained n-equipartitions, the values of $f(n, \boldsymbol{C}_{A,w})$ allow us to identify arbitrarily many places in the decimal expansion of r. And every real number will correspond to a unique decimal expansion of this kind. (The relationship between decimal expansions and real numbers isn't quite one-to-one, since, e.g., 1.0 and $0.\overline{9}$ correspond to the same real number. But the manner of identifying decimal expansions using using $f(n, \boldsymbol{C}_{A,w})$ as described above *will* be unique, since it never yields the latter (...$\overline{9}$) kinds of decimal expansions.)

Now, this only shows that we can identify the unique real number $r = ch_{A,L_w}(C)$ if $\boldsymbol{C}_{A,w}$ is a member of the rich cluster R, since we've only shown that all of the relevant n-equipartitions exist in R (lemma 17). But axiom 4 entails that every $\boldsymbol{C}$ is on a par with some $\boldsymbol{C'_R}$, and it follows from (iii) above that the numerical chance that gets assigned to $\boldsymbol{C'_R}$ must be the same as the chance assigned to $\boldsymbol{C}$. So we can use this technique in R to identify the relevant numerical chances for any $\boldsymbol{C}$ in NS.

• Given lemma 18, we can now prove lemma 16 as follows. Suppose $A \supseteq A'$ and $ch_{A,L}(C \mid A')$ and $ch_{A',L}(C)$ are well-defined. It follows from the definition of conditional probability that $ch_{A,L}(C \cap A')$ and $ch_{A,L}(A')$ are well-defined, and thus (by lemma 15) that $ch_{A',L}(C \cap A')$ and $ch_{A',L}(A')$ are well-defined. Thus $ch_{A',L}(C \mid A')$ is well-defined as long as $ch_{A',L}(A') \neq 0$. And since $ch_{A',L}(A') = 1 \neq 0$ (by lemmas 10 and 14)), it follows that $ch_{A',L}(C \mid A')$ is well-defined.

If $ch_{A',L}(C \mid A') \neq ch_{A,L}(C \mid A')$ then $\frac{ch_{A',L}(C \cap A')}{ch_{A',L}(A')} \neq \frac{ch_{A,L}(C \cap A')}{ch_{A',L}(A')}$. Given lemma 18, we can identify the unique real numbers that each of those four terms correspond to, and thus identify the real numbers the ratios of these chances on the left and right hands sides correspond to, by looking at the values of the corresponding fs for increasingly large ns. If the left and right hand sides differ, then for some large enough m, for all $n > m$, $\frac{f(n, \mathbf{C} \cap \mathbf{A'}_{A',w})}{f(n, \mathbf{A'}_{A',w})}$ will differ from $\frac{f(n, \mathbf{C} \cap \mathbf{A'}_{A,w})}{f(n, \mathbf{A'}_{A,w})}$. But (assuming neither of the denominators stay at 0 for arbitrarily large n) axiom 12 forbids this. So $ch_{A',L}(C \mid A') = ch_{A,L}(C \mid A')$. And since ch is probabilistic and $ch_{A',L}(A') = 1$ (by lemmas 10 and 14), it follows that $ch_{A',L}(C) = ch_{A,L}(C \mid A')$.

What if either of the denominators of $\frac{f(n, \mathbf{C} \cap \mathbf{A}_{A',w})}{f(n, \mathbf{A'}_{A',w})}$ or $\frac{f(n, \mathbf{C} \cap \mathbf{A'}_{A,w})}{f(n, \mathbf{A'}_{A,w})}$ do stay at 0 for arbitrarily large n? Then the real number representing these values is 0, and $ch_{A,L}(C \mid A')$ or $ch_{A',L}(C \mid A')$ will be undefined. But as we've shown, both of these values are well-defined. So this is impossible.

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