Logical Maximalism in the Empirical Sciences

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1 Popper's Logical Maximalism

Karl R. Popper (1947a, b) took the central topic of logic to be the theory of deductive inference and the main problem he was concerned with in his early writings on deductive logic was to give a satisfactory definition for the notion of "deductive valid inference". Although his definition was supposed to be a generalization of Tarski's definition of logical consequence, Popper showed that the notion of "truth" can be avoided, even though its use is not objectionable. In addition, unlike Tarski's model-theoretic (or, better, group-theoretic) criterion, he proposed an inferential criterion to draw a line between the formative (i.e., logical) and the non-formative signs (i.e., non-logical) and defined the validity of deductive inferences on the basis of inferential definitions.[1]

Although the notion of *truth* can be avoided in this inferential foundational approach to deductive inferences, in some latter writings Popper acknowledged that the validity of deduction goes beyond the signs and the rules that govern their use, and emphasized the constitutive role that the notion of *truth* has for logical deduction:

> Deduction, I contend, is not valid because we choose or decide to adopt its rules as a standard, or decree that they shall be accepted; rather, it is valid because it adopts, and incorporates, the rules by which truth is transmitted from (logically stronger) premises to (logically weaker) conclusions, and by which falsity is re-transmitted from conclusions to premises. (This re-transmission of falsity makes formal logic the *Organon of rational criticism* –that is, of refutation). (Popper 1962, 64)

In addition, Popper et al. (1970, 18) took the notion of truth to be crucial for the applications of logic in the other areas of inquiry. With regard to these applications,

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[1]For a detailed and extensive analysis of Popper's account on deductive inference and logical constants see Schroeder-Heister (1984).

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he distinguished between two main uses of logic: a demonstrational use, for proofs, in the mathematical sciences, and a derivational use, for critical discussions, mainly in the empirical sciences. These two uses of logic mirror two essential model-theoretic features of logical consequence, namely, the transmission of truth from the premises to the conclusion (in proofs), and the retransmission of falsity from the conclusion to at least one of the premises (in critical discussions)—feature that actually makes logic *the organon of rational criticism*. This retransmission of falsity, modelled by *modus tollens*, places thus logic at the very centre of the methodology of the empirical sciences:

> The transmission of truth from premises to the conclusion means also the re-transmission of falsity from the conclusion to (at least one of) the premises. This is, from the pragmatic point of view, just as important an aspect of a valid deduction as the obtaining of reliable secondary information. It enables us to reject prejudices by falsifying their consequences; and it allows us to test a hypothesis by the method of trying to refute some of the conclusions which follow from it; for, if one of these is not true, the hypothesis cannot be true either. (Popper 1947a, 266)

The application of logic in the mathematical sciences and in critical discussions is supposed to be made in agreement to, what I will call, the requirements of *logical minimalism* and *logical maximalism*. Logical maximalism is the idea that in the derivational use of logic, in critical contexts, and in particular in the empirical sciences, one ought to use the strongest logic at our disposal. If we use a weaker logic, Popper contends that we are not critical enough:

> If we want to use logic in a critical context, then we should use a very strong logic, the strongest logic, so to speak, which is at our disposal; for we want our criticism to be severe. In order that criticism should be severe we must use the full apparatus; we must use all the guns we have. Every shot is important. It doesn't matter if we are over-critical: if we are, we shall be answered by counter-criticism. Thus we should (in the empirical sciences) use the full or classical or two-valued logic. If we do not use it but retreat into the use of some weaker logic - say, the intuitionist logic, or some three-valued logic (as Reichenbach suggested in connection with quantum theory) - then, I assert, we are not critical enough. (Popper et al. 1970, 18)

Conversely, logical minimalism is the idea that in the demonstrational use of logic, in mathematical proofs, one ought to use minimal logical means, such as intuitionistic, minimal or positive logic. So, in a mathematical proof it seems rational to attempt to minimize the spending of logical resources. Popper seems to regard logical minimalism as being actually embedded in the practice of the working mathematicians:

> Now, let us look, by contrast, at proofs. Every mathematician knows that considerable interest lies in proving a theorem with the help of *minimum apparatus*. A proof which uses stronger means than necessary is mathematically unsatisfactory, and it is always interesting to find the weakest assumptions or minimum means which have to be used in a proof. In other words, we want the proof not only to be sufficient, that is to say valid, but we want it if possible to be necessary, in the sense that a minimum of assumptions have been used in the proof. (Popper et al. 1970, 19)

Both logical minimalism and logical maximalism are two challenging assumptions. For instance, logical minimalism sounds more like a cost–benefit analysis of mathematical proofs. It aims at finding the minimal logical cost for proving a mathematical theorem. Certainly, it is not very clear whether Popper would prefer minimizing logical cost even if this incurred any mathematical or epistemological expenses. Logical minimalism by itself requires a detailed analysis, but my interest below, however, will be with its correlate, logical maximalism. The problem that I will be concerned with in the next section is whether the logical maximalism requirement makes the logic used in the empirical sciences immune to revision.

2 Is Logical Maximalism Compatible with a Revision of Logic?

If we consider logical maximalism strictly, with no other qualifications, it simply tells us that we *ought to* use the strongest logic at our disposal. In this case, if we already know which is the strongest logical theory on the theoretical market, then we have no other option left for a change of logic. Haack (1996, 37–38) describes Popper's motivation for logical maximalism, in relation to his falsificationist methodology, as follows:

> Popper's position seems to be like this: logic is a tool employed in the programme of attempted falsification. Since it is methodologically desirable that a test of an hypothesis should be as stringent as possible, the strongest possible logic should be used in deriving consequences from the hypothesis, so that its class of potentially falsifiers may be as inclusive as possible. This viewpoint is particularly forcibly expressed when Popper discusses the proposal that logic be modified in order to avoid certain 'anomalies' allegedly arising in quantum physics; if there are anomalies, Popper argues, they show that there is something wrong with quantum theory and modifying logic to avoid them is a dangerous evasion. [...] He wants to rule out altogether the possibility of *ever* resorting to change of logic rather of other beliefs.

Hence, according to Haack's interpretation, logical maximalism is a consequence of Popper's falsificationist methodology. Since we want to have empirical tests as severe as possible, then we should use the logic that allows us to deduce as many consequences as possible from the hypotheses, i.e., the strongest logic. If things are so, then a change in logic should not be preferred over a change in other beliefs. For instance, in the proposal of modifying logic in order to deal with certain 'anomalies' that seem to arise from quantum physics, logical maximalism advices us to look closely to the quantum theory itself and not to the logic used to derive from it consequences formulated in observational terms. For Popper, thus, the revision of logic seems to be contrary to progress in the empirical sciences. This is so because the weaker the logic, the less consequences one can deduce from hypotheses; the less consequences, the more chances to hold on to false hypotheses. By contrast, the stronger the logic, the more consequences one can deduce from hypotheses; the more consequences, the more chances to discard false hypotheses.[2] In this sense, strictly viewed, logic is not a proper part of science, since logical statements cannot be inconsistent with basic statements, that is, they are not scientific, i.e., falsifiable in the same way in which the empirical hypotheses are. However, as we shall see below, logic and physics are, in a certain sense, in the same boat for Popper.

It should be noted however that Popper's logical maximalism is a general requirement that applies to every rational dispute, not only to those from scientific contexts. The derivational use of logic is actually present in every critical discussion. Popper's main tenet is that disputing the logic that underlies a critical discussion may undermine the method of critical discussion itself. Thus, answering criticism by changing the logic is not, at least *prima facie*, an adequate attitude:

> What I should wish to assert is (1) that criticism is a most important methodological device; and (2) that if you answer criticism by saying, 'I do not like your logic: your logic may be all right for you, but I prefer a different logic, and according to my logic this criticism is not valid', then you may undermine the method of critical discussion. (Popper et al. 1970, 18)

One of the main objections that Haack raises to Popper's logical maximalism is that it would leave logic *totally immune* from criticism. So to say, we are not critical enough if we adopt logical maximalism; some parts of the web of knowledge will be accepted *dogmatically*, or only on the basis of a pragmatic justification through their usefulness in the process of critical discussions.

In order to see whether Popper had a non-critical attitude towards logic in the derivational contexts, it is fruitful to take a look at an exchange that he had with W.O.V. Quine on the relation between logic and physics at the *International Colloquium on Logic, Physical Reality, and History*, held in Denver in 1966. In his comments to Popper's paper 'A Realist View of Logic, Physics, and History', Quine agreed with many aspects embedded in Popper's view on logic (for instance, the non-conventionalism, the function of truth transmission and falsity re-transmission), but he also had some critical remarks in connection with the status of logic and physics in the overall web of knowledge:

> I am a logical realist, as Popper is, but I put logic and physics on the same level. They are truths about the world in the broadest sense of the word. But in the case of a disagreement

[2]Tennant (1985) takes the deductive structure of the *experimental refutation* of scientific hypotheses, present in Popper's analysis of scientific method, to have the following schema (P): from hypotheses and boundary conditions we obtain predictions which, together with observational reports, lead to contradiction. The role of logic in the Popperian methodology is taken by Tennant to consist in the retransmission of falsity from the contradiction encountered to at least one of the premises. After he proves that every classically inconsistent set of first order sentences (expressed in ~ , v, &, → , and ∃) is intuitionistically inconsistent and (expressed in ~ , v, & and ∃) is minimally inconsistent, Tennant concludes that both intuitionistic and minimal logic are adequate for *Popperian science*. I think that the reduction of the role of logic *only* to the downward direction in Schema P is not a good adviser for judging which logic is adequate for *Popperian science*. Logic is also needed for deriving the predictions from hypotheses and boundary conditions and this is an important reason for which we have to use *all the guns we have,* i.e., we have to use all the logical rules available in order to deduce consequences from the empirical hypotheses. From this point of view, classical logic has an advantage, namely, it provides us with more logical tools than the other systems of logic.

> that is so fundamental as a disagreement on logical principles, the only thing we can do is talk about talking. But this, again, isn't peculiar to logic and it doesn't make logic a matter of convention. The same thing happens in physics. The same thing happens whenever an issue takes the form of disagreement over extremely fundamental matters, matters so fundamental to the conceptual scheme that if you walk at that level you keep begging the question by simply saying, "Well, it is obvious from what you said that this follows." This happens not only in logic. There is something of this kind when we start talking about relativity theory or quantum mechanics. One has to stand off, talk about the system, and appeal to the pragmatic value of this or that system. One says: Here is a simpler formulation that takes in all the data that the old formulations took in, and, moreover, does it more simply. (In Popper et al. 1970, 30–31)

Quine's main point is that Popper's "view about the function of logic draws no clear boundary between logic and other sciences". For instance, in an argument with three premises and a conclusion, logic is used to transmit truth from premises to the conclusion, but this argument could be also rewritten such that it has two premises and the third one is absorbed as the antecedent of a material conditional in the new conclusion. This material conditional obtained in actually a sentence of "physics or biology, or whatever the subject matter may have been". Quine also points out that the properties of logical consequence of transmitting truth and retransmitting falsity do not provide a criterion of delineating the province of logic. Moreover, if a disagreement on fundamental matters occurs, the main thing that has to be primarily done is critical discussion. But this is common both in logic and in physics. This is one of the reasons for which Quine takes logic and physics to be in the same boat. The web of beliefs confronts experience as a whole and a conflict with experience may bring changes even in the central parts of the web. Logic is thus also exposed to revision, as the physical hypotheses are.

In his reply to Quine, Popper admitted that, up to a point, logic and physics are in the same boat, in the sense that both of them are criticizable. Nevertheless, he immediately recognized that a weakening of classical logic, as suggested by Birkhoff and von Neumann, or by Reichenbach, is not an adequate response for the difficulties from quantum physics:

> There seems to be complete - or almost complete - agreement between Quine and myself. Where I thought that we disagreed was on Birkhoff and von Neumann's "Logic of Quantum Mechanics": I thought (perhaps mistakenly) that he had taken this famous paper to show that logic and physics are in the same boat. Up to a point no doubt they are: both are criticizable. But a mere weakening of classical (or Boolean) logic, as suggested by Birkhoff and von Neumann, and also by Reichenbach, does not seem to me an adequate response to a difficulty in one of the empirical sciences. It is a strategy which we ought to exclude if we want to learn from experience, because every sufficient weakening of the underlying logic will make any empirical theory for ever secure. It will make it uncriticizable, irrefutable. These remarks show that in some cases at least we can critically discuss a proposed change in logical theory. (Popper et al. 1970, 35)

Popper's reply is made in accordance with the logical maximalism requirement. If we weaken the underlying logic, then we are in danger of being insufficiently critical with the scientific hypotheses. A sufficient weakening will make the scientific hypotheses uncriticizable and, thus, irrefutable.[3] We can easily understand that there is a tension between the criticizability of logic and the criticizability of empirical hypotheses. If one increases, the other one decreases. Certainly, the strongest logic appears indeed to be the one which allows the maximum criticizability of empirical hypotheses, which is the main target of the falsificationist methodology. However, since Popper admits that both logic and physics are criticizable, this means that logic is not taken dogmatically to be immune to revision. Although Popper does not develop in details the sense in which logic itself is criticizable in its derivational use, some useful remarks on this matter are made by Miller (1994, 90–91):

> In addition to the point *a* above, that logical truths are not criticizable, there is much more serious point concerning the uncriticizability of our system of rules of logical inference. These are not, like tautologies, mere consequences of other positions we adopt but vital constituents of the rationalist position itself. […] Critical argument certainly cannot be carried on without some system of logic. You cannot in this sense abandon logic and remain a rationalist. […] But the system of logic employed can –despite Lewis Carroll- be taken as one of the things under investigation in a critical discussion. […] The rules, after all, are universal, and if they were systematically to lead to error in a way that in fact they do not, we might eventually decide that we have got them wrong. We must not be misled by the fact that we have not got them wrong.

Miller operates the distinction between logical truths and rules of logical inference. The rules, of course, are very important for the critical rationalist, being central in the process of critical argumentation. As Tarski (1941/1995, 47) emphasized, "the rules of inference, which must not be mistaken for logical laws, amount to directions as to how sentences already known as true may be transformed so as to yield new true sentences." The rules, thus, allow us to see explicitly the consequences of the beliefs that we hold and to evaluate them accordingly. Miller recognizes that we can evaluate in a critical discussion even the rules of inference themselves. Being universal sentences, any *counterexample* would suffice to invalidate them. However, it is not very clear in what a *counterexample* would consist of.[4] Their revision would be motivated "if they were systematically to lead to error *in a way that in fact they do not*" (my emphasis). Once again, the requirement of logical maximalism is not theoretically incompatible with the revision of logic, although it is not quite clear what kind of evidence would make us to revise the system of logic used in critical discussions. The expression "in a way that in fact they do not" seems to suggest that,

[3]Mortensen and Burgess (1989, 48–49), in reply to Popper et al. (1970), have proposed a sort of logical minimalism in the empirical sciences, i.e., "prefer a weaker logic". Their main reason for this principle is that a weaker logic has a larger number of theories and a theory criticized from a weaker logical base excludes the option of modifying it to enter in dispute with those of a stronger logic. This idea, however, in a different shape, is still debated nowadays. Williamson (2017, 337) briefly dismisses the idea that weakness is a strength in logic, since almost every logical principles has been subjected to criticism. Thus, a weaker logic is not necessarily in a better position. Bell (2019, 213–217) offers a detailed analysis of this idea, but he concludes that Williamson's argument for logical strength does not work, unless we assume a sort of Quinean conservatism principle (see also footnote 10 below).

[4]This problem will be discussed in the last section of this paper.

at least until present, the system of logic used in critical discussions, presumably the classical one, is not open to refutation.

To sum up the above discussion, logical maximalism tells us that we *ought to* use the strongest logic at our disposal in critical discussions, but if we *ought to* use the strongest logic in these contexts, then logic seems to remain *immune* to criticism. However, the critical rationalists[5] admit that we can critically discuss a change in logical theory. Therefore, the logical maximalism requirement can be itself subjected to criticism. The question that arises at this point is the following: what kind of *evidence* would lead the critical rationalist to revise the system of logic that underlies a physical theory, such as quantum physics. Is it the same kind of evidence as in the case of the empirical theories or it is more like a pragmatic decision of changing the conceptual framework in which the empirical theories are formulated? A better understanding of this question will be gained once we analyze the abductivist view on logic and compare Popper's critical rationalism with it.

3 The Abductivist Assessment to Logic

It is a widespread conviction nowadays among logicians and philosophers that logic should be assessed abductively, i.e., on the basis of empirical evidence and theoretical virtues. The abductive methodology is a rational reconstruction of the way in which the scientists select and justify their hypotheses and theories. In its modern use, abduction is better known as the inference to the best explanation. Roughly expressed, a scientific theory is selected as the best one, according to the abductivists, on the basis of its theoretical features and of its explanatory and predictive power. For instance, Williamson (2017, 334) synthesizes the abductive methodology as follows:

> We make the standard assumption that scientific theory choice follows a broadly abductive methodology. Scientific theories are compared with respect to how well they fit the evidence, of course, but also with respect to virtues such as strength, simplicity, elegance, and unifying power.

Williamson takes this methodology to be 'the best one science provides' and for this reason it should also be applied in selecting a logical theory as being the best one.[6] He does not provide us with many clarifications as to what would constitute evidence for a logical theory, but he gives us, however, a negative determination, namely: 'Evidence is not confined to observations. We may use anything we know

[5]Critical rationalism is defined by Miller (2012, 93) as "the generalization, from the empirical sciences to the whole of our knowledge, of the methodological falsificationism (or deductivism) proposed by Karl Popper in *Logic der Forschung*".

[6]The abductivist approach to logic is characterized by Russell (2019, 550) as consisting in two main claims about the *justification* of logic. The first one is the holism about justification, namely, that it is entire logics that are subject to justification, and not particular claims of logical consequence—logic being taken as a theory of the relation of logical consequence. The second one is that the justification is given by adequacy to the data, and the possession of virtues and absence of vices.

as evidence.' Williamson (2017, 335). That *evidence* is not restricted to observations means that it can also incorporate our intuitions in certain contexts, as the modal ones. Nevertheless, these remarks on *evidence* are very general and could be accommodated with different philosophical views. For instance, we may understand them in Quine (1951)'s sense, namely, that certain empirical observations might determine us to revise even some sentences situated in the central parts of the web, as the logical principles are. But we may also interpret them in Carnap (1950)'s sense, namely, that some empirical observations could suggest us to change the conceptual framework we work in and thus the logic constitutive of that framework. Nevertheless, letting these interpretations aside, it is a clear point that for the abductivist the *empirical* evidence is relevant for choosing or changing a logical theory. Certainly, it remains the same question as for the critical rationalists: in what way empirical evidence may determine us to change a logical theory already at use in a certain area of inquiry?

Beside evidence, the other criteria that are used by the abductivist to choose a logical theory are the theoretical virtues. One of the most important virtues is taken by Williamson (2017) to be *strict logical strength*.[7] In this sense, a logical theory T is stronger than T* if and only if every theorem of T* is a theorem of T, but not conversely. Since two internally consistent theories may be inconsistent with each other, Williamson (2017, 336) introduces a looser sense of strength, i.e., *looser scientific strength*. In this looser sense, a theory T is stronger than a theory T*, with which it may be externally inconsistent, on the ground that 'the former is more specific or informative than the latter'. Both senses have applications in his view to logical theories. Williamson claims that if a theory T is stronger that T* in the strict sense, then is also stronger in the looser sense, but not conversely.[8] In addition, and this is the point in which he and Popper converge, Williamson (2017, 337, 338) emphasizes that logical strength is one of the virtues that makes classical logic the most adequate instrument in inquiry:

[7]Russell (2019, 556) argues that logical strength is theoretically neutral in the selection process of a logical theory, i.e., it is neither a virtue nor a vice. However, Russell discusses the problem outside a specific context and thus she concludes that "logical strength is something that a logic is supposed to *get right*, rather than something that it is always good to have more of". Roughly expressed, the idea is that for certain purposes a weaker logic may be better than a stronger one. Certainly, Popper would agree with this idea, since he actually emphasized that, in the demonstrational use of logic, "the weaker the logical means we use, the less is the danger of consistency" (Popper et al. 1970, 20). Nevertheless, it is worth mentioning that the distinction between the demonstrational use of logic and its derivational use probably does not make much sense for the abductivists, who work, so to say, in a Quinean holist framework.

[8]In reply to Williamson, Russell (2019, 557–562) argues that logical strength does not entail scientific strength since, among other reasons, weaker and stronger logics are on a par if informativeness refers to the classification of arguments in valid or invalid. Certainly, this sense of informativeness is *internal* to the province of a certain logical theory, i.e., the validity of an argument is relative to a certain logical theory. However, as Williamson (2017, 337) exemplifies, if we extend the classical and the intuitionistic propositional calculi to include quantification into sentence position, ⌜(∀p)(p v ~ p)⌝ will be a theorem in the extended PC, while ⌜ ~ (∀p)(p v ~ p)⌝ will be assertable in the extended IC. Since a universal generalization is more informative than its negation, there is at least one clear sense in which a logical theory is more informative than another. (For an elaborate discussion on the relation between logical and scientific strength see Incurvati and Nicolai (2020)).

> Once we assess logics abductively, it is obvious that classical logic has a head start on its rivals, none of which can match its combination of simplicity and strength. Its strength is particularly clear in propositional logic, since PC is Post-complete, in the sense that the only consequence relation properly extending the classical one is trivial (everything follows from anything). First-order classical logic is not Post-complete, but is still significantly stronger than its rivals, at least in the looser scientific sense, as well as being simpler than they are; likewise for natural extensions of it to more expressive languages. In many cases, it is unclear what abductive gains are supposed to compensate us for the loss of strength involved in the proposed restriction of classical logic.

Although Williamson prefers classical logic and its extensions, in particular higher-order modal classical logic, he lets open the possibility that some non-classical logic "can overcome the initial advantages of classical logic once we move to a wider setting, by considering fit with evidence or with other scientific theories, or by treating more expressions as logical constants" Williamson (2017, 338). The paradigmatic historical case where the use of classical logic in the empirical sciences was challenged is the proposal of introducing quantum logic in order to simplify and get a better understanding of quantum phenomena. The point of view of the abductivist approach is to compare the theory generated from the principles of quantum mechanics by using classical logic and the theory generated from them by using quantum logic. Relying on Quine (1970, 85–86)'s and Putnam (2012)'s analyses, Williamson (2017, 338) concludes that 'in practice, rejecting classical logic just does not seem to help us understanding the nature of quantum reality'.

Williamson's attitude towards classical logic is not being based primarily just on strength and conservatism.[9] He considers that it 'has been tested *far* more severely than any other logic in the history of science, most notably in the history of mathematics, and has withstood the tests remarkably well' Williamson (2017, 338). His position seems to converge here to Miller (1994)'s view expressed above, namely, that logical rules would be changed if they were systematically to lead to error in a way that in fact they do not, since the tests to which they have been subjected thus far have corroborated them.[10]

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[9] See Bell (2019) for a very good analysis and criticism of Williamson (2017). He argues, however, that the conservatism principle, or something similar to it, underlies "Williamson's new Quinean argument".

[10] The severe tests that Williamson refers to are interpreted by Bell (2019, 220) in the sense that the theories which have successfully passed the testing process appear to be closed under the *classical* relation of logical consequence. However, Bell does not consider this fact as a disadvantage of weaker logical theories, since a theory closed under classical logic is also closed under a sub-classical one. Moreover, the successful testing of classical logic in the history of mathematics appears so since in mathematics the theoretical possibilities are usually reduced in practice to classical models. Certainly, Popper would agree with Bell here, since, in the demonstrational use of logic, using weaker means is a better option.

4 Final Remarks: Critical Rationalism Versus Abductivism

We have seen thus far that the critical rationalism methodology and the abductivist one (as it is described and used by Williamson) lead to the same result concerning the selection of a logical theory for being used in the empirical sciences[11]: classical logic is the best candidate for doing scientific research. Although both Popper and Williamson consider that the condition of logical maximalism is not an absolute one and that the classical logic could itself be replaced, they are not very informative as to what would constitute a clear case of evidence for changing a logical theory.

Despite this fact, however, there is a fundamental point in which the two methodologies go apart. The critical rationalists have a neutral attitude towards theoretical constructs (concepts, ideas, theories, conceptions), namely, they do not look for justifications for them, but rather they do their best in order to criticize them as much as possible. To look for justifications for a certain idea means to already have a certain *attachment*[12] towards it, which may incline you to interpret as evidence something that actually might not be so. For instance, if you already have a certain attachment towards classical logic (since you are very familiar with it, you taught it, you worked a lot with it), then you will be inclined to see it confirmed by a lot of things, in a lot of cases. In contrast, if you simply accept it because it resisted to the criticisms to which it has been exposed so far, and remain open to further criticisms that may, *in principle*, overthrown it, then you have a more adequate theoretical and practical attitude with regard to the powers and the results of our reason. The distinction between justification and criticism was very nicely emphasized by Miller (2012, 95):

> Central to the philosophy of rational criticism is the realization that the process of reasoning can never provide justification, but it may provide criticism; and indeed, that the rational attitude consists wholly of openness to criticism, and of appropriate responses to criticism. Justification, conclusive or inconclusive, is revealed as neither possible, nor useful, nor necessary.

Consequently, from the perspective of critical rationalism, anything can count as evidence for revising a logical theory. Whatever constitutes itself in a strong criticism for revising a theoretical construct will do the job. The only thing that the logical maximalism requirement imposes, in my understanding, is a certain order in the processes of testing and revision, namely: we should not try *prima facie* to revise the logical statements, since this action may have enormous consequences in the total system of science, due to their central role in the system and to the logical interconnections among the other statements of the system. What we should rather do

[11] Williamson does not seem to make a clear-cut distinction between the use of logic in the empirical sciences and in the mathematical ones, a thing quite natural if we consider the Quinean background of the abductive methodology. Nevertheless, since he selects classical logic as the best candidate *simpliciter*, its use in the empirical sciences is also presupposed here.

[12] By this I mean both epistemic and psychological attachment. Our ideas are embedded in the overall web of our mind and are important tools for the accommodation in natural and social environment. Since they are *ours*, we do not have all the time an objective and critical attitude towards them.

is to use the strongest logic in order to identify the problematic parts of the scientific theories.

The abductivist approach to logic, being focused on justification, I think that is exposed to have a less critical attitude to logic. In my understanding, justification is just a particular instance of criticism. By this I mean that we label a certain idea to be justified if it resisted, thus far, to the criticisms to which it has been subjected. Someone may say that this is just a terminological issue. However, I think that it is not just so because the function of the justification concept in the economy of our thinking induces to us the tendency of being attached to the beliefs for which we think we posses *justification*—thing that makes us less receptive to good criticisms that may be *prima facie* ignored due to this attachment. Certainly, this is a psychological aspect, but it should not be neglected.

Another important point that is relevant for the understanding of the revision process of logic in the derivational contexts was made by Popper in his contribution to the symposium *Why Are the Calculuses of Logic and Arithmetic Applicable to Reality?*. Popper (1946, 42–44) analyzed the distinction between rules of inference and formulas of logical calculi in connection with their applicability to, what he called, *reality*. In his view, logical rules of inference should not be confused with the sentences of a logical calculus. The former are about sentences, i.e., are meta-sentences, and assert unconditionally that every sentence of a certain form is inferable from a set of sentences of another form, while the latter are about the semantic values of the sentences and the relations among them. The same rules of inference may be used for constructing different logical calculi (for instance *modus ponens*).

Popper (1946, 48) defines a rule of inference such that it cannot have a counterexample. It always leads from true premises to a true conclusion: "we call a rule of inference 'valid' if, and only if, no counterexample to this rule exists; and we may be able to establish that none exists". In this sense, by definition, a rule of inference cannot be refuted.[13] So to say, it is *analytic*. Yet, he accepts that we can have alternative logics by formulating "alternative rules of inference with respect to more or less different languages" Popper (1946, 51). Popper's assumption here is that the rules of inference are '*to a certain extent,* always relative to a language system'. Consequently, it seems that for Popper, as for Carnap and Quine, to say that a specific rule of inference is invalid simply amounts at changing the *subject,* that is, changing the conceptual scheme in which we work, scheme that provide the logical constants with their actual meaning at use.

Regarding the logical calculi, Popper has a quite similar attitude. Logical calculi[14] are constructed on the basis of the rules of inference from primitive formulas. In a

[13]To have a clear cut understanding of this idea we may think to the decision procedures in a logical system. Think for instance to the method or normal truth tables in classical propositional logic. We can easily decide I this case that there is no *permissible* possibility to have true premises and a false conclusion for modus ponens once we precisely disambiguate all the premises and the conclusion and accept the principle of bivalence. (See also Miller (2012, 100–103) for a discussion of the status of the logical rules of inference from a critical rationalist perspective).

[14]Popper takes a logical calculus to be either a derivational logic or a demonstrational one. The former is a system of logic "intended from the start to be a theory of inference in the sense that

way that resemblances quite well Einstein (1921, 233)'s view on the relation between geometry and experience,[15] Popper (1946, 54) concluded that the relation between logical calculi and *reality* has the following dynamics:

> in so far as a calculus is applied to reality, it loses the character of a *logical* calculus and becomes a descriptive theory *which may be empirically refutable*; and in so far as it is treated as irrefutable, i.e., as a system of *logically true* formulae, rather than a descriptive scientific theory, it is not applied to reality.

We see thus that, as in the case of the rules of inference, logical formulas are taken to be true by definition. In this sense, as far as the calculus to which they belong is considered *in itself*, we have no counterexamples to them. Once the calculus is applied, i.e., it is used in a certain descriptive theory (physical, for instance), it becomes subject of philosophical criticism. The formulas of the calculus are treated now as hypotheses. However, the logical maximalist constraint advices us to criticize them only after we have used all the *shots* these formulas provide us for criticizing the empirical hypotheses.

All these being said, it seems to me that Popper's logical maximalism requirement is an adequate rational attitude, both theoretically and practically, on the use of logic in critical discussions, since it advices us to derive as many consequences as possible from a given hypothesis or theory in order to confront them with experience through observations and experiments. Once this process has been performed, if problems still persist, of course, we can critically discuss the logical instrument at use. The process is here, so to say, dialectical. More precisely, it proceeds by the trial and error method.[16] By this I mean that we start to criticize the logical rules of inference or the logical formulas[17] that seem more problematic (as are the law of distributivity or the law of excluded middle in the case of quantum phenomena) and we keep using the other rules of inference and logical formulas. In the long run, the output of the criticism process may reside in changing some parts of the mathematical formalism

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it allows us to derive from certain informative (non-logical) statements other informative statements" Popper (1947b, 230). This logic contains rules of inference for drawing consequences from hypotheses. A system of natural deduction fits very well this role. In contrast, "most systems of modern logic are not purely derivational, and some (for example, in the case of Hilbert-Ackerman) are not derivational at all." Popper (1947b, 230). These systems are demonstrational logics and are serving better in the demonstrational use of logic. The derivations conducted in them usually start from logical axioms, definitions or theorems. It should be noted however that this distinction refers primarily to the deductive format of the system and not to its strength.

[15] "As far as the propositions of mathematics refer to reality, they are not certain; and as far as they' are certain, they do not refer to reality".

[16] We apply the trial and error method whenever we are faced with a problem, we tentatively advance a solution, and then criticize it as much as possible in order to eliminate the possible errors. A detailed analysis between the dialectic and the trial and error methods is offered by Popper (1940). As he emphasized, the dialectic development could be actually explained on the basis of the trial and error method, which has a wider application.

[17] Certainly, if the deduction theorem holds for the system of logic we use, this distinction is less relevant.

of the physical theory, in providing new interpretations for the formalism,[18] or in abandoning certain logical principles. If we look at the quantum mechanics case, it seems that, thus far, changing the logical theory was not a fruitful path to follow.[19]

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[18]It is important to mention that Birkhoff and von Neumann's proposal, and likewise Reichenbach's proposal, of introducing a new logic to deal with quantum phenomena was not for *testing* the quantum mechanics, but rather for obtaining a better understanding of it. See Putnam (2012) for a captivating story of the philosophical development of quantum logic.

[19]In addition, there are also some general considerations which suggest that logic cannot be treated in the same manner in which the physical hypotheses are, although both of them are criticizable. For a philosophical discussion on the status of classical logic, viewed from a metalogical point of view, see Brîncuş (2019).

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