net ist[17], zunichte werden und nur der Mensch übrigbleibt[18], dann wird das generische Lebewesen zunichte[19], ohne daß das universale Lebewesen zunichte würde. Weiterhin werden dadurch, daß das generische Lebewesen zunichte wird[20], nicht alle Arten zunichte, solange nicht[21] das universale Lebewesen zunichte wird; die Arten werden dadurch, daß das generische Lebewesen zunichte wird, überhaupt nicht zunichte.

Damit ist klar erwiesen, daß sich die Arten zur Gattung nicht verhalten wie der Teil zum Ganzen und daß die Art nicht vor der Gattung ist, wie Xenokrates annahm.

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17 Zur zugrundeliegenden griechischen Terminologie vgl. o. S. (134) Anm. (75).

18 Pines konjiziert, wahrscheinlich zu Recht, wa-baqiya statt wa-hiya. Die Hs. liest eindeutig wa-hiya. Jedoch mag in ihrer Vorlage oder in einer ihrer Vorlagen das Wort baqiya unpunktiert geschrieben gewesen sein; die Verlesung lag dann sehr nahe. Die Hs. stammt aus dem Jahre 558/1162 (vgl. Badawī [51]); bis zu Abū ʿUṯmān ad-Dimašqī (gest. Anfang 10. Jh.), der den Text „edierte" (die eigentliche Übersetzung ist vermutlich ein Jahrhundert älter; vgl. G. Endress, Die arabischen Übersetzungen von Aristoteles' Schrift De caelo, Diss. Frankfurt 1966, S. 124 ff.), besteht also ein Zwischenraum von etwa 250 Jahren.

19 Die von Pines vorgeschlagene Konjektur baṭala für bal scheint in der Tat die nächstliegende Lösung; die Hs. hat eindeutig bal.

20 Ich ergänze bi-buṭlān in der Lücke, entsprechend der folgenden Zeile.

21 Wörtlich: „ohne daß".

A Mathematical Model of Aristotle's Syllogistic

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1. Our purpose in the present article is to present a mathematical model designed to reflect certain structural aspects of Aristotle's logic. Accompanying the presentation of the model is an interpretation of certain scattered parts of the *Prior* and *Posterior Analytics*. Although our interpretation does not agree in all respects with those previously put forth, the present work would have been impossible without the enormous ground work of previous scholars — especially Jenkinson, Lukasiewicz and W. D. Ross — to whom we are deeply grateful.

Our interpretation restores Aristotle's reputation as a logician of consumate imagination and skill. Several attributions of shortcomings and logical errors to Aristotle are seen to be without merit. Aristotle's logic is found to be self-sufficient in several senses. In the first place, his theory of deduction is logically sound in every detail. (His indirect deductions have been criticized, but incorrectly on our account.) In the second place, Aristotle's logic presupposes no other logical concepts, not even those of propositional logic. In the third place, the Aristotelian system is seen to be complete in the sense that every valid argument statable in his system admits of a deduction within his deductive system, i. e. every semantically valid argument is deducible.

In the present paper we consider only Aristotle's theory of non-modal logic which has been called "the theory of the assertoric syllogism" and "Aristotle's syllogistic." Aristotle presents the theory almost completely in Chapters 1, 2, 4, 5 and 6 of the first book of *Prior Anylatics*, although it presupposes certain developments in previous works — especially the following two: first, a theory of form and meaning of propositions having an essential component in *Categories* (Ch 5, esp. 2a 34—2b 7); second, a doctrine of opposition (contradiction) more fully explained in *De Interpretatione* (Ch. 7, and cf. Ross, p. 3). Bochenski has called this theory "Aristotle's second logic" because it was apparently developed after the relatively immature logic of *Topics* and *Sophistical Refutations* but before the rather complicated theory of modal logic appearing mainly in chapters 3 and 8 through 22 of *Prior Analytics*. On the basis of our own investigations we have come to accept the essential correctness of Bocheński's chronology and classification of the *Organon* (Bochenski, p. 43; Lukasiewicz, p. 133; Tredennick, p. 185)[1].

Although the theory is rather succintly stated and developed (in the space of five chapters), the system of logic envisaged by it is discussed at some length and detail throughout the first book of *Prior Analytics* (esp. chapters, 7, 23 through 30, 42 and 45) and it is presupposed (or applied) in the first book of *Posterior Analytics*. Book II of *Prior Analytics* is irrelevant to this study and contains doctrines which may be incompatible with those of Book I.

1.1 *Theories of Deduction Distinguished From Axiomatic Sciences.* We agree with Ross (p. 6), Scholz (p. 3) and many others that the theory of the categorical syllogisms is a logical theory concerned in part with deductive reasoning (as this term is normally understood). Because a recent challenge to this view has gained wide popularity (Lukasiewicz, preface to 2nd ed.), a short discussion of the differences between a theory of deduction (either "axiomatic" or "natural") and an axiomatic science is necessary.

A theory of deduction puts forth a number of principles (logical axioms and rules of inferences) used in the construction of deductive proofs of conclusions from premises. All principles of a theory of deduction are necessarily metalinguistic — they describe certain constructions involving object language sentences. However, a theory of deduction is one part of a theory of logic (which deals with grammar and meaning as well[2]). Theories of deduction (and, of course, deductive systems) have been classified as "natural" or "axiomatic" by means of a loose criterion based on the prominence of logical axioms as opposed to rules — the more rules the more natural, the more axioms the more axiomatic. On one extreme one finds the so-called Jaskowski-type systems which have no logical axioms and which are therefore most properly called "natural". On the other extreme there are the so-called Hilbert-type systems which employ infinitely many axioms though but one rule and which are most properly called "axiomatic". The reason for the choice of the term "natural" may be attributed to the fact that our normal reasoning seems better represented by a system in which rules predominate whereas axiomatic systems of deduction seem contrived in comparison (cf. Corcoran, "Theories", pp. 162—171).

[1] In order to avoid excessive footnotes bracketed expressions are used to refer by author (and/or by abbreviated title) and location to items in the list of references at the end of this article.

[2] These ideas are scattered throughout Church's introductory chapter, but in Schoenfield (q. v.) sections 2.4, 2.5 and 2.6 treat respectively, languages, semantic systems and deductive systems.

A science, on the other hand, deals not with reasoning but with a certain universe or domain of objects insofar as certain properties and relations are involved. For example, arithmetic deals with the universe of numbers in regard to certain properties (odd, even, prime, perfect, etc.) and relations (less than, grater than, divides, etc.). Aristotle was clear about this (*Post. An.*, I, 10, 28) and modern efforts have not obscured his insights (Church, pp. 57, 317—341). The laws of a science are all stated in the object language whose non-logical constants are interpreted as indicating the required properties and relations and whose variables are interpreted as referring to objects in the universe of discourse. From the axioms of a science other laws of the science are deduced by *logical reasoning*. Thus an axiomatic science, though not itself a logical system, presupposes a logical system for its deductions (cf. Church, pp. 57, 317). The logic which is presupposed by a given science is called the *underlying logic* of the science.

It has been traditional procedure in the presentation of an axiomatic science to leave the underlying logic implicit. For example, neither in Euclid's geometry nor in Hilbert's does one find any codification of the logical rules used in the deduction of the theorems from the axioms and definitions. It is also worth noting that even Peano's axiomatization of arithmetic and Zermelo's axiomatization of set theory were both presented originally without explicit description of the underlying logic (cf. Church, p. 57). The need to be explicit concerning the underlying logic developed late in modern logic.

1.2 *Preliminary Discussion of the Present Interpretation:* Our view is that in the above-mentioned chapters of *Prior Analytics*, Aristotle developed a logical theory which included a theory of deduction for deducing categorical conclusions from categorical premises. We further hold that the logic thus developed was treated by Aristotle as the underlying logic of the axiomatic sciences discussed in the first book of *Posterior Analytics*. The relation of the relevant parts of *Prior Analytics* to the first book of *Posterior Analytics* is largely the same as the relation of Church's chapter 4, where first order logic is developed, to the part of chapter 5 where the axiomatic science of arithmetic is developed with the preceding logic as its underlying logic. This interpretation is in accord with the traditional view (cf. Ross, p. 6 and Scholz, p. 3) which is supported by reference to the *Analytics* as a whole as well as to crucial passages in the *Prior Analytics* where Aristotle tells what he is doing (*Pr. An.*, I, 1; and cf. Ross, p. 2). In these passages Aristotle gives very general definitions, in fact, ones which may seem to have more generality than he ever uses (cf. Ross, p. 35).

In this article the term 'syllogism' is not restricted to arguments having only two premises. Indeed, were this the case either here or throughout the Aristotelian corpus, then the whole discussion would amount to an elaborate triviality. That Aristotle did not so restrict his usage *throughout* is suggested by the form of his definition of syllogism (24b 19—21), by his statement that every demonstration is a syllogism (25b 27—31), by the content of Chapter 23 of *Prior Analytics* I, and by several other circumstances to be mentioned below. Unmistakable evidence that Aristotle applied the term in cases of more than two premises is found in *Prior Analytics* II, 17, 18 and 19 (esp. 65b 14, 66a 18 and 66b 2). However, it is equally clear that in many places Aristotle does restrict the term to the two-premise case. It may be possible to explain Aristotle's emphasis on two-premise syllogisms by means of reference to Aristotle's discovery (*Pr. An.*, I, 23) that *if* all two-premise syllogisms are deducible in his system *then* all syllogisms without restriction are so deducible. As mentioned above, in this article the term has the more general sense. Thus sorites are syllogisms (but, of course, enthymemes are not).

The *Analytics* as a whole forms a treatise on scientific knowledge (24a, 25b 28—31). On Aristotle's view every item of scientific knowledge is either known in itself by experience (or some other non-deductive method) or else it is deduced from items known in themselves (*Post. An.*, passim, esp. II, 19). The *Posterior Analytics* deals with the acquisition and deductive organization of scientific knowledge. It is the earliest general treatise on the axiomatic method[3] in sciences. The *Prior Analytics*, on the other hand, develops the underlying logic used in the inference of deductively known scientific propositions from those known in themselves. (But the logic of the *Prior Analytics* is not designed solely for such use; cf., e.g., 53b 4—11; Kneale and Kneale, p. 24.)

According to Aristotle's view, once the first principles have been discovered all subsequent knowledge is gained by means of "demonstrative syllogisms", syllogisms having antecedently known premises, and it is only demonstrative syllogisms which lead to "new" knowledge (*Post. An.*, I, 2). However, the knowledge thus gained is in a sense not "new" because it is already implicit in the premises (*Post. An.*, I, 1).

According to more recent terminology (Mates, *Elementary Logic*, p. 3) a *premise-conclusion argument* (P-c argument) is simply a set of sentences called the *premises* together with a single sentence called the *conclusion*. Of course the conclusion need not follow from the premises, but if it does then the argument is said to be *valid*. If the conclusion does not follow the argument is *invalid*. It is obvious that even a valid argument with known premises does not *prove anything* — one is not expected to come to know the conclusion by reading the argument because there is no reasoning expressed in a P-c argument. For example, take the premises to be the axioms and definitions in geometry and take the conclusion to be any complicated theorem which actually follows. Such a valid argument, far from demonstrating anything, is the very kind of thing which needs "demonstrating". *In "demonstrating" the validity of an argument one adds more sentences until one has constructed a chain of reasoning proceeding from the premises and ending with the conclusion.* The result of such a construction is called a *deductive argument* (premises, conclusion, plus a chain of reasoning) or, more briefly, a *deduction*. If the reasoning in a deduction actually shows that the conclusion follows from the premises it is said to be *sound*, otherwise *unsound*. Given this terminology we can say that by *perfect syllogism* Aristotle meant precisely what we mean by *sound deduction* and that Aristotle understood the term *syllogism* to include both valid P-c arguments and sound deductions[4] (cf. 24b 19—32). For Aristotle an invalid premise-conclusion argument is not a syllogism at all (cf. Rose, pp. 27—28). In an imperfect syllogism the conclusion follows but it is not evident that it does. An imperfect syllogism is "potentially prefect" (27a 2, 28a 16, 41b 33 and Patzig, p. 46) and it is made perfect by adding more propositions which express a chain of reasoning from the premises to the conclusion (24b 22—25, 28a 1—10, 29a 15, passim). Thus a demonstrative syllogism for Aristotle is a sound deduction with antecedently known premises (71b 9—24, 72a 5, passim).

[3] On the basis of the best evidence of the respective dates of the *Analytics* (Ross, p. 23) and Euclid's *Elements* (Heath, pp. 1, 2) one can infer that the former was written in the neighborhood of fifty years before the latter. The lives of the two authors probably overlapped; Aristotle is known to have been teaching in Athens from 334 until 323 (Edel, pp. 40, 41) and it is probable both that Euclid received his mathematical training from Aristotle's contemporaries and that he flourished c. 300 (Heath, p. 2). In any case from internal evidence Ross (p. 56) has inferred that Euclid was probably influenced by the *Analytics*. Indeed, some scholarship on the *Elements* makes important use of Aristotle's theory of the axiomatic organization of science (cf. Heath, pp. 117—124). However, it should be admitted that Hilbert's geometry (q. v.) is much more in accord with Aristotle's principles than is Euclid's. For example, Hilbert leaves some terms "undefined" and he states his universe of discourse at the outset whereas Euclid fails on both of these points which are already clear Aristotelian requirements.

[4] Aristotle may have included deductive arguments which would be sound were certain intermediate steps added, cf. Section 5.1 below.

That "a demonstrative syllogism", for Aristotle, is not simply a valid P-c argument with appropriately known premises is already obvious from his view that such syllogisms are productive of knowledge and conviction (*ibid.*, 73a 21; Ross, pp. 508, 517; also cf. Church, p. 53). *A fortiori*, a syllogism cannot be a single sentence of a certain kind, as other interpreters have suggested (see below).

Aristotle is quite clear throughout that treatment of scientific knowledge presupposes a treatment of syllogisms (in particular, of perfect syllogisms). In order to be able to produce demonstrative syllogisms one must be able to reason deductively, i. e., to produce perfect syllogisms. Demonstration is a kind of syllogism but not vice versa (25b 26—31, 71b 22—24). According to our view outlined above, Aristotle's syllogistic includes a theory of deduction which, in his terminology, is nothing more than a theory of perfecting syllogisms. More specifically and in more modern parlance, Aristotle's syllogistic includes a *natural deduction system* by means of which categorical conclusions are deduced from categorical premises. The system countenances two types of deductions (direct and indirect) and, except for "conversions", each application of a rule of inference is (literally) a first figure syllogism. Moreover, as will be clear below, *Aristotle's theory of deduction* is fundamental in the sense that it *presupposes no other logic*, not even propositional logic[5]. It also turns out that the Aristotelian system (cf. Section 5 below) is complete in the sense that every valid P-c argument composed of categorical sentences can be "demonstrated" to be valid by means of a formal deduction in the system. In Aristotelian terminology this means that every imperfect syllogism can be perfected by Aristotelian methods.

As will become clear below in section 4, our interpretation is able to account for the correctness of certain Aristotelian doctrines which previous scholars have had to adjudge incorrect. For example, both Lukasiewicz (p. 57) and Patzig (p. 133) agree that Aristotle believed that all deductive reasoning is carried out by means of syllogisms, i. e., that imperfect syllogisms are perfected by means of perfect syllogisms, but they also hold that Aristotle was wrong in this belief (Lukasiewicz, p. 44; Patzig, pp. 135). Rose (p. 55) has wondered how one syllogism can be used to prove another but he

[5] This will account somewhat for the otherwise inexplicable fact already noted by Lukasiewicz (p. 49) and others that there are few passages in the Aristotelian corpus which could be construed as indicating an awareness of propositional logic.

did not make the mistake of disagreeing with Aristotle's view. Indeed, in the light of our own research one can see that Rose was very close (p. 53) to answering his own question. We quote in part:

> We have seen how Aristotle establishes the validity of ... imperfect [syllogisms] ... This amounts to presenting an extended argument with the premises of the imperfect [syllogism] ... as ... premises ... using several intermediate steps, ... finally reaching as the ultimate conclusion the conclusion of the imperfect [syllogism] ... being established. A natural reaction ... is to think of the first figure [syllogisms] ... as axioms and the imperfect [syllogisms] ... as theorems and to ask to what extent Aristotle is dealing with a formal deductive system.

This would be natural indeed to someone not concerned with "natural" formal deductive systems. To someone concerned with the latter, it would be natural to consider the first figure syllogisms as "applications" of rules of inference and the imperfect syllogisms as derived arguments, and to scrutinize chapters 2 and 4 (*Pr. An.*, I) in search of parts needed to complete the specification of a natural deductive system. What Rose calls "an extended argument" is simply a deduction or, in Aristotle's terms, a discourse got by perfecting an imperfect syllogism. Rose had already seen the relevance of pointing out (p. 10) the fact that the term "syllogism" had been in common use in the sense "mathematical computation". One would not normally apply the term "computation" to mere data and answer reported in the form of an equation, e. g. (330 + 1955 = 2285). It would seem that the "*sine qua non*" of a computation would be the intermediate steps and one might be inclined to call the mere data-plus-answer complex an "imperfect computation" or a "potential computation". A "perfect" or "completed" computation would then be the entire complex of data, answer and intermediate steps. At one point Patzig seems to have been closer to our view than Rose. We quote from Patzig (p. 135) who sometimes uses 'argument' for "syllogism".

> ... the odd locution "a potential argument" (synonymous with "imperfect argument" ...) which, as was shown, properly means "a potentially *perfect* argument" ... has no clear sense unless we assume that Aristotle intended to state a procedure by which 'actual' syllogisms could be produced from these 'potential' ones, i. e., actually evident syllogisms produced from potentially evident ones.

Although Rose seems to have missed our view by failing to consider the possibility of a natural deduction system in Aristotle, Patzig was diverted in less subtle ways as well. In the first place Patzig uncritically accepted the false conclusion of previous interpreters that all perfect syllogisms are in the first figure and thus arrives at the strange view that imperfect syllogisms are "as it were disguised first figure syllogisms" (loc. cit.). Secondly, and surprisingly, Patzig (p. 136) seems to be unaware of the distinction between a valid P-c argument and a sound deduction having the same premises and conclusion.

1.3 *The Lukasiewicz View and Its Inadequacies:* In order to contrast our view with the Lukasiewicz view it is useful to represent categorical statements with a notation which is mnemonic for readers of twentieth century English.

Amd	*A*ll m are d.
Smd	*S*ome m is d.
Nmd	*N*o m is d.
$md	Some m is *not* d.

Lukasiewicz holds that Aristotle's theory of syllogistic is an axiomatic science which presupposes a theory of deduction unknown to Aristotle (pp. 14, 15, 49). The universe of the Lukasiewicz science is the class of secondary substances (man, dog, animal, etc.) and the relevant relations are those indicated above by A, N, S, and $, i. e., the relations of inclusion, disjointness, partial inclusion and partial non-inclusion respectively (pp. 14, 15). Accordingly, Lukasiewicz understands Aristotle's schematic letters (alpha, beta, gamma, mu, nu, xi, pi, rho and sigma) as variables ranging over the class of secondary substances and he takes A, N, S and $ as non-logical constants *(ibid.)*. Some of the axioms of the Lukasiewicz science correspond to Aristotelian syllogisms stated as single sentences (not as arguments) and generalized with respect to the schematic letters (see Mates, *op. cit.*, p. 178). For example the argument scheme

All Z are Y
All X are Z
So All X are Y

corresponds to the following sort of axiom in the Lukasiewicz system

$\forall xyz$ ([Azy & Axz] > Axy).

The Lukasiewicz view is ingenious and his book represents a wealth of intricate, insightful and useful scholarship. Indeed it is worth emphasizing that without his book the present work could not have been done in even twice the time. Despite the value of the book, its viewpoint must be adjudged incorrect for the following reasons. In the first place, as mentioned above, Lukasiewicz (p. 44) does not take seriously Aristotle's own claims that imperfect syllogisms are proved by means syllogisms. He even says that Aristotle was wrong in this claim. In the second place, he completely overlooks the many passages in which Aristotle speaks of perfecting imperfect syllogisms (e. g. *Pr. An.*, 27a 17, 29a 15, 29a 30, 29b 1–25). Lukasiewicz (p. 43) understands "perfect syllogism" to indicate only the [valid] syllogisms in the first figure. This leads him to neglect the crucial fact that chapters 4, 5 and 6 of *Prior Analytics* deal with Aristotle's theory of deduction. Thirdly, Aristotle is clear in *Posterior Analytics* (I, 10) about the nature of axiomatic sciences and he nowhere mentions syllogistic as a science (Ross, p. 24), but Lukasiewicz still wants to regard the syllogistic as such. (Lukasiewicz does seem uneasy (p. 44) about the fact that Aristotle does not call his basic syllogisms "axioms".) Indeed, as has already been noticed by Scholz (p. 6), Aristotle could not have regarded the syllogistic as a science because to do so he would have had to take the syllogistic as its own underlying logic. Again, were the Lukasiewicz system to be a science in Aristotle's terms, then its universe of discourse would have to form a genus (e. g., *Post. An.*, I, 28) — but Aristotle nowhere mentions the class of secondary substances as such. Indeed, on reading the tenth chapter of the *Posterior Analytics* one would expect that *if* the syllogistic were a science *then* its genus would be mentioned on the first page of *Prior Analytics*. Not only does Aristotle fail to indicate the subject matter required by the Lukasiewicz view, he even indicates a different one — viz. demonstration — but not as a genus (*Pr. An.*, first sentence)[6]. In the fourth place, if the syllogistic were an axiomatic science and A, N, S and $ were relational terms, as Lukasiewicz must have it, then awkward questions ensue. (a) Why are these not mentioned in *Categories*, Chapter 7, where relations are discussed? Are they unimportant relations? (b) Why did Aristotle

[6] In a doubly remarkable passage (p. 13) Lukasiewicz claims that Aristotle did not reveal the object of his logical theory. It is not difficult to see that Likasiewicz is correct in saying that Aristotle nowhere admits to the purpose which Lukasiewicz imputes to him. However, other scholars have had no difficulty in discovering passages which do reveal Aristotle's true purpose (cf. Ross, pp. 2, 24, 288; Kneale and Kneale, p. 24).

not seek for axioms the simplest and most obvious of the propositions involving these relations, i. e., "Everything is predicated of all of itself" and "Everything is predicated of some of itself". In fact Aristotle seems to have deliberately avoided self-predication although he surely knew of several reflexive *relations* (identity, equality, congruence). Lukasiewicz counts this as an oversight and adds the first of the above self-predications as a "new" axiom. In connection with the above questions we may also note that the relations needed in the Lukasiewicz science are of a different "logical type" from those considered by Aristotle in *Categories* — the former relate secondary substances whereas the latter relate primary substances. Fifth, if indeed Aristotle is axiomatizing a system of true relational sentences on a par with the system of relational sentences which characterize the ordering of the numbers, as Lukasiewicz must and does claim (pp. 14, 15, 73), then again awkward questions ensue. (a) Why is there no discussion anywhere in the second logic of the general topic of relational sentences? (b) Why does Aristotle axiomatize only one such system? The "theory of congruence" (equivalenze relations) and the "theory of the ordering of numbers" (linear order) are obvious, similar systems and nowhere does Aristotle even hint at the analogies. Sixth, as Lukasiewicz himself implicitly recognizes in a section called "Theory of Deduction" (pp. 79—82), if the theory of syllogisms is understood as an axiomatic science then, as indicated above, it would presuppose an underlying logic (which Lukasiewicz supplies). But all indications in the Aristotelian corpus suggest not only that Aristotle regarded the theory of syllogistic as the most fundamental sort of reasoning (Kneale and Kneale, p. 44, and even Lukasiewicz, p. 57) but also that he regarded its logic as *the* underlying logic of all axiomatic sciences[7]. Lukasiewicz himself says, "It seems that Aristotle did not suspect the existence of a system of logic besides his theory of the syllogism" (p. 49). Seventh, the view that syllogisms are sentences of a certain kind and not extended discourses is incompatible with Aristotle's occasional but essential reference to *ostensive* syllogisms and to *per impossibile* syllogisms (41a 30—40, 45a 23, 65b 16, e. g.). These references imply that some syllogisms have internal structure even over and above "premises" and "conclusion". Finally, although Lukasiewicz gives a mathematically precise

[7] This point has already been made by Kneale and Kneale (pp. 80—81) who point out further difficulties with Lukasiewicz's interpretation. For yet further sensitive criticism see Austin's review and also Iverson, pp. 35—36.

system which obtains and rejects "laws" corresponding to those which Aristotle obtains and rejects, the Lukasiewicz system neither justifies nor accounts for the methods that Aristotle used. Our contention is that the method is what Aristotle regarded as most important. In this connection, besides the systematic results there are metamathematical results obtained by Aristotle using methods which are clearly accounted for by the present interpretation but which must remain a mystery on the Lukasiewicz interpretation[8].

As will be shown below, Aristotle's theory of deduction presents a self-sufficient natural deduction system which presupposes no other logic.

2. *The Language L:* In the second logic Aristotle dealt only with propositions of the above four forms and only with those whose subject and predicate terms are different. In place of the "terms" we take a non-empty set U of characters which we call *non-logical constants* or *content* words. The characters A, N, S, and $ are *logical constants*. L is defined as the set of all strings formed by prefixing a logical constant to the left of a string of two (distinct) non-logical constants.

The omission of sentences containing only one term (Axx, Nxx, Sxx, $xx) may require comment. In the passages comprising the second logic there are no appearances of such "self-predications". The only appearance of such in *Analytics* is in the second book of *Prior Analytics* (63b 40—64b 25) which was written later.

[8] Although we have no interest in giving an account of how Lukasiewicz may have arrived at his view, it may be of interest to some readers to note the possibility that Lukasiewicz was guided in his research by certain attitudes and preferences not shared by Aristotle. The Lukasiewicz book seems to indicate the following. (1) Lukasiewicz preferred to consider logic as concerned more with truth than with either logical consequence or deduction (e. g., pp. 20, 81). (2) He understands "inference" in such a way that correctness of inference depends on starting with true premises (e. g., p. 55). (3) He feels that propositional logic is somehow objectively more fundamental than quantificational or syllogistic logic (e. g., pp. 47, 79). (4) He tends to concentrate his attention on axiomatic deductive systems to the neglect of natural systems. (5) He tends to underemphasize the differences between axiomatic deductive systems and axiomatic sciences. (6) He places the theory of the syllogism on a par with a certain branch of pure mathematics (pp. 14, 15, 73) and he believes that logic has no special relation to thought (pp. 14, 15). Indeed, he seems to fear that talk of logic as a study of reasoning necessarily involves some sort of psychologistic view of logic. (7) He believes that content words or non-logical constants cannot be introduced into logic (pp. 72, 96). The Lukasiewicz attitudes are shared by several other logicians notably, in this context, by Bochenski (q. v.). It may not be possible to argue in an objective way that the above attitudes are incorrect but one can say with certainty that they were not shared by Aristotle.

Thus, it would seem that inclusion of self-predications would be an interpolation and not necessarily a rectification of an oversight (as Lukasiewicz claims, p. 45). The system works out perfectly well without them. Besides, one scholar has presented a rather involved argument to the effect that Aristotle deliberately excluded them (J. Mulhern, pp. 111—115). Moreover, absence of self-predications may help explain the absence of a doctrine of logically necessary truths in Aristotle: Axx and Sxx are the only sentences of the above sort which are true under all interpretations.

Although Aristotle does not seem to have excluded all but common nouns and adjectives from the set of non-logical constants of the language of a science (but cf. 43a 25—44), his "second logic" does not explicitly handle anything else. The omission of relatives is notorious. The rules of the "second logic" (see below) do not suffice to deduce 'some man is wise' from 'Pittacus is a man' and 'Pittacus is wise'. So proper nouns play a muted role. In addition, the so-called indefinite propositions also seem to be relegated to a secondary status. Inclusion of proper nouns, relatives or indefinite propositions would imply *only additions* to our model; no other changes would be required. Thus our system seems to be a subsystem, at worst, of any faithful analogue of Aristotle's system[9].

It is necessary at this point to define a few concepts which depend only on the language and which are independent of semantical notions to be presented below. Define P + s to be the result of adjoining the sentence s with the set P. Define C(s) to be the Aristotelian contradictory of s.

$$C(Axy) = \$xy$$
$$C(\$xy) = Axy$$
$$C(Nxy) = Sxy$$
$$C(Sxy) = Nxy.$$

It may be worth noting that there is no truth-functional sign of negation in the system. This role is played by the notion of contradictory. Note also that arguments are composed of sentences[10] of L.

3. *The Semantic System S:* Aristotle seems to have regarded the truth-values of the non-modal sentences as being determined extensionally (*Pr. An.*, 24a 26ff). Accordingly we define an *interpretation* to be an assignment of a non-empty set to each non-logical constant in U and we define the truth-value of a sentence under an interpretation in accord with the normal understanding of Aristotle's words. We require the extension of each term to be non-empty

[9] Exclusion of proper names, relatives, and indefinite propositions is based more on a reading of the second logic as a whole rather than on specific passages (cf. 43a 25—40).

[10] Rose (p. 39) has criticized the Lukasiewicz view that no syllogisms with content words are found in the Aristotelian corpus. Our view goes beyond in holding that *all* Aristotelian syllogisms have content words, i. e., that Aristotle nowhere refers to argument forms or propositional functions as such. All apparent exceptions are best understood as metalinguistic reference to "concrete syllogisms". This view is in substantial agreement with the view implied by Rose at least at one place (p. 25).

because this gives the best fit with Aristotle's inferences and because he seems to require that each meaningful common noun subsume at least one individual (*Categories*, 2a 34—b7[11]. [Some readers conversant with modern logic may notice the absence of a concept of universe of discourse. This does not appear in Aristotle and it plays no role in our development. Besides, addition of it would entail no mathematical consequences[12].]

It is necessary here to define a few semantic notions. As usual, a *true interpretation* of a set P of sentences is simply an interpretation which makes every sentence in P true. If all true interpretations of P make a sentence s true, then P is said to *imply* s and s is said to be a *logical consequence*[13] of P. When P implies s the argument (P, s) is *valid*, otherwise *invalid*. A *counter interpretation* of an argument (P, s) is a true interpretation of P which makes s false.

By reference to the definitions just given one can show the following important semantic principle — which is suggested by Aristotle's "contrasting instances" method of establishing inva-

[11] This would explain the so-called existential import of A and N sentences. As far as we have been able to determine, this is the first clear *theoretical account* of existential import based on textual material.

[12] Jaskowski (q. v.) claims that the universal set and the null set are excluded but he gives no textual grounds. There are, however, some passages (e. g., 98b 22) which imply that the class of all existent individuals is not a genus. In subsequent developments of "Aristotelian logic" which include "negative terms", exclusion of the universe must be maintained to save exclusion of the null set.

[13] This is the mathematical analogue of the classical notion of logical consequence which is clearly presupposed in traditional work on so-called "postulate theory". It is important to notice that we have offered only a mathematical analogy of the concept and not a definition of the concept itself. The basic idea is this. Each interpretation represents a "possible world". To say that it is logically impossible for the premises to be true and the conclusion false is to say that there is no possible world in which the premises actually are true and the conclusion actually is false. The analogue, therefore, is that no true interpretation of the premises makes the conclusion false. Church (p. 325) attributes this mathematical analogy of logical consequence to Tarski (pp. 409—420) but Tarski's notion of true interpretation (model) seems too narrow (at best too vague) in that no mention of alternative universes of discourse is made or implied. In fact the limited Tarskian notion seem to have been already known even before 1932 by Lewis and Langford (p. 342), to whom, incidentally, I am indebted for the terms "interpretation" and "true interpretation", which seem heuristically superior to the Tarskian terms "sequence" and "model" the latter of which has engendered category mistakes — a "model of set of sentences" in the Tarskian sense is by no means a model (in any ordinary sense) of a set sentences.

lidity of arguments[14] (below and cf. Ross, pp. 28, 292—313 and Rose, pp. 37—52).

(3.0) Principle of Counter Interpretations: A premise-conclusion argument is invalid if and only if it has a counter interpretation.

The import of this principle is that whenever an argument is invalid it is possible to interpret its content words in such a way as to make the premises true and the conclusion false. It is worth remembering that the independence of the parallel postulate from the other "axioms" of geometry was established by construction of a counter interpretation — an interpretation of the language of geometry in which the other axioms were true and the parallel postulate false.

Perhaps the most important semantic principle underlying Aristotle's logical work is the following, also deducible from the above definitions.

(3.1) Principle of Form: An argument is valid if and only if every argument in the same form is also valid.

Aristotle tacitly employed this principle throughout the *Prior Analytics* in two ways. First, in order to establish the validity of all arguments in the same form as a given argument he establishes the validity of an arbitrary argument in the question (i. e. he establishes the validity of an argument leaving its content words unspecified). Second, in order to establish the invalidity of all arguments in the same form as a given argument he produces a specific argument in the required form for which the intended interpretation is a counter interpretation. The latter, of course, is the method of "contrasting instances." In neither of these operations, which are applied repeatedly by Aristotle, is it necessary to postulate either alternative interpretations or argument forms (over and above individual arguments).

[14] The method of "contrasting instances" is a fundamental discovery in logic which may not yet be fully appreciated in its historical context. Because Lukasiewicz (p. 71) misconstrued the Aristotelian framework he said that modern logic does not employ this method. It is obvious however that all modern independence (invalidity) results from Hilbert (pp. 30—36) to Cohen (see Cohen and Hersh) are based on developments of this method. Indeed there were essentially no systematic investigations of questions of invalidity from the time of Aristotle until Beltrami's famous demonstration of the invalidity of the argument whose premises are the axioms of geometry less the parallel postulate and whose conclusion is the parallel postulate itself (Heath, p. 219). Although there is not a single invalidity result in the *Port Royal Logic* or in Boole's work, for example, modern logic is almost characterizable by its wealth of such results — all harking back to Aristotle's method of contrasting instances.

4. *The Deductive System D:* Reiterating what was said above concerning theories of deduction, we observe that such a theory is intended to specify the steps of deductive reasoning performed in order to come to know that a certain proposition s follows logically from a certain set P of propositions. Aristotle's theory of deduction is his theory of perfecting syllogisms.As al ready said, our view is that a perfect syllogism is a discourse which expresses correct reasoning from premises to conclusion. In case the conclusion is immediate, nothing need be added to make clear the implication (24a 22). In case the conclusion does not follow immediately, then additional sentences must be added (24b 23, 27a 18, 28a 5, 29a 15, 29a 30, 42a 34, etc.). A valid argument by itself is only potentially perfect (27a 2, 28a 16, 41b 33) and it is "made perfect" (29a 33, 29b 5, 29b 20, 40b 19, etc.) by, so to speak, filling its interstices.

According to Aristotle's theory there are only two general methods[15] for perfecting an imperfect syllogism — either directly (ostensively) or indirectly *(per impossibile)* (e. g., 29a 30—29b 1, 40a 30, 45b 5—10, 62b 29—40, passim). In constructing a direct deduction of a conclusion from premises one interpolates new sentences by applying conversions and first figure syllogisms to previous sentences until one arrives at the conclusion. Of course, it is permissible to repeat an already obtained line. In constructing an indirect deduction of a conclusion from premises one adds to the premises, as an additional hypothesis, the contradictory of the conclusion and then one interpolates new sentences as above until both of a pair of contradictory sentences have been reached.

Our deductive system D to be defined presently is a syntactical mathematical model of Aristotle's system of deductions which we have found in his theory of perfecting syllogisms.

Definition of D

First restate the laws of conversion and perfect syllogism as rules of inference. Use the terms 'a D-conversion of a sentence' to indicate

[15] One is impressed with the sheer number of times that Aristotle alludes to the fact that there are but two methods of perfecting syllogisms — and this makes it all the more remarkable that an apparent third method occurs, the so-called method of *ecthesis*. There are two ways of explaining the discrepancy. In the first place, *ecthesis* is not a method of proof on a par with the direct and indirect methods but rather it consists in a class of rules of inference on a par with the class of conversion rules and the class of perfect syllogism rules (see below). In the second place, and more importantly, *ecthesis* is clearly extrasystematic relative to Aristotle's logical system (or systems). It is only used three times (Lukasiewicz, p. 59): once in a clearly metalogical passage (25a 17) and twice redundantly (28a 23, 28b 14).

the result of applying one of the three conversion rules to it. Use the terms 'D-inference from two sentences' to indicate the result of applying one of the perfect syllogism rules to the two sentences.

A *direct deduction in D of c from P* is defined to be a finite list of sentences ending with c, beginning with all or some of the sentences in P and such that each subsequent line (after those in P) is either (a) a repetition of a previous line, (b) a D-conversion of a previous line or (c) a D-inference from two previous lines.

An *indirect deduction in D of c from P* is defined to be a finite list of sentences ending in a contradictory pair, beginning with a list of all or some of the sentences in P followed by the contradictory of c, and such that each subsequent additional line (after the contradictory of c) is either (a) a repetition of a previous line, (b) a D-conversion of a previous line or (c) a D-inference from two previous lines.

All examples of deductions will be annotated according to the following scheme. (1) Premises will be prefixed by '+' so that '+ Axy' can be read "assume Axy as a premise." (2) After the premises are put down we interject the conclusion prefixed by '?' so that '?Axy' can be read "we want to show why Axy follows". (3) The hypothesis of an indirect *(reductio)* deduction is prefixed by 'h' so that 'hAxy' can be read "suppose Axy for purposes of reasoning". (4) A line entered by repetition is prefixed by 'a' so that 'aAxy' can be read "we have already accepted Axy". (5) Lines entered by conversion and syllogistic inference are prefixed by 'c' and 's' respectively. (6) Finally, the last line of an indirect deduction has 'B' prefixed to its other annotation so that 'BaAxy' can be read "but we have already accepted Axy", etc. We define an *annotated deduction in D* to be a deduction in D annotated according to the above scheme. In accordance with now standard practice we say that c is *deducible from* P in D to mean that there is a deduction of c from P in the system D. It is also sometimes convenient to use the locution "the argument (P, c) is deducible in D". This completes the definition of the deductive system D.

The significance of D is as follows. We claim that D is a faithful mathematical model of Aristotle's theory of perfect syllogisms in the sense that every perfect syllogism (in Aristotle's sense) corresponds in a direct and obvious way to a deduction in D. Thus what can be added to an imperfect syllogism to render it perfect corresponds to what can be "added" to a valid argument to produce a deduction in D. In the case of a direct deduction the "space" between the premises and conclusion is filled up in accordance with the given rules.

In order to establish these claims as well as they can be established (taking account of the vague nature of the data), the reader may go through the deductions presented by Aristotle and convince himself that each may be faithfully represented in D. We have given four examples below; three of direct (or ostensive) proofs and one of an indirect (or *per impossibile*) proof. The others raise no problems. Below we reproduce two of Aristotle's deductions (27a 5—15; Rose, p. 34), each followed by the corresponding annotated deductions in D.

(1)

Let M be predicated of no N
and of all X.
(conclusion omitted in text)
Then since the negative premise converts N belongs to no M.
But it was supposed that M belongs to all X.
Therefore, N will belong to no X.

+ Nnm
+ Axm
(? Nxn)
cNmn
aAxm
sNxn

(2)

Again, if M belongs to all N
and to no X,
X will belong to no N.
For if M belongs to no X,
X belongs to no M.
But M belonged to all N.
Therefore, X will belong to no N...

+ Anm
+ Nxm
?Nnx
aNxm
cNmx
aAnm
sNnx

Below we reproduce Aristotle's words (28b 8—12) followed by the corresponding annotated deduction in D.

(3)

For if R belongs to all S,
P to some S,
Since the affirmative P must belong to some R.
statement is convertible S will belong to some P:
Consequently since R belongs to all S,
and S to some P,
R must also belong to some P:
Therefore P must belong to some R.

+ Asr
+ Ssp
?Srp
cSps
aAsr
aSps
sSpr
cSrp

To exemplify an indirect deduction we do the same for 28b 17—20.

(4)

For if R belongs to all S,
but P does not belong to some S,
it is necessary that P does not belong to some R.
For if P belongs to all R,
and R belongs to all S,
then P will belong to all S:
but we assumed that it did not.

+ Asr
+ $sp
? $rp
hArp
aAsr
sAsp
Ba$sp

Readers can verify the following (by "translating" Aristotle's proofs of the syllogisms he proved, using ingenuity in the other cases): All valid arguments in any of the four traditional figures are deducible in D.

4.1 *Some Metamathematical Results in Aristotle:* Generally speaking, a metamathematical result is a mathematical result concerning the structure of a logical or mathematical system. Such results can also be called metasystematic. The point of the terminology is to distinguish the results codified by the system from results concerning the system itself. The latter would necessarily be stated in the metalanguage and codified in a metasystem. It is also convenient (but sometimes artificial) to distinguish intrasystematic and intersystematic results. The former would concern mathematical relations among parts of the given system whereas the latter would concern mathematical relationships between the given system and another system. The artificiality arises when the "other" system is actually a part of the given system.

There are several metasystematic results in the "second logic", none of which have been given adequate explanation previously. We regard an explanation of an Aristotelian metasystematic result to be adequate only when it accounts for the way in which Aristotle obtained the result.

4.1.1 *Aristotle's Second Deductive System D2.* As already indicated above the first five chapters of the "second logic" (*Pr. An.*, I, 1, 2, 4, 5, 6) include a general introductory chapter, two chapters presenting the system and dealing with the first figure and two chapters which present deductions for the valid arguments in the second and third figures[16]. The next chapter (7) is perhaps the first substantial metasystematic chapter in the history of logic.

The first interesting metasystematic passage begins at 29a 30 and merely summarizes the work of the preceding three chapters. It reads as follows:

It is clear too that all the imperfect syllogisms are made perfect by means of the first figure. All are brought to conclusion either ostensively or *per impossibile.*

From the context it is obvious that by "all" Aristotle means "all second and third figure" syllogisms. Shortly thereafter begins a long passage (29b 1—25) which states and proves a substantial metasystematic result. We quote.

It is possible to reduce all syllogisms to the universal syllogisms in the first figure.

Again "all" is used as above, "reduce to" *here* means "deduce by means of", and "universal syllogism" means "one having an N or A

[16] For an interesting solution to "the mystery of the fourth figure" (the problem of explaining why Aristotle seemed to stop at the third figure) see Rose, *Aristotle's Syllogistic*, pp. 57—79.

conclusion". What Aristotle has claimed is that all of the syllogisms previously proved can be established by means of deductions which do not involve the "particular" perfect syllogistic rules. Aristotle goes on to explain in concise, general but mathematically precise terms exactly how one can construct the twelve particular deductions which would substantiate the claim. Anyone can follow Aristotle's directions and thereby construct the twelve formal deductions using our system D.

In regard to the validity of the present interpretation, these facts are significant. Not only have we accounted for the content of Aristotle's discovery but we have also been able to reproduce exactly the methods that he used to obtain them. Nothing of this sort has been attempted in previous interpretations (cf. Lukasiewicz, p. 45).

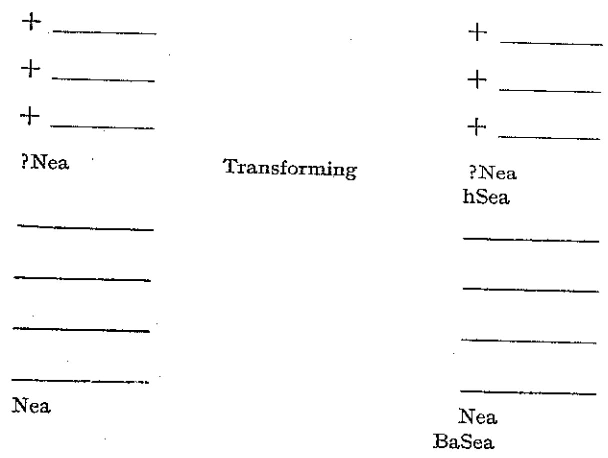
Let D2 indicate the deductive system obtained by deleting the "particular" rules from D. Aristotle's metaproof shows that the syllogisms formerly deduced in D can also be deduced in D2. On the basis of the next chapter (*Pr. An.*, I, 23) of the "second logic" (cf. Bochenski, p. 43; Lukasiewicz, p. 133; Tredennick, p. 185) it becomes clear that Aristotle thinks that he has shown that *every* syllogism deducible in D can also be deduced in D2. On reading the relevant passages (29b 1—25) it is obvious that Aristotle has *not* proved the result. However, Aristotle's claim *is* correct (Corcoran, "Completeness"). But regardless of the correctness of his proof, one must credit Aristotle with conception of the first significant hypothesis in proof theory.

4.1.2. *Redundancy of Direct Deductions:* Among indirect deductions it is interesting to distinguish two subclasses on the basis of the role of the added hypothesis. Let us call an indirect deduction *normal* if a rule of inference is applied to the added hypothesis and *abnormal* otherwise. In many of the abnormal cases, one reasons from the premises, ignoring the added hypothesis until the desired conclusion is reached, and then one notes "but we have assumed the contradictory".

Aristotle begins chapter 29 (*Pr. An.*, I) by stating that whatever can be proved directly can also be proved indirectly. He then gives two examples of normal indirect deductions for syllogisms he has already deduced directly. Shortly thereafter (45b 1—5) he says,

"Again if it has been proved by an ostensive syllogism that A belongs to no E, assume that A belongs to some E and it will be proved *per impossibile* to belong to no E. Similarly with the rest."

The first sentence means that by interpolating the added hypothesis Sea into a direct deduction of Nea one transforms it to an indirect deduction of the same conclusion. See the diagram below.



The second quoted sentence is meant to indicate that the same result holds regardless of the form of the conclusion. In other words Aristotle has made clear the fact that whatever can be deduced by a direct deduction can also be deduced by an abnormal indirect deduction, i. e., that direct deductions are redundant from the point of view of the system as a whole[17].

We feel that this is additional evidence that Aristotle was actually and self-consciously studying interrelation among deductions — exactly as is done Hilbert's "proof theory" (e. g., cf. van Heijenoort, p. 137).

4.2 *Indirect Deductions or A Reductio Rule?* Aristotle considered indirect reasoning to be a certain style of deduction. After the premises are set down one adds the contradictory of what is to be proved and then proceeds by "direct reasoning" to each of a pair of contradictory sentences. Imagine, however, the following situation: one begins an indirect deduction as usual and immediately gets bogged down. Then one sees that there is a pair of contradictories, say s and C(s), such that (1) s can be got from what is already assumed by indirect reasoning and (2) C(s) can be got from s together with what is already assumed by direct reasoning.

In a normal context of mathematics there would be no problem — the outlined strategy would be carried out without a second thought. In fact the second situation is precisely what is involved in a common proof of "Russell's Theorem" (no set contains exactly the sets which do not contain themselves). It involves using *reductio* reasoning as a structural rule of inference (cf., e. g., Corcoran, "Theories", pp. 162ff.). The trouble is that the strategy requires ability to add a *second* hypothesis and this is not countenanced by the Aristotelian system (*Pr. An.*, I 23, 41a 33—36).

The salient differences between a system with indirect deductions and a system with a *reductio* rule are the following. In the case of indirect deductions one can add but one additional hypothesis (viz. the contradictory of the conclusion to be reached) and one cannot in general use an indirectly obtained conclusion later on in a deduction — once the indirectly obtained conclusion is reached the indirect deduction is, by definition, finished. An indirectly obtained conclusion is never written as such in the deduction. In the case of the *reductio* rule one can add as many additional hypotheses as desired and once an indirectly obtained conclusion is reached it is written as an intermediate conclusion usable in subsequent reasoning.

The deductive system of Jeffrey (q. v.) consists solely of indirect deductions whereas the system of Anderson and Johnstone (q. v.) has a *reductio* rule.

Metamathematically one important difference is the following. Where one has a *reductio* rule it is generally easy to prove the metamathematical result that C(d) is (indirectly) deducible from P whenever each of a pair of contradictories is separately deducible from P + d. This result can be difficult in the case where one does not have a *reductio* rule—especially when each of the pair of contradictories was reached indirectly.

In order to modify the system (or systems) to allow such "iterated or nested *reductio* strategies" one would abandon the distinction between direct and indirect deductions and in the place of the indirect deductions one would have (simply) deductions which employ one or more applications of a *reductio* rule. Statements of such *reductio* rules are in general easily obtained, but they involve several ideas which would unnecessarily complicate this article. Let us assume that D2 has been modified[18] to permit iterated and nested *reductio* deductions and let us call the new system D3.

Now we have two final points to make. In the first place it is clear that nothing is gained by adding the *reductio* rule, i. e., since D2 is known to be complete, every argument deducible in D3 is already deducible in D2. In the second place, Aristotle may well have been thinking of *reductio* as a rule of inference but either lacked the motivation to state it as such or else actually stated it as such only to have his statements deleted or modified by copyists. Third, it should be obvious both that indirect deductions are logically sound and that a sound *reductio* rule is consistent with Aristotle's writings in the second logic. In any case, it is clear that Aristotle was not confused about indirect proof[19].

5. *The Mathematical Logics I and II:* In the previous three sections we considered the components of several mathematical logics, any one of which could be taken as a reasonably faithful model of the system (or systems) of logic envisaged by Aristotle's theory (or theories) of syllogistic. The two models which we take to be especially important both have L as language and S as semantics. The first model we explicitly define is the mathematical logic I with D as its deductive system. The second is II with D2 as deductive system[20]. It is our view that I is the system most closely corresponding to Aristotle's explicit theory and that II is another system which Aristotle studied.

Concerning any mathematical logic there are two kinds of questions. In the first place, there are *internal* questions concerning the mathematical properties of the

[17] It is in the interest of accuracy that we reluctantly admit that Aristotle also *seems* to claim the converse. It is germane also to observe that although the above claim is substantiated not only by examples but also by a general formula, the converse is false.

[18] In regard to the reductio rule, the system D3 is like the one proposed by Smiley (q. v.) which was brought to my attention by Prof. Bas van Fraassen in February of 1972. The systems D, D2 and D3 had been presented in lectures at SUNY/Buffalo, the Johns Hopkins University, University of Pennsylvania, University of Montreal and Laval University all in 1971. The completeness result for D was announced at the December 1971 meeting of the Association for Symbolic Logic (Corcoran, "Natural Deduction").

[19] As an indication that this is no mean achievement one may note with Iverson (p. 36) that Lukasiewicz (p. 55) misunderstood indirect proof.

[20] Of course one should not overlook the possible historical importance of III (the logic having components L, S and D3). In this connection we have been asked whether there are deductive systems other than D, D2 and D3 implicit in the second logic. This question is confidently answered negatively, even though Patzig (p. 47) alleges to have found other systems in *Pr. An.*, Bk. I, Ch. 45. It is clear that this chapter merely investigates certain interrelationships among the three figures without raising any issues concerning alternative deductive systems. Although Aristotle speaks of "reducing" first figure syllogisms to the other figures there is no mention of "perfecting" first figure syllogisms (or any others for that matter) by means of syllogisms in the other figures. Indeed, because of Aristotle's belief that syllogisms can be perfected only through the first figure one should not expect to find any deductive systems besides those based on first figure syllogistic rules. In addition, one may note that Bochenski (p. 79) alleges to have found other deductive systems outside of the second logic in *Pr. An.* Bk. I, Ch 45 which we just discussed.

system itself. For example, we have compared the deductive system D to the semantics S by asking whether every deducible argument is valid (problem of soundness) and conversely whether every valid argument is deducible (problem of completeness). Both of these questions and all other internal questions are perfectly definite mathematical questions concerning the logic as a mathematical object. And if they are answered, then they are answered by the same means used to answer any mathematical questions — viz. by mathematical reasoning from the definitions of the systems together with the relevant mathematical laws. In the second place, there are external questions concerning the relationship of the model to things outside of itself. In our case the most interesting question is a fairly vague one — viz. how well do our models represent "the systems" treated in Aristotle's theory of the syllogism?

As the various components of the models were developed, we considered the external questions in some detail and the overall conclusion is that both systems can be used to account for many important aspects of the development of Aristotle's theory as recorded in the indicated parts of *Analytics* (see above). Moreover, neither logic adds anything to what Aristotle wrote except for giving an explicit reference to interpretations and formulating a systematic definition of formal deductions. It is especially important to notice that neither deductive system involves anything different in kind from what Aristotle explicitly used — no "new" axioms were needed and no more basic sort of reasoning was presupposed.

As far as internal questions are concerned it is obvious that both I and II are sound, i. e., that all arguments deducible in either D or D2 are valid. This is clear from section 3 above. The questions of completeness have been settled affirmatively (Corcoran, "Natural Deduction", "Completeness"), i. e., we have been able to demonstrate as a mathematical fact concerning the above logics that every argument valid according to the semantics S can be obtained by means of a formal deduction in D. Thus not only is Aristotle's logic self-sufficient in the sense of not presupposing any more basic logic, but it is also self-sufficient in the sense that no further sound rules can be added without redundancy.

5.1 *The Possibility of a Completeness Proof in Prior Analytics:* According to Bochenski's view (p. 43), in which we concur, chapter 23 follows chapter 7 in *Prior Analytics*, Bk. I. As already indicated chapter 7 shows that all syllogisms in the three figures are "perfected by means of the universal syllogisms in the first figure". Chapter 23 begins with the following words (Oxford translation by A. J. Jenkinson).

"It is clear from what has been said that the syllogisms in these figures are made perfect by means of universal syllogisms in the first figure and are reduced to them. That every syllogism without qualification can be so treated will be clear presently, when it has been proved that every syllogism is formed through one or the other of these figures."

The same chapter (41b 3—5) ends thus.

"But when this has been shown it is clear that every syllogism is perfected by means of the first figure and is reducible to the universal syllogisms in this figure."

From these passages *alone* one would naturally infer that the intermediate material contained the main part of a completeness proof for D2 which depended on a "small" unproved lemma. One would further infer that the imagined completeness proof had the following three main parts. First, it would define a new deduction system which had the syllogisms in all three figures as rules. Second, it would prove the completeness of the new sytem. Third it would show that every deduction in the new system can be transformed into a deduction in D2 having the same premises and conclusion.

Unfortunately, the text will not support this interpretation. Before considering a more adequate interpretation one can make a few historical observations. In the first place, even raising a problem of completeness seems to be a very difficult intellectual achievement. Indeed, neither Boole nor Frege nor Russell asked such questions[21]. Apparently no one stated a completeness problem[22] before it emerged naturally in connection with the underlying logic of modern Euclidean geometry in the 1920's (Corcoran, "Classical Logic", Part III) and it is probably the case that no completeness result (in this *exact* sense) was printed before 1951 (cf. Corcoran, "Theories", p. 177, for related results) although the necessary mathematical tools were available in the 1920's. In the second place, it does not seem to be the case that Aristotle was clear enough about his own semantics to understand the problem. If he had been then he could have solved the problem definitively for any finite "topical sublogic" by the same methods employed in *Prior Analytics* (I, 4, 5, 6). In fact, in these chapters he "solves" the problem for the "topical sublogic" having only three content words.

In the intervening passages of chapter 23 Aristotle seems to argue not that *every* syllogism is deducible in D2 but rather that *any syllogism deducible at all* is deducible in D2. And, as indicated in his final sentence, he does not believe that he has completed his argument. He reasons as follows. In the first place, he asserts without proof that any syllogism deducible by means of all syllogisms in the three figures is deducible in D2 (but here he is overlooking the problem of iterated reductio mentioned in section 4.3 above). In any case, granting him that hypothesis, he then argues that any syllogism deducible at all is deducible by means of the syllogisms in the three figures thus. Every deduction is either direct or hypothetical — the latter including both indirect deductions and those involving *ecthesis* (see above). He considers the direct case first. Here he argues that every direct deduction must have at least two premises as in the three figures and in the case two premises the conclusion has already been proved. Then he simply aserts that it is "the same if several middle terms should be necessary" (41a 18). In considering the hypothetical deductions he takes up indirect deductions first and observes that after the contradictory of the conclusion is also assumed one proceeds as in the direct case — concluding that the reduction to D2 is evident in this case also (41b 35 ff). Finally, he simply asserts that it is the same with the other hypothetical deductions. But the latter he has immediate misgivings about (41b 1). He leaves the proof unfinished to the extent that the nonindirect hypothetical deductions have not been completely dealt with.

[21] Mates (*Stoic Logic*, pp. 4, 81, 82, 111, 112) has argued that the Stoics believed their deductive system to be complete. But had the Aristotelian passage (from 40b 23 up to but not including 41b 1) been lost Mates would have equivalent grounds for saying that Aristotle believed his system complete. There are no grounds for thinking that the problem was raised in either case.

[22] Unfortunately, the Lukasiewicz formulation makes it possible to confuse these problems with the so-called decision problems. The two types of problems are distinct but interrelated to the extent that decidable logics are generally (but not necessarily) complete. It is hardly necessary to mention the fact that ordinary first order predicate logic is complete but not decidable (Jeffrey, pp. 195 ff; Kneale and Kneale, pp. 733—734).

6. *Conclusion:* As a kind of summary of our research we present a review of what we take to be the fundamental achievements of Aristotle's logical theory studied above. In the first place, he clearly distinguished the role of deduction from the role of experience (or intuition) in the development of scientific theories. This is exemplified by means of his strong distinction between the axioms of a science and the logical apparatus used in deducing the theorems. Today this would imply a distinction between logical and non-logical axioms, but Aristotle had no idea of logical axioms (but cf. 77a 22—25), and, indeed, he has no systematic discussion of logical truth (Axx is not even mentioned once). In the second place, Aristotle developed a natural deduction system which he exemplified and discussed accurately and at great length. Moreover, he was able to formulate fairly intricate metamathematical results relating his centralsystem to a simpler one. It is also important to notice that Aristotle's system is sound and strongly complete. In the third place, Aristotle was clear enough about logical consequence so that he was able to discover the method of counter instances for establishing invalidity. This method is the cornerstone of all independence or invalidity results (though it probably had to be rediscovered in modern times, cf. Cohen and Hersh). In the fourth place, the distinction between perfect and imperfect syllogisms suggests a clear understanding of the difference between deduction and validity — a distinction which modern logicians believe to be their own (cf. Church, p. 323, fn. 529). In the fifth place, Aristotle used principles concerning form repeatedly and accurately although it is not possible to establish that he was able to state them nor is it even clear that he was consciously aware of them as logical principles.

The above are all highly theoretical points — but it is to be emphasized that Aristotle did not merely theorize; he carried out his ideas and programs in amazing detail despite the handicap of inadequate notation. In the course of pursuing details Aristotle originated many important discoveries and devices. He described indirect proof. He used syntactical variables (alpha, beta, etc.) to stand for content words — a device whose importance in modern logic has not been underestimated. He formulated several rules of inference and discussed their interrelations.

Philosophers sometimes say that Aristotle is the best introduction to philosophy. This is perhaps an exaggeration. One of the Polish logicians once said that the *Analytics* is the best introduction to logic. My own reaction to this was unambiguously negative — the

severe difficulties in reading the *Analytics* form one obstacle and I felt then that the meager results did not warrant so much study. After carrying out the above research I can compromise to the following extent. I now believe that Aristotle's logic is rich enough, detailed enough, and sufficiently representative of modern logics that a useful set of introductory lectures on mathematical logic could be organized around what I have called the main Aristotelian system.

From a modern point of view there is perhaps only one mistake which could sensibly be charged to Aristotle: his theory of propositional forms is very seriously inadequate. That he did not come to discover this for himself is remarkable, especially in view of the fact that he mentions specific proofs from arithmetic and geometry[23]. If he had tried to reduce these to his system he might have seen the problem. But once the theory of propositional forms is taken for granted there are no important inadequacies which are chargeable to Aristotle given the historical context. Indeed, his work is comparable in completeness and accuracy with that of Boole and it seems incomparably more comprehensive than the Stoic or medieval efforts. It is tempting to speculate that perhaps it was the over-simplified theory of propositional forms which made possible the otherwise comprehensive system, because a more adequate theory of propositional forms would have required a much more complicated theory of deduction — indeed, one which was not developed until the present era.

As a final remark I would like to emphasize the tentative nature of my results and the incompleteness of my research. I have left several questions unanswered and there are many interesting issues which have not been discussed at all. Cardinal among the latter is the question of Aristotle's views on the issue of whether reasoning is "natural" or "conventional". I suspect that Aristotle believed that reasoning is natural (and objective) in the sense that without regard to any conventionally established system of deduction it is still meaningful to say in an objective sense that a given deduction is correct or incorrect — but essentially nothing has been done on this issue, either by way of explication or by way of historical scholarship.

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[23] Mueller (q. v.) raises this question in a broader context.

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