

## *Awareness of Abstract Objects*<sup>1</sup>

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Awareness is a two-place determinable relation some determinates of which are seeing, feeling, hearing, etc.<sup>2</sup> If you see a fire truck, or if you hear a fire truck, then you are aware of that fire truck—i.e., you stand in the awareness relation to the fire truck you see, or hear, and do so because you see it or hear it. Abstract objects are items, such as universals, propositions, numbers, functions, and sets, which contrast with concrete objects, such as solids, liquids, gases, particles, and organisms. It is a nice question what defines the category of abstract objects.<sup>3</sup> Typical abstract objects lack spatiotemporal locations and do not stand in causal relations. Saying this much should suffice for my purposes here.

It is uncontroversial that we have experiences that make us aware of concrete objects. My topic is the more controversial one of experiences that make us aware of abstract objects. It is important to distinguish two questions pertaining to this topic.

First, are there any such experiences? I am not interested in forms of experience beyond our reach; nor am I interested in particular cases. Rather, my question is: among the sorts of experiences that we sometimes have, are there any that make us aware of abstract objects? Call this the Existence Question.

Suppose the answer is yes, and that we have fixed on a particular kind of experience that makes us aware of abstract objects. The natural next question is: in virtue of what does such an experience make its subject aware of an abstract object when it does so? That is, the second question to be distinguished is: supposing subject S is aware of abstract object o by having an experience e of the particular sort we have assumed, then in virtue of what does e make S aware of o? Call this the Grounding Question.

It is important to distinguish the Grounding Question from some other questions with which it might be confused. Consider another question about grounding. Suppose I am illegally parked. We might ask: in virtue of what am I illegally parked? And the right answer might be: I am illegally parked in virtue of being parked next to a fire hydrant. Notice that this answer does not give a conceptual analysis of the concept of being illegally parked, does not reduce illegal parking to something else, and does not give necessary and sufficient conditions for illegal parking. Similarly, an answer to the Grounding Question need not take the form of a conceptual analysis, of a reduction, or of giving necessary and sufficient conditions. To ask for one of those is to ask a different question. If I am illegally parked in virtue of being parked next to a fire hydrant, then the fact that I am parked next to a fire hydrant is what makes it the case that I am illegally parked. In general, if the fact that *p* obtains in virtue of the fact that *q*, then the fact that *q* is what makes it the case that *p*. According to Gideon Rosen, if the fact that *p* obtains in virtue of the fact that *q*, then the fact that *q* necessitates that *p*.<sup>4</sup> Suppose this is so. Then the fact that I am parked next to a fire hydrant is only a partial ground. The full ground of the fact that I am illegally parked must include the fact that the parking laws prohibit parking next to fire hydrants. In my view there are special cases in which a full ground does not necessitate what it grounds, but I will not appeal to this view here.<sup>5</sup> My aim in answering the Grounding Question, then, will be to identify those facts that necessitate and make it the case that *e* makes *S* aware of *o*, given that *e* does make *S* aware of *o*.

In this paper I will defend the view that some intuitions—more specifically, some mathematical intuitions—make their subjects aware of abstract objects. In section 1, I consider reasons for thinking that this is so. Their persuasiveness hinges on the availability of a plausible account of the ground of intuitive awareness. Instead of developing such an account directly, however, I first consider two arguments against the view that some intuitions make their subject aware of abstract objects. I present these arguments in section 2. I reply to the first in section 3, and the second in section 4. Responding to these arguments involves putting into place a number of ideas that provide the ingredients for a plausible account of the ground of intuitive awareness. I develop this account in section 5.

### 1. The Existence Question

Some philosophers think that some sensory experiences make us aware of abstract objects. Mark Johnston, for example, has argued that in sensory hallucinations we are aware of what he calls sensible profiles, which are structured universals, and so abstract objects.<sup>6</sup>

A more traditional view is that awareness of abstract objects is a distinctively intellectual achievement that occurs in intuitions. In one of the most well known expressions of this view, Kurt Gödel writes:

But, despite their remoteness from sense experience, we do have something like a perception also of the objects of set theory, as is seen from the fact that the axioms force themselves upon us as being true. I don't see any reason why we should have less confidence in this kind of perception, i.e., in mathematical intuition, than in sense perception . . .<sup>7</sup>

Edmund Husserl also argued that there is such a thing as intuitive awareness of abstract objects, analogous to sensory awareness of concrete objects:

In an act of abstraction, which need not necessarily involve the use of an abstract name, the universal *itself is given to us*; we do not think of it merely in significative fashion as when we merely understand general names, but we apprehend it, *behold it*. Talk of an *intuition* and, more precisely, of a *perception of the universal* is in this case, therefore, well-justified.<sup>8</sup>

Husserl draws an important distinction. It is one thing to think of an abstract object—to have it in mind “in significative fashion.” It is another thing to be intuitively aware of that abstract object—to “apprehend it, behold it.” The distinction parallels that between thinking about a concrete object and being sensorily aware of it. The claim that both Gödel and Husserl endorse is not just that we have thoughts about abstract objects, but that in intuition we stand in the awareness relation to them.

What motivation is there for this claim? The quoted passages suggest the following argument by phenomenological analogy. Take a concrete object, a tree. Now consider Husserl's contrast between thinking of that tree and being aware of that tree. These two experiences differ with respect to the phenomenon Gödel emphasizes, the phenomenon of having a proposition force itself on you as being true. When you think about the tree, propositions about it do not force themselves on you as being true. When you are aware of it, however, propositions about it do force themselves on you as being true. Suppose, for example, you see the tree. Then your visual experience will force on you—i.e., make seem true—various propositions about the tree's shape, size, color, etc. Plausibly, what accounts for the difference is that in this case you are visually aware of the tree, not just thinking about it. Now consider a set theoretical axiom:

Extensionality Axiom: Sets A and B have exactly the same members if, and only if,  $A = B$ .

The Extensionality Axiom might occur to you while you think about sets. In some cases it might not force itself on you as being true. But in other cases, when you carefully reflect on what it is for something to be a set, the Extensionality Axiom might very well force itself on you as being true. Call such an experience an intuition. Why is it that when you have the intuition, not the mere thought, the Extensionality Axiom forces itself on you as being

true? It is natural to answer by analogy with the case of the tree: in your intuition, you are intuitively aware of the property of being a set, not just thinking about it.

One might challenge the phenomenological analogy.<sup>9</sup> Suppose you have an experience in which it visually seems that this tree here is barren. The proposition forced on you is obviously about the tree and immediately suggests it as the object of your visual awareness. Suppose, on the other hand, you have an experience in which the Extensionality Axiom intuitively seems true. The proposition forced on you is not obviously about the property of being a set and so does not immediately suggest it as the object of your intuitive awareness. When you just consider the proposition—i.e., the Extensionality Axiom—that intuitively seems true, there is this difference. But if you consider those reflections that contribute to making the Extensionality Axiom intuitively seem true, they do appear to focus on the property of being a set. One way such reflections might go is this. You consider committees and orderings. They have members. But they are not wholly determined by their members. Two committees with the same members but different functions or defining goals are distinct. Two orderings with the same members but in a different order are distinct. Sets, on the other hand, are different from committees and orderings in that they are wholly determined by their members; that is just what it is to be a set. These reflections should suffice to make the Extensionality Axiom intuitively evident. Below I argue that reflections of this sort are parts of intuition experiences. If this is so, then what they focus on—in this case the property of being a set as opposed to the property of being a committee or an ordering—is indeed suggested as an object of intuitive awareness.

Even if the argument by phenomenological analogy carries some weight, it is not decisive. When you hallucinate a tree, various propositions about it will force themselves on you as being true, but this is not because you are aware of a tree. So when the Extensionality Axiom forces itself on you as being true, this might not be because you are aware of the property of being a set, but because you have an experience as of such awareness, an intuitive hallucination.

The phenomenological considerations Gödel and Husserl suggest require supplementation. Here I will consider one sort of supplementation—an epistemological supplementation.<sup>10</sup> The idea is simple. Some intuition experiences put you in a position to know the propositions that intuitively seem true in them. Consider one of these, in which the Extensionality Axiom, for example, intuitively seems true to you. This intuition experience couldn't put you in a position to know that the Extensionality Axiom is true if it were an intuitive hallucination. So it must not be an intuitive hallucination. That is, it must genuinely make you aware of the property of being a set. In general, then, at least some intuition experiences, namely those that put their subjects

in a position to know what intuitively seems true in them, make their subjects aware of abstract objects.

One might resist this argument by denying the premise that some intuition experiences put you in a position to know the propositions that intuitively seem true in them. The motivation for this premise comes from reflection on examples. Consider a non-set-theoretical one. At some point in my life I didn't know that circles are symmetrical about their diameters. I just hadn't thought about it. Now I do know that circles are symmetrical about their diameters. This seems to me as certain as anything I believe. What happened? From what I can tell, all I did was imagine folding some circles over some of their diameters whereupon I had an intuition that circles are symmetrical about their diameters, which intuition I took at face value. If this account is accurate, then my belief that circles are symmetrical about their diameters amounts to knowledge because it is based on my intuition. So it is my intuition that put me in a position to know that circles are symmetrical about their diameters.

Here are two different ways of challenging my story. First, you might argue that my intuition merely accompanies my gaining knowledge that circles are symmetrical about their diameters and is no part of the basis on which I have this knowledge. One reason I find this alternative view unattractive is this. If I reflect on the case and consider what *justifies* me in believing that circles are symmetrical about their diameters, all I can point to is my intuition experience. But surely that which justifies my belief should at least be part of the basis on which my belief amounts to knowledge. Suppose, then, my intuition is at least part of the basis on which I know that circles are symmetrical about their diameters. A second way of challenging my story is this. You might argue that my intuition is *only* a part of the basis on which I know, that it requires supplementation by something else.<sup>11</sup>

There are a few different potential reasons why one might think my intuition requires supplementation. One potential reason is that intuitions are unreliable.<sup>12</sup> A second potential reason is that even if intuitions are often veridical, for all we know about how intuitions work, it could be that veridical intuitions are veridical just by accident.<sup>13</sup> A third potential reason is that we lack a good account of what could ground the fact that an intuition experience makes its subject aware of, or otherwise gives its subject access to, the abstract objects it is about.<sup>14</sup> I believe the first potential reason can be undermined by drawing a principled distinction between those intuitions that are reliable and those that are unreliable, and limiting the scope of the claim that some intuitions put their subjects in a position to know the propositions that intuitively seem true in them to intuitions of the reliable kind.<sup>15</sup> The third potential reason can be undermined by giving an account of the ground of intuitive awareness, i.e., an answer to the Grounding Question. I pursue this task below. In my view an account of the ground of intuitive awareness

would also put us in a position to undermine the second potential reason for thinking intuitions are not sufficient bases for knowledge. The idea is that in favorable circumstances a veridical intuition is veridical *because* it makes its subject aware of the abstract objects it is about. I must leave developing this idea further to another occasion.<sup>16</sup>

The foregoing motivates taking up the Grounding Question. Instead of approaching this question directly, I will first consider two arguments against the view that some intuition experiences make their subjects aware of abstract objects. Responding to these arguments involves putting into place a number of ideas that provide the ingredients for a plausible account of the ground of intuitive awareness.

## 2. Dependence and Differentiation

In thinking about intuitive awareness, it is natural to look toward the more developed literature on visual awareness for ideas. Here are two prominent ones:

Causal Dependence: If S sees o by having visual experience e, then e causally depends on o.<sup>17</sup>

Visual Differentiation: If S sees o by having visual experience e, then e's phenomenology differentiates o from its background.<sup>18</sup>

Instead of helping, unfortunately, Causal Dependence and Visual Differentiation initially appear to hurt the case for intuitive awareness of abstract objects. They suggest two arguments against the view that some intuition experiences make their subjects aware of abstract objects.

I present them here without commentary, leaving discussion for subsequent sections.

### Dependence Argument

- (1) If S is aware of o by having experience e, then e depends on o.
- (2) Causal dependence is the only kind of dependence.
- (3) Nothing causally depends on an abstract object.

Therefore,

- (4) No intuition experience causally depends on an abstract object, and so no intuition experience makes its subject aware of an abstract object.

### Differentiation Argument

- (1) If S is aware of o by having experience e, then e's phenomenology differentiates o from its background.

- (2) Unlike the objects of visual awareness, the objects of intuitive awareness—e.g., properties and functions—do not have backgrounds.
- (3) Unlike visual phenomenology, intuitive phenomenology is not the sort that could differentiate an object from its background.

Therefore,

- (4) For no intuition experience *e* and abstract object *o* does *e*'s phenomenology differentiate *o* from its background, and so no intuition experience makes its subject aware of an abstract object.

Before addressing these arguments, I want to take up a preliminary point about the notion of awareness.<sup>19</sup> So far I have been using the notion as if it were clear what is at issue. But a response one might have to the foregoing arguments is to wonder whether there really is a determinable relation of awareness one determinate of which is seeing, another determinate of which is intuitive awareness, and on which there are common substantive requirements, such as those articulated in the first premises of the Dependence and Differentiation arguments. I think there is.

Consider Susanna Siegel's characterization of object-seeing:

I've said that playing a role in anchoring *de re* mental states to their objects provides a positive characterization of object-seeing. The role is this: if one sees an object *o*, one can form a *de re* mental state about *o*, or demonstratively refer to *o*, just by exercising whatever general apparatus is needed for *de re* mental states or demonstrative reference.<sup>20</sup>

Siegel's characterization permits the following natural generalization:

Playing a role in anchoring *de re* mental states to their objects provides a positive characterization of awareness. The role is this: if one is aware of an object *o*, one can form a *de re* mental state about *o*, or demonstratively refer to *o*, just by exercising whatever general apparatus is needed for *de re* mental states or demonstrative reference.<sup>21</sup>

I believe this characterization helps to pick out the relevant determinable relation of awareness. It is a relation for which the first premises in the Dependence and Differentiation arguments are plausible. It clearly has seeing as a determinate. And, bracketing challenges to the possibility of intuitive awareness, reflection on examples suggests that intuitive awareness is also a determinate of it.<sup>22</sup> Consider intuiting that circles are symmetrical about their diameters. When I reflect on this experience it seems to me like it enables to me to entertain thoughts of the sort, "Figures of *that* shape are symmetrical about lines of *that* kind." Consider intuiting the Extensionality

Axiom. When I reflect on this experience it seems to me like it enables me to entertain thoughts of the sort, “Items of *that* kind are determined by their members.” This is just how it seems. Whether actual *de re* reference to circles, diameters, and sets is secured in these cases depends on whether they really are cases of intuitive awareness. Now I return to developing the view that they can be.

### 3. Intuitive Dependence

In this section I respond to the Dependence Argument. The weak link in the argument is Premise 2, that causal dependence is the only kind of dependence. Some things non-causally depend on other things. Xantippe’s widowhood depends on Socrates’ death. But Socrates’ death does not cause Xantippe’s widowhood. Rather, Socrates’ death constitutes Xantippe’s widowhood.<sup>23</sup>

A proponent of the argument might concede that Premise 2 is too strong, but insist that the argument can get along with out it. The premise might be revised to something weaker, such as: Causal dependence is the only kind of dependence *relevant to awareness*. Indeed, the Xantippe’s widowhood/Socrates’ death example doesn’t shed much light on how intuition experiences might depend on abstract objects. So there is a legitimate challenge here. We need some model of how an intuition experience might (non-causally) depend on an abstract object.

There already is one in the literature on perceptual experience: naïve realism. Consider Michael Martin’s characterization of naïve realism in the following:

According to naïve realism, the actual objects of perception, the external things such as trees, tables and rainbows, which one can perceive, and the properties which they can manifest to one when perceived, partly constitute one’s conscious experience, and hence determine the phenomenal character of one’s experience . . . . Furthermore, it is of the essence of such states of mind that they are partly constituted by such objects, and their phenomenal characters are determined by those objects and their qualities.<sup>24</sup>

Naïve realism is a view about the nature of perceptual experience according to which if a perceptual experience makes its subject aware of an object, then that perceptual experience depends on that object, not (just) causally, but constitutively. We might put it this way:

If S is sensorily aware of o by having sensory experience e, then e depends on o, in that: in accordance with the essence of e, o partly constitutes e and thereby determines e’s phenomenal character.

There are three components to the naïve view of e’s perceptual dependence on o: (i) o partly constitutes e; (ii) o thereby determines e’s phenomenal



character; (iii) it lies in the essence of *e* for (i) and (ii) to obtain. All are important. Component (i) relates *e* to *o*. Without (ii), however, it would be unclear why (i) should bear on awareness: partial constitution in general has nothing to do with awareness. Finally, component (iii) grounds (i) and (ii) in the essence of *e*, thereby ensuring that *S* couldn't have *e*—that very experience—in the absence of *o*, which is a hallmark commitment of naïve realism.

It is easy to formulate an analogous naïve realist view about intuitive dependence:

If *S* is intuitively aware of an abstract object *o* by having intuition experience *e*, then *e* depends on *o*, in that: in accordance with the essence of *e*, *o* partly constitutes *e* and thereby determines *e*'s phenomenal character.

Just like naïve realism about perceptual dependence, naïve realism about the intuitive dependence has three components—one about partial constitution, one about phenomenal character, and one about the essentiality of the other two.

As stated, naïve realism about intuitive dependence is puzzling. The reason why has to do with the component about phenomenal character. Consider, first, the claim about phenomenal character associated with naïve realism about perceptual experience. John Campbell spells it out this way:

The phenomenal character of your experience, as you look around the room, is constituted by the actual layout of the room itself: which particular objects are there, their intrinsic properties, such as colour and shape, and how they are arranged in relation to one another and to you.<sup>25</sup>

Consider, next, the analogous claim about intuition experiences. Suppose you are working out some simple properties of adding natural numbers. Then, following Campbell, we might say:

The phenomenal character of your experience, as you reflect on adding natural numbers, is constituted by the actual features of the operation itself: which particular objects it is defined over, their properties, such as oddness and compositeness, and how they are ordered in relation to one another.

Whereas Campbell's claim about the phenomenal characters of perceptual experiences possesses some *prima facie* believability, the analogous claim about the phenomenal characters of intuition experiences does not. Color, shape, and arrangement in space seem apt to constitute phenomenal character. Oddness, compositeness, and order in the number sequence do not seem apt to constitute phenomenal character. The worry remains even if we set aside arithmetical intuitions and consider, instead, geometrical intuitions,

such as the intuition that circles are symmetrical about their diameters. While circularity is a shape, it doesn't have a shape; it seems no more apt than oddness or compositeness to constitute phenomenal character. One might try to resist the worry by invoking non-sensory phenomenology.<sup>26</sup> Intuition experiences, like conscious thoughts, one might argue, possess non-sensory phenomenology. Perhaps properties of numbers and operations over them can constitute non-sensory phenomenology. Saying just this much, however, does not make the view any less puzzling.

The right way to address this puzzlement, I believe, is to focus attention on what it is for an object *o* to partly constitute a subject *S*'s experience *e*. So far I have been taking this notion as basic. But there are importantly different ways of understanding it.

One proposal is to define it in terms of awareness:

Object *o* partly constitutes *S*'s experience *e* =<sub>df</sub> *e* is a complex state one of whose parts is *S*'s awareness of *o*.<sup>27</sup>

Plugging this understanding into the naïve realist schema yields a view I will call **Primitivist Naïve Realism**:

**Primitivist Naïve Realism:** If *S* is intuitively aware of an abstract object *o* by having intuition experience *e*, then *e* depends on *o*, in that: in accordance with the essence of *e*, *S*'s awareness of *o* is a part of *e* and thereby determines *e*'s phenomenal character.

I call this view **Primitivist Naïve Realism** since it takes *S*'s awareness of *o* as basic.

One of its virtues is that it sheds light on the component of the view about phenomenal character. Facts about what *S* is aware of in having *e* seem apt to ground facts about *e*'s phenomenal character. The main problem with the view, for me at least, is that it grounds intuitive dependence on intuitive awareness. Eventually I will propose that intuitive awareness is grounded in intuitive dependence. But this is inconsistent with **Primitivist Naïve Realism**—at least assuming, as I will, that grounding is anti-symmetric.

Another way to understand what it is for an object *o* to partly constitute a subject *S*'s experience *e* is in terms of straightforward parthood:

Object *o* partly constitutes *S*'s experience *e* =<sub>df</sub> *o* is a part of *e*.

Plugging this understanding into the naïve realist schema yields a view I will call **Material Naïve Realism**.

**Material Naïve Realism:** If *S* is intuitively aware of an abstract object *o* by having intuition experience *e*, then *e* depends on *o*, in that: in accordance

with the essence of *e*, *o* is a part of *e* and thereby determines *e*'s phenomenal character.

One attraction of Material Naïve Realism is that it invokes only antecedently familiar notions with wide application, such as parthood and phenomenal character.

There are two main problems with the view. First, it is unclear whether experiences can have objects and properties as parts. Perhaps they can in some indirect fashion: compositeness might count as part of an experience whose essence involves awareness of compositeness. Unlike Primitivist Naïve Realism, however, Material Naïve Realism does not link experiences to objects and properties through awareness of those objects and properties, but directly through the parthood relation. A second problem is that Material Naïve Realism exacerbates the mystery in the component of the view about phenomenal character. Suppose an experience does somehow have compositeness as a part. Why should that determine that it has one phenomenal character rather than another? This seems like a mystery to me.

There is one other way of understanding what it is for an object *o* to partly constitute a subject *S*'s experience *e* that I will discuss. Consider a bicycle. It has parts. And its parts are arranged in a certain way: they are arranged so as to allow locomotion on two wheels by peddling. Mark Johnston calls this arrangement a "principle of unity." According to him it is not another part of the bicycle, but a relation in which the parts of the bicycle must stand if they are to compose it.<sup>28</sup> Kathrin Koslicki calls this arrangement a "formal part." According to her it is another part of the bicycle, though it is not a "material part" in the way that the peddles, wheels, and seat are.<sup>29</sup> I will remain neutral between these two ways of understanding the arrangement of the bicycle's (material) parts. What is important for my purposes is to note that many complex items do, often as a matter of their essence, require their (material) parts to fall into a certain arrangement or structure, exhibit a certain form, or instantiate a certain principle of unity—I will not distinguish between these.

The third way of understanding what it is for an object *o* to partly constitute a subject *S*'s experience *e*, then, can be put this way:

Object *o* partly constitutes *S*'s experience *e* =<sub>df</sub> *o* is part of the principle of unity that *e*'s (material) parts instantiate.

Note that *o* need only be part of the principle of unity that *e*'s (material) parts instantiate. Here is how I will understand this. A principle of unity is a condition. Conditions are the sorts of things that some items satisfy or fail to satisfy, and so are apt to be expressed by open sentences. Just as a closed sentence might express a Russellian proposition—a truth-evaluable complex composed of objects and properties—so an open sentence might express an

open Russellian proposition—a satisfiable complex composed of objects and properties. Principles of unity, then, can be identified with open Russellian propositions, and an object *o* will count as part of it just in case it is among the objects and properties that compose it.<sup>30</sup>

Plugging this understanding of what it is for *o* to partly constitute *S*'s experience *e* into the naïve realist schema yields a view I will call Formal Naïve Realism.

Formal Naïve Realism: If *S* is intuitively aware of an abstract object *o* by having intuition experience *e*, then *e* depends on *o*, in that: in accordance with the essence of *e*, *o* is part of the principle of unity that *e*'s (material) parts instantiate and thereby determines *e*'s phenomenal character.

Formal Naïve Realism possesses the attractions of Primitivist and Material Naïve Realism, but lacks their problems.<sup>31</sup>

First, like Primitivist Naïve Realism, Formal Naïve Realism sheds light on the component of the view about phenomenal character. The structure of an experience seems like just the right sort of thing to make a difference to its phenomenal character. And according to Formal Naïve Realism it is by bearing on the structure of—the principle of unity instantiated by—an intuition experience that an abstract object can determine that experience's phenomenal character. Second, like Material Naïve Realism, Formal Naïve Realism invokes notions such as parthood, principle of unity, and phenomenal character, which are antecedently familiar and have widespread application. Third, unlike Primitivist Naïve Realism, Formal Naïve Realism does not ground intuitive dependence on intuitive awareness; it leaves open the possibility of grounding intuitive awareness on intuitive dependence.

Formal Naïve Realism provides us with a model of intuitive dependence—that is, a model of how intuition experiences might depend on the objects they make their subjects aware of. It is, however, incomplete. It does not specify what the (material) parts of an intuition experience are, nor does it specify just how an abstract object might figure in an intuition experience's principle of unity. I return to these gaps in section 5.

#### 4. Intuitive Differentiation

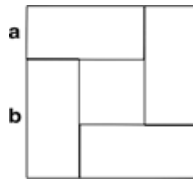
The aim of this section is to address two questions. First, what could constitute the background of an abstract object, such as a property or a function? Second, what in the phenomenology of an intuition experience could differentiate an object from its background? The answers will provide a basis for rejecting premises 2 and 3 in the Differentiation Argument.

It will help to have some example intuitions to reflect on.

A. You consider the proposition that circles are symmetrical about their diameters. In order to tell whether it is true, you need to get clear on which

chords on a circle are diameters, and which chords on a circle are axes of symmetry. What you do is imagine an arbitrary circle, and imagine folding it over various chords that divide it into equal parts. These chords are its diameters, and it is clear from your imaginative endeavor that the circle is symmetrical about them.

B. You consider the proposition that  $(a + b)^2 \geq 4ab$  and  $(a + b)^2 = 4ab$  just when  $a = b$ . This is a complicated claim, and it doesn't immediately strike you as true, or as false. Setting aside how, you light on the following idea. First, you imagine the following figure:



The area of the big square represents  $(a + b)^2$  and the area of the four little rectangles together represents  $4ab$ , and this makes it clear to you that  $(a + b)^2 \geq 4ab$ . Now you imagine expanding each of the little rectangles so that the little middle square shrinks to nothing. As you do this  $4ab$  approaches  $(a + b)^2$  and is equivalent to it when the little middle square disappears, i.e. when  $a = b$ .

C. You consider the proposition that  $\max(a, b) = [(a + b) + |a - b|]/2$ —in other words, the bigger of two numbers is always the average of their sum and difference.<sup>32</sup> You think of things this way. Averaging the sum and difference first involves taking the bigger number, adding it to the smaller number, and then adding however much the smaller number falls short of the bigger number to that. This can be thought of a walking the length of the bigger number, backtracking by the length of the smaller number, and then covering the remaining length to your starting point. This gives you twice the bigger number—you've walked that length twice. So dividing by 2 gives you the bigger number.

What abstract objects are you aware of in the foregoing examples? The following seem plausible: in (A) you are aware of the property of being a diameter; in (B) you are aware of the function  $f(a,b) = |(a + b)^2 - 4ab|$ ; in (C) you are aware of the function  $g(a,b) = [(a + b) + |a - b|]$ .<sup>33</sup> In each case you are likely aware of additional abstract objects, but these can serve as representatives.

Now recall our first question. What could constitute the background of an abstract object, such as a property or a function? A simple idea is: other properties and functions. In (A), you differentiate the property of being a diameter from various other properties chords on a circle instantiate. In (B) and (C), you differentiate  $f(a,b)$  and  $g(a,b)$  from other functions of two

numbers. In my view, this simple idea correctly answers the question asked. But there is a question in the vicinity that it does not answer. This is: what *determines* which items constitute the background of a given property or function?

Consider the analogous question about concrete objects of visual awareness. What determines which items constitute the background of a given concrete object? A very simple idea is: a distance—so the items that constitute the background of a given concrete object are just those that lie within a certain distance of it. This very simple idea cannot be the whole story, since the distance will vary across cases: for an ant it is small, for a star it is large, and for a tree it is somewhere in-between. A still simple idea is: a visual field—so the items that constitute the background of a given concrete object relative to a given subject are just those that lie within that subject's visual field.<sup>34</sup> Changes in the visual field change the relevant distance. This simple idea can be generalized.

Take sounds, odors, and tastes. Suppose you are with a companion near a noisy stream. Sometimes you hear him; sometimes you do not. When you hear him, your auditory phenomenology differentiates his voice from its background. In this case the background consists of other sounds. Which sounds? Those in your auditory field.<sup>35</sup> Suppose you sip some wine. You might, or might not, taste apricot in it. When you do, your gustatory phenomenology differentiates the taste of apricot from its background. In this case the background consists of other tastes. Which tastes? Those in your gustatory field. It is easy to devise a similar example for odor.

The natural generalization to intuitive awareness is this. In case (A), for example, you differentiate the property of being a diameter from its background. This background consists of other properties. Which properties? Those in your *cognitive* field. Your cognitive field consists of all the things you are thinking about—in this case it includes all the different sorts of chords on a circle. To say that you are thinking about all the different sorts of chords on a circle is not to say that each one stands out clearly before your mind. Only one does: the diameter. The other sorts of chords are present in what is sometimes called the fringe of your consciousness; they are “mere nascencies of cognition” to use one of William James' choice phrases.<sup>36</sup> And this is precisely why they form the background against which the property of being a diameter stands in relief.<sup>37</sup>

Our second question was: what in the phenomenology of an intuition experience could differentiate an object from its background? The two most popular views about intuition experiences are doxasticism and sui generism:

Doxasticism: Intuition experiences are judgments, or inclinations to make judgments.<sup>38</sup>

Sui Generism: Intuition experiences are sui generis propositional attitudes, intuitive (or intellectual) seemings.<sup>39</sup>

Doxasticists and standard *sui generis*ists do agree on one thing. This is that the structure of an intuition experience is exhausted by its propositional attitude structure. Call this the PA Thesis:

PA Thesis: The structure of an intuition experience is exhausted by its propositional attitude structure—i.e., it is exhausted by its attitude and its propositional content.

Doxasticists and *sui generis*ists disagree over the nature of the attitude. There has been some controversy in the literature over what the contents of intuition experiences are.<sup>40</sup> But the PA Thesis itself is contemporary orthodoxy.

I reject the PA Thesis. In my view intuition experiences are composed of other mental states, such as thoughts and imaginings.<sup>41,42</sup> This is compatible with the view that they are *sui generis*, since it does not entail that intuition experiences are identical to, or reducible to, these other mental states. It is also compatible with the view that intuition experiences have propositional attitude structure. On the face of it they do: in (A), for example, it intuitively seems to you that circles are symmetrical about their diameters. So I count as a *sui generis*ist. What I add to standard *sui generis*ism is the view that intuition experiences have other mental states as parts, and these are typically the cognitive and imaginative experiences we call “reflections.” In reflecting on the proposition that circles are symmetrical about their diameters, for example, you imagine an arbitrary circle. In my view, this imaginative experience is a part of the intuition experience you have when it intuitively seems to you that circles are symmetrical about their diameters.

Why think that the reflections associated with a given intuition experience are parts of it, rather than just causes of it? This question brings us back to our starting question about the phenomenology of intuition experiences. There is reason to think that the reflections associated with a given intuition experience contribute their phenomenology to that experience. Let  $p$  be the proposition that  $(a + b + c)^3 \geq 9abc$  and  $(a + b + c)^3 = 9abc$  just when  $a = b = c$ . Now consider two intuition experiences. In the first it intuitively seems to you that  $p$  because you imaginatively manipulate a diagram that is a three-dimensional analogue of the two-dimensional diagram from example (B). In the second it intuitively seems to you that  $p$  because you reflect as follows: just as a big square of side  $a + a$  exactly contains four  $a \times a$  squares, so a big cube of side  $a + a + a$  exactly contains nine  $a \times a \times a$  cubes; the only thing that making the sides of those nine inner cubes unequal can do is leave spaces between them. Someone very good at visualizing might have the first experience. For the rest of us, the second is more likely. The two intuition experiences are phenomenally different: in each there is a distinctive way that  $p$  intuitively seems true to you. This suggests that each includes more than an attitude (= intuitive seeming) and content (=  $p$ ). The view that each includes its associated reflections tells us what more. And it does so in a way that accounts for our phenomenological observations, since, on the face

of it, the difference between the two intuition experiences consists precisely in the fact that the first includes imaginatively manipulating a complicated three-dimensional figure and the second includes drawing a simple analogy between squares and cubes.<sup>43</sup>

The foregoing provides some motivation for the view that intuition experiences have associated thoughts and imaginings as parts. I will assume that this view is correct, though I recognize that it requires much more defense than I have been able to give it here.<sup>44</sup> The reason why it is important for my purposes is this. It implies that the phenomenal character of a given intuition experience will include, perhaps somewhat modified because of how they are organized together into a unified experience, the phenomenal characters of those reflections that compose it. And this is the sort of phenomenology that can differentiate an object from its background.

Consider the examples. In (A) and (B) your intuition experiences have differentiating phenomenology because of the imaginative endeavors that partly compose them. In (A), for example, you imagine a chord that divides the circle into two equal halves. This chord instantiates the property of being a diameter, and so highlights that property, rather than others that a chord might instantiate. In (B), you imagine a concrete operation on areas. The operation is that of covering a square of side  $(a + b)$  with four  $a \times b$  rectangles, and checking to see what portion of the square remains uncovered. This operation mirrors  $f(a,b) = |(a + b)^2 - 4ab|$ : the result of the concrete operation is non-zero just in case  $|(a + b)^2 - 4ab|$  is non-zero.<sup>45</sup> One might worry that both cases suffer from indeterminacy. Consider (B). Why, one might worry, should your imaginative endeavor differentiate  $f(a,b)$  from its background rather than some other function that is non-zero just when the result of the concrete operation you imagine is non-zero? The answer is your imaginative intention. In imagining the concrete operation, you intend to illustrate  $f(a,b)$  for yourself, not some other function. Compare Christopher Peacocke's example of imagining a suitcase and imagining a suitcase with a cat hidden behind it.<sup>46</sup> In both cases you have the same mental imagery. What accounts for the difference between them is your imaginative intention.<sup>47</sup>

Let us suppose that in (C), you do not imagine anything. In this case, then, your intuitive phenomenology has a differentiating character because of the thoughts that compose it. You think about how  $g(a,b) = [(a + b) + |a - b|]$  works, and your thoughts make  $g(a,b)$  vivid to you in a way that other functions of  $a$  and  $b$  are not. One might worry that assigning this role to your thoughts falls afoul of the distinction between thinking and awareness, which I emphasized in section 1. It does not. Thinking about  $o$  is distinct from being aware of  $o$  since it is possible to think about  $o$  and not be aware of  $o$ . Some ways of thinking about  $o$ , however, can endow an intuition experience with phenomenology that differentiates  $o$  from its background. If—as I will propose—intuitive awareness is grounded in intuitive differentiation and—as example (C) suggests—some cases of intuitive differentiation are grounded



in thought, then some cases of intuitive awareness are grounded in thought. This does not imply that thinking about *o* is identical to being aware of *o*, nor that just any way of thinking about *o* is apt to ground awareness of *o*.<sup>48</sup>

According to premises 2 and 3 of the Differentiation Argument,

unlike the objects of visual awareness, the objects of intuitive awareness—e.g., properties and functions—do not have backgrounds, and

unlike visual phenomenology, intuitive phenomenology is not the sort that could differentiate an object from its background.

Neither claim holds up under examination. The objects of intuitive awareness do not have the same sorts of backgrounds as the objects of visual awareness, and these backgrounds are not determined in the same way. But so much is to be expected. There is no reason to deny that they have backgrounds at all. Further, if, as I've proposed, intuition experiences are not just attitudes toward propositions, but include reflections, then their phenomenology, though different from visual phenomenology, does seem rich enough to be of the sort that could differentiate an object from its background.<sup>49</sup>

### **5. The Ground of Intuitive Awareness**

The Dependence Argument challenges the proponent of the view that some intuition experiences make their subjects aware of abstract objects to explain how an intuition experience might depend on an abstract object. The proposal I ended with in section 3 was this:

Formal Naïve Realism: If *S* is intuitively aware of an abstract object *o* by having intuition experience *e*, then *e* depends on *o*, in that: in accordance with the essence of *e*, *o* is part of the principle of unity that *e*'s (material) parts instantiate and thereby determines *e*'s phenomenal character.

This proposal is incomplete in two ways. First, it does not say what the (material) parts of an intuition experience are. Second, it does not say just how an abstract object might figure in an intuition experience's principle of unity.

The discussion of intuitive differentiation in section 4 suggests how to fill these gaps. The (material) parts of an intuition experience are the reflections, such as thoughts and imaginings, that compose it. An abstract object *o* might figure in an intuition experience *e*'s principle of unity this way: *e*'s principle of unity demands that *e*'s (material) parts be so arranged that *e*'s phenomenology differentiates *o* from its background.

Filling the gaps as suggested completes the response to the Dependence Argument. But it does more. It suggests a view about the ground of intuitive awareness:

Ground of Intuitive Awareness: If S is intuitively aware of an abstract object *o* by having intuition experience *e*, then *e* makes S aware of *o* *because*: in accordance with *e*'s essence, *e*'s (material) parts are so arranged that *e*'s phenomenology differentiates *o* from its background, and *o* thereby plays a role in determining *e*'s phenomenal character.

Here are a few reasons why I find this proposal attractive.

First, it has the right implications. It implies that if S is intuitively aware of abstract object *o* by having intuition experience *e*, then (i) *e* depends on *o* and (ii) *e*'s phenomenology differentiates *o* from its background. Both implications count in favor of the proposal. Suppose—as we have been—there are necessary dependence and differentiation conditions on any kind of awareness, including intuitive awareness. Then, one would expect, the correct account of the ground of intuitive awareness should imply that these conditions are met in any case of intuitive awareness. The proposal meets this expectation.<sup>50</sup>

Second, it fits introspection. What seems to me to happen when I become intuitively aware of an abstract object, for example the function  $f(a,b) = |(a + b)^2 - 4ab|$ , is this. Initially I have some scattered reflections, in which I hazily flit about lots of different functions of *a* and *b*. Then these reflections coalesce into a new experience in which my thoughts and imaginings are lined up so as to make  $f(a,b)$  stand out from the other functions of *a* and *b* I had in mind. It could turn out that introspection is a poor guide to what grounds intuitive awareness. But absent a reason to think so, fitting introspection should count in favor of an account of intuitive awareness. And the proposal does seem to me to do that.

Third, it possesses metaphysical familiarity and credibility. It possesses metaphysical familiarity because it invokes notions such as essence, part-hood, phenomenology, etc., which, though often the objects of philosophical controversy, have wide application. It possesses metaphysical credibility because it does not imply that we, or abstract objects, are any different from what is widely supposed. We are parts of the natural world with no supernatural abilities. Abstract objects are causally inert. One might build an account of intuitive awareness that is incompatible with one or both of these claims. The proposal is compatible with both of them.

Fourth, it possesses generality. Throughout, I've focused on the sort of awareness that occurs in mathematical intuition. This seems to me the easiest case to get a handle on. But nothing in the proposal restricts its application to mathematical intuition.<sup>51</sup> If we are aware of properties such as goodness, knowledge, and logical implication in moral, philosophical, and logical intuitions, then there must be some facts in virtue of which such intuitions make us aware of these abstract objects, when they do so. The proposal suggests what facts these might be.<sup>52</sup>

The forgoing considerations do not demonstrate that the proposal is correct. Doing that is beyond the scope of this paper. What they do, however, is

show that the proposal is plausible: it is a serious theoretical option, worthy of further exploration. Note, however, that it just tells us how intuitive awareness might be grounded, if intuitive awareness ever occurs. It does not tell us that intuitive awareness ever occurs. But, as we saw in section 1, there are considerations in favor of thinking that some intuition experiences do make their subjects aware of abstract objects. The persuasiveness of these considerations hinged on the availability of a plausible account of the ground of intuitive awareness. But now we have one in hand. So the considerations developed in section 1 should move us—they do provide us with some reason to think that some intuition experiences make their subjects aware of abstract objects.

### Notes

<sup>1</sup> I thank Uriah Kriegel, Richard Tieszen, and an anonymous reviewer for helpful comments on an earlier draft of this paper.

<sup>2</sup> (Dretske 1999) distinguishes between object-awareness and fact-awareness. My focus in this paper is on object-awareness. Dretske also discusses property-awareness. From what I can tell, this is just object-awareness restricted to objects of a certain kind, namely properties. Dretske's notion of fact-awareness, on the other hand, is not just object-awareness restricted to objects of a certain kind, namely facts. We should allow the possibility of bearing the object-awareness relation to facts; this wouldn't be fact-awareness in Dretske's sense, however. (Schellenberg forthcoming) distinguishes between an extensional and an intensional sense of "awareness." My usage in this paper is restricted to the extensional sense.

<sup>3</sup> For an overview and references to the literature see (Rosen 2001).

<sup>4</sup> See (Rosen 2010), pg. 118.

<sup>5</sup> I defend it and explore its philosophical significance in (Chudnoff forthcoming-a and ms).

<sup>6</sup> See (Johnston 2004). See also (Dretske 1999), (McGinn 1982), and (Tye 2002, 2010).

<sup>7</sup> (Gödel 1947), pg. 268 in (Gödel 2001).

<sup>8</sup> (Husserl 2001), pg. 292. Italics in the original.

<sup>9</sup> I thank an anonymous reviewer for pressing me to deal with the present challenge.

<sup>10</sup> Another sort of supplementation is semantic. According to Russell, acquaintance with, or awareness of, abstract objects grounds our ability to think about them; see (Russell 1992, 1997). Suppose the relevant kind of awareness is intuitive awareness. If an intuition experience grounds your ability to think about *o*, then, one might argue, it must make you aware of *o*, and so not be an intuitive hallucination. This is not a line of argument I am inclined to press.

<sup>11</sup> The claim that some intuitions play partial roles in putting their subjects in a position to know those propositions that intuitively seem true in them is not sufficient to patch up the argument by phenomenological analogy. The reason why is that the partial role might just be that of a justifier, and, at least according to most epistemological views, hallucinations that fail to make you aware of anything can still justify beliefs.

<sup>12</sup> (Hahn 1956) presses this challenge against mathematical intuition. Some work in the experimental philosophy tradition presses this challenge against thought experiment intuitions; see (Knobe and Nichols 2008).

<sup>13</sup> Cf. (Field 1989).

<sup>14</sup> (Boghossian 2001) forcefully presses this challenge. The *locus classicus* is (Benacerraf 1973).

<sup>15</sup> For examples of this strategy see (Bealer 1998), (Bengson forthcoming), (Feferman 2000), (Ludwig 2007), and (Sosa 2007). Feferman is particularly concerned to address the challenge to mathematical intuition pressed by Hahn. Bealer, Ludwig, and Sosa draw the principled distinction between intuitions called for by this strategy on etiological grounds. My preference would

be to draw it on phenomenological grounds. I discuss what I take to be the phenomenology of reliable intuitions in (Chudnoff 2011a and 2011b).

<sup>16</sup> (Bengson 2010) develops this idea in detail and, to my mind, persuasively.

<sup>17</sup> (Grice 1961), (Strawson 1979), (Tye 1982).

<sup>18</sup> (Dretske 1969), (Siegel 2006), (Tye 2010).

<sup>19</sup> I thank an anonymous reviewer for pressing me to address the present concern.

<sup>20</sup> (Siegel 2006), pg. 432. Paul Snowdon gives a similar characterization; see (Snowdon 1990), pg. 67 in the reprint in (Byrne and Logue 2009).

<sup>21</sup> Michael Tye gives a similar characterization of what he calls “consciousness of things,” which I take to be the same as the awareness relation I am discussing: “The general suggestion, then, is as follows: If a phenomenally conscious state of mine is such that at a minimum it at least enables me to ask ‘What is that?’ with respect to some entity, and it does so directly on the basis of its phenomenal character alone, then I am conscious of that entity.” See (Tye 2010), pg. 14.

<sup>22</sup> One important difference between intuitive awareness and seeing is that in many cases intuitive awareness occurs in experiences one couldn’t have without exercising cognitive apparatus that goes beyond “whatever general apparatus is needed for *de re* mental states or demonstrative reference.” So intuitive awareness is not as cognitively primitive as seeing. This is to be expected. Note that even if you must be able to think about an abstract object in order to have an experience that makes you aware of that abstract object it could be that this experience is what first enables you to entertain *de re* thoughts about that abstract object. A similar phenomenon can occur in the sensory case: you might be able to think of Hume’s missing shade of blue—as the shade between two others, say—and rely on this cognitive capacity in directing yourself to imagine the missing shade, which imaginative experience then puts you in a position to entertain *de re* thoughts about it. I thank Nick Wiltsher for discussion of this point.

<sup>23</sup> Cf. (Kim 1974), (Ruben 1992), pg. 223.

<sup>24</sup> (Martin 1997), pg. 93 in reprint in (Byrne and Logue 2009).

<sup>25</sup> (Campbell 2002), pg. 116.

<sup>26</sup> For more on non-sensory phenomenology see (Pitt 2004) and (Horgan and Tienson 2002).

<sup>27</sup> Cf. (Fish 2009).

<sup>28</sup> (Johnston 2006).

<sup>29</sup> (Koslicki 2008).

<sup>30</sup> A worry one might have about the foregoing explication of partial constitution is this. Consider its application to ordinary objects, such as bicycles. According to it, a bicycle is not just partly constituted by its wheels, but also by the property of being a wheel, which is a part of the bicycle’s principle of unity. This sounds strange. I think it is acceptable, however. Once we distinguish partial constitution from parthood as ordinarily understood, it does not obviously violate any of our ordinary commitments to say that a bicycle is partially constituted by the property of being a wheel. Further, our ordinary commitments are defeasible. Recall, Koslicki defends the view that their principles of unity themselves are parts of ordinary objects; see (Koslicki 2008). And Laurie Paul has defended a view on which all the properties of ordinary objects are parts of them; see (Paul 2002). I do not think we should rule out these views just because they sound strange.

<sup>31</sup> It is important to distinguish Formal Naïve Realism from a truncated version of the view that drops the last clause, “and thereby determines *e*’s phenomenal character.” Consider the truncated view: If *S* is intuitively aware of an abstract object *o* by having intuition experience *e*, then *e* depends on *o*, in that: in accordance with the essence of *e*, *o* is part of the principle of unity that *e*’s (material) parts instantiate. Picking up on the discussion from footnote 31, it turns out that a bicycle depends on the property of being a wheel in the sense of dependence captured by the truncated view. This suggests that this sense of dependence is unhelpful in explicating awareness. Formal Naïve Realism proper does not suffer from the same worry. Cf. the above discussion of the different components in Martin’s characterization of naïve realism.

<sup>32</sup> To avoid complication, I stipulate that if  $a$  and  $b$  are identical, then “the bigger” just picks out that number  $n = a = b$ .

<sup>33</sup> About (B), why  $f(a,b) = |(a + b)^2 - 4ab|$ ? Your imaginative manipulation of the figure illustrates the way the difference between  $(a + b)^2$  and  $4ab$  changes as  $a$  and  $b$  change. About (C), I focus on  $[(a + b) + |a - b|]$  not  $[(a + b) + |a - b|]/2$ . The reason why is that when I project myself into the case, my experience seems to me to break down into an intuitive part, where I am aware of  $[(a + b) + |a - b|]$  and appreciate that it yields twice the bigger of  $a$  and  $b$ , and an inferential part where I conclude that dividing by 2 yields the bigger of  $a$  and  $b$ .

<sup>34</sup> I am taking a visual field to be a volume of space, not something mental.

<sup>35</sup> The example is adapted from (Koffka 1935), pgs 200 – 201.

<sup>36</sup> The phrase comes from (James 1983), pg. 552.

<sup>37</sup> For more on cognitive fields and their structure see (Gurwitsch 1964) and (Chudnoff forthcoming-b).

<sup>38</sup> See, for example, (Williamson 2008) and (Earlenbaugh and Molyneux 2009). I criticize doxasticism in (Chudnoff 2011b).

<sup>39</sup> See, for example, (Bealer 1998) and (Huemer 2001). I defend a form of sui generism in (Chudnoff 2011b).

<sup>40</sup> See, for example, (Williamson 2008) and (Ichikawa and Jarvis 2009).

<sup>41</sup> In this paper I focus on thoughts and imaginings; I do not intend to rule out the possibility that other mental states might compose an intuition experience as well.

<sup>42</sup> The view that I am defending is inspired by Husserl’s view that intuitions are “founded” on other mental states; See (Husserl 2001), especially page 286 and ff. Charles Parsons and Richard Tieszen have also developed views of intuition inspired by Husserl’s view; see (Parsons 1979, 2007) and (Tieszen 1989, 2005a, 2005b).

<sup>43</sup> The argument in this paragraph is similar to arguments in favor of the view that perceptual experiences have mental paint, qualia, or sensational properties; see, e.g., (Block 1996), (Peacocke 1983). In those arguments two perceptual experiences representing that  $p$  are supposed to differ phenomenally. My argument, however, is compatible with the view that phenomenology supervenes on intentional content, since in the case I am considering it can be the intentional contents of the thoughts and imaginings that make the phenomenal difference between the two intuitive appearances

<sup>44</sup> See (Husserl 2001), (Parsons 1979, 2007), and (Tieszen 1989, 2005a, 2005b) for additional argumentation in favor of thinking that intuitions are “founded” in some sense on other mental states. See (Chudnoff 2011a, 2011b) for further elaboration and defense of my own interpretation of what this amounts to. In these papers I address two worries that might have occurred to the reader, but that I cannot take up here. The first worry is that experiences that are composed in the way that I am suggesting are really inferences, not intuitions. The second worry is that there are unreflective intuitions, which do not seem to be composed of other mental states.

<sup>45</sup> I am assuming that ‘ $a$ ’ and ‘ $b$ ’ range over natural numbers.

<sup>46</sup> (Peacocke 1985).

<sup>47</sup> The role of imaginative intentions requires further discussion. Here are three quick observations. First, you might intend to imaginatively illustrate an abstract object that does not exist. This intention misfires; you do not succeed in differentiating “that” abstract object from its background. Second, if you intend to imaginatively illustrate  $o$ , you must be able to think about  $o$ . So if—as I will propose—intuitive awareness of  $o$  depends on differentiation of  $o$  and in some cases differentiation of  $o$  depends on intending to imaginatively illustrate  $o$ , then in those cases your ability to think about  $o$  must not depend on your intuitive awareness of  $o$ ; this is compatible with the point made in footnote 22 that your ability to entertain *de re* thoughts about  $o$  might derive from your intuitive awareness of  $o$ . Third, your imaginative intention must mesh with what you imagine. You cannot succeed in imagining an instance of circularity, say, by imagining a square and intending that it be a circle.

<sup>48</sup> The intuition of the Extensionality Axiom discussed in section 1 provides another example in which differentiation is grounded in thought. In that case what you do is

differentiate the property of being a set from other kinds of things with members, such as committees and orderings.

<sup>49</sup> In his work on mathematical intuition Richard Tieszen has explored how we might “construct” various mathematical objects in reflections of the sort I have suggested compose intuition experiences. In (Tieszen 1989) he focuses on natural numbers and finite sets; in the papers collected in (Tieszen 2005b) he explores geometrical kinds and transfinite sets, among other sorts of mathematical objects. This work is congenial to my view of just how rich the phenomenology associated with intuition can be.

<sup>50</sup> One worry I have about Tieszen’s account of awareness of abstract objects (supposing its target is awareness in the sense I have been exploring) is that it fails to imply that if S is intuitively aware of *o* by *e*, then *e* depends on *o*. The reason the implication fails is that Tieszen subscribes to the following two theses (Tieszen 1989 *passim*): whether S is intuitively aware of *o* depends wholly on the phenomenology of S’s experiences; the phenomenology of S’s experiences depends wholly on intrinsic properties of S. Tieszen’s account might be adequate for a different target, for example awareness of abstract objects in what Schellenberg calls the intensional sense; see (Schellenberg forthcoming).

<sup>51</sup> Even in the case of mathematical intuition, the account is distinguished by its generality. Parsons’ account of intuitive awareness, for example, exploits special properties of what he calls quasi-concrete abstract objects, such as word-types, in a way that makes it unclear whether, or how, it might be extended to intuitive awareness of abstract objects that are not quasi-concrete, such as functions of the sort discussed above; see (Parsons 1979, 2007). I should note that in the case of quasi-concrete abstract objects the two accounts are compatible.

<sup>52</sup> Though nothing in the account restricts its application to mathematical intuition, actually applying it to other sorts of intuition requires additional substantive work. In particular, it requires exploring what differentiating properties such as goodness, knowledge, and logical implication from a background might consist in.

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