

On a Simple Derivation of the Effect of Repeated Measurements on Quantum Unstable Systems by Using the Regularized Incomplete β -Function

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Abstract: a simple derivation of the effect induced from repeated measurements on quantum unstable systems is obtained by using the regularized incomplete β -function .

Keywords: quantum mechanics, quantum Zeno effect. Regularized incomplete β -function

Introduction

The quantum Zeno effect is a name that was introduced by George Sudarshan and Baidyanath Misra in 1977 when they analyzed the physical condition that an unstable quantum system (as example the decay of a radioactive source) , if observed continuously, will never decay [1] . The meaning of the term has since expanded, leading to a more technical definition in which time evolution can be suppressed not only by measurement. Actually , the quantum Zeno effect seems to be the suppression of the unitary time evolution caused by quantum decoherence in quantum systems provided by a variety of sources: measurement, interactions with the environment, stochastic fields, and so on [2]. As an outgrowth of study of the quantum Zeno effect, it has become clear

that applying a series of sufficiently strong and fast pulses with appropriate symmetry can also decouple a system from its decohering environment [3] After the initial approach of George Sudarshan and Baidyanath Misra a lot of phenomena has been discussed in literature and relating such quantum baic effect. In order to cover all of these phenomena (including the original effect of suppression of quantum decay), the Zeno effect can be defined as a class of phenomena in which some transition is suppressed by an interaction . It allows the interpretation of the resulting state in the terms of a transition that did not yet happen and transition has already occurred, or by the statement that the evolution of a quantum system is halted if the state of the system is continuously measured by a macroscopic device to check whether the system is still in its initial state.[3]

Closely related (and sometimes not distinguished from the quantum Zeno effect) is the so called watchdog effect, in which the time evolution of a system is affected by its continuous coupling to the environment.[4,5]

The aim of the present paper is to present a very simple but consistent derivation of the effect induced on unstable quantum systems using repeated observations . In our previous studies we attempted to consider the possible advances in the application of quantum mechanics in neuroscience and psychology. As outlined in particular by Stapp , human cognition may be directly involved in arising of such so much extraordinary quantum effect.

Theoretical Elaboration

Consider an unstable quantum system described by the usual quantum superposition of states

$$|\psi(t)\rangle = c_1(t)|\varphi_{undecayed}\rangle + c_2(t)|\varphi_{decayed}\rangle \quad (1.1)$$

with

$$p_1(t) = |c_1(t)|^2 \quad (1.2)$$

probability for the system to be in the undecayed state , and

$$p_2(t) = |c_2(t)|^2 \quad (1.3)$$

probability for the system to be in the decayed state, and

$$p_1(t) + p_2(t) = 1 \quad (1.4)$$

Let us admit that we perform (n) observations , being thus (n) an integer value..

Consider the trivial expression

$$[p_1(t) + p_2(t)]^n \quad (1.5)$$

The first term in the expansion of this formula gives the probability to find the system undecayed after (n) observations at a time t , the second term represents that one decay has occurred , and the (r+1) term gives the probability that r decays have occurred.

We have that

$$y(t) = p_1^n(t) + np_1^{n-1}(t)(1 - p_1(t)) + \dots + {}_n C_r p_1^{n-r}(t)(1 - p_1(t))^r = \sum_{i=0}^r {}_n C_i p_1^{n-i}(t)(1 - p_1(t))^i \quad (1.6)$$

The interesting and unexpected feature of the elaboration is that we are interested now to calculate the derivation of the calculated $y(t)$ function respect to $p_1(t)$.

We obtain that

$$\frac{dy(t)}{dp_1(t)} = (n - r) {}_n C_r p_1^{n-r-1}(t)(1 - p_1(t))^r \quad (1.7)$$

The integration of the (1.3) gives

$$y(t) = (n - r) {}_n C_r \int_0^{p_1(t)} p_1^{n-r-1}(t)(1 - p_1(t))^r dp_1(t) \quad (1.8)$$

This is the incomplete β -function.

The complete β - function is

$$\beta(p, q) = \int_0^1 p_1^{p-1}(t)(1 - p_1(t))^{q-1} dp_1(t) = \frac{(p - 1)!(q - 1)!}{(p + q - 1)!} \quad (1.9)$$

so that , being

$$\beta(n - r, r + 1) = \frac{(n - r - 1)!r!}{n!} \quad (1.10)$$

and

$$(n - r) {}_n C_r = \frac{n!}{(n - r - 1)!r!} \quad (1.11)$$

we have finally that in the computation of the corresponding surviving function , when surviving no more than r decays , is given from the regularized incomplete β - function

$$y(t) = \frac{\beta_x(n - r, r + 1)}{\beta(n - r, r + 1)} \quad (1.12)$$

with $x = p_1(t)$ and $y(t)$ becoming thus a function of n and of r .

We have obtained an elegant derivation on the manner in which observations affect the decay of the considered quantum unstable system by using the regularized β -function.

Interestingly we may remember here that the regularized incomplete beta function is the cumulative distribution function of the Beta distribution, and is related to the cumulative distribution function of a random variable X from a binomial distribution, where the "probability of success" is p and the sample size is n :

$$F(k; n, p) = \Pr(X \leq k) = I_{1-p}(n - k, k + 1) = 1 - I_p(k + 1, n - k).$$

To complete the exposition we intend to give briefly some comment in the field of applied physics. In the last decade we have advanced a number of results in the direct application of quantum mechanics at cognitive level. In detail, we have explored the possible logical origin of quantum mechanics , consequently its possible role in

explaining human cognitive performance , and we have also given a number of experimental confirmations evidencing that quantum interference arises during perception- cognition in humans [6, 23].

It seems that an ulterior evidence is reached by finding the induced effect of repeated measurements on unstable quantum systems. In fact , the quantum Zeno effect (with its own controversies related to the problem of measurement) is becoming a central concept in the exploration of controversial theories of quantum mind consciousness within the discipline of cognitive science. In his book *Mindful Universe* (2007), Henry Stapp [24] claims that the mind holds the brain in a superposition of states using the quantum Zeno effect. He advances that this phenomenon is the principal method by which the conscious can effect change, a possible solution to the mind-body dichotomy.

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Received: August, 2012