Does the number sense represent number?

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Abstract

On a now orthodox view, humans and many other animals are endowed with a "number sense", or approximate number system (ANS), that represents number. Recently, this orthodox view has been subject to numerous critiques, with critics maintaining either that numerical content is absent altogether, or else that some primitive analog of number ('numerosity') is represented as opposed to number itself. We distinguish three arguments for these claims – the arguments from *congruency*, *confounds*, and *imprecision* – and show that none succeed. We then highlight positive reasons for thinking that the ANS genuinely represents numbers. The upshot is that proponents of the orthodox view should not feel troubled by recent critiques of their position.

Keywords: number sense; numerosity; approximate number system; analog magnitude system.

Introduction

On a now orthodox view, humans (and many non-human animals) possess a primitive and pre-linguistic capacity to represent number. This is sometimes referred to as our "number sense" (Dehaene, 1997).

At the heart of this theory lies the postulation of an *approximate number system* (ANS). This is a well-studied psychological system that enables organisms to efficiently intuit gross numerical quantities. It's not perfect. Most notably, its performance conforms to Weber's Law. So, while the ANS enables organisms to intuit (sometimes quite large) numerical quantities, ANS-governed numerical discriminations remain imprecise and ratio sensitive. 8 is easier to discriminate from 10 than 10 is from 12. Conversely, 8 is discriminated from 10 as easily as 16 is discriminated from 20. In each case, absolute difference in number has little effect on performance: it is the ratio between sets which matters (the further from 1:1 the better).

It bears emphasizing that the postulation of an ANS has proven extremely productive. Countless studies have served to support its existence, with the postulation of an ANS generating empirical predictions that have been borne out repeatedly in carefully controlled experiments (for reviews, see Anobile, Cicchini & Burr, 2019; Odic & Starr, 2018). In spite of this, there has been growing discontent with the suggestion that humans (and other animals) possess an ANS which genuinely represents number. Instead, critics have argued that (strictly speaking) the relevant systems/processes merely represent non-numerical magnitudes, like size and density, or primitive analogs of number (like "numerosity"). Either way, critics have denied that number is literally represented by an ANS, thereby calling the number sense theory into question.

In the present treatment, we address these critiques. We begin by distinguishing three arguments that have motivated scepticism about the ANS, and its capacity to represent number, and show that none succeed. We then highlight positive, and previously unarticulated, reasons for thinking that the ANS genuinely represents numbers. The upshot is that proponents of a number sense should not feel troubled by recent critiques of their position.

The ANS

To begin, let us consider a tiny sample of the many studies that have purported to show that humans possess an ANS with genuine numerical content. To be clear, it is the interpretation of these studies that is at issue in this paper. Nevertheless, it is useful to have some evidence in view before considering the skeptic's critiques.

One particularly important source of evidence emerged in the early 2000s, when studies yielded evidence that young human infants could track the numerical properties of large sets, albeit imprecisely and in accord with Weber's Law. For instance, Xu and Spelke (2000) habituated six-month-old infants to seen arrays containing either 16 or 8 dots. When habituated to an 8-dot array, infants recovered interest when presented with a 16- or 4-dot array, but not a 12-dot array. Meanwhile infants who were habituated to a 16-dot array would dishabituate to a 32- or 8-dot array, but not a 24-dot array. Since obvious confounds were controlled for, these findings were interpreted as showing that six-month-old infants can represent and discriminate the approximate number of items in large sets provided they differ by a suitably large ratio (e.g. 1:2). Subsequent studies then suggested that these basic discriminative capacities persist and improve into development. For instance, nine-month-old infants were found to reliably discriminate numerical quantities in comparable tasks provided these differed by a ratio of just 2:3 (Wood & Spelke, 2005), with adult humans pre-attentively discriminating numerical quantities with tighter ratios still (Halberda, Ly, Wilmer, Naiman & Germine, 2012). In each case, performance decreases as the numerical ratio nears 1:1, irrespective of precise cardinal values.

Cross-modal studies bolster the suggestion that these results reflect a genuine sensitivity to number or numerical quantity. In one study, Izard, Sann, Spelke and Steri (2009) even found neonates capable of matching numerical quantities across modalities – i.e. matching a number of seen items with a number of heard sounds in a sequence of tones. Note that studies of this sort complicate attempts to explain the preceding results in terms of non-numerical confounds. After all, neonates in Izard et al.'s study could not have relied on (say) the size of the dots, or the total area of the seen set, when identifying a match, since properties of this sort could not have been heard. In this way, cross-modal studies support the conclusion that even pre-linguistic infants are genuinely sensitive to numerical quantities. And, since their failures were (again) a function of ratio, these results implicate a system with the performance profile of an ANS as outlined above.

Of course, infants are notoriously difficult to study, requiring the use of indirect measures such as looking time. But preschoolers can be directly asked which of two stimuli has "more" dots or tones, and since they're still too young to reliably count, they use their ANS to answer. For instance, Barth, La Mont, Lipton and Spelke (2005) found not only that preschoolers could reliably answer which of two visual stimuli had "more" (e.g. whether there were more red dots or blue dots), but also that they were roughly *as good* at doing this across modalities (e.g. vision to audition) as they were at matching intra-modal stimuli (e.g. the numerosities of two seen displays). Similar results are found in adults (Arrighi Togoli & Burr, 2014). In each case, the numerical ratio between sets continues to predict patterns of success and failure, further implicating a system with the performance profile of an ANS.

Using a quite different paradigm, Franconeri, Bemis and Alvarez (2009) and He, Zhang, Zhou and Chen (2009) discovered additional evidence for an ANS. In their studies, subjects were presented with visual arrays containing sets of dots. What they showed was that connecting pairs of dots with a thin line (effectively turning pairs of dots into single dumbbell-shaped items) substantially reduces subjects' intuitive estimates of numerosity. Franconeri et al. also showed that introducing a small break in the lines would immediately eliminate this "dumbbell effect". Since displays with small breaks and displays with dots connected by thin lines differ only very slightly with respect to their total surface area, spatial frequency, and other non-numerical magnitudes, but dramatically with respect to their numerical quantity (displays of the former type contain *twice as many items*) these studies support the hypothesis that humans possess an operational ANS that provides a genuine sensitivity to number itself.

Finally, all of these results have been linked to findings at the level of neural implementation. Individual neurons in the intraparietal sulcus (IPS) respond selectively to specific numbers (Nieder, 2016). Thus, specific neurons in the IPS respond preferentially when one sees or hears an array or sequence containing (say) *seven* items, Crucially, this response profile is noisy. Thus, a neuron which is tuned to 7 will fire when one observes 6 or 8 items and occasionally when one observes 5 or 9 items. Indeed, noise levels *increase* with number. As various theorists have observed, this is precisely what we would expect of neurons implementing an ANS, for it would seem to explain the system's conformity to Weber's Law.

Recent Critiques

The preceding remarks do not provide a comprehensive overview of work on the ANS. Nor do they definitively establish its existence. They merely provide readers with an initial sense of the vast, and seemingly convergent, evidence which speaks in favor of the ANS's existence and capacity to represent number. With this in mind, we will now consider three arguments that have been levied against such proposals. For brevity, we will call these the arguments from *Congruency, Confounds,* and *Imprecision*. In showing that all three fail, we gain a deeper appreciation for why we really should posit an ANS which genuinely represents numbers.

The Argument from Congruency

An initial reason why some continue to doubt the existence of an ANS, with even broadly numerical content, stems from the existence of *numerical congruency effects*. These are cases in which numerical judgments are influenced by the perception of irrelevant magnitudes. For instance, when subjects compare Arabic numerals and decide which picks out a larger number, their reaction times are influenced by font size. So, when the larger numeral is printed in a larger font (a "congruent" trial) they answer more quickly than when the numerals are identical in font size (a "neutral" trial). And when the smaller numeral has a larger font (an "incongruent" trial), they're slower and less accurate (Henik & Tzelgov 1982; Gebuis, Cohen Kadosh, de Haan & Henik, 2009; Gebuis & Reynvoet 2012c). Similar effects occur in non-symbolic tasks. Thus, subjects who are tasked with determining whether one display of dots is more or less numerous than another are influenced in comparable ways by things like average dot diameter, dot density, convex hull, and dot brightness (Cohen Kadosh & Henik 2006; Dakin Tibber, Greenwood, Kingdom & Morgan, 2011; Gebuis & Reynvoet, 2012a, 2012b; Leibovich and Henik 2014).

The *argument from congruency* proposes that these effects undermine the ANS hypothesis (Leibovich, Katzin, Harel & Henik, 2017; Gebuis, Cohen Kadosh & Gevers, 2016). Here, it is claimed that if there were an ANS with genuine numerical content we would expect relevant numerical judgments to be based *entirely* on its outputs. In other words, continuous magnitudes should be ignored. But the existence of congruency effects shows that (often) they're not, so the ANS hypothesis is problematic.

Gebuis et al. (2016, p. 22) put the objection like this: if the relevant numerical judgments are influenced by the perception of non-numerical magnitudes then:

> why would there be an ANS system that can extract "pure numerosity"? What would be the use of having a system that can tell us exactly which cue at the passport control contains less [*sic*] people when it in the end adjusts this accurate answer in a possibly incorrect answer [*sic*] when for instance the length of the people in the cue [*sic*] is taken into account?

From the perspective of optimal design, Gebuis et al. propose that it makes little sense for an ANS to exist if its outputs are influenced by confounding variables.

An initial problem with this argument is that it overgeneralizes. It is well known that congruency effects affect our judgements of uncontroversially perceptible magnitudes. For instance, judgments of duration exhibit congruency effects on size (Lourenco & Longo, 2010), luminance (Xuan, Zhang, He & Chen, 2007), length (Casasanto & Boroditsky 2008), and distance (Sarrazin, Giraudo, Pailhous & Bootsma, 2004). So, if congruency effects demonstrate that numerical quantity isn't represented in studies of the above sort, then by parity of reasoning they would demonstrate that magnitudes such as duration and distance aren't perceptually represented either.

To compound matters, congruency effects tend to be symmetric. For while numerical judgments are influenced by area and density, judgments of area and density are likewise influenced by number. Indeed, number appears to influence judgments of area and density *more* than vice-versa (Cicchini, Anobile & Burr, 2016). So, if the fact that numerical judgments are influenced by area and density shows that number isn't represented by an ANS, there should be equal or greater evidence that area and density aren't perceptually represented either. In this way, the argument from congruency leads to an implausible skepticism about the perceptual representation of magnitudes quite generally.

These considerations suggest that the argument from congruency fails, but where exactly does it go wrong? We believe that the argument errs in assuming congruency effects are even in tension with the ANS hypothesis. If there is an ANS dedicated to representing number, the perception of continuous magnitudes might introduce biases at the initial encoding stage, influencing inputs to the ANS, or at the decision/response stages, altering outputs of the system. At the encoding stage, numerical and non-numerical magnitudes such as density may be computed on the basis of overlapping features, such as spatial frequency (Dakin et al. 2011) or object size (Dehaene & Changeux 1993). Thus, altering non-numerical magnitudes might alter numerical judgments by altering features that are inputs to the ANS (Odic 2018). Alternatively, congruency effects might be a Stroop-like byproduct of competition for a single response. And given its scalar variability, the ANS is not a perfectly precise instrument. So, if it's true that certain magnitudes typically correlate with number, it might make sense to adjust the outputs of the system in accord with those correlations. Thus, congruency effects might reflect an optimal strategy for counteracting the imprecision of the ANS.

The Argument from Confounds

The argument from congruency is unpersuasive. However, a related and comparatively troubling objection stems from the observation that numerical quantity is never presented independently of all confounding variables. For instance, a visual display containing nine dots will also contain dots with an average diameter, cumulative area, convex hull, and density. Similar points apply to heard or felt sets. Consequently, there is always the worry that number isn't really being represented or tracked in studies of the above sort, only confounding variables. The *objection from confounds* claims that this undermines experimental attempts to evince an ANS with genuine numerical content (Leibovich & Henik 2013; Leibovich et al. 2017; Gebuis et al. 2016).

There are actually two readings of this objection. On a *strong* reading, it is deemed impossible to empirically adjudicate the hypothesis that subjects represent numerical quantities (in addition to various sensory confounds) against the hypothesis that they merely represent sensory confounds. According to a *weak* reading of the objection it may not be impossible to empirically distinguish these hypotheses, but it is sufficiently difficult that there is, at present, no empirical justification to favor one hypothesis over the other – studies that have been conducted so far are, thus, equivocal.

We see no reason to accept the argument in its stronger incarnation. Theories in science are always underdetermined by the data, and the selection of one theory over another requires an inference to the best explanation (Duhem, 1914). So, in psychology, there will never be a single experiment that eliminates all potential confounds. Instead, we must consider multiple studies that, cumulatively, support one hypothesis over the other. The postulation of an ANS with genuine numerical content is not special in this regard. For while number is an abstract property that cannot be observed in isolation, the same is true of many other properties which are plausibly represented by the pre-linguistic mind, such as causation (Kominsky & Carey, 2018) and animacy (Gergely & Csibra, 2003). In each case, these hypotheses are assessed against plausible alternatives, for to the extent that viable alternatives are ruled out in controlled experiments we can reasonably increase our faith in the relevant conjectures.

This leaves the weaker reading of the objection. To appreciate its force, consider studies that examine our visual perception of number by presenting arrays of dots on a screen. Some such studies choose one potential confound – say, total surface area, and keep it constant while number varies. As Leibovich et al. (2017, p. 4) observe, this always leaves other continuous magnitudes uncontrolled for. For example, if the total surface area of the dots is kept constant while numerical quantity increases, then the average size of each dot will need to decrease. So, if subjects succeed in discriminating a difference, we don't know if that's because they're tracking number or average dot-size.

Other studies take turns varying non-numerical magnitudes across trials, such that no one confound correlates with number throughout the whole experiment. Thus, half the trials might keep total surface area constant while the other half keep average dot-size constant. Alternatively, each of a range of non-numerical magnitudes might be varied across trials such that, throughout the whole experiment, they are congruent on half of the trials and incongruent on the other half. But while these controls suggest that subjects do not rely on a single confounding magnitude, Gebuis et al. (2016, pp. 23–24) object that subjects could still be switching between cues throughout the experiment or relying on multiple confounds (see also Leibovich et al. 2017, pp. 4-6).

This may be possible, but in and of itself this fails to provide reasonable grounds for doubt. This is because a reasonable skepticism about the ANS cannot be *ad hoc*. It cannot rest on a piecemeal strategy of finding one set of continuous cues to account for behavior in one experiment, a second set of continuous cues to account for behavior in another experiment, and so on. What is needed is a positive proposal that explains how some particular function of continuous cues, or some principled strategy for switching among cues, could account for what appears to be number-tracking behavior across a wide range of studies. Skeptics of the ANS have not provided one. Instead, they simply observe that numerical judgments are influenced by non-numerical magnitudes – that is, that they are subject to congruency effects. But as we have already seen, congruency effects are fully compatible with the existence of an ANS and fail to provide grounds for doubt.

These points undermine the argument from confounds, indicating that it is theoretically undermotivated. But, before moving on, it is important to stress that *empirical* findings speak directly against the objection. Take the dumbbell effects discussed when introducing evidence for the ANS. There, we noted that connecting items with a thin line substantially reduces judgments of number, while introducing a small break in these lines eliminates this effect (Franconeri et al. 2009; He et al. 2009). Given that displays with and without a small break are nearly identical with respect to sensory confounds, but differ substantially in number, these studies indicate that number is itself tracked and represented and that performance doesn't simply involve tracking non-numerical confounds. We have found no discussion of these studies by skeptics of the ANS.

Additionally, some studies reveal that our sensitivity to number differs markedly from our sensitivity to non-numerical magnitudes. For example, DeWind, Adams, Platt and Brannon (2015) compared how the number, size, and spacing of dots in a display affect numerical judgments and found that judgments were more sensitive to actual number than to size or spacing, suggesting that number itself is being tracked. (For a reply, see Leibovich et al. 2017, p. 10, and for a rebuttal to the reply see Park, DeWind & Brannon, 2017). Similarly, Cicchini et al. (2016) had subjects judge the area, density, and number of dots in a visual display, and found that number judgments were more sensitive than area and density judgments. Again, this suggests that subjects do not simply represent area and density, but also numerical quantity.

Finally, we have already seen that cross-modal studies naturally eliminate potential confounds. For as was discussed when introducing evidence for the ANS, a static array of seen dots and a sequence of heard tones will seem to lack properties in common that could serve as a plausible crutch on which to base numerical comparisons. For while the dots will have a cumulative area, average diameter, and convex hull, the tones will have none of those properties. Since numerous cross-modal studies demonstrate success in numerical discrimination tasks, this further undermines the objection.

Skeptics of the ANS do sometimes recognize this latter point. Leibovich et al. (2017) note that cross-modal studies provide "[a] very strong line of evidence supporting the ANS" (p. 5). But while they proceed to question whether cross-modal studies of newborns and infants show that the ANS is already operational at (or near) birth, we can bracket these worries here since we aren't focused on the issue of innateness. The important point is that there are numerous cross-modal studies of adults (Barth, Kanwisher & Spelke, 2003), preschoolers (Barth et al. 2005), and animals (Church and Meck 1984) that don't face the same worries. In each case, these cross-modal comparisons display the distinctive performance profile of an ANS – discriminability in accord with Weber's Law – thereby supporting the system's existence.

A more relevant critique for our purposes comes from Gebuis et al. (2016). They acknowledge the existence of studies in human adults demonstrating cross-modal adaptation (Arrighi et al., 2014) and cross-modal comparisons (Barth et al., 2003), but claim that such studies "do not present a clear result" (p. 27). They reason that if number is represented across modalities, there should be no cost to cross-modal comparison. But while the existence of some such cost remains a matter of dispute (contrast Barth et al., 2003, with Gebuis et al. 2016), its potential discovery should not alarm proponents of the ANS. In intra-modal tasks, numerical comparisons are probably facilitated by congruent continuous magnitudes while inter-modal tasks leave no such opportunity for facilitation. In any case, it is the fact that cross-modal numerical comparisons can be successfully executed at all that speaks in favor of an ANS with numerical content. So, while none of these studies are immune to criticism, they collectively constitute a compelling reply to the argument from confounds.

The Argument from Imprecision

The preceding remarks highlight empirical findings that are hard to make sense of without an ANS that affords genuine sensitivity to number. Nevertheless, a final argument – the argument from imprecision – raises a conceptual worry with this suggestion.

Once again, this argument comes in two varieties. In its weaker incarnation, the argument states that the imprecision of the ANS's numerical discriminations reveals that it cannot literally be representing the numbers that mathematicians recognize and discuss. At best, it can be representing numerosities, which are primitive analogs of number but not literally numbers themselves. In its stronger incarnation the argument purports to establish that the imprecision of the ANS prevents it from having any numerical content whatsoever – even primitive numerosity content.

The stronger incarnation of the argument appears to be endorsed by Núñez (2017). He introduces his worry by noting that when the ANS discriminates numerical quantities its discriminations are "rarely exact" (p.417) – as we have seen, they are imprecise and conform to Weber's Law. But as he sees it:

> A basic competence involving, say, the number 'eight', should require that the quantity is treated as being categorically different from 'seven', and not merely treated as often – or highly likely to be – different from it. (ibid.)

In this way, Núñez proposes that the ascription of genuine numerical content to the ANS would require that it quantify "in an exact and discrete manner" lest this amount to nothing more than "loose" talk (p.418). Since this is something that the ANS does not do (to reiterate, its discriminations conform to Weber's Law), Núñez proposes that the ANS does not represent numerical quantities at all.

As we interpret him, Núñez is not merely proposing that the ANS is an *approximate* number system which represents imprecise or primitive analogs of number. He is denying that it produces any numerical content whatsoever. This is apparent in his "crucial distinction" between cognition that is genuinely "numerical" and cognition that it is merely "quantical" (a theoretical term Núñez introduces). Among other things, quantical cognition concerns "quantity-related capacities" that do not meet the requisite level of precision to qualify as genuinely numerical. Thus, Núñez argues that unless an ANS meets the requisite level of precision, it is inappropriate to suppose it could represent anything more than non-numerical quantities.

In so doing, Núñez effectively lumps the ANS's representations in with the representation of other magnitudes, such as duration, brightness, distance, and chemical concentrations. All of them are on a par. They are all "quantical." But this obscures an important difference: numerical quantities are higher-order (Frege, 1884). Thus, numbers can only be assigned relative to a sortal—a criterion for individuating the entities that are being counted. The question, "How many things are in this room?" is ill-posed. The type of entity that's being counted needs to be specified. When researchers study the ANS, they are investigating a system which tracks a property that is higher-order in this sense.

To see why this matters, consider a recent study by Plotnik, Brubaker, Dale, Tiller, Mumby and Clayton (2019). Here, elephants were presented with pairs of buckets containing sunflower seeds. These had opaque, perforated lids, allowing elephants to smell (but not see) their contents. Plotnik et al. found that elephants would preferentially select the bucket containing a greater quantity of sunflower seeds, albeit imprecisely and in accord with Weber's Law. On this basis, they took their results to corroborate studies that have been seen to indicate the existence of a numerical ANS in these creatures (e.g. Irie, Hiraiwa-Hasegawa & Kutsukake, 2019). But note that while this might be so, it neglects a simpler possibility: that the elephants were merely sensitive to the intensity of the odor emanating from the buckets, leading them to approach the bucket with the stronger odor (and hence more seeds). On this account, Plotnik et al.'s findings would be orthogonal to the presence or absence of an ANS with genuine numerical content; they would simply provide further demonstration of these creatures' formidable capacity for olfaction.

Plotnik et al.'s study fails to distinguish between these possibilities. But it is a substantial question which is correct. And, it is a question we might wish to answer whether or not the relevant discriminations are intrinsically imprecise. This is because there is a basic distinction between a mere sensitivity to a non-numerical first-order magnitude such as odor, and an ability to abstract away from these to represent a higher-order numerical magnitude. Núñez's approach obscures this important distinction, perhaps because he assumes that numerical magnitudes must be represented precisely. But just as first-order, non-numerical magnitudes can be represented precisely or imprecisely (you can represent someone's height as "72 inches" or as "approximately the length of a sofa") so too can higher-order numerical magnitudes (you can represent the number of coins in your pocket as "exactly five" or as "several").

Aware of this, and the fact that various studies would be hard to make sense of unless the ANS were genuinely tracking and representing numerical quantities, many theorists embrace a modest version of the argument from imprecision. They side with Núñez in thinking that the system's imprecision precludes it from literally representing integers or other numbers that mathematicians typically recognize. But they maintain that this does not preclude it from representing numerical quantities of some variety. To this end, they introduce an intermediate category – "numerosity". Numerosities are higher-order magnitudes but are intrinsically imprecise and indeterminate in a way that integers and other numbers of the sort discussed in the math class are not. Thus, proponents of the argument from imprecision, in its weaker variety, reject the suggestion that the ANS represents number (on account of its imprecision) but they allow that the system still represents imprecise numerosities of this sort.

Interestingly, this position is often defended by theorists who are otherwise sympathetic to a number sense. For instance, Carey (2009, p. 295) writes that the ANS is "not powerful enough to represent the natural numbers" despite strongly advocating for the existence and workings of an ANS throughout the lifespan. Similarly, Spelke and Tsivikn (2001) claim that the ANS merely represents "a blur on the number line" given its imprecision, the suggestion being that rather than represent precise numbers, like integers, the ANS is representing an imprecise analog of these. Indeed, we suspect that most contemporary researchers working on the ANS implicitly accept a conclusion of this sort. Why else would they carefully avoid the term "number" in favor of the neologism "numerosity" when discussing the outputs of the system?

Carey provides two arguments for the conclusion that the ANS represents numerosity rather than number. Neither succeeds. First, she notes that ANS representations "fail to capture small numerical differences between large sets of objects" (e.g. 58 vs. 59), the implication being that if the ANS were to represent these numbers it must be sensitive to these. This seems to be a mistake. The visual system fails to discriminate small differences between distances (e.g. 58 vs. 59 meters), but it doesn't follow that it fails to represent distance. This is because, there is nothing problematic in the thought that a precise quantity (like a precise distance) might be represented imprecisely. As in the example above, someone's height might be represented imprecisely as "about the length of my sofa". There is no reason why the ANS, or its capacity to represent precise numbers, should be any different. To suggest otherwise is to mistake *what* the system is representing (e.g. precise integers) for how it represents this (e.g. precisely or imprecisely).

Carey's second argument moves beyond her first. She argues that because the ANS treats 5 and 6 as more similar than 4 and 5, it obscures the successor relation, and thus cannot represent precise numbers like the integers. Here, the suggestion is that a capacity to represent numbers, like the integers, ought to require a sensitivity to properties or features of these that are essential (or in some way central) to our conception of them (as the successor relation plausibly is with respect to number). But note, this is not true of the general case. Short of assuming an outdated and widely discredited descriptivism about mental content (which Carey would be at pains to reject) a capacity to represent some property does not require a sensitivity to any or all of its essential properties. So, while the successor relation is central (perhaps essential) to a mature grasp of number, a capacity to represent number (e.g. precise integers) does not depend on our capacity to represent this.

To illustrate, note that without extensive training in chemistry, few would be able to distinguish genuine cases of gold from cases of fool's gold. This is true despite essential differences in their chemical makeup. But it would be absurd to suppose that, for this reason, we are unable to represent and (e.g.) think about gold *as such* prior to gaining a chemistry degree. Indeed, the point applies to any given feature of the kind (Burge, 2010). So, in the same way that one can represent gold, despite an insensitivity to any specific differences between gold and other chemicals (e.g. fools' gold), there is no reason why an ANS could not represent precise integers, like 7 or 8, despite an insensitivity to their precise categorical boundaries or successive relationship.

Of course, even if Carey's arguments don't succeed, that doesn't show that the ANS represents number rather than numerosity. But we think that this should be the default view for two reasons. First, it avoids the awkward question of what a numerosity is. This is a good thing since so far as we can tell no one really has any idea.

Second, positing that the ANS genuinely represents number allows us to avoid a curious double standard that has plagued discussions of the ANS. To appreciate this, note that positing genuine number representations allows for greater consistency with our treatment of non-numerical magnitudes. For instance: we have already noted that perceptual representations of distance are imprecise, but we have not come across a single passage which concludes that we thereby represent "distancosity" as opposed to distance. So, pending a convincing argument to the contrary, it is natural to hold on to the hypothesis that humans possess an ANS that genuinely represents numbers.

Conclusion

This paper has considered a now orthodox view according to which humans have an approximate number system (ANS), that represents number. This orthodox view has faced resistance in the form of three objections – the arguments from congruency, confounds and imprecision. But, upon close inspection, none are persuasive. To compound matters, there are reasons to hold onto the suggestion that the ANS genuinely represents numbers, pending a convincing argument to the contrary. So, as things stand, proponents of the orthodox view have nothing to fear from recent critiques of their position.

Acknowledgements

Both authors were supported in writing this paper by York University's Vision: Science to Applications Program, thanks in part to funding from the Canada First Research Excellence Fund. Jacob Beck was additionally supported by a Social Sciences and Humanities Research Council of Canada Insight Grant. This paper benefited from presentation to the Language and Development Lab at UCSD. We are grateful to David Barner and other audience members for constructive feedback on that occasion.

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