A model of the ontology of time

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A. Introduction

I this paper I give minimal axioms for the ontology of time, especially A-theories and B-theories and I derive philosophically interesting lemmas. The exercise is set-theoretical, defining all notions and indicating assumptions and philosophical points of disagreement, while being easy to translate to other formal expressions[1]. The issue of a logic for A-theories of time is treated towards the end, where I sketch 'copresent' operators for capturing the idea of temporal passage. The main conclusion will be that, while circularity threatens both families of theories, the growing block theory (belonging to the A-theory) is the least objectionable philosophically.

But first, some early decisions:

a) Time may be made up of events, moments, states and so on, per different philosophers. Moments[2] (in a dense order) are needed to be able to count time as we usually do, by fractions as well as integers. Events[3] are also very natural to accept, since moments are instantaneous and there is a very good sense in which some entities 'take time', have duration. Therefore, I opted to have events, which include both instantaneous moments, entities such as myself enduring and maybe propositions. The only constraint is to have a start and an end.

b) Against philosophers[4] who hold that descriptions such as 'one year ago' and 'yesterday' belong to the A-series, I can see no substantial difference between those expressions and 'one year before now' and 'one day before now' who rightly belong to the B-series. Then, the counting of time is not differentiated between the modeled ontological theories.

c) Since the topology and epistemology of time are outside the scope of this paper, I opted to have linearity of moments for simplicity. But one can change that if one wills.

d) Besides counting times, we define instantaneity, persistence, and simultaneity[5] both between moments and events, and an intuitive sense of past, present, and future. That's why we start with a common construction which is neutral between ontological theories of time. But there is no

[1] Such as programming code.
[2] Also called *instants* sometimes.
[3] Also called *intervals* sometimes.
[4] Power, *Philosophy of Time*, 32.
[5] Simultaneity between events which can have different duration is an interesting matter which is clarified here by the introduction of starts and ends.

good way to model the difference between tensed and tenseless theories without referring to tenses, namely past, present, or future, so I introduce them as objects, leaving to theories to specify how events connect to them.

e) By the same reasoning, I admit the real and the unreal as objects.

Then, the main difference between tensed and tenseless theories will be that in the former some event both *is* of a tense (e.g., present) and *was* of a tense (e.g., past), while in the latter it can be said only that it *is* of a tense, the relation of being of a tense being reduced to a comparison on the linear order. Change of an event for tense theory will be defined similarly as both *being* a tense and *having been* a tense, while change for tenseless theory will reside in the fact that persistent events simultaneous with the indexical present have distinct moments. The difference between eternalism and presentism will be that all events *are* real for the former, while present events *are* real but *were* unreal for the latter. The growing block is presentism with the real extended over the past.

There are four main sections: B. Common framework, C. A-theory, D. B-theory, E. Temporal logic and temporal passage. Discussions are in each section.

B. Common framework

B.1. Metaphysics

(1.1) **Entities**
E - the events, a set
T - the tenses, a 3-tuple $<\pi, \rho, \varphi>$, π - $v_1(T)$ is the past, ρ - $v_2(T)$ is the present, φ - $v_3(T)$ is the past[6].
F = the fundaments: a pair $<\xi, \omega>$, ξ - $v_1(T)$ is the real, ω - $v_2(T)$ is the unreal.
E, T and F are disjoint.

(1.2) ***τ* is the *tense function*** from events to specified tuples of tenses: either a singleton, a pair (only $<\rho, \varphi>$ allowed) or a triple (only $<\pi, \rho, \varphi>$ allowed).
τ: E $\rightarrow$ $\{\{x\} \mid x \in T\} \cup \{<\rho, \varphi>\} \cup \{<\pi, \rho, \varphi>\}$.

Remark that any event will map singly to past, present, or future, or jointly to the past and present, or jointly to the past, present and future. We call $v_1(\tau(a))$ the *most recent tense* of a, e.g. if $\tau(a)$ is $<\pi, \rho, \varphi>$, then it will give π the past. We'll write $\tau_n(a)$ instead of $v_n(\tau(a))$.

[6] v_n is the function that returns the n-th element of a tuple, with 1 the first.

(1.3) ***ψ* is the *fundament function*** from events to specified tuples of fundaments: either singleton or pair (only <ξ, ω> allowed) or triple (only <ω, ξ, ω> allowed).
$\psi: E \rightarrow \{\{x\} \mid x \in F\} \cup \{<\xi, \omega>\} \cup \{<\omega, \xi, \omega>\}$

Remark that any event will map singly to the real or unreal, or jointly to the real and unreal, or jointly to the unreal, real, and unreal. We call $v_1(\psi(a))$ the *most recent fundament* of a, e.g., if $\psi(a)$ is <ω, ξ, ω>, then it will give ω the unreal. We'll write $\psi_n(a)$ instead of $v_n(\psi(a))$.

(1.4) ***γ* is the *change function*** from events to truth values.
$\gamma: E \rightarrow \{0,1\}$

(1.5) **We say:**
a) An event a *is past/present/future* if $\tau_1(a) = \pi / \rho / \varphi$.
b) An event a *was past/present/future* if $\tau_2(a) = \pi / \rho / \varphi$ or $\tau_3(a) = \pi / \rho / \varphi$
c) An event a *is copresent* with event b if $\tau_1(a) = \pi$ and $\tau_1(b) = \pi$.
d) An event a *is real/unreal* if $\psi_1(a) = \xi / \omega$.
e) An event a *was real/unreal* if $\psi_2(a) = \xi / \omega$ or $\psi_3(a) = \xi / \omega$.
f) An event *has changed* if $\gamma(a) = 1$.

B.2. Moments, events, simultaneity

(2.1) M - the **set of moments** of at least two moments, included in events[7],
There is $x, y \in M$, $x \neq y$, $M \subseteq E$.

(2.2) $<_M$ - **Linear ordering** of moments
If $x, y \in M$ and $x \neq y$, then $x <_M y$ or $y <_M x$. $<_M$ is also transitive and asymmetric.

When either $x = y$ or $x <_M y$, we can write $x \leq_M y$. Remark that $\leq_M$ is a total preorder.

(2.3) **Dense ordering** of moments, countability
For $x, y \in M$ and $x \neq y$, if $x <_M y$, then $\exists z \in M\ \ x <_M z <_M y$.

That is, M and E are infinite, without deciding whether they have an end, and whether they have a start. In what follows I also assume that M is countable, to introduce later a bijection to $\mathbb{Q}$, but one could switch to uncountable M mutatis mutandis.

(2.4) **Events start and end at moments.**

[7] Moments are events, but events are not moments, since events may extend along more than one moment.

Define function $\Subset$, read "*to start at*", written $\Subset a$, $\Subset: E \rightarrow M$.
Define function $\Supset$, read "*to end at*", written $\Supset a$, $\Supset: E \rightarrow M$.

For any $x, y \in M$ and $z \in E$, if $x = \Subset z$ and $y = \Supset z$, then $x \leq_M y$.

That is, any event has a single moment as a start, a single moment as an end and the start of an event is either the end or before the end of the event[8].

(2.5) **Moments have themselves as start and end**.
For any $x \in M$, $\Subset x = x$ and $\Supset x = x$.

(2.6) **Events are simultaneous**, written $\approx$ iff there is a moment between their starts and ends.
$\approx \overset{def}{=} \{<x, y> \mid x, y \in E \text{ and there is a } z \in M \text{ so that } \Subset x \leq_M z \leq_M \Supset x, \Subset y \leq_M z \leq_M \Supset y\}$

That is, simultaneity is "sharing a moment" for events and identity for moments.

(2.7) **We say:**
a) An event *a is before* event *b* if $\Supset a <_M \Subset b$.
It follows that a moment *a* is before moment *b* if $a <_M b$.

b) An event *a is after* event *b* if $\Supset b <_M \Subset a$.
It follows that a moment *a* is after moment *b* if $b <_M a$.

c) An event *a* is *at the same time* with event *b* if they are simultaneous $a \approx b$.
It follows that a moment *a* is at the same time with moment *b* if $a = b$.

d) An event *a is instantaneous* if $\Subset a = \Supset a$.
It follows that any moment is instantaneous.

e) An event *a is persistent* iff not instantaneous.
It follows that any moment is not persistent.

(2.8) **Lemmas for events**[9]**:**
(L1) $\Subset$ and $\Supset$ are identical with $<_M$ for moments.
(L2) The starts of all events form a total preorder $\preccurlyeq_s$.
$\preccurlyeq_s \overset{def}{=} \{<x, y> \mid x = \Subset z, z \in E \text{ and } y = \Subset w, w \in E \text{ and } x \leq_M y\}$
(L3) The ends of any events form a total preorder $\preccurlyeq_e$.

[8] We allow events to be instantaneous (start and end with one moment) without being a moment.
[9] Proofs are in the Appendix.

$\preccurlyeq_e \overset{def}{=} \{<x, y> \mid x = \ni z, z \in E \text{ and } y = \ni w, w \in E \text{ and } x \leq_M y\}$
(L4) Moments are simultaneous just if identical.
(L5) Simultaneity is not transitive, because of persistent events. Then, the relation *to be before or at the same time* is an interval order.

B.3. Counting time

(3.1) Define **a *time slice*** as any subset S of at least two elements of M which is continuous under $<_M$.
$S \subseteq M$. There is $x, y \in S$, $x \neq y$.
For any $x, y, z \in M$ so that $x <_M z <_M y$, if $x, y \in S$, then $z \in S$.

Note that M is itself a time slice, as well as any interval on M continuous under $<_M$ (i.e., a period of time), or the set started at any point and continuing under $<_M$ (e.g. future moments)

(3.2) There is **a *counting function***, from any time slice S to rationals which is bijective, monotonous with $<_M$ and whose range contains $\mathbb{Z}$. Note the function for M with ***θ.***

$\theta: S \rightarrow \mathbb{Q}$ with S a time slice of M, $\rightarrow$ bijective.
For any $x, y \in S$, $x = y$ iff $\theta(x) = \theta(y)$ and $x <_M y$ iff $\theta(x) < \theta(y)$.
For any $x \in \mathbb{Z}$, there is a $y \in M$ so that $\theta(y) = x$.

Existence is immediate from the countability, linearity and density of $<_M$. Remark that now we can assign unique intuitive numbers to moments, such as -1,1, 2, 3.5 etc.

(3.3) Define ***an interval* of M** as any time slice with a minimum and maximum element.
S is an interval iff S a time slice of M and there is $x \in S$ such that for any $y \in S$, $x \leq_M y$ and there is $z \in S$ such that for any $w \in S$, $w \leq_M z$.
Note the minimum with *inf* and the maximum with *sup.*

(3.4) Define ***the duration of an interval* S,** written **δ:**
For S an interval of M, $\delta(S) = \theta(\sup(S)) - \theta(\inf(S))$
Existence follows from the existence of sup and inf and that of θ.

(3.5) **Lemmas for counting:**
(L6) The duration of any interval S is positive: $\delta(S) > 0$.
(L6.1) The start and end of any persistent event define an interval of M, which has a duration.

(3.6) **We say:**

a) An event a *has duration* n if n = $\delta(S)$ where S is the interval of M defined by the start and end of the event.
It follows from (L6) and (L6.1) that *all persistent events have positive duration*.

b) An event *has no duration* if it is instantaneous (non-persistent)
This follows as a lemma from (L6.1) and a). It follows that *moments have no duration.*

B.4. Further constraints and time models

(4.1) **Constraints on the tense function τ**
a) If a moment is before another which is past, it is past
For any x, y $\in$ M, if $x <_M y$ and $\tau_1(y) = \pi$, then $\tau_1(x) = \pi$

b) If a moment is after another which is future, it is future
For any x, y $\in$ M, if $x <_M y$ and $\tau_1(x) = \varphi$, then $\tau_1(y) = \varphi$

c) No moment before the present is future
For any x, y $\in$ M, if $x <_M y$ and $\tau_1(y) = \rho$, then not $\tau_1(x) = \varphi$

d) No moment after the present is past
For any x, y $\in$ M, if $x <_M y$ and $\tau_1(x) = \rho$, then not $\tau_1(y) = \pi$

e) If the start of an event is past and its end is not past, the event is present
For any x $\in$ E, if $\tau_1(\Subset x) = \pi$ and $\tau_1(\Supset x) \neq \pi$, then $\tau_1(x) = \rho$

f) If the start of an event is not future and its end is future, the event is present
For any x $\in$ E, if $\tau_1(\Subset x) \neq \varphi$ and $\tau_1(\Supset x) = \varphi$, then $\tau_1(x) = \rho$

g) If the start and end of an event is past/future, the event is past / future
For any x $\in$ E, if $\tau_1(\Subset x) = \pi$ and $\tau_1(\Supset x) = \pi$, then $\tau_1(x) = \pi$
For any x $\in$ E, if $\tau_1(\Subset x) = \varphi$ and $\tau_1(\Supset x) = \varphi$, then $\tau_1(x) = \varphi$

Remark that the conditions are compatible with more than one moment being in the present and with any two of the past, present and future having no events.

(4.2) **There are real events** under fundament function ψ.
There is x $\in$ E, $\psi_1(x) = \xi$

(4.3) **There are changed and non-changed events** under change function γ
There is x, y $\in$ E, $\gamma(x) = 1$ and $\gamma(x) = 0$.

As with (4.2), we expect the theories that follow to explain how this comes to be.

(4.4) Define **a *time model*** as any tuple <E, T, F, τ, ψ, γ, M, $<_M$, ⋐, ⋑, ≈ , θ, δ[10]> respecting the axioms and definitions given up to now.
We can subscript the name of the model when not clear from the context, e.g., E_Π will be the events of time model Π.

(4.5) **Lemmas for tenses:**
(L7) Any event is either past, present, or future.
(L8) If the end of an event is in the past, it is past.
(L9) If the start of an event is in the future, it is future.

B.5. Discussion

The construction up to now is minimal. Yet:
a) The admission of persistent events makes simultaneity and related relations such as *to be at the same time or before* not transitive but in fact, interval orders (L5). There may be some philosophical relevance here, namely that the ideal linearity of time is more about what can be counted, i.e., moments, not about events as they interest us (people, contracts, etc.) which have overlapping durations.

b) A good sense can be given to non-persistent events *not having* a duration (3.6), instead of having duration zero. This may correspond to philosophies inclined to the study of duration.

c) There are some relational constraints between tenses which are intuitive, but don't force us to populate the present, past and future with events (4.1), leaving that to various ontologies of time.

We turn now to such theories, in order to better define the tenses, real and unreal, and change.

[10] E - the events, T - the tenses, F - the fundaments, τ - the tense function, ψ - the fundament function, γ - the change function, M - moments, <M - linear dense ordering on moments, ⋐ - event start function, ⋑ - event end function, ≈ - simultaneity relation, θ - counting function, δ - duration function.

C. A-Theory

The common thesis of the A-theory can be called the *substantiality of tense*[11], that there is such a thing as having a tense in a way that does theoretical work. Since any model of B-theory will call some events present (indexically), there will be in any case a definable `present` predicate. But A-theory insists that we need tense to explain such metaphysical aspects as change and that tense cannot be eliminated. Our solution is then to take the substantiality of tense as the thesis that there can be combination of tenses, namely that any present event was future and that any past event was present and future, thus having a single explanation of both tense and change.

But we would like this essential thesis of the A-theory to be a lemma proven from something else. The best candidate would be a function that would describe tense change, call it *temporal passage*: it would prefix the tense of present on a future event and the tense of past on a present event. Such a function would intuitively start from the edge of the future and make it present, but under a dense ordering, there is no first future moment, just like there is no next moment from a presumably unique present moment. We seem to have an incompatibility between temporal passage and the density of time. Two options are:

a) Assume time is discrete - has the disadvantage of being quite counterintuitive.

b) Assume temporal passage is done in discrete intervals - a time slice (as defined above) of moments become present at once - it would have the advantage of allowing for some intuitions regarding the inconsistency of change under A-theory but the disadvantage of having copresent moments which are not simultaneous.

I choose the second option because it subsumes the first in a way. Then we face the difficulty of where to start temporal passage. The solution will be to assume that there is a starting state in the history of time which is consistent with the basics of A-theory: the present has been future.

C.1. Temporal passage

(1.1) ***A tensed model*** is a tuple $\langle E, T, F, \tau, \psi, \gamma, M, <_M, \Subset, \Supset, \approx, \theta, \delta, j\rangle$ where $\langle E, T, F, \tau, \psi, \gamma, M, <_M, \Subset, \Supset, \approx, \theta, \delta$[12]$\rangle$ a time model and:

a) ***j*** is a positive rational number

b) Tense function τ **assigns tense in slices** of duration j under counting function θ starting from any integer:

[11] I avoid “reality” to avoid confusion with the real and unreal.

[12] E - the events, T - the tenses, F - the fundaments, τ - the tense function, ψ - the fundament function, γ - the change function, M - moments, <M - linear dense ordering on moments, ⋐ - event start function, ⋑ - event end function, ≈ - simultaneity relation, θ - counting function, δ - duration function.

For any $z \in \mathbb{Z}$ and x, y, $w \in M$ so that $\theta(y) = \theta(x) + j$ and $\theta(x) = z * j$ and $x <_M w <_M y$, $\tau(x) = \tau(w)$.
That is, with j=0.5, all moments having θ-value in each of …, [-0.5,0), [0,0.5), [0.5, 1), … will have the same tenses.

c) **An event is changed** iff it is of a tense and was of a tense
For any $x \in M$, $\gamma(x) = 1$ iff $\tau_1(x)$ and $\tau_2(x)$ exist, 0 otherwise.

We call All_{ATM} the class of all tensed models.

(1.2) **The *temporal passage*** is a function ***p*** defined from tensed models to tensed models. *p*: $All_{ATM} \rightarrow All_{ATM}$ with the conditions:
a) All members are the same between Π and p(Π), except tense functions: $\tau_\Pi \neq \tau_{p(\Pi)}$

b) For any present moments in Π, $\tau_{p(\Pi)}$ assigns $<\pi, \rho, \varphi>$. That is, makes them past.

c) Makes the next time slice under j_Π present:
For any $x \in M_\Pi$, if y is the minimum future moment and x is future and has $\theta_\Pi(x) < \theta_\Pi(y) + j_\Pi$, $\tau_{p(\Pi)}$ assigns $<\rho, \varphi>$ to x. That is, makes them present.

(1.3) **Lemmas for any tensed model Π**:
(L10) *Non-simultaneity in the present:* If there is a present moment, there are copresent non-simultaneous moments.
(L11) *Existence of the past or present:* There is a present moment or a past moment.
(L12) *Existence of the bounds of present and future:* If there is a future moment, there is a minimum future moment. And similarly for a minimum present moment.

Now we can compose the temporal passage function since it will always output a new model.

(1.4) Define ***a history of time h*** as any natural chain of compositions of *p* (temporal passage) started from a tensed model with no past moments and a consistent present.
A natural chain of compositions of a function f started from e is the ordered set {e, f(e), f ∘ f (e) , f ∘ ... ∘ f(e), … } where ordering is given by the number of ‘f’. Remark they start from 0,1,...

h is a function from specified tensed models to {Π, *p*(Π), *p* ∘ *p*(Π) , *p* ∘ ... ∘ *p*(Π), …} where Π is the tensed model.

The specifications on Π are[13]:
a) there is no past: no $x \in E_\Pi$ so that $\tau_1(x) = \pi$.
b) the present is consistent with A-theory: for any $x \in E_\Pi$, if $\tau_1(x) = \rho$ then $\tau_2(x) = \varphi$. That is, we avoid the situation allowed by τ of having only $<\rho>$ on some events.

Call the ordering function *o*, used as *o(h(Π))* and remark that it is a bijection with ℕ.

(1.5) **Lemmas for any tensed model Π in a history of time H:**
(L13) *Substantiality of tense*: any not-future moment before a future moment was future, and any non-present moment before a present moment was present.
For any x, y ∈ M, if $x <_M y$ and $\tau_1(y) = \varphi$ and $\tau_1(x) \neq \varphi$, then $\tau_2(x) = \varphi$ or $\tau_3(x) = \varphi$
For any x, y ∈ M, if $x <_M y$ and $\tau_1(y) = \rho$ and $\tau_1(x) \neq \rho$, then $\tau_2(x) = \rho$
(L13.1) *The bounds of change*: all present and past events were changed, the future was not changed and there is a first not-changed moment.

C.2. Presentism

To the axioms in (C.1) we add two axioms.

(2.1) The present and only the present is real.
For any x ∈ E, $\psi_1(x) = \xi$ iff $\tau_1(x) = \rho$

(2.2) Any present events were unreal, and any past events were both real and unreal.
For any x ∈ E, if $\tau_1(x) = \rho$ then $\psi(x) = <\xi, \omega>$ and if $\tau_1(x) = \pi$ then $\psi(x) = <\omega, \xi, \omega>$

That is, we tie temporal passage to reality as well: when a future moment becomes present, it becomes real, but it was unreal; past events are unreal, were real and unreal.

(2.3) **Lemmas for presentism in any tensed model:**
(L14) There is a present event.
(L15) Any past or future events are unreal.
(L16) *If a persistent event is present or real, its start or end are unreal***.**
(L17) *Bound of reality*: there is a minimum real moment.

[13] These are necessary to be able to start an induction. Thua, A-theory requires either a start of time (which we avoided), or that there should have been always the case that the present events were previously future. It may seem a petitio principii, but it is subtly different than the following lemmas.

C.3. Growing block

To the axioms in (C.1) we add two axioms.

(3.1) An event is real just in case it is present or past.
For any $x \in E$, $\psi_1(x) = \xi$ iff $\tau_1(x) = \rho$ or $\tau_1(x) = \pi$

(3.2) Any present or past events were unreal.
For any $x \in E$, if $\tau_1(x) = \rho$ or π then $\psi(x) = \langle \xi, \omega \rangle$

That is, we allow for the same mechanism as for tense: when a future moment joins the growing block, it becomes real, but it was future, and it was unreal.

(3.3) **Lemmas for the growing block in any tensed model:**
(L18) Any future events are unreal.
(L19) If a persistent event ends in the future, its end is unreal.
(L20) *No bound of reality*: there is not a minimum real moment.

C.4. Eternalism

To the axioms in (C.1) we add one axiom.

(4.1) Everything is solely real.
For any $x \in E$, $\psi(x) = \{\xi\}$

(4.2) **Lemmas for eternalism in any tensed model:**
(L21) Any past or present events are real, and no event was unreal.
(L22) The future is real and has not changed.

C.5. Discussion

Some intermediary conclusions for the A-theory:
a) If time does not have a start and there should be change, there is no way of modelling the possibility that every moment is future (L11)[14].

[14] Except perhaps to introduce more tenses such as *far future*.

b) Temporal passage cannot happen by sliding from a next to a previous moment if time is dense. It happens mysteriously, perhaps as here by slices, with the consequence that events which are not precisely simultaneous still share together the present (L10).

c) However, this same characteristic ensures that there are some bounds of present, future and change (L12 and L13.1), which is something to hang our intuition of separation between past present and future on. Note that folk time has both separation of tenses and density of time.

d) The debate between eternalists and presentists may turn on persistent events: by (L16) presentism holds that persistent events (such as people) have parts which are unreal. But the lemmas of the growing block are more advantageous than those of both: fewer parts of persistent events are unreal and the real does not have a minimum element (L17, L19, L20)[15].

Philosophically, A-theory is a natural locus of contention about change and any variant will face the issue of the misfit between temporal passage and simultaneity. Note that Sextus Empiricus's classical argument against time posited that something changes from hot to cold in the present, but it's hard to accept that hotness and coldness are simultaneous, since one is quite clearly before the other. That means that maybe b) is as it should be, and we'll inquire at section E. below for the accommodation of such a construction in temporal logic.

D. B-theory

Things are simpler with B-theory. Understood negatively[16], B-theory is the claim that we don't need the A-theory. We need to eliminate what we took as the latter's essential thesis, namely the substantiality of tense. And show that change and the real can still be explained in an intuitive manner. The present is indexical to any moment and the past and the future are reduced to the ordering of moments in function of the present. Events as we defined them will follow moments, since they have a start and an end.

We create a tenseless model of time to include the indexical present, then reduce the other contentious theoretical entities to it. Most contentious is change, which for B-theorists is "any variation in anything's properties between different times"[17]. That "anything" is our persistent events, which have distinct start and end and span moments. But we could understand the quote in two ways. First, understand the variation as caused only by the move of the indexical present,

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[15] I suggest that there may be another variant of the growing block which reduces the present to a line of demarcation indexed to $\mathbb{N}$, starting from the intuition that while time is dense, any numbering of the present seems to be in natural numbers.
[16] Power, *Philosophy of Time*, 52–53.
[17] Power, 53.

in which case we would need to introduce the old indexical present into the model, in order to define change. Or second, to understand the variation as simply the fact that there are different properties between the moments making up the persistent event. For example, one such property of any moment in an event is *distance from the start of containing event*. I think the latter suffices, but we'll adopt the first solution, since it gives a richer way to model change.

D.1. Tenseless model

(1.1) ***A tenseless model*** is a tuple $\langle E, T, F, \tau, \psi, \gamma, M, <_M, \Subset, \Supset, \approx, \theta, \delta, i, j\rangle$ where $\langle E, T, F, \tau, \psi, \gamma, M, <_M, \Subset, \Supset, \approx, \theta, \delta$[18]$\rangle$ a time model and:

a) *i*, the *indexical present*, a moment, $i \in M$

b) *j*, the *old indexical present*, a moment, $j \in M$, $i \neq j$

c) Tense function τ assigns solely present to *i*, solely past to moments before and solely future to moments after
$\tau(i) = \{\rho\}$. For any $x \in M$, if $x <_M i$ then $\tau(x) = \{\pi\}$ and if $i <_M x$ then $\tau(x) = \{\varphi\}$
The constraints in (B4.1) will suitably classify all events.

d) *An event has changed* iff it is simultaneous with the indexical present, distinct from it, and the distance from its start to it is different than the distance to the old indexical present[19]
For any $x \in M$, $\gamma(x) = 1$ iff $x \approx i$, $x \neq i$ and $\theta(\Subset x) - \theta(i) \neq \theta(\Subset x) - \theta(j)$, 0 otherwise.

e) *Everything is solely real*
For any $x \in E$, $\psi(x) = \{\xi\}$

D.2. Lemmas for B-theory:

(L23) Any past or present events are real, and no event was unreal.
(L24) The future is real and has not changed.
(L25) *There is a present persistent event.*

[18] E - the events, T - the tenses, F - the fundaments, τ - the tense function, ψ - the fundament function, γ - the change function, M - moments, <M - linear dense ordering on moments, ⋐ - event start function, ⋑ - event end function, ≈ - simultaneity relation, θ - counting function, δ - duration function.
[19] The last part looks like petitio principii, because of course, i ≠ j. But that would happen to any definition of change for B-theory and the present one has the advantage of allowing for change only during the present, otherwise all persistent events (or metaphysically all non-abstract object) would count as changed.

D.3. Discussion

Some conclusions for B-theory:
a) B-theory is eternalism without tenses (L23, L24). The future is real but has not changed and no event is unreal[20].

b) As A-theory in accounting for temporal passage, this modelling of B-theory can be accused of circularity: the formal construction partially assumes what it has to show, namely the indexical present has moved, by being distinct from the old. But I think that this corresponds to how B-theorists account for change as simply the switching of properties as indexical presents switch, i.e., the difference between the latter is the root explanation.

Philosophically, B-theory either require that the present always has a persistent event, in order to have something changed at all (L25), or it will have a plenitude of change, by taking any persistent entity to be changed (at least if not an immutable entity) since it has different properties at different moments.

E. Temporal logic and temporal passage

The models sketched above should be researched further to see how they fit with various types of temporal logics. I now just give a rough sketch of adding temporal passage to a temporal logic.

We saw that, were it an operator moving along events thought of as propositions, it cannot advance moment by moment given the density of time. But we can adopt a solution of using intervals, as in a metric temporal logic. The temporal passage function *p* above creates discrete presents (each a time slice, hopefully tiny). What can be the intuitive justification for this? I think there are a few reasonable ones. First, that reflected in (L20), namely, to express the metaphysical idea that there is non-simultaneity in the present. This operator would be useful for making sure that we're outside the changeable present, where we can trust iterations of tense operators. Or similarly for making sure that we are inside the changeable present, where they are not so reliable: states differently ordered by <P> or <F> are still copresent, just as Sextus's hot and cold. Second, it can have the secondary role of allowing a switch from dense to discrete, directly in the language. And thirdly it could function like an 'epoch' operator, indicating that, say, humanity has passed through Stone Age, Bronze Age and so on, all times belonging to one epoch. But here we call it 'copresent'.

[20] A solution may be to treat reality as indexical too: the closest we get to the indexical present, the more we may be justified in calling events real.

I will write K^{t+} for Priest's $K^{t}_{\tau\eta\eta'\delta\varphi\beta}$[21], which describes a linear dense accessibility relation. We call the logic sketched below K^{C}., without any pretense of originality.

E.1. The language

Language of K^{t+}: propositional language plus operators [P], <P>, [F], <F>.

We add five operators:

>C< *copresent*,

\C/ *previous copresent*,

/C\ *next copresent*,

C> *future copresent*

<C *past copresent*[22]

The grammar of all is the same as for [P].

E.2. The semantics

We define *t* a function from rational numbers to their half-open intervals defined by integers.

For any a ∈ ℚ, t(a) = [m, m+1) where m ≤ a < m+1 and m ∈ ℤ.

The function will give the *thresholds*, i.e., on which the copresent operators will work.

An interpretation is a pair <W, v> with:

a) W a set of rational numbers, possibly empty. These are the *worlds*, but they must be given as numbers so as to assume linearity.

b) v is a function, such that for any world w ∈ W it assigns propositional parameters 0 or 1.

[21] I follow Priest, *An Introduction to Non-Classical Logic: From If to Is*.

[22] More operators could be defined through negation.

For an interpretation v assigns each formula in the language 0 or 1 following standard rules for ¬, ∧ and ∨. For modal operators:

i. Standard rules with ≤ instead of the specifiable accessibility relation:

$\nu_w(<P>A) = 1$ if, for some $w' \in W$ such that $w' \leq w$, $\nu_{w'}(A) = 1$; and 0 otherwise.

$\nu_w([P]A) = 1$ if, for all $w' \in W$ such that $w' \leq w$, $\nu_{w'}(A) = 1$; and 0 otherwise.

$\nu_w(<F>A) = 1$ if, for some $w' \in W$ such that $w \leq w'$, $\nu_{w'}(A) = 1$; and 0 otherwise.

$\nu_w([F]A) = 1$ if, for all $w' \in W$ such that $w \leq w'$, $\nu_{w'}(A) = 1$; and 0 otherwise.

ii. Rules for the new operators:

$\nu_w(>C<)$ $= 1$ if, for some $w' \in W$ such that $w' \in t(w)$, $\nu_{w'}(A) = 1$; and 0 otherwise.

A proposition will be *copresent* iff it is true in a world in the threshold interval of w.

$\nu_w(\backslash C/)$ $= 1$ if, for some $w' \in W$ such that $w' \in t(w-1)$, $\nu_{w'}(A) = 1$; and 0 otherwise.

A proposition will be in the *previous copresent* iff it is true in a world in the previous threshold interval as compared to w.

$\nu_w(/C\backslash)$ $= 1$ if, for some $w' \in W$ such that $w' \in t(w+1)$, $\nu_{w'}(A) = 1$; and 0 otherwise.

A proposition will be in the *next copresent* iff it is true in a world in the next threshold interval as compared to w.

$\nu_w(C>)$ $= 1$ if, for some $w' \in W$ and some $x \in \mathbb{Q}$, $x \geq w+1$, $w' \in t(x)$, $\nu_{w'}(A) = 1$; and 0 otherwise.

A proposition will be *future copresent* iff it is true in any copresent following the current one.

$\nu_w(<C)$ $= 1$ if, for some $w' \in W$ and some $x \in \mathbb{Q}$, $x \leq w-1$, $w' \in t(x)$, $\nu_{w'}(A) = 1$; and 0 otherwise.

A proposition will be *past copresent* iff it is true in any copresent preceding the current one.

E.3. Discussion

(3.1) Tentative remarks on the operators

a) All five operators (by their reciprocals) respect the Kripke schema.

b) C> *future copresent* and <C *past copresent* are very similar with <F> *future* and <P> *past*, since they select (mostly) any world in the future/past, with the single difference that there may be future events in the copresent (the non-simultaneous ones) which, then, are not in a future copresent. Same for past events. Thus, for any statement from the copresent on, they coincide with <F> and <P>:

C><F>A ⊨ <F>C>A and <F>C> ⊨ C><F>A

<C <P>A ⊨ <P><C A and <P> <C A ⊨ <C<P> A

C> and <C can be used to say that we're outside the metaphysically challenging zone, if we wanted to say that.

c) \C/ *previous copresent* and /C\ *next copresent* have the mirror image property[23], they could be switched if the -1 and +1 were switched in the rules above. Also, they cancel each other (as they advance/recede by one):

/C\ \C/ A ⊨ A and the reverse

From *previous copresent*, you can deduce *past*, but not the reverse. Same for *next copresent* and *future*. Similarly for C> and <C.

\C/ A ⊨ <P> A but <P>A ⊭ \C/ A

/C\ A ⊨ <F> A but <F>A ⊭ /C\ A

<C A ⊨ <P> A but <P>A ⊭ <CA

C> A ⊨ <F> A but <F>A ⊭ C> A

\C/ and /C\ can be used to describe the step-by-step movement of *now*, under the metaphysical assumption above, that each *now* may contain more than one moment. But I give an interpretation below where there is one moment per present (giving up density), in which case I think they match the steps of any discrete ordering.

c) >C< only picks up the current copresent. Its negation allows *future* and *future copresent* behave the same and similarly for *past* and *past copresent*.

<F>A ∧ ¬ >C< A ⊨ C> A and the reverse

<P>A ∧ ¬ >C< A ⊨ <C A and the reverse

(3.2) An interpretation

Dictionary:

[23] Priest, *An Introduction to Non-Classical Logic: From If to Is*, 51.

- S – Sextus Empiricus was alive
- H – The iron was hot

I = $<W, v>$ with W = $\mathbb{Q}$. We read the integer thresholds as number of seconds from 1 CE.

We note *m* = -4.6B, *n* = -4.7B, *o* = -4.8B (correspond to ~130/140/150 BC when Sextus was active). We superscript a number when we iterate an operator.

Function v assigns 1 just to: i) S at all $x \in \mathbb{Z}$, $m \leq x \leq o$, ii) H at *n* and at 0.

Then, with w the current world (w=0), the following will be 1 under v_{w}:
$\backslash C/^{m}S \wedge \backslash C/^{n}S \wedge \backslash C/^{o}S$ – Sextus was alive in ~130 and ~140 and ~150 BC.
$<C$ S – There was a present when Sextus was alive.
$\backslash C/^{n} S \wedge \backslash C/^{n} H \wedge \backslash C/^{n} \neg H$ – In ~140 BC it happened together that Sextus was alive, and the iron was hot and not hot.
$>C< H \wedge >C< \neg H$ – In the present the iron is hot and in the present it is not hot.

The last two hold because of the interval [n, n+1) since v assigns H 1 only at n and at 0.

Of course, everything above is a rough sketch. The construction can only model a discrete movement of *now*, by equating it with an interval.

F. Conclusions

The ontology of time was discussed in this minimal construction. Some conclusions include:
a) The natural *to be at the same time or before* is not transitive, being in fact an interval order, because of some events having duration.

b) The logic of tenses (past, future, present) need not be tied strongly to the linearity of moments.

c) Both the A-theory and the B-theory face an issue of circularity. For the former, in order to account naturally for temporal passage, we need to assume that the starting state was tensed. For the latter, the definition of change comes down to the difference between indexical presents, which is even worse.

d) A-theory is richer, but temporal passage and density of time do not fit well together. I showed how to treat this misfit as "copresent" interval operators.

For philosophical treatment of time, the least-objectionable theory is the growing block, while B-theory seem to me the best theory of logical treatment of time, because of its simplicity.

Annex: Sketches of proofs

L1. Immediate from (B2.5).

L2. For transitivity, suppose $\Subset a \preccurlyeq_s \Subset b$ and $\Subset b \preccurlyeq_s \Subset c$. Then by the definition, $a \leq_M b$ and $b \leq_M c$. Since $\leq_M$ is transitive, then $a \leq_M c$ which by the definition gives $\Subset a \preccurlyeq_s \Subset c$. Totality is immediate from the definition and (B2.2). Transitivity and totality give reflexivity.

L3. As above.

L4. Immediate from (B2.5) and (B2.6).

L5: By an example: suppose $x, y \in M$, $x <_M y$ and suppose $w \in E$ with $\Subset w = x$ and $\Supset x = y$. Then $x \approx w$ and $w \approx y$ but not $x \approx y$. To see that they form an interval order, remark they any event can be mapped to intervals of $\mathbb{Q}$.

L6. Immediate from (B3.2), (B3.3) and (B3.4).

L6.1. Immediate from (B2.8.d), (B3.1), (B3.3) and (B3.4).

L7. From (B1.2) the definition of function $\boldsymbol{\tau}$ and (B1.5).

L8. By (B2.4), (B4.a) and (B4.g).

L9. As above.

L10: Take any present moment a, $\theta(a) = m$. By the Archimedean property of $\mathbb{Q}$, there are $s, t \in \mathbb{N}$, $t = s + 1$, so that $s * j_\Pi \leq m \leq t * j_\Pi$. Choose any i, $0 < i < j_\Pi$. By the bijectivity and monotonicity of θ in (B3.2), there is a moment b so that $\theta(b) = i$ and $a <_M b$. By (C1.1b), since a is present, b is present. So, a and b are copresent but not simultaneous: not $a \approx b$.

L11. By (B4.3) there is a changed event. By (C1.1.c) any such changed event a has $\tau_1(a) \neq \tau_2(a)$. By inspection of the range of τ in (B1.2), a is either present or is past.

L12. Take any future moment b, $\theta(b) = n$. Take the first case of (L10), there is a present moment, call it a, $\theta(a) = m$. By the Archimedean property of $\mathbb{Q}$, there are $s, t, u, w \in \mathbb{N}$, $t = s+1$, $w = u+1$, so that $s*j_\Pi < m < t*j_\Pi$ and $u*j_\Pi < n < w*j_\Pi$. By (C1.1b), the slice [s, t) corresponds to present events and [u, w) to future. Reason now by induction on $t + 0$, $t + 1$ and so on, there is a last $t + z$, $z \in \mathbb{N}$ that corresponds to a present event. Then $t + z + 1$ is the θ-value of the minimum future moment. Similarly for the other case, start from the past moment existing by (L11).

L13. By induction on *o*. For Π, there are no past moments and by (C1.4.b) the present moments respect the condition. For the inductive step, we inspect *p* the temporal passage function in (C1.2): it makes present events past, adding to them the present and future tense and makes some future events present but adds them the future tense, so each application of *p* respects the condition.

L13.1. For present and past events, use (L13), constraint on change (C1.1.c) and the definition of *was changed* in (B1.5). For future events, immediate by inspection of the range of τ_{Π} in (B1.2), (C1.1.c) and (B1.5). From (L12) it follows that there is a first not-changed moment.

L14. Immediate from (C2.1), (B4.3), and (L11).

L15. Immediate from (C2.1) and (B1.5).

L16. Immediate from (C2.1) and the definitions of the terms.

L17. Immediate from (C2.1) and (L12).

L18. Immediate from (C3.2) and (B1.5).

L19. Immediate from (C3.2) and the definitions of the terms.

L20. Immediate from (C3.1) and lack of a start of time.

L21. Immediate from (C3.2) and definitions.

L22. Immediate from (C3.2) and (C1.1c).

L23. Identical with (L21).

L24. Identical with (L22).

L25. By (B4.2) there is at least one changed event. By (1.1d) such event is distinct from *i* yet simultaneous with it, so it is a persistent event by (B2.6) which is present.

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