Children’s number judgments are influenced by connectedness

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Research Highlights

- By five-years of age, children are susceptible to the numerical ‘connectedness’ illusion, previously observed in adults.
- The magnitude of the illusion increases in strength during development and into adulthood.
- At all ages, the magnitude of the illusion is positively correlated with visual number acuity.
- These results suggest that the distorting effects of the connectedness illusion are a side effect of the visual number system’s optimal functioning.

Abstract

Visual illusions provide a means of investigating the rules and principles through which approximate number representations are formed. Here, we investigated the developmental trajectory of an important numerical illusion – *the connectedness illusion,* wherein connecting pairs of items with thin lines reduces perceived number without altering continuous attributes of the collections. We found that children as young as 5 years of age showed susceptibility to the illusion and that the magnitude of the effect increased into adulthood. Moreover, individuals with greater numerical acuity exhibited stronger connectedness illusions after controlling for age. Overall, these results suggest the approximate number system expects to enumerate over bounded wholes and doing so is a signature of its optimal functioning.

Keywords: numerical perception; approximate number system; connectedness illusion; development; visual system

1. Introduction

Visual illusions provide a non-invasive means of investigating the rules and assumptions that guide our visual systems' computations (Eagleman, 2001). For instance, a visual system which assumes that light comes from above will systematically misrepresent shape under atypical lighting conditions, resulting in a host of familiar shape-from-shading illusions (Ramachandran, 1988; Johnston & Curran, 1996). Meanwhile, a visual system which updates its assumptions based on experience, or general maturation, will prove differentially sensitive to illusions at different stages of development (Tsurumi et al., 2023). Accordingly, visual illusions have offered a window into the rules and assumptions used by human visual systems in their computational inferences, and the contexts under which those rules and assumptions emerge or develop over time. In the present study, we investigated the developmental trajectory of an important numerical illusion – *the connectedness illusion* (Franconeri et al., 2009; He et al., 2009).

Humans possess a well-known ability to visually discriminate approximate numerical quantities (e.g., Dehaene, 2001; Clarke & Beck, 2021). For instance, adult humans can quickly compare or estimate the approximate number of seen dots in an array, even when they are prevented from explicitly counting these (Barth et al., 2003; Cordes et al., 2001). Infants (e.g., Xu & Spelke, 2000) who have not yet learnt to count, and even newborns (Izard et al., 2009) discriminate collections based on number, and studies using diverse brain imaging methods suggest that the same neural systems are recruited in numerical discrimination throughout development. For instance, the intraparietal sulcus of neurotypical human adults responds selectively to changes in the number of elements in an array (Piazza et al., 2004) and young children exhibit number selective neural responses in the same brain region (Cantlon et al., 2006). Yet, like all visual attributes, perceived number is subject to illusion (Frith & Frith, 1972; Ginsburg, 1976; DeWind et al., 2020; Qu et al., 2022; Burr & Ross, 2008, c.f. Yousif et al., 2024).

In *the connectedness illusion*, arrays containing dots that are connected into pairs by task-irrelevant lines (effectively turning pairs of items into single dumbbell-shaped objects) are perceived as less numerous than otherwise identical arrays of unconnected dots (Franconeri et al., 2009; He et al., 2009). Remarkably, thin or even illusory lines suffice to elicit this effect (Adriano et al., 2021), and the introduction of small breaks in these lines can eliminate the effect entirely (Franconeri et al., 2009). Indeed, the connectedness illusion is robust across a range of experimental paradigms. These include simultaneous ordinal comparison tasks, in which participants must choose the more numerous of two collections that are presented concurrently on a screen (e.g., He et al., 2009; Qu et al., 2024), sequential ordinal comparisons in which the two collections are presented consecutively (e.g. Franconeri et al., 2009; Fornaciai et al., 2016), and estimation tasks in which participants must estimate the total number of items in a single array (e.g. He et al., 2015). In each case, the illusion persists even when subjects are explicitly told to ignore the connecting lines, demonstrating the automaticity of the connectedness effect. A major implication of the connectedness effect is that the approximate number system (ANS) appears to operate on bounded objects by default (Franconeri et al., 2009). Despite the virtually identical continuous attributes of arrays with connections vs arrays with unconnected lines (e.g., Figure 1) arrays are perceived as having dramatically different numerical values. This suggests that rather than deriving approximate number from non-numerical properties of the displays, such as the total surface area, average surface area, or spatial density of the displays, connections substantially reduce perceived number because the ANS functions to enumerate discrete individuals (He et al. 2009; *pace* Allick & Tuulmets 1991; Durgin 2008) or sortals more broadly (Clarke & Beck 2021).

Despite its theoretical significance, little is known about the developmental origins of the connectedness illusion. Previous work has yielded mixed results about the influence of development on the strength of other visual illusions. For instance, Hadad (2018) found that while four-year-old children were susceptible to the Ponzo and Ebbinghaus illusions, they were fully immune to illusions that required greater levels of visual integration, such as the rectangle and 3D-cube illusions. Consistent with these observations, the strength of other visual illusions increases with age, and does not reach adult levels until somewhere between 6 and 15 years of age (Weintraub, 1979; Zannuttini, 1996; Bondarko & Seminov, 2004; Brosvic et al., 2002), perhaps because these effects result from the visual system's optimization to statistical regularities in the environment (Weiss et al., 2002). The Ebbinghaus illusion has been reported to be virtually identical in magnitude in children and adults (Duemmler et al., 2008; Hanisch et al., 2001; but see Doherty, Campbell, Tsuji, & Phillips, 2010), although this finding is complicated by the suggestion that the illusion results from multiple mechanisms, developing along distinct trajectories (Porac & Coren, 1981; Doherty et al., 2010). Meanwhile, other visual illusions, such as the Muller-Lyer illusion, have been reported to be stronger in children, and to decrease in strength throughout development (e.g., Binet, 1895; Pintner & Anderson, 1916; Werner, 1957; Sun, 1964).

Since the connectedness illusion remains unexplored in children, all of these developmental trajectories are hypothetically viable. If the connectedness illusion increases in strength over development, this would parallel other numerical illusions such as the regular-random illusion (Ginsburg & Deluco, 1979) and the coherence illusion (DeWind et al., 2020; Qu et al., 2022). For instance, Qu et al. (2022) found that the coherence illusion was present in children as young as five but increased in strength into adulthood. If instead the magnitude of the connectedness illusion decreases over development this might reflect maturation in executive functioning skills which allows inhibition of the irrelevant lines (e.g., Gilmore et al., 2013). Finally, the strength of the connectedness illusion could prove stable over development. To adjudicate these possibilities, we quantified the strength of the connectedness illusion in children, aged 5- 12 years, and in adults.

2. Methods

Our hypotheses, procedures, and main analyses for this experiment were pre-registered at https://osf.io/d7k6e (for both children and adults). All data and materials can be located at https://osf.io/rydvx/?view_only=e236b6aea32b412dab25d82a5e2fb45f. (Note that this is an anonymous link for peer review.)

2.1 Participants

We preregistered a targeted sample size of 30 children and 30 adults. But, because our recruitment system was asynchronous, we accrued more participants in both samples than expected. In addition, although we preregistered that we were targeting children aged 6-9 siblings outside the age range sometimes completed the study. Likewise, we include all participants who met our preregistered exclusion criterion, but we also analyzed the first 30 participants in each sample to ensure that there were no differences in the results. The final samples included 43 children aged from 5.1 to 12.0 years old (M_{age} = 7.9 years; 25 female participants, 18 male participants, 0 non-binary participants) and fifty-seven adults (M_{age} = 20.17 years; 33 female participants, 24 male participants, 0 non-binary participants). One child and six adults were excluded due to mean accuracy falling more than 1.5 interquartile ranges below the first quartile (Q1) of the overall sample distribution. All participants reported normal or corrected-to-normal visual acuity and normal color vision. Children were recruited via the XXXXXXXX developmental database and compensated with Amazon gift cards. Adults were recruited from a large university community in exchange for course credit. Adult participants and parents of children participants provided informed consent to a protocol approved by the XXXXXXX Institutional Review Board before starting the experiment.

2.2 Stimuli and Design

This experiment utilized an online PsychoPy routine (Peirce, 2007). Due to the online nature of the study, we were unable to control monitor sizes and screen resolutions. PsychoPy software packages offer diverse device-independent units for stimuli description (Peirce, 2007). We utilized the 'height' units in PsychoPy builder to specify stimulus units relative to the window's height. This ensured that stimuli scaled naturally with the window size, allowing participants using different browsers to view the full-screen window with an appropriate pixel size for the display.

Each stimulus array was composed of 8-32 blue [RGB: 66, 133, 244] dots on a white background with slender lines that either connected pairs of dots or did not. Lines were generated with random orientations and spatial positions and matched the colors of the dots. Figure 1 displays the two types of stimuli. In *unconnected* stimuli, the lines of adjacent dots did not converge and were oriented in distinct directions, forming isolated lollipop-shaped configurations. In *connected* stimuli, pairwise dots were connected, merging the lines of adjacent dots into unified, continuous, and straight lines, which thereby resulted in dumbbell-shaped configurations. If a stimulus had an odd number of dots to be connected, one lollipop-shaped object remained unconnected. For instance, a connected stimulus with 11 blue dots would result in five dumbbell-shaped objects and one lollipop-shaped object. We ensured that lines, whether connected or unconnected, did not cross each other or extend beyond the boundaries of the array.

There were three different trial types presented with equal frequency: (1) two unconnected arrays, (2) a connected array on the left side and an unconnected array on the right side, (3) an unconnected array on the left side and a connected array on the right side.

Following previous research using numerical discrimination tasks, the numerical values ranged from 8-32 and were chosen to approximate equal spacing on a logarithmic scale while rounding to whole numbers (8, 10, 11, 13, 16, 19, 23, 27, 32) (DeWind et al., 2020; Qu et al., 2022). The log scale ensured a uniform ratio between any two adjacent numerical values. To illustrate, the ratio between arrays of 10 and 13 dots mirrored that between arrays with 16 and 23 dots, both pairs being two units apart on the log scale. While rounding introduces minor discrepancies, these ratios maintain the intended logarithmic spacing. The numerical ratios between the two arrays were evenly distributed in a base-2 log space, spanning from 0 to 1 and produced 5 distinct ratio values: 1, 1.19, 1.41, 1.68, and 2. Each participant was tested with 450 trials divided into three 150-trial blocks. There were approximately 150 trials for each of the three trial types, and there were approximately 90 trials for each of the five ratios. Note that these numbers are approximate because both numerical ratio levels and trial types were pseudorandomly generated.

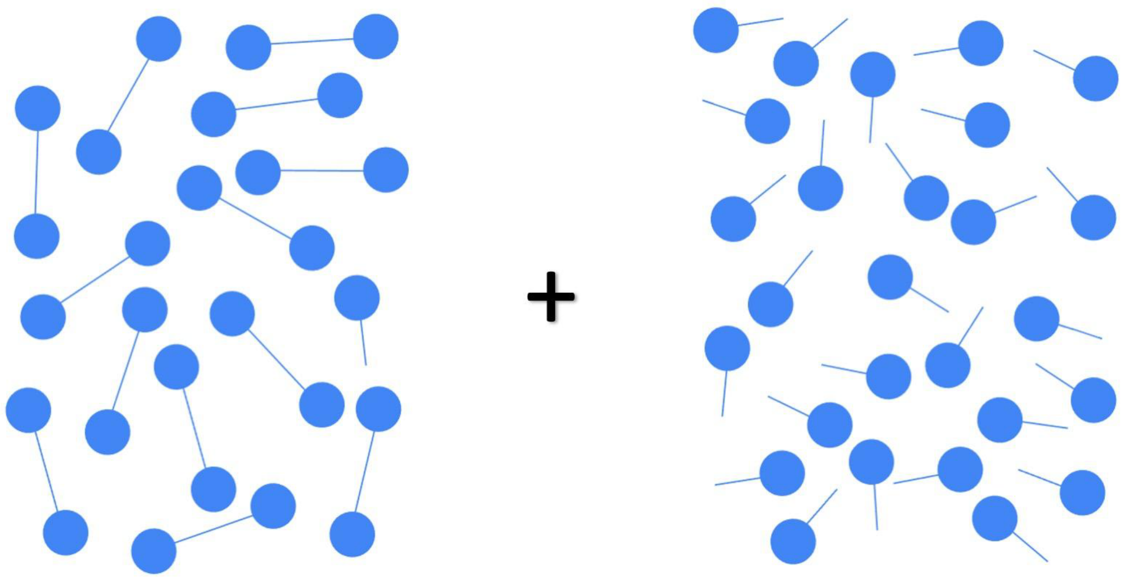

Figure 1. Examples of stimulus arrays. The connected array (left side) with 27 dots has 13 dumbbell-shaped connected pairs and one lollipop-shaped object. The unconnected array (right side) with 27 dots has 27 lollipop-shaped objects.

2.3 Procedure

At the beginning of the experiment, all participants were given the following child-friendly instructions:

> *Help Whiskers the cat collect as many blueberries as possible! In each trial, Whiskers has two piles of blueberries to pick from. We need to decide which pile has more!*
>
> *Sometimes, the stems of two blueberries will be touching. Ignore the stems! Even when the stems (lines) are touching each other, the amount of blueberries (blue circles) remains the same! All we care about is* ***which pile has more blueberries, regardless of what the stems do.***

After the instructions, participants were given three practice trials with feedback to ensure that they understood that their task was to decide whether the left or right pile had more blueberries (dots) while ignoring the stems (lines). Participants were then tested with three 150-trial blocks which took approximately 15-30 minutes to complete. Participants were reminded of the instructions before the start of the second and third blocks.

Each experimental trial began with a 500ms central fixation cross followed by two arrays presented simultaneously for 750ms. Participants were then presented with a response cue that instructed them to press “f” if the left array had more dots and “j” if the right array had more dots. Participants were given unlimited time to respond but were instructed to respond as quickly and accurately as possible. Responses during the presentation of the stimulus arrays were permitted and aborted the stimulus presentations.

2.4 Data analyses – generalized linear model (GLM)

To quantify the effect of connectedness on participants' number judgments, we employed a Generalized Linear Model (GLM) and modeled each participant's response data separately. This model was fitted to each participant's binary response data using a probit link function and a binomial error distribution. The GLM incorporated a constant term and regressors for the logarithm of the numerical ratio and the difference in connectedness between the two arrays. Derived from prior research on numerical illusions, this GLM enables the dissection of trial-level choice data into acuity and bias (DeWind et al., 2020; Qu, Bonner, DeWind, & Brannon, 2023; Qu et al., 2022). This modeling approach provides a numerical estimate of the degree to which connectedness influenced numerical judgments for each individual that is independent from all other factors influencing accuracy.

$$p(choose\ right) = \Phi(\beta_{side} + \beta_{num} log_2(r_{num}) + \beta_{connect} DiffConnect) \quad (1)$$

In the given formula, $p(choose\ right)$ represents the proportion of trials where participants selected the right side. Φ denotes the cumulative normal distribution, and β_{side} is a constant indicating the side bias of participants toward choosing one side over the other. The variable r_{num} is the ratio of the number of elements on the right array to the number on the left array. The $DiffConnect$ regressor signifies the difference in connectedness between the two arrays, with three possible values (-1, 0, and 1). These values are coded as follows: 1 if the right side is connected and the left side is unconnected, -1 if the left side is connected while the right side is unconnected, or 0 if both sides are unconnected.

The coefficients β_{num} and $\beta_{connect}$, fitted to the numerical ratio and the difference in connectedness regressors for each participant, serve as indicators of participants' numerical discrimination acuity and the effect of connectedness on their number judgments. The magnitude of β_{num} quantifies numerical discrimination acuity. If participants were completely insensitive to connectedness, the $\beta_{connect}$ fitted to the $DiffConnect$ regressor would be zero. Conversely, any consistent effect of connectedness on number perception would result in a non-zero $\beta_{connect}$. A negative $\beta_{connect}$ would indicate that the side with connected elements is perceived as less numerous than the side with no connections whereas a positive value would indicate the reverse.

To better illustrate the magnitude of the connectedness effect, we calculated the numerical ratio required to offset the difference in perceived number between connected and unconnected arrays. Specifically, we solved Equation 2 to determine r_{num} using the average coefficient estimates (β_{num} and $\beta_{connect}$) from the regression model (Equation 1) across all participants, for children and adults respectively. The ratio r_{num} was converted to a percentage for easier interpretation.

$$\beta_{num} log_2(r_{num}) = \beta_{connect} DiffConnect \quad (2)$$

3. Results

3.1 Choice data

In the final samples, 42 children (Accuracy M = 83.6%, SD = 0.10, 95% CI [0.80, 0.87], t(41) = 21.48, p < 0.001; One sample t-test) and 51 adults (Accuracy M = 93.3%, SD = 0.05, 95% CI [0.93, 0.94], t(50) = 112.44, p < 0.001; One-sample t-test) completed the tasks with above-chance accuracy. The binary response data of each participant was fit to a generalized linear model (Equation 1). The psychometric curves in Figure 2 represent the smooth fit of the regression model to the pooled data, which depict the proportion of trials that participants chose the right side as a function of the logarithm of the numerical ratio at different connectedness conditions for both children and adults. We then ran a one-sample t-test for each of the three regression coefficients (β_{side}, β_{num}, $\beta_{connect}$) to determine if the regression coefficients were significantly different from zero.

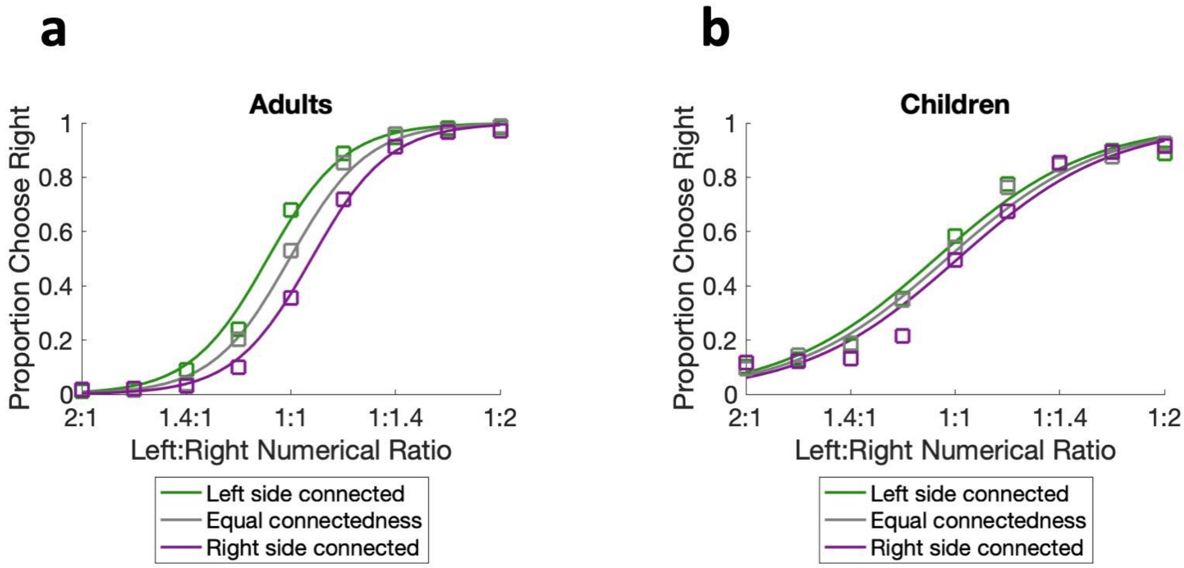

Figure 2. Connectedness reduces perceived number for both adults and children. The psychometric curves represent the smooth fit of the regression model (Equation 1) to the pooled data. The squares represent the mean proportion of choosing the right at each combination of left-to-right numerical ratios and connectedness conditions across participants. The clear distinction between the curves under different connectedness conditions indicates that participants more often chose the right side when the left side was connected and the right side was unconnected.

β_{side} was significantly greater than zero for children (β_{side} M = 0.084, SEM = 0.02, 95% CI [0.04, 0.13], t(41) = 3.82, p < 0.001) but not for adults (β_{side}M = 0.041, SEM = 0.03, 95% CI [-0.02, 0.11], t(50) = 1.28, p = 0.21), indicating that children showed a very slight preference for the right side (the mean proportion of choosing right is 51.3%), whereas adults did not show a bias for selecting one side over the other. Note that no participants met the pre-registered exclusion criterion for an extreme side bias (greater than 65% of responses to one side).

$\beta_{num}$was significantly greater than zero for both children (β_{num} M = 1.998, SD = 0.16, 95% CI [1.68, 2.32], t(41) = 12.67, p < 0.001) and adults (β_{num} M = 3.44, SD = 0.15, 95% CI [3.13, 3.75], t(50) = 22.58, p < 0.001) indicating that both age groups reliably chose the larger numerical value. β_{num} was significantly greater in adults compared to children (t(91) = -6.53, 95% CI [-1.88, -1.00], p < 0.001). As depicted in Figure 3a, further analysis collapsing the two age groups revealed a positive correlation between age and β_{num} (r = 0.59, t(91) = 6.90, 95% CI [0.43, 0.71], p < .001) consistent with prior literature demonstrating improvements in numerical acuity with age (Halberda & Feigenson, 2008; Halberda, Ly, Wilmer, Naiman, & Germine, 2012).

$\beta_{connect}$ was significantly less than zero for both children ($\beta_{connect}$ M = -0.099, SD = 0.04, 95% CI [-0.18, -0.02], t(41) = -2.52, p = 0.016) and adults ($\beta_{connect}$ M = -0.36, SD = 0.04, 95% CI [-0.44, -0.28], t(50) = -8.98, p < 0.001), indicating that for both age groups connectedness yielded a reduction in perceived number. According to the GLM (Equation 1), a negative value of $\beta_{connect}$ indicates that the side with connected objects is perceived as less numerous than the side with unconnected objects. As shown in Figure 2, there was distinct separation of the psychometric curves based on connectedness conditions whereby participants tended to choose the right side more frequently when the left side was connected, and the right side was unconnected, and this separation was more pronounced for adults (Figure 2a) than children (Figure 2b). According to Equation 2, on average children perceived unconnected arrays as 3.36% more numerous than connected arrays, while for adults unconnected arrays were perceived as 7.01% more numerous than connected arrays.

In order to more intuitively gauge the magnitude of the connectedness illusion, we inverted the value of $\beta_{connect}$, so that larger values ($-\beta_{connect}$) reflected a greater magnitude of the connectedness effect. The effect of connectedness was statistically larger in adults compared to children (U = 460, p < 0.001, Mann-Whitney U test), demonstrating that adults were more strongly influenced by the presence of connections between dots compared to children. Further analysis combining both age groups revealed a positive correlation between age and connectedness (r = 0.44, $t(91) = 4.61$, 95% CI [0.25, 0.59], $p < .001$), indicating that the strength of the connectedness illusion increases into adulthood[1] (see Figure 3b).

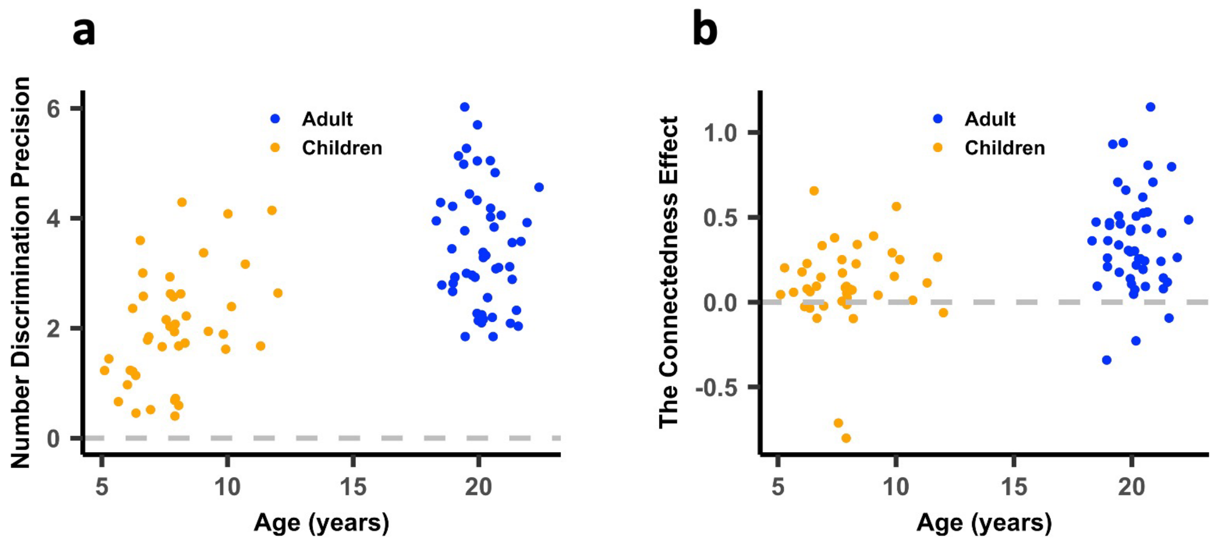

Figure 3. Scatter plots of age effect. (a) The precision of numerical discrimination, quantified as β_{num}, improved with age. **(b)** The strength of the connectedness effect (quantified as $-\beta_{connect}$) was higher in adults compared to children, indicating that the magnitude of the connectedness illusion increased over development. The grey dashed line denotes the baseline where there was no effect of connectedness.

We next examined the relationship between numerical discrimination acuity and the magnitude of the connectedness illusion controlling for age. As depicted in Figure 4, after partialling out the variance related to age, the partial correlation between β_{num} and the strength of the connectedness effect remained significantly positive (r = 0.34, t(91) = 3.47, 95% CI [0.15, 0.51], p < .001). This indicates that individuals with sharper ANS acuity exhibited a greater susceptibility to the connectedness illusion independent of age.

[1] When we removed the two child data points with very low Beta connectedness values from analysis, a significant positive correlation between age and connectedness remained (r = 0.40, t(95) = 4.28, CI [0.25, 1.0], p < 0.000).

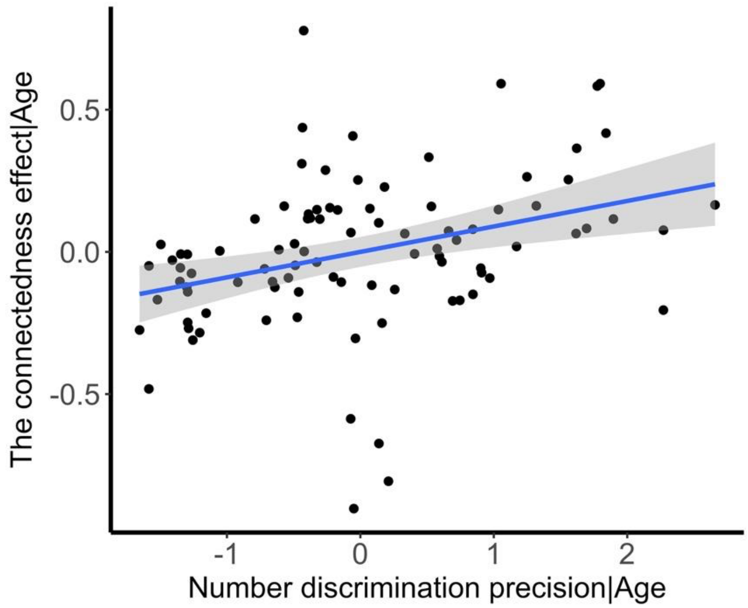

Figure 4. Partial correlation between numerical discrimination precision and the strength of the connectedness illusion when controlling for age. The grey band region represents 95% confidence intervals.

3.2 Exploratory analyses on inspection time and reaction time

Although we did not pre-register analyses on reaction-time, we conducted exploratory analyses to assess whether differences in inspection time might account for developmental differences in the magnitude of the connectedness illusion. We first eliminated outliers by excluding reaction times that exceeded or fell below three standard deviations from each participant's median reaction time (Leys, Ley, Klein, Bernard, & Licata, 2013). Following this procedure, 1,455 out of the total 22,950 trials (6.34%) were excluded in adults, and 1,348 out of the total 16,200 trials (8.32%) were excluded in children. Figure 5a displays a frequency distribution of reaction time for children and adults. The median reaction time for children was significantly longer than that of adults ($Mdn_{children}$ = 0.835s, Mdn_{adults} = 0.535s, U = 1493, $p < .0001$, Mann-Whitney U test).

To examine whether reaction-time was ratio dependent, we first calculated the median reaction time at each numerical ratio for each individual. We then fit the median reaction times by a linear mixed model with numerical ratio entered as a fixed effect and participants treated as a random effect. As shown in Figure 5b, the fixed effect of the numerical ratio on median reaction times was significant for both children (β = -0.195, SE = 0.02, $t(143)$ = -8.23, $p < .0001$) and adults (β = -0.13, SE = 0.01, $t(203)$ = -18.08, $p < .0001$).

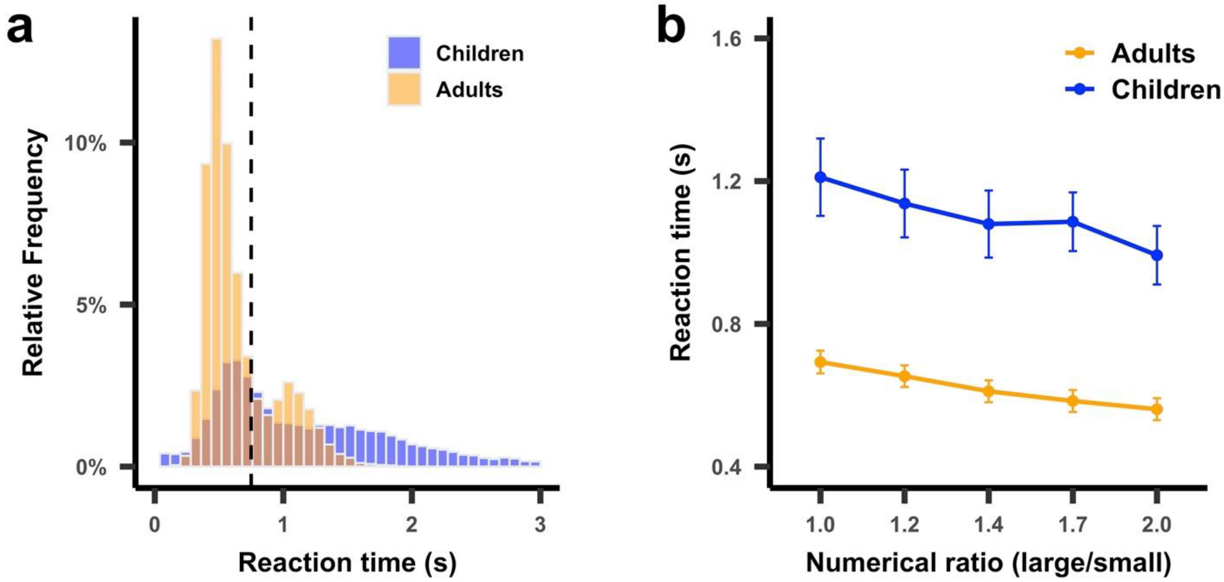

Figure 5. Reaction time analyses. (a) Frequency distribution of reaction time for children and adults. The dashed line represents the presentation time of the stimuli (750ms). **(b)** The ratio effect on reaction time for both children and adults. The dots represent the mean reaction time averaging across participants' median reaction time at each numerical ratio. All error bars are SEM.

Given that the stimuli were presented for a maximum of 750ms, but early responses aborted the stimulus presentation, we conducted a few additional analyses to compare inspection time for children and adults. Overall, 14 out of 36 children (38.9%) and 42 out of 51 adults (82.4%) exhibited a median reaction time less than the presentation time of the stimuli (750-ms), which indicates that adults and to a lesser extent children often responded before the offset of the stimuli presentation. To compare the time children and adults spent inspecting the stimuli, we set inspection time to 750ms for any trial for which responses occurred after 750ms and set inspection time equal to reaction-time for any trial that ended with a response before the maximum 750ms. stimulus presentation time. The mean inspection time for children was significantly longer than that of adults ($M_{children}$ = 0.663s, M_{adults} = 0.572s, U = 1372, $p < .0001$, Mann-Whitney U test). Furthermore, there was no correlation between the magnitude of the connectedness illusion and the average inspection time among participants (Pearson test: $r = -0.07$, $t(85) = -0.62$, $p = 0.54$; Spearman rank correlation test: rho = -0.04, $p = 0.72$). This indicates that variations in inspection time do not explain the individual differences observed in the connectedness illusion.

4. Discussion

We found that children as young as 5 years of age are susceptible to the connectedness illusion and that the illusion increases in strength during development. This developmental trend parallels prior work on other numerical illusions (Ginsburg & Deluco, 1979; Qu et al., 2022) and contrasts with the influence of continuous variables, such as surface area and perimeter on numerical judgments which declines from 4-6 years of age and remains stable from 6 years into adulthood (Starr et al., 2017).

The connectedness illusion sheds light on the controversy over whether nonverbal number representations are indirectly or directly constructed. Direct models of number extraction posit that the visual system performs an initial stage of 'normalization' wherein bounded objects are individuated independently of confounding variables (Franconerri et al. 2009) or discerned via basic principles of perceptual organization (He et al. 2009; see also Green 2020). This enables them to then be tallied up by a secondary 'accumulator' stage of processing (Dehaene & Changeux, 1993). By contrast, indirect models posit that number is secondarily calculated from representations of continuous variables, like area or density (e.g., Leibovich et al., 2017).

The connectedness illusion bears on our evaluation of these models, for despite having an identical surface area or pixel number, arrays with connected pairs are judged as less numerous than arrays with free floating lines. This presents a problem for indirect models of numerical perception which posit that number is computed from continuous variables like area (Allick & Tuulmets 1991). Indirect models which place emphasis on the density of the displays (Durgin 2008) might fair better in this regard, at least insofar as density is construed in terms of the *frequency of bounded items per spatial unit.* This is because connectedness reduces the density of the displays, thus construed. However, this adds complexity to an account of visual number discrimination since the extraction of density now depends upon the prior parsing of visual items into bounded wholes. In other words, number discrimination is now construed as three stage process (bounded wholes -> density -> number), where direct models posit just two (bounded wholes -> number). In any case, it is worth noting that recent studies conducted on rhesus monkeys have found a "reverse" connectedness effect wherein monkeys overestimate arrays with connected pairs (Beran et al. 2024). This is precisely what we would expect if these animals were using non-numerical properties, like surface area, as a crutch for number, just as indirect models would predict. Further work is necessary to explore this difference in that it could indicate that fundamentally distinct algorithms underwrite numerical discriminations in different species.

What remains puzzling in the human case, is that direct models of visual numerical estimation seemingly predict that connections among dots would approximately *half* perceived number, yet the effects of the connectedness illusion are far weaker than this. For instance, in our study we observed a mean 7.01% reduction in perceived number in adults as a function of connecting all the dots in an array into pairs. Future work should seek an explanation for the relatively weak effect of connectedness. One possibility would be to appeal to some capacity-limited role for attention in the integration of features into dumbbell-shaped wholes (see Pomé et al. 2021). Alternatively, an impure model of visual number estimation might rely on both direct and indirect mechanisms of enumeration or posit distinct direct mechanisms which individuate items according to principles of their own. This would liken number perception to the perception of other magnitudes, like distance, which involve myriad mechanisms, operating according to idiosyncratic principles of their own, before having their conclusions integrated and/or weighted against one another, perhaps in a Bayes optimal manner (Ernst 2006).

Previous work has found that other visual illusions result from myriad mechanisms, working in tandem, each of which develop along their own independent trajectories (Porac & Coren 1981). This raises the possibility that the strength of a visual illusion at a given stage of development will vary depending on the context under which it is investigated and depending on subtle details of the stimuli used. Consistent with this possibility, Bressan and Kramer (2013, 2021) demonstrated that group differences in the strength of visual illusions can often be attributed to differences in stimulus inspection time (Bressan & Kramer 2021). However, our post-hoc analyses indicate that inspection time cannot account for the increase in the strength of the illusion over development. Nevertheless, this does not preclude the possibility that future work will uncover other important variables which affect the strength of the illusion in a given population.

Developmental changes in the connectedness illusion may reflect maturation of the visual system, maturation of the ANS, or an interaction between the two.

Like many classic visual illusions, there is considerable inter-individual variation in the magnitude of the connectedness illusion at all ages. What predicts and explains this variability? Individual differences in classic visual illusions are generally not predicted by differences in visual acuity (Cretenoud et al., 2021) but are sometimes predicted by other cognitive variables (e.g., Binet, 1985). For instance, Coren and Porac (1987) found that illusions of linear extent (e.g., the Muller-Lyer illusion) and illusions of direction and area (e.g., the Delboeuf illusion) showed distinct relationships with spatial ability. Higher levels of spatial ability predicted lower strengths of visual illusions of direction and area, but the reverse relationship held for illusions of linear extent. That individuals with better ANS acuity exhibit stronger connectedness illusions might suggest that bounded wholes play an important functional role in number perception. The reduction in perceived number as a function of connectedness may best be thought of as a feature rather than a glitch of the ANS.

5. Conclusion

Helmholtz (1896) argued that "It is especially those cases in which our impressions evoke in us representations which do not correspond to reality that are particularly informative for finding the laws of processes and ways through which normal percepts are established" (p. 96; quoted in Todorovic 2020, p.1192). Our findings show that children as young as 5 years of age exhibit the connectedness effect, that the effect increases in magnitude with age, and that it is positively correlated with the precision with which participants make numerical discriminations. Collectively, these results suggest that the visual system expects to enumerate over bounded wholes and doing so is a signature of its optimal functioning.

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