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A Mereological Reading of the *Dictum de Omni et Nullo*

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Abstract: When Aristotle introduces the complete moods, he refers back to the *dictum de omni et nullo*, a semantic condition for universal affirmations and negations. There recently has been renewed interest in the question whether the *dictum* validates the assertoric syllogistic. I rehearse evidence that Aristotle provides a mereological semantics for universal affirmations and negations, and note that this semantics entails a nonstandard reading of the *dictum*, under which the *dictum*, in the presence of a minimal logical apparatus, indeed validates the assertoric syllogistic. I argue that this mereological validation offers advantages over recent discussions in Morison, Malink, Ebert and Vlasits.

Keywords: syllogistic, Aristotelian logic, ancient logic, history of logic

1 Introduction

Early in the *Prior Analytics*, Aristotle introduces certain inferences with a 'callback' to a previous passage. It will prove fruitful to consider this callback within its immediate context.

A1 Whenever three terms so stand to each other that the last is wholly in the middle and the middle is either wholly in or wholly not in the first, it is necessary for there to be a complete syllogism of the extremes.

Ὅταν οὖν ὅροι τρεῖς οὕτως ἔχωσι πρὸς ἀλλήλους ὥστε τὸν ἔσχατον ἐν ὅλῳ εἶναι τῳ μέσῳ καὶ τὸν μέσον ἐν ὅλῳ τῳ πρώτῳ ἢ εἶναι ἢ μὴ εἶναι, ἀνάγκη τῶν ἄκρων εἶναι συλλογισμὸν τέλειον.

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A2	I call ‘the middle’ that which both is itself in another and has another in it; this is also middle in position; the extremes are the terms which are solely in another or solely have another in them.	καλῶ δὲ μέσον μὲν ὃ καὶ αὐτὸ ἐν ἄλλῳ καὶ ἄλλο ἐν τούτῳ ἐστίν, ὃ καὶ τῇ θέσει γίνεται μέσον· ἄκρα δὲ τὸ αὐτό τε ἐν ἄλλῳ ὂν καὶ ἐν ᾧ ἄλλο ἐστίν.
A3	If A is predicated of all B and B is predicated of all C, then it is necessary for A to be predicated of all C.	εἰ γὰρ τὸ Α κατὰ παντὸς τοῦ Β καὶ τὸ Β κατὰ παντὸς τοῦ Γ, ἀνάγκη τὸ Α κατὰ παντὸς τοῦ Γ κατηγορεῖσθαι·
A4	For it was said earlier how to read ‘predicated of all.’	πρότερον γὰρ εἴρηται πῶς τὸ κατὰ παντὸς λέγομεν.
A5	Similarly, if A is predicated of no B and B is predicated of all C, A will belong to no C. (APr 1.4, 25b32–26a2)[1]	ὁμοίως δὲ καὶ εἰ τὸ μὲν Α κατὰ μηδενὸς τοῦ Β, τὸ δὲ Β κατὰ παντὸς τοῦ Γ, ὅτι τὸ Α οὐδενὶ τῳ Γ ὑπάρξει.

In this passage, Aristotle introduces the moods known by their medieval mnemonics, Barbara and Celarent. **A1** gives a formulation equivalent to the two moods and **A2** glosses the terminological distinction between middle and extreme terms used in that formulation. **A3** and **A5** state canonical formulations of Barbara and Celarent, respectively. Unlike the formulation in **A1**, these formulations use Aristotle’s preferred terminology for affirmations and negations, with either the locution ‘predicated of’ (*katēgoreisthai* with *kata*) or ‘belongs to’ (*huparchei* with the dative). Here he speaks of universal affirmation (*kata pantos katēgoreisthai*) and universal negation (*oudeni huparchei*); in the translation, I take ‘predicated’ to be understood in **A3** and **A5** from its one occurrence in **A3**. But what is the role of **A4** and to what earlier passage does it refer? A *gar* clause, **A4** purports to give a reason for accepting Barbara, as stated in **A3**, as a valid mood by reference to an earlier clarification of the *kata pantos* terminology used for expressing universal predication; again, I take ‘predicated’ to be understood in **A4**.

Most scholars view **A4** as referring to 24b28–30, a passage where Aristotle appears to explicate the meaning of this terminology. It will again prove fruitful to consider this passage within its immediate context.

1 Translations mine and based on Smith (1994), except as noted.

B1 A *syllogism* is an argument in which, some things having been supposed, something other than what has been supposed *results of necessity* from their being so. I mean by ‘from their being so’ resulting through them, and by ‘resulting through them,’ needing no term from outside for the necessity to arise.

συλλογισμὸς δέ ἐστι λόγος ἐν ᾧ τεθέντων τινῶν ἕτερόν τι τῶν κειμένων ἐξ ἀνάγκης συμβαίνει τῷ ταῦτα εἶναι. λέγω δὲ τῷ ταῦτα εἶναι τὸ διὰ ταῦτα συμβαίνειν, τὸ δὲ διὰ ταῦτα συμβαίνειν τὸ μηδενὸς ἔξωθεν ὅρου προσδεῖν πρὸς τὸ γενέσθαι τὸ ἀναγκαῖον.

B2 I call a syllogism *complete* if it stands in need of nothing else besides the things taken in order for the necessity to be evident. I call it *incomplete* if it still needs either one or several additional things which are necessary because of the terms assumed, but yet not taken by means of premises.

τέλειον μὲν οὖν καλῶ συλλογισμὸν τὸν μηδενὸς ἄλλου προσδεόμενον παρὰ τὰ εἰλημμένα πρὸς τὸ φανῆναι τὸ ἀναγκαῖον, ἀτελῆ δὲ τὸν προσδεόμενον ἢ ἑνὸς ἢ πλειόνων, ἃ ἔστι μὲν ἀναγκαῖα διὰ τῶν ὑποκειμένων ὅρων, οὐ μὴν εἴληπται διὰ προτάσεων.

B3 ‘One thing is wholly in another’ means the same as ‘one thing is predicated universally of another.’

τὸ δὲ ἐν ὅλῳ εἶναι ἕτερον ἑτέρῳ καὶ τὸ κατὰ παντὸς κατηγορεῖσθαι θατέρου θάτερον ταὐτόν ἐστιν.

B4 And we say ‘one thing is predicated universally of another’ whenever none of [those of] the subject can be taken of which the other cannot be said, and we use ‘predicated of none’ likewise. (APr 1.1, 24b18–30)

λέγομεν δὲ τὸ κατὰ παντὸς κατηγορεῖσθαι ὅταν μηδὲν ᾖ λαβεῖν [τῶν] τοῦ ὑποκειμένου καθ’ οὗ θάτερον οὐ λεχθήσεται· καὶ τὸ κατὰ μηδενὸς ὡσαύτως.

Here Aristotle characterizes the syllogism as a special kind of argument or inference (**B1**), distinguishes between complete and incomplete syllogisms (**B2**) and makes two remarks about universal affirmations and negations (**B3** and **B4**). **B4** is the source of the traditional *dictum de omni et nullo*, and the venerable tradition of reading **A4** as referring to **B4** goes back at least to Alexander (Alex. APr. 125.33–126.8).[2]

2 Aristotle makes a callback, similar to **A4**, when he introduces the assertoric Darii and Ferio at 1.4, 26a23–28. Aristotle views some of the first figure moods of the apodeictic and problematic syllogistic, like the assertoric first figure moods, as also evidentally valid and so complete—that is to say, Let us put **A1–5** and **B1–4** together. The apparent cameo appearance of the *dictum* when introducing Barbara and Celarent, through the callback in **A4** to **B4**, has suggested to many that the *dictum* is intended to justify the complete syllogisms. Indeed some, Maier (1936, 149) and Keynes (1906, 301) among them, took the syllogistic to rest entirely on the *dictum*. This view of the *dictum* is surely overstated. And as Barnes (2007) notes, on the orthodox way of reading the *dictum*, the *dictum* does not validate the syllogistic, for reasons that I will discuss later in the paper. There has been, however, recent renewed interest in viewing the syllogistic as at least partly grounded in the *dictum*: see, for example, Patterson (1993 and 1995), Morison (2008 and 2015), Malink (2008, 2009, 2013 and 2020), Batit (2011), Gili (2015), Marion and Rückert (2016), Crubellier, Marion, McConaughey and Rahman (2019), and Ludlow and Živanović (2022). This resurgence has gained momentum due to several factors: the accepted view that **A4** refers to **B4**; the recognition that there is an alternative way of reading the *dictum*, and the optimism that this heterodox version of the *dictum* is not vulnerable to certain objections; and the appreciation that under the heterodox reading, and in the presence of a minimal logical apparatus in the background, the *dictum* does indeed validate the syllogistic.

This essay is a contribution to heterodoxy. Relying on the work of Mignucci, Morison, Malink, Smith (1982) and others, I will endorse a version of the heterodox *dictum* which does partly validate the syllogistic. However, some of the factors driving the resurgence of interest in the *dictum* are due for reassessment. First, the accepted view that **A4** refers just to **B4** should be questioned. I will sketch a mereological semantic theory based on **B3** and, developing an observation of Mignucci (1996 and 2000), note that the mereological semantics entails a heterodox reading of the *dictum*. So we might view the heterodox *dictum* as a consequence of the mereological semantics, and this opens the option of reading **A4** as referring to **B3** and **B4** taken together. Second, the optimism in some of the current literature that the heterodox *dictum* is not vulnerable to certain objections is premature. Barnes (2007) recognizes the advantages of the heterodox reading but raises several criticisms,

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not requiring a proof that makes use of additional premises. And Aristotle makes several elliptical callbacks when noting the validity of some of these moods: for example, when Aristotle introduces in the apodeictic syllogistic Barbara with two necessary premises at 30a2–3, and when noting the validity of Darii with two contingent premises at 33a24–25. There is a tradition of viewing these passages as a reference to a suitably modalized *dictum de omni*: see for example, Patterson (1995, 220) and Malink (2013, 52); and, for a dissenting view, Smith (1989, 120). Other than these callbacks, Aristotle explicitly and uncontroversially uses the *dictum* to validate an inference in surprisingly few passages, if any. Smith (1989, 128–29) views the validation of Barbara with two contingent premises at 32b38–33[a]1 as appealing to a modalized *dictum*, and Striker (2009, 137) views the discussion at 33a1–33a5, in the validation of Celarent with two contingent premises, to be reminiscent of the *dictum*.

to which Morison and Malink respond. I will argue that these responses are less than fully satisfactory, so the heterodox *dictum*, as it is presented in this literature, is still open to several objections. However, viewing the heterodox *dictum* as a consequence of the mereological semantics offers an alternative presentation of the *dictum* that is not vulnerable to these objections. And so the mereological reading of the *dictum* yields the advantages of the heterodox *dictum* semantics without certain of its disadvantages.

2 Logical Background

It might be helpful to begin by reminding the reader of the broad outlines of the syllogistic. Moods are widely accepted today to be two-premise arguments with categorical propositions as the premises and conclusion. I will start by considering just the assertoric syllogistic, where the object language is restricted to expressing only the assertoric categorical propositions. These propositions have one of the following forms:

AaB: A belongs to all B
AeB: A belongs to no B
AiB: A belongs to some B
AoB: A does not belong to some B.

I will call these universal affirmations or a-predications, universal negations or e-predications, particular affirmations or i-predications, and particular negations or o-predications, respectively. The moods are classified into three figures, which have the following format. The premises contain the two terms of the conclusion respectively and a common or middle term: in the first figure, the middle term is in the predicate position of the first premise and in the subject position of the second premise; in the second and third figures, the middle is the predicate or the subject, respectively, of both premises. In *Prior Analytics* 1.4-7, Aristotle considers various combinations for the three figures of syllogisms and shows which are valid and which invalid. The first figure moods are complete or, as they are sometimes called, perfect: that is to say, they are taken in some way to be evidently valid. The complete moods are then used to prove the validity of the moods of the second or third figures. One proof method involves the use of the conversion rules:

e-conversion: From A belongs to no B infer B belongs to no A.
i-conversion: From A belongs to some B infer B belongs to some A.
a-conversion: From A belongs to all B infer B belongs to some A.

For example, Aristotle takes the first figure mood, Celarent

> A belongs to no B; B belongs to all C; so A belongs to no C

as evidently valid. He then establishes the validity of Cesare

> M belongs to no N; M belongs to all O; so N belongs to no O

by converting the first premise of Cesare to

> N belongs to no M

by means of the conversion rule e-conversion and then using Celarent to infer the conclusion. Aristotle also uses indirect proof and ecthesis, which I will discuss in more detail later.

A few initial remarks may be in order. First, the reader might be surprised at the claim that Aristotle justifies the complete moods at all. Aristotle glosses completeness and incompleteness in **B2**, recall, in this way: “I call a syllogism *complete* if it stands in need of nothing else besides the things taken in order for the necessity to be evident. I call it *incomplete* if it still needs either one or several additional things which are necessary because of the terms assumed, but yet not taken by means of premises.” And this might suggest to the reader that the validity of complete moods are self-evident and not in need of justification. Under a perhaps not uncommonly held view, Aristotle must take the first figure moods to be obviously or self-evidently valid. So that in virtue of which the first figure moods are valid is left unaddressed. He then shows that the narrow syllogisms of the second and third figure are valid, only under the unexamined assumption that the first figure syllogisms are valid.[3] Recent work on completeness, however, challenges this view. Morison (2015) argues that a mood is complete if it does not require additional premises, over and above the two premises of the mood itself, for the validity to become apparent; incomplete moods, by contrast, are shown to be valid by additional premises brought in through the methods of conversion, ecthesis or indirect proof. This is consistent with a complete mood being amenable to validation.

3 See, for example, Lear (1980: 3), who writes that Aristotle “simply states that it is evident that the first figure syllogisms are perfect. No argument is given for their validity. For if the syllogisms are perfect, no argument need be given.” Compare Rose (1968: 27), who writes that Aristotle’s “way of handling validity is to take the valid moods of the first figure as basic and to establish the validity of moods in the remaining figures by reducing them to moods of the first figure.”

What would be Aristotle's motivation for justifying the basic inferences? A comparison with contemporary logic may be helpful. We reason with a concept of logical consequence intuitively grasped but merely imprecisely expressed by textbook commonplaces such as 'the conclusion follows necessarily from the premises' or 'it's impossible for the premises to be true and the conclusion false.' As Etchemendy (1990, 5), Priest (1995, 283) and others observe, to give a precise account of logical consequence is the fundamental task of logic, just as we have an intuitive appreciation of an effective procedure and to give a precise account of computability is the fundamental task of recursion theory.

Contemporary logicians typically define two notions of logical consequence. According to a syntactic relation of deducibility among uninterpreted schemata, an inference is valid (roughly) if its conclusion can be derived from its premise by a series of applications of one or more members of a set of primitive inference rules. According to a semantic property of satisfiability, an inference is valid (again, roughly) if there is no interpretation where its premises are all true and its conclusion false or, equivalently, the conjunction of the premises and the negation of the conclusion is unsatisfiable. We establish the extensional equivalence between these two notions by proving soundness, the thesis that any argument derivable from the inference rules is counted as valid by the semantic theory, and completeness, the converse theory that any argument valid by the semantic theory is derivable by the inference rules. Dummett (1973) influentially viewed soundness and completeness as together providing a justification for a deductive system. On this view, soundness validates the primitive inference rules: the choice of such rules does not *overgenerate* the validities, according to what the semantic theory counts as valid. And completeness shows that any validity, so counted, can be derived by some sequence of these rules: so the choice of primitive inference rules does not *undergenerate* the validities.

Is Aristotle engaged in a comparable project? Aristotle's syllogistic resembles a derivation system. He takes a handful of inferences as basic and deduces a class of derivative inferences from this primitive set. I will show that he does defend his choice of the primitive inferences, in part by appeal to the *dictum*. Furthermore, I will support the view that the *dictum* is a semantic condition, a corollary following from certain decisions Aristotle makes on what the categorical propositions mean. And so, if Aristotle indeed appeals to the *dictum* to at least partly validate the syllogistic, then his methodology resembles the justification of primitive derivation rules by appeal to what inferences a semantic theory licenses.

This is not to say that Aristotle is engaged in a contemporary project. And Aristotle's validation of the syllogistic falls short of being a soundness proof for at least two reasons. First, contemporary proof theories concern the derivation patterns of uninterpreted schemata, and so rest on the distinction between syntax and semantics being cashed out in terms of the contrast between interpreted and uninterpreted expressions. It is a commonplace among historians of logic to observe that Aristotle either lacks a clear-cut syntactic/semantic distinction, or does not draw this distinction in terms of interpretation. Moods and conversion rules are presented with capitalized letters standing for terms, but these are, in Kirwan's (1978, 1–8, 33) coinage, dummy letters—that is to say, not variables or uninterpreted letters but rather letters the interpretation of which is left unspecified, since the specific referent of the letter is irrelevant.[4] A second reason why Aristotle's validation is not a soundness proof is that Aristotle, as we will see, assumes without comment the validity of a *reductio* rule. So the validation of the basic inferences of the syllogistic is partial. And although Aristotle arguably intends partly to justify a syntactic notion of derivability by appeal to semantic conditions, he does not attempt a full justification of the concept of deductive validity.

With these remarks in mind, I will say that a semantic theory *validates* the assertoric syllogistic if the first figure moods, Barbara and Celarent and the three conversion rules can be derived from that semantics, when supplemented by a *reductio* rule. The incomplete moods can be derived from the first figure moods either by conversion or through indirect proof. We will also consider the proof method called ecthesis, but its role in proving the incomplete moods is dispensable. And as Aristotle himself recognizes at 1.7 (29^b6–11), two first figure moods, Darii and Ferio, are superfluous, and can be themselves derived from Barbara and Celarent. So if Barbara, Celarent and the three conversion rules can be derived from a semantic theory, when supplemented by a *reductio* rule, then that theory might reasonably be said to validate all of the moods in the assertoric syllogistic.

4 In addition to Kirwan, the point is discussed in Mignucci (1965, 156–58), Frede (1974, 113), Lear (1980, 2), Barnes (1990, 20), and Barnes and Bobzien (1991, 116n71). Lear (1980, 2) rightly notes that if a proof theory is expressed in terms of uninterpreted schemata, there is a pressing need to justify the inference rules with a semantic theory: "it has become too easy to assume that a syntactic inference must be justified by some form of semantical soundness proof. This is because logicians have tended to treat formal systems as uninterpreted, as a safeguard against theoretical assumptions remaining hidden in the underlying logic." But, as Lear goes on to observe, one might recognize the desideratum to justify the choice of valid inferences, even if these are not uninterpreted rules.

3 Options for Interpreting the *Dictum*

Let us turn to a discussion of the *dictum*. There are several choice points in interpreting the *dictum de omni et nullo* that I can quickly set aside. One choice concerns whether the *dictum de omni et nullo* is a semantic condition or an inference schema. The standard view of the *dictum* is that it provides truth conditions for a- and e-propositions. Morison (2015) has argued for an alternative reading of the *dictum* as an inference schema. On this view, for example, the *dictum de omni* licenses the conclusion that the major predicate holds of all of the minor subject, underwriting Barbara. A challenge for Morison's reading, however, is that **B4,** the textual source of the *dictum*, strongly resembles a semantic condition. Recall **B4**:

B4	And we say 'one thing is predicated universally of another' whenever none of [those of] the subject can be taken of which the other cannot be said, and we use 'predicated of none' likewise. (24b28–30)	λέγομεν δὲ τὸ κατὰ παντὸς κατηγορεῖσθαι ὅταν μηδὲν ᾖ λαβεῖν [τῶν] τοῦ ὑποκειμένου καθ᾽ οὗ θάτερον οὐ λεχθήσεται· καὶ τὸ κατὰ μηδενὸς ὡσαύτως.

The condition is for our stating (*legomen*) a universal affirmation. And the condition, "none of [those of] the subject can be taken of which the other cannot be said," is introduced by *hotan*, an adverb of time translated by 'whenever' but ranging over instances of usage. Generally, inferences can be suggested by asserting a semantic condition, but such assertions retain their character as semantic conditions: a conclusion is licensed when the condition obtains. Going forward, I will follow the standard view that the *dictum* is a semantic condition.

A second choice concerns whether the *dictum de omni et nullo* ought to be supplemented by a *dictum de aliquo et aliquo non*. **B4** states a *dictum de omni* and suggests a *dictum de nullo*. It would be natural to hold that Aristotle intends to imply truth conditions for particular affirmations and negations akin to the *dictum de omni et nullo*. Morison (2015, 143–45) has shown that the *dictum de omni et nullo* without supplementation can underwrite all four first figure assertoric moods. But the standard view has been to take **B4** to imply a semantics for all four assertoric categorical propositions. To give a few recent examples, Patterson (1995), Barnes (2007) and Malink (2006 and 2013) all take this approach. In this essay, I will follow the standard interpretative decisions that the *dictum* is a semantic condition and includes a *dictum de aliquo et aliquo non*. As such, the central question asked in the paper might be viewed as a conditional: if the *dictum* indeed provides a semantics for the assertoric categorical propositions, how ought it be read?

With these assumptions in place, let us now take a closer look at **B4**. Most manuscripts lack *tōn* but the insertion is accepted by many commentators; for discussion, see Malink (2013, 34n2). The plural genitive of the article here suggests that the *dictum de omni* is concerned with a plurality associated with the subject. Assuming that 'the other' refers to the predicate, the *dictum de omni* might be read as stating that every member of the plurality associated with the subject is a member of the plurality associated with the predicate. Malink (2013, 36) helpfully offers a formulation of the *dictum* employing this expression 'member of the plurality associated with' or '*mpaw*' for short, and making use of classical propositional logic. Recall that A, B, … are dummy letters for terms. We use the capitals X, Y, Z, … as variables ranging over the denotations of these terms; unlike dummy letters, variables can be bound by quantifiers. With these conventions in mind, we might state the *dictum* as follows.

AaB iff $\forall Z(Z\ \mathit{mpaw}\ B \supset Z\ \mathit{mpaw}\ A)$
AeB iff $\forall Z(Z\ \mathit{mpaw}\ B \supset \neg\ Z\ \mathit{mpaw}\ A)$
AiB iff $\exists Z(Z\ \mathit{mpaw}\ B \wedge Z\ \mathit{mpaw}\ A)$
AoB iff $\exists Z(Z\ \mathit{mpaw}\ B \wedge \neg\ Z\ \mathit{mpaw}\ A)$

Following Malink, call this the 'abstract *dictum* semantics.' As Malink notes, these truth conditions are neutral as to what the plurality associated with a term is and what the members of this plurality are.

One option is to take the plurality associated with a term to be its extension. On this reading the *dictum de omni* states that the extension of the subject is a subset of the extension of the predicate. This is what Barnes calls the orthodox *dictum de omni*. Again availing himself of classical predicate logic, Malink offers the following reading of the orthodox *dictum* semantics. The *dictum de omni* asserts that A belongs to all B just in case there is no *individual* B of which A cannot be predicated. Let us take small x, y, z, … as variables ranging over individuals, and use 'Az' to stand for 'z is A.' Then we can say that, for example, the orthodox *dictum de omni* asserts that A belongs to all B just in case for any z, if Bz, then Az. On the orthodox *dictum* semantics, Malink (2013, 46) notes that the categorical propositions have the following semantic profiles:

AaB iff $\forall z(Bz \supset Az)$
AeB iff $\forall z(Bz \supset \neg Az)$
AiB iff $\exists z(Bz \wedge Az)$
AoB iff $\exists z(Bz \wedge \neg Az)$

I will discuss the orthodox *dictum* semantics in more detail soon, but let me first lay out the interpretative alternative.

A rival view of the plurality associated with a term is that it consists of exactly those items of which the term is a-predicated. This yields what Barnes calls the heterodox *dictum de omni*: for every item of which the subject is a-predicated, the predicate is a-predicated of that item as well.[5] Malink (2013, 63) offers the following reading of the heterodox *dictum* semantics:

AaB iff $\forall Z(BaZ \supset AaZ)$
AeB iff $\forall Z(BaZ \supset \neg AaZ)$
AiB iff $\exists Z(BaZ \wedge AaZ)$
AoB iff $\exists Z(BaZ \wedge \neg AaZ)$

As a terminological point, let us distinguish the informal heterodox *dictum* (the claim that for every item of which the subject is a-predicated, the predicate is a-predicated of that item as well) from Malink's formal heterodox *dictum* semantics. I will soon consider the advantages of the informal heterodox *dictum* semantics for the validation of the syllogistic. But first, let me present an alternative formal semantic presentation of the informal heterodox *dictum*. Underappreciated in the recent discussion of the heterodox *dictum* is Mignucci's (1996, 4) observation that a mereological reading of the categorical propositions yields a heterodox reading of the *dictum de omni et nullo*.

Developing this observation will require some set-up, so let us start with a semantic claim operative in both **A1–5** and **B1–4**. **A1** and **B3** associate predication and mereology. I am interested here primarily in the suggestion of a mereological semantics on the basis of **A1** and **B3**. But Aristotle elsewhere also appears to think of predication in mereological terms. For example, his terminology for universal and particular propositions—*katholou*, according to the whole, and *kata meros*, according to the part, respectively—are etymologically related to mereology. The view might strike the reader as open to an obvious objection. For example, since I am human, I am a part of the whole designated by 'human' and, since **A1** commits Aristotle to the transitivity of containment—more on this momentarily—it may seem that Aristotle must say that my parts are thereby human, but of course I am not composed of homunculi. Is Aristotle vulnerable to this objection? Aristotle distinguishes between what became known as quantitative and distributive parts in the following passage:

5 See Malink (2013, 47n2) for the recent history of this line of interpretation. Gili (2015) argues that Alexander takes the heterodox reading.

C We call a 'part' in one sense that into which a quantity can in any way be divided; for that which is taken from a quantity qua quantity is always called a 'part of it,' e.g., two is called in a sense a 'part of three.' In another sense it means, of the parts in the first sense, only those which measure the whole; this is why two, though in one sense it is, in another is not, a part of three. Moreover, apart from the quantity, the things into which the species might be divided are also called 'parts of it'; for this reason, we say that the species are 'parts of' the genus. (*Metaphysics* 5.25, 1023^{b}12–19; translation mine based on Ross (1949))

μέρος λέγεται ἕνα μὲν τρόπον εἰς ὃ διαιρεθείη ἂν τὸ ποσὸν ὁπωσοῦν (ἀεὶ γὰρ τὸ ἀφαιρούμενον τοῦ ποσοῦ ᾗ ποσὸν μέρος λέγεται ἐκείνου, οἷον τῶν τριῶν τὰ δύο μέρος λέγεταί πως), ἄλλον δὲ τρόπον τὰ καταμετροῦντα τῶν τοιούτων μόνον: διὸ τὰ δύο τῶν τριῶν ἔστι μὲν ὡς λέγεται μέρος, ἔστι δ᾽ ὡς οὔ. ἔτι εἰς ἃ τὸ εἶδος διαιρεθείη ἂν ἄνευ τοῦ ποσοῦ, καὶ ταῦτα μόρια λέγεται τούτου: διὸ τὰ εἴδη τοῦ γένους φασὶν εἶναι μόρια.

Aristotle here distinguishes among quantitative parts those which do measure their wholes from those which do not; and he contrasts quantitative and distributive parts. The distributive part relation relates a species to its genus and arguably an individual to its species. This might suggest that, unlike quantitative parts, distributive parts are the same in kind as their wholes. This distinction might mitigate the worry behind the objection, since it allows for transitivity failure over distinct types of part-whole relations. I am a distributive part of humanity; my limbs are quantitative parts of me; and it does not follow from the transitivity of the distributive part relation that my limbs are a part of humanity and so human. The distinction may or may not alleviate the worry for the reader; certainly, more would have to be said about the distinction. And, even if the distinction alleviates worries about transitivity failure, this is not to say that Aristotle's views are unproblematic. But my concern here is with what validation of the assertoric syllogistic is available to Aristotle, not with the success of his project.

How might we develop Aristotle's move? Here is one precisification. An informal presentation, drawing on work such as Simons (1987), Koslicki (2008) and Varzi (2016), will suffice for our purposes. We define a system $\mathbb{M}$ that we will use to interpret terms and stipulate truth conditions for the categorical propositions. We assume a domain of entities denoted by terms A, B, … and for convenience let us again use the capitals X, Y, Z, … as variables ranging over this domain. We also take the improper part relation P as a primitive relation obtaining among these denotations and assume classical propositional and quantificational logic. We can then define the proper part relation. Let PPXY $=_{df}$ PXY ∧ ¬PYX. The choice of the improper part relation, as opposed to the proper part relation PP, as primitive is arbitrary and the two are interdefinable. To be clear, the choice is arbitrary for the characterization of 𝕄. As we will see, the choice has significance for the validation of the syllogistic. It will be convenient to also define *Overlap* as OXY $=_{df}$ ∃Z(PZX ∧ PZY) and *Disjointedness* as DXY $=_{df}$ ¬OXY. 𝕄 has the following axiom schemata:

(Reflexivity) PXX
(Antisymmetry) (PXY ∧ PYX) ⊃ X =Y
(Transitivity) (PXY ∧ PYZ) ⊃ PXZ
(Weak Supplementation) PPXY ⊃∃Z(PZY ∧ DZX).

The improper part relation is intuitively a partial ordering, a relation that is reflexive, antisymmetric and transitive: everything is a part of itself; if a first thing is a part of a second, distinct from the first, then the second is not a part of the first; and any part of a part is itself a part. There are other natural (if not universally held) mereological intuitions. A whole intuitively has at least one part; at very least, one improper part. And a whole cannot have just one proper part: if a whole has a proper part, it also has a remainder, another part of the whole disjoint from the first. 𝕄 reflects these intuitions. Notice that 𝕄 lacks a null individual and exhibits weak supplementation. 𝕄 is arguably the weakest system that tracks our intuitions about parts and wholes.

This sketch of the mereological semantics leaves many questions unanswered. For example, is the relevant mereology stronger than 𝕄? Does it exhibit strong supplementation or extensionality? Notice that, in going forward, I only make the semantic claim that categorical propositions have mereological truth conditions; does Aristotle hold the metaphysical thesis that universals are sums of individuals? Fuller discussion of a mereological interpretation of Aristotle's semantic or corresponding metaphysical views can be found in Tweedale (1987), Mignucci (1996 and 2000), Koslicki (2008) and Corkum (2015 and 2018). But for our present purposes, we need not pursue these questions further. And indeed 𝕄 is stronger than we need, since weak supplementation will play no role going forward. Could Aristotle's talk of parts and wholes in this context, then, be referencing relations weaker than mereological relations? Vlasits (2019, 10–11) views the mereological terminology as merely metaphorical, since "the language of parthood is the closest that Aristotle could have come to describing preorders." Preorders are reflexive and transitive relations. As we will see, Vlasits shows that the assertoric syllogistic can be validated by appeal to a preorder semantic theory. But mereological notions provide for Aristotle's readers an attractive and accessible interpretation of universal predication, one that motivates such features as reflexivity and transitivity; and as an interpretative default position, there is reason to take Aristotle's talk of parts and wholes at face value.

We can use 𝕄 to provide schematic truth conditions for the four types of categorical propositions in terms of inclusion, disjointedness and overlap. For example:

AaB iff P_{BA}
AeB iff D_{BA}
AiB iff O_{BA}
AoB iff $\neg P_{BA}$

We can now return to Mignucci's (1996, 4) observation that a mereological reading of the categorical propositions yields a heterodox reading of the *dictum de omni et nullo*. We can offer a somewhat more precise observation: 𝕄 entails the informal heterodox *dictum* semantics. And so the informal heterodox *dictum* semantics can be viewed as a corollary of a mereological semantics for the object language of the syllogistic. Suppose that A belongs to all B. Then, by the mereological semantics, B is a part included in the whole A. And, by the transitivity of the part relation, any given part of B, say X, is also a part of A. So A belongs to all X. To precisify this gloss somewhat, we can formulate the heterodox *dictum de omni et nullo* in terms of the mereological semantics:

AaB iff $\forall_X (P_{XB} \supset P_{XA})$
AeB iff $\forall_X (P_{XB} \supset \neg P_{XA})$
AiB iff $\exists x (P_{XB} \wedge P_{XA})$
AoB iff $\exists x (P_{XB} \wedge \neg P_{XA})$

To distinguish this from Malink's presentation, call this the 'mereological *dictum* semantics.' To be clear, the mereological *dictum* semantics is a heterodox reading. The mereological *dictum* semantics entails a heterodox reading of the *dictum* since the plurality associated with a term is not necessarily an individual but may instead be a sum—indeed, we do not specify that they are sums of individuals; that is to say, we have not stipulated that the relevant mereology is extensional—and the relevant notion of predication on the RHS of the equivalence is a-predication. But the mereological *dictum* semantics offers a presentation of heterodoxy which is an alternative to Malink's, and I will reserve the term 'heterodox *dictum* semantics' for the Malink presentation. I will argue in § 5 that the mereological *dictum* semantics is preferable to the heterodox *dictum* semantics. But first, let us consider the validation of the assertoric syllogistic.

4 The Validation of the Syllogistic

We have before us five semantic theories: the abstract, orthodox, heterodox and mereological *dictum* semantics, as well as the mereological semantics. Under the abstract *dictum* semantics, for example, an a-predication is true just in case any member of the plurality associated with the subject is a member of the plurality associated with the predicate. Under the orthodox *dictum* semantics, an a-predication is true just in case the extension of the subject is a subset of the extension of the predicate. Under the heterodox *dictum* semantics, an a-predication is true just in case for every item of which the subject is a-predicated, the predicate is a-predicated of that item as well. Under the mereological semantics, an a-predication is true just in case the denotation of the subject is a part of the denotation of the predicate; and under the mereological *dictum* semantics, an a-predication is true just in case any part of the denotation of the subject is a part of the denotation of the predicate. Recall that to show that a semantic theory underwrites the syllogistic, it would suffice to show that the theory, in the presence of a *reductio* rule, justifies Barbara, Celarent and the e-, i- and a-conversion rules. We will consider the prospects for each semantic theory for each case in turn. Let us begin with Barbara and Celarent, and the mereological semantics.

As we have seen, Aristotle explicitly appeals to the transitivity of inclusion at **A1** when introducing Barbara and Celarent. And indeed, in $\mathbb{M}$ Barbara is equivalent to (Transitivity). Even the weakest standard mereologies take the part relation to be a partial ordering, and so reflexive, antisymmetric and transitive, with the additional condition of weak supplementation. These features are not mere technical conveniences. The part relation is intuitively transitive. If Aristotle is appealing to mereological notions to interpret the categorical propositions, as he appears to be doing, it would be surprising if he did not take the part relation to be transitive.

At this point in the essay, I am largely concerned with what is available to Aristotle, and less concerned with his intentions. But I have often heard in conversation the following objection to reading **A1** as an intended justification of Barbara, and it may be helpful to the reader to address this objection now. The objection goes like this. **A1** only draws an *equivalence* between the mood and the transitivity of mereological inclusion; and so there is no more reason to hold that the mereological claim explains the meaning of the predicative claim than there is reason to hold that the predicative claim explains the meaning of the mereological claim. However, although equivalences are symmetrical, it is not uncommon to see equivalences in philosophy where there is a tacit assumption of an *explanatory* asymmetry. For example, identity theorists in the philosophy of mind draw an equivalence between mental states and brain states. The equivalence is typically taken to support the physicalist thesis that there are no mental states over and above physical states (and not the idealist thesis that there are no physical states over and above mental states). For such theorists hold that the physical state is more tractable than the mental state, and that we are already committed to the existence of physical states. In this context, an equivalence has the force of an explanatory asymmetry.

There is good reason to hold that there is a similar explanatory asymmetry tacitly assumed in **B3**. As we have seen, Aristotle expresses the categorical propositions with several locutions. For example, for universal affirmations, he uses 'belongs to all' (*pantos huparchein*) and 'predicated of all' (*kata pantos katēgoreisthai*). He uses these expressions interchangeably and must believe that they all have the same meaning: otherwise the use of 'belongs to all' in **A5** would not be parallel to the uses of 'predicated of all' in **A3**, as it is apparently meant to be. The appeal of the terminology, as many have noted, is perhaps that the validity of the first figure syllogisms is perspicuous when they are expressed with these locutions. But these are all terms of art in need of explanation.

I next show that the mereological semantics validates Celarent (AeB, BaC so AeC). At the risk of pedantry, a Fitch-style natural deduction may provide a helpful presentation.

1	AeB	
2	BaC	
3	D_{AB}	1, mereological semantics
4	$\neg\exists Z\,(P_{ZA} \wedge P_{ZB})$	3, $\mathbb{M}$ (definition of D)
5	P_{CB}	2, mereological semantics
6	$\exists Z(P_{ZC} \wedge P_{ZA})$	assumption for *reductio*
7	$P_{DC} \wedge P_{DA}$	6, existential instantiation
8	P_{DC}	7, conjunction elimination
9	P_{DA}	7, conjunction elimination
10	P_{DB}	5, 8, $\mathbb{M}$ (transitivity of P)
11	$P_{DA} \wedge P_{DB}$	9, 10
12	$\exists Z\,(P_{ZA} \wedge P_{ZB})$	11, existential generalization
13	$\neg\exists Z(P_{ZC} \wedge P_{ZA})$	4, 12, *reductio*
14	AeC	13, mereological semantics

Notice the key role played by the transitivity of the part relation on line 10. Aristotle's cryptic remark in **A5** gives us too little to confidently reconstruct Aristotle's intended justification of Celarent. Recall **A5**: "Similarly, if A is predicated of no B and B of all C, A will belong to no C." The adverb 'similarly' (*homoiōs*) that introduces this sentence may merely indicate that the passage referred back to in **A4** also gives us a reason to take Celarent to be valid. But the range of the adverb may extend back to **A1**. The passage **A1–A4** arguably draws an equivalence between Barbara and the transitivity of inclusion in the context of making the validity of Barbara vivid to the reader. **A5** may serve to indicate that this equivalence is also pertinent to appreciating the validity of Celarent. If so, then Aristotle might be thinking of a proof similar to the one above.

Ebert (2015) also argues that the transitivity of the relation expressed by the 'belongs to all' expression suffices to validate Celarent. It will be instructive to look briefly at Ebert on this point. Ebert treats a- and e-predications as generalized conditionals in classical predicate logic: a-predications have the form $\forall x(Ax \supset Bx)$, and e-predications $\forall x(Bx \supset \neg Ax)$. Inverting the usual premise order for the mood makes the role of transitivity clearer. Ebert (2015, 358) writes:

> It is easy to see that this formulation of Celarent is quite close to a presentation of this syllogism as an inference in predicate logic style, starting with the minor premiss:
> $\forall x(Cx \supset Bx)$ & $\forall x(Bx \supset \neg Ax) \vdash \forall x(Cx \supset \neg Ax)$
> Or, replacing the variable by an arbitrary name:
> $(Ca \supset Ba)$ & $(Ba \supset \neg Aa) \vdash Ca \supset \neg Aa$
> Here it is the transitivity of the implication, the if-then connection symbolically expressed by the horseshoe, which is responsible for the transitivity of Celarent.

As Ebert notes, the transitivity of the 'belongs to all' relation follows from the transitivity of the material conditional in Ebert's representation of categorical propositions.

The abstract, orthodox and heterodox *dictum* semantics, as presented in section 3, have a similar reliance on the transitivity of the material conditional. As Malink (2013, 38) notes, the abstract *dictum de omni* entails that a-predication is transitive, and so validates Barbara. For if ∀Z(Z *mpaw* B ⊃ Z *mpaw* A) and ∀Z(Z *mpaw* C ⊃ Z *mpaw* B), then ∀Z(Z *mpaw* C ⊃ Z *mpaw* A). Notice that, as with the Ebert presentation, this result follows simply from the transitivity of the material conditional, and so does not depend on the interpretation of what relation among terms is expressed by '*mpaw*.' Let me for the moment just flag and bracket this reliance; I will return in the next section to the transitivity of the material conditional in these presentations. For the reasons just rehearsed, the transitivity of a-predication under the abstract *dictum* semantics also validates Celarent. The abstract *dictum* semantics thus validates both Barbara and Celarent and, since the abstract *dictum* is entailed by either the orthodox or the heterodox *dictum* semantics, these too validate Barbara and Celarent. Finally, as we have seen, $\mathbb{M}$ entails the mereological *dictum* semantics and, since $\mathbb{M}$ validates Barbara and Celarent, the mereological *dictum* semantics does as well.

We next need to consider the conversion rules. Let us begin with e- and i-conversion which, recall, are the rules licensing the inference from A belongs to no B to B belongs to no A, and from A belongs to some B to B belongs to some A. The mereological semantics validates e-conversion. Recall that on this semantics, e-propositions express mereological disjointedness and the e-conversion rule is equivalent to the symmetry of mereological disjointedness. Recall also that i-propositions express overlap. One proof of the symmetry of mereological disjointedness relies on the contradictoriness of disjointedness and overlap, which is equivalent to e-i contradictoriness. The proof is straightforward.

> Recall that we might define *overlap* as $OXY =_{df} \exists Z(PZX \wedge PZY)$ and *disjointedness* as $DXY =_{df} \neg OXY$. Disjointedness is symmetric; that is to say, DXY iff DYX. To show the LTR direction, let DXY, then by the definitions of disjointedness and overlap $\neg OXY$, i.e. $\neg\exists Z(PZX \wedge PZY)$. It follows from a theorem of classical logic, an application of the commutativity of conjunction within the scope of the negation operator and existential quantifier, that $\neg\exists Z(PZY \wedge PZX)$ and so $\neg OZY$, i.e. DYZ; and similarly for the other direction.

Analogous comments can be made about i-conversion, since the i-conversion rule is equivalent to the symmetry of mereological overlap, a thesis of standard mereology that can be straightforwardly shown by a proof similar to the one just sketched for the symmetry of mereological disjointedness.

The abstract *dictum* also validates e- and i-conversion. The validation of e-conversion, for example, is easily seen and again follows from the features of the material conditional. According to the abstract *dictum*, recall, e-predication is read as a generalized conditional: AeB iff $\forall Z$(Z *mpaw* B $\supset\neg$ Z *mpaw* A). If an antecedent materially implies a consequent, then the negation of that consequent materially implies the negation of that antecedent. Similar comments can be made for i-conversion.

The justification of e- and i-conversion is available to Aristotle. Is some such method his intention? Before examining the evidence, it will be helpful to discuss a lemma. As Smith (1982 and 1989, xxiii-xxv) shows, the *dictum de omni et nullo* underwrites ecthesis. Ecthesis is a proof method that sets out a witness to represent a term. For example, if AiB then we are licensed to posit some C where AaC and BaC. That is to say, if A belongs to some B, there is something of which both A and B can be predicated. For example, Aristotle proves Bocardo first with a *reductio* proof; but he notes that Bocardo can be proven by ecthesis, and so without appealing to a *reductio* proof, writing at 28b20–21 that "this [i.e., Bocardo] can also be proven without the leading away [i.e., a *reductio* proof], if some one of the Ss should be chosen to which P does not belong." The proof rests on taking the major premise of Bocardo, PoS, as licensing us to take some member of S to which P does not belong. PoS is of course the contradictory of PaS, and the licensed move is the negation of the *dictum de omni*. For these considerations, the abstract *dictum* semantics validates ecthesis, and so all of the *dictum* semantics (orthodox, heterodox and mereological) do as well. Aristotle's own proof of e-conversion makes use of ecthesis within an indirect proof:

D If A belongs to none of the Bs, then neither will B belong to any of the As. For if it belongs to some, for instance to C, it will not be true that A belongs to none of the Bs, since C is one of the Bs. (APr. 1.2, 25a15–17)

εἰ οὖν μηδενὶ τῷ Β τὸ Α ὑπάρχει, οὐδὲ τῷ Α οὐδενὶ ὑπάρξει τὸ Β· εἰ γάρ τινι, οἷον τῷ Γ, οὐκ ἀληθὲς ἔσται τὸ μηδενὶ τῷ Β τὸ Α ὑπάρχειν· τὸ γὰρ Γ τῶν Β τί ἐστιν.

Aristotle's intention is to justify e- and i-conversion with a proof procedure equivalent to even the weakest *dictum* semantics.

The last conversion rule to consider is a-conversion. Aristotle holds that universal affirmations carry existential import: a universal affirmation entails a certain particular affirmation. In classical logic, a universal affirmation does not entail a particular affirmation: ∀xFx can be true even when F is empty. A different way of putting the same point: Aristotle holds that a- and e-propositions are contraries. We can see this reliance on a-e contrariety in Aristotle's own proof of a-conversion

E If A belongs to all B, then B will belong to some A. For if it belongs to none, then neither will A belong to any B; but it was assumed to belong to all. (APr. 1.2, 25a17–19)

εἰ δὲ παντὶ τὸ Α τῶι Β, καὶ τὸ Β τινὶ τῶι Α ὑπάρξει· εἰ γὰρ μηδενί, οὐδὲ τὸ Α οὐδενὶ τῶι Β ὑπάρξει· ἀλλ᾽ ὑπέκειτο παντὶ ὑπάρχειν.

Aristotle's proof that AaB licenses BiA assumes AaB and takes as an assumption for *reductio* the contradictory of BiA, BeA. Then we derive AeB from BeA through the just established e-conversion. AaB and AeB are contraries, completing the indirect proof. The proof relies on e-i contradictoriness and a-e contrariety.

The difficulty with the assumption of a-e contrariety is this: in classical logic, universal affirmations and corresponding universal negations are, with empty terms, both true. This problem of motivating the existential import of universal affirmations in Aristotle's logic has generated a large secondary literature. For a recent discussion of this literature, see Corkum (2018). Some scholars within this literature restrict the object language of the syllogistic to non-empty terms. A difficulty with this approach is to provide a motivation for the restriction. Taking Aristotle to intend to give a mereological semantics for the categorical propositions provides a satisfactory motivation. Mereologies typically lack a null individual. This is not a merely technical observation. It is intuitively appealing to hold that there is a sum only if there is at least one (perhaps improper) part. Universal and particular affirmations and negations express notions of collectivity. It would be natural to cash out this notion of collection in terms of parts and wholes. And if one thus associates categorical propositions with mereological concepts, the existential import of universal affirmations would be an expected corollary. I will return to a more detailed discussion of a-conversion in the next section when we have a rival motivation for the existential import of universal affirmations on the table.

Neither the abstract nor the orthodox *dictum* semantics validate a-conversion. Recall that a-conversion is equivalent to the contrariety of a- and e-propositions. But both ∀Z(Z *mpaw* B ⊃ Z *mpaw* A) and ∀Z(Z *mpaw* B ⊃¬ Z *mpaw* A) are vacuously true when A is empty. And so both a-predications and corresponding e-predications are true when the subject term is empty, under the abstract *dictum* reading. Similar comments can be made about the orthodox *dictum* semantics. But, as Malink (2013, 68) notes, “[i]n the heterodox *dictum* semantics, the plurality associated with a term is the set of those items of which the term is a_x -predicated. [Malink uses ‘a_x – predication’ to stand for assertoric universal affirmations.] Since a_x -predication is reflexive, the plurality associated with any term A has at least one member, namely, A itself.” Here is Malink’s (2013, 67) argument that the heterodox *dictum* semantics validates a-conversion:

1. Aa_xB (premise)
2. Ba_xB (by reflexivity of a_x-predication)
3. $Aa_xB \wedge Ba_xB$ (from 1, 2)
4. $\exists Z(Aa_xZ \wedge Ba_xZ)$ (from 3)
5. Bi_xA (from 4; by heterodox *dictum de aliquo*)

Notice the role in this proof of the reflexivity of a-predication. Malink, recall, takes reflexivity to be, like transitivity, a primitive and unexplained feature of a-predication.

Let us sum up. Recall that under the orthodox *dictum* semantics, an a-predication, for example, is true just in case any one of the extension of the subject is also one of the extension of the predicate; by contrast, under the heterodox *dictum* semantics, an a-predication is true just in case anything of which the subject is a-predicated is also something of which the predicate is a-predicated. We have seen that the orthodox *dictum* semantics, the heterodox *dictum* semantics and the mereological *dictum* semantics all validate Barbara and Celarent, e-conversion and i-conversion. But unlike the orthodox *dictum* semantics, the heterodox *dictum* semantics and the mereological *dictum* semantics also validate a-conversion and so, in the presence of classical propositional and quantificational logic and a *reductio* rule, either of these *dictum* semantics may be said to underwrite the assertoric syllogistic. This is a good reason for preferring one of the heterodox *dictum* semantics or the mereological *dictum* semantics over the orthodox reading.

5 In Favor of the Mereological *Dictum* Semantics

The heterodox *dictum*, under the Malink presentation, is open to several objections. I will run through these objections with the aim of showing that the mereological *dictum* semantics, as a rival presentation of the heterodox *dictum*, offers interpretative advantages.

The first objection is from Barnes (2007, 412), who objects that **B4** cannot be read the way that the heterodox *dictum de omni* requires.[6] Recall that **B4** states that "none of [those of] the subject can be taken of which the other cannot be said." This suggests that the predicate is predicated of every member of the plurality associated with the subject, but Aristotle does not specify that the relevant kind of predication is a-predication, as the heterodox *dictum* requires. Morison (2008, 214) and Malink (2006 and 2013, 64–65) defend the heterodox *dictum* semantics against this objection, noting that Aristotle occasionally uses the 'said' locution, *legein* and its cognates (here: *lechthēsetai*), to indicate specifically a-predication. This observation may strike the reader as tenuous. Morison and Malink are surely right that taking the relevant kind of predication to be a-predication is an interpretative decision that is not *ruled out* by the Greek. But the reading is certainly the less natural one.

The mereological *dictum* provides a more satisfying reading of **B4**. Recall that, under the mereological *dictum* semantics, an a-predication, for example, is true just in case any part of the subject is a part of the predicate. This mereological *dictum* semantic condition follows from the corresponding mereological semantics condition under which an a-predication is true just in case the predicate is a part of the subject, along with some natural axioms of the underlying mereology $\mathbb{M}$. Aristotle implies a mereological interpretation for the categorical propositions in **B3**, and then expresses in **B4** the *dictum* in terms which are neutral between the orthodox and heterodox readings. Taking the two contiguous passages together, the reader can supply the heterodox *dictum* herself, as the right way to read **B4**, given its context. But Aristotle does not need to explicitly state the heterodox *dictum* for the reader to do so. And in the passage, "none of [those of] the subject can be taken of which the other cannot be said," a general reference to predication suffices. So, *contra* Barnes, the heterodox *dictum* does not require specific reference to a-predication in **B4**; and, *contra* Morison and Malink, we do not need to ascribe to Aristotle an uncommon use of *legein* in this passage.

The second objection is also from Barnes (2007, 409–12), who objects that the heterodox reading of the *dictum de omni* does not provide a *definition* of univer-

6 Mignucci (1996, 3) holds that there is no local evidence in the *Prior Analytics* in favor of the mereological or heterodox readings of the *dictum* over the orthodox reading.

sal predication, since the *dictum*, so read, *employs* such predications. In response, Morison (2008, 214) rightly observes that the heterodox reading could be viewed as a *characterization* of universal predication and not a definition. Similarly, Malink (2009, 116) notes that, although the heterodox reading employs predication on both sides of the equivalence, the *dictum* nonetheless is informative on this reading since it specifies certain logical properties of a-predication, namely, its reflexivity and transitivity. We have seen that the abstract *dictum* entails that a-predication is transitive (it was for this reason that the abstract *dictum* validates Barbara and Celarent) and reflexive. Reflexive and transitive relations, recall, are preorders. And Malink takes a-predication to be a primitive relation not defined in more basic terms, but further characterized by the *dictum* as a preorder.

In agreement with Morison and Malink, we do not need to view the *dictum* as providing a definition of universal predication in order to view the *dictum* as informative. But, in disagreement with Malink, we do not thereby need to take a-predication as primitive, or to read Aristotle as providing no guidance on how to read universal predications beyond the observation that they are preorders. For there is a third alternative to either viewing **B4** as a definition of universal predication or viewing universal predication as undefined. The mereological semantics suggested in **B3** also yields a characterization of what it is to say that one thing belongs of all of another. But this characterization provides an account of such predication fuller than the observation that the relation tying together the one thing and the other is a preorder. Some such account surely would be welcome in the context of introducing a logical system with the categorical propositions as its object language. Expressions of quantity such as ‘every’ and ‘all’ demand explication, and are ill-suited as primitives. To characterize a-predication merely as a preorder is to leave the relation heavily underspecified. Any equivalence or partial ordering is a preorder, and so the characterization would not distinguish a-predication from a vast range of relations, the relations of numerical identity, and of = and ≤ on the real numbers, among them. Although the characterization of a-predication as a preorder suffices to validate the syllogistic, it leaves mysterious to what a-predication amounts. By contrast, on the reading put forward in this essay, the reflexivity and transitivity of a-predication flows from intuitive and pre-theoretic features of the part-whole relation.[7]

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7 This observation provides an opportunity to remark on another recent discussion of Aristotle’s logic. Vlasits (2019) shows that a preorder semantics, with, for example, a-predication interpreted as a preorder, validates the syllogistic—justifying Barbara, Celarent, the conversion rules and the *reductio* rule—independently of the *dictum*. This result shows that something weaker than either the mereological semantics or the heterodox *dictum* semantics validates the syllogistic. However, mereological notions provide for Aristotle’s readers an attractive and accessible interpretation of

Let us return to a point I flagged and bracketed in the previous section. Recall that the validation of Barbara and Celarent hinges on the transitivity of a-predication. We saw that in Ebert's presentation, where a-predication has the form $\forall x(Ax \supset Bx)$, the transitivity of a-predication is ensured by the transitivity of the material conditional, represented here by the horseshoe; the validity of Barbara, the inference $\forall x(Cx \supset Bx)$ & $\forall x(Bx \supset Ax) \vdash \forall x(Cx \supset Ax)$, follows immediately. A similar story is told for the heterodox *dictum* semantics. In making use of classical propositional and predicate logic to formulate this semantics, as well as the abstract *dictum* semantics, Malink chooses material implications. Recall that in the heterodox *dictum* semantics, A belongs to all B just in case $\forall Z(BaZ \supset AaZ)$. On this interpretation, the transitivity of a-predication is, as in the Ebert interpretation, ensured by the transitivity of the material conditional. And again the validity of Barbara, the inference $\forall Z(BaZ \supset AaZ)$ & $\forall Z(AaZ \supset CaZ) \vdash \forall Z(BaZ \supset CaZ)$, follows immediately. This raises the question: might Aristotle have thought of the semantic conditions for a- and e-propositions as employing material conditionals?

It is controversial whether conditionals express material implications in English. Some of the so-called paradoxes of material implication arise from the fact that the falsity of the antecedent suffices for the truth of the implication, and it is counterintuitive to read many conditional expressions in English in this way. For example, many speakers would deny that 'If you do that, you will be sorry' is true when you do not do that, since the truth of the conditional depends on such variables as whether the action is regrettable. Moreover, the material implication is explicitly recognized arguably only after Aristotle, by Philo of Megara (fl. 300 BC), and the connective denoted by the horseshoe in classical logic is largely due to its re-discovery by Frege (1879).[8]

There are other interpretative options. For example, we might view conditional expressions in Aristotle as denoting entailments. Entailment arguably better represents many conditional expressions in English. An entailment is a non-truth functional connective, and lacks a determinate truth value when its antecedent is false. And, although controversial, it is natural to view entailment as non-transitive.[9] Similar comments can be made about e-conversion: there are relevant logics

universal predication, one that motivates such features as transitivity; and as an interpretative default position, there is reason to take Aristotle's talk of parts and wholes at face value.

8 For discussion, see for example Sanford (2003). As Bobzien (2016) notes, the assimilation of Philo's truth functional conditional to the material implication in classical logic is rough and ready, since Philo allows propositions to change truth values over time.

9 Those who view entailment as non-transitive include Geach (1958), Smiley (1959), MacLachlan (1972) and Epstein (1979); those who view entailment as transitive include Jackson (1970) and Anderson and Belnap (1975).

where if an antecedent entails a consequent, it does not follow that the negation of that consequent entails the negation of that antecedent.

To be clear, I do not claim that either Malink or Ebert wrongly *ascribe* to Aristotle an anachronism by deriving the transitivity of a-predication, used to validate Barbara and Celarent, from the material conditional. Rather, Malink and Ebert *represent* Aristotle's use of a transitivity principle through employing the material conditional within the semantic condition for a-predication. The approach leaves open the question: for Aristotle what is the source of this principle? This is a question that falls outside the aims of Malink (2013) and Ebert (2015); Malink (2013, 20 and 66, recall, takes a-predication to be a primitive relation with no further explanation to be given why it is reflexive and transitive.

But to answer the questions whether Aristotle views the formal features of a-predication as primitive or as grounded and, if grounded, grounded in what, is one of the goals of this essay. An advantage of the mereological *dictum* semantics is that the transitivity of a-predication is not left unexplained but instead flows from attractive and widely held intuitions about parts and wholes. For perspicuity, I have assumed classical logic in presenting the mereological semantics and the mereological *dictum* semantics. But this aspect of the presentation is eliminable, and were the semantics expressed in, say, relevant logic the transitivity of the part relation would suffice for the transitivity of a-predication.[10]

6 Conclusion

In closing, it may be worth emphasizing again the points of connection with the secondary literature. The thesis that Aristotle would be concerned to justify the first figure moods at all draws on Morison (2015), among others. That Aristotle views categorical propositions as having mereological truth conditions, and that the mereological semantics entails a heterodox reading of the *dictum de omni et nullo*, draws

10 Thanks to an anonymous reader for the following objection. Both the mereological semantics (suggested by **B3** and under which, for example, AaB iff B is a part of A) and the mereological *dictum* (suggested by **B4** and under which for example AaB iff every part of B is a part of A) suffice to validate the syllogistic. The question arises: why would Aristotle have multiple validation strategies? Notice that, if the availability of multiple validation strategies is a problem, it is not a problem unique to the interpretation put forward in this paper. Recall that Vlasits (2019) shows that a preorder semantics also validates the syllogistic, so anyone who views a-predication as transitive and reflexive ought to accept that Aristotle need not appeal to the *dictum* to validate the syllogistic, and as we have seen, heterodox *dictum* semanticists such as Malink rightly hold that a-predication is transitive and reflexive.

on Mignucci (2000), among others. The observation that the mereological semantics validates Barbara, Celarent and the conversion rules is anticipated by Vlasits' (2019) proof that a weaker semantic theory, the preorder semantics, validates the syllogistic. The observation that the heterodox *dictum* validates the syllogistic draws on Malink (2013). And I have relied partly on Barnes' (2007) criticism of the heterodox *dictum* to argue that viewing the heterodox *dictum* as a consequence of the mereological semantics offers interpretative advantages. My aim has been to support the work of these authors. The paper lends support to some of the work of Morison, and others who see completeness and justification as compatible, by fleshing out a specific proposal for the justification of the first figure moods. The paper lends support to the work of Mignucci, and others who take seriously Aristotle's mereological terminology in discussing categorical propositions, by showing that a mereological semantics validates the first figure moods and conversion rules of the syllogistic. I have motivated a more satisfactory reading of **B4**, under which the passage only states the abstract *dictum* but is to be read in context as the mereological *dictum*. And I have provided an informative account of a-predication, neither implausibly taking **B4** to be a definition nor viewing a-predication as a primitive. All this supports the work of Mignucci, Morison, Malink, and others who advocate a heterodox interpretation of the *dictum*.

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Bibliography

Alex. APr. Alexander of Aphrodisias. 1883. *Aristotelis Analyticorum priorum librum unum*. Ed. M. Wallies. Berlin: Reimer.

APr. Aristotle. 1964. *Analytica priora*. Ed. W. D. Ross. Oxford: Oxford University Press.

Anderson, A. R. and Belnap, N. D. 1975. *Entailment: The Logic of Relevance and Necessity*, vol. I. Princeton: Princeton University Press.

Barnes, J. 1990. 'Logical Form and Logical Matter.' In A. Alberti (ed.), *Logica, Mente et Persona*. Florence: Olschki, 7–119.

—. 2007. *Truth Etc*. Oxford: Clarendon Press.

Barnes, J. and Bobzien, S. 1991. *Alexander of Aphrodisias: On Aristotle* Prior Analytics 1.1–7 (Ancient Commentators on Aristotle) (Bk.1). London: Duckworth Press.

Bobzien, S. 2016 'Ancient Logic.' In E. Zalta (ed.), *The Stanford Encyclopedia of Philosophy* (Winter 2016 Edition). https://plato.stanford.edu/archives/win2016/entries/logic-ancient/.

Bonitz, H. 1955. *Index Aristotelicus*. 2nd ed. Graz: Akademische Druck- u. Verlagsanstalt.

Corkum, P. 2015. 'Aristotle on Predication.' *European Journal of Philosophy* 23, 793–813.

—. 2018. 'Empty Negations and Existential Import in Aristotle.' *Apeiron* 51, 201–19.

Crubellier, M., Marion, M., Mcconaughey, Z., and Rahman, S. 2019. 'Dialectic, the *dictum de omni* and *ecthesis*.' *History and Philosophy of Logic* 40, 207–233.

Dummett, M. 1973. 'The Justification of Deduction.' *Proceedings of the British Academy*, 59, 201–231.

Ebert, T. 2015. 'What is a Perfect Syllogism in Aristotelian Syllogistic?' *Ancient Philosophy* 35, 351–74.

Epstein, R. L. 1979. 'Relatedness and Implication.' *Philosophical Studies* 36, 137–173.

Etchemendy, J. 1990. *The Concept of Logical Consequence*. Cambridge, MA: Harvard University Press.

Frede, M. 1974. 'Stoic vs. Aristotelian Syllogistic.' *Archiv für Geschichte der Philosophie* 56(1), 1–32.

Frege, G. 1879. *Begriffsschrift: Eine der arithmetischen nachgebildete Formelsprache des reinen Denkens*. Halle an der Saale: Verlag von Louis Nebert.

Geach, P. T. 1958. 'Entailment.' *Aristotelian Society Proceedings* supp. vol. 32, 157–172.

Gili, L. 2015. 'Alexander of Aphrodisias and the Heterodox *dictum de omni et nullo*.' *History and Philosophy of Logic* 36, 114–128.

Jackson, F. 1970. 'The Transitivity of Entailment.' *Philosophical Quarterly* 20, 385–388.

Keynes, J. N. 1906. *Formal Logic*. London: Macmillan.

Kirwan, C. 1993. *Aristotle's Metaphysics, Gamma, Delta, Epsilon*, 2nd ed. Oxford: Clarendon Press.

Koslicki, K. 2008. *The Structure of Objects*. Oxford: Oxford University Press.

Lear, J. 1980. *Aristotle and Logical Theory*. Cambridge: Cambridge University Press.

Ludlow, P. and Živanović, S. 2022. *Language, Logic and Form*. Oxford: Oxford University Press.

Maclachlan, D. L. C. 1972. 'The Transitivity of Entailment.' *Philosophical Quarterly* 22, 57–61.

Maier, H. 1936. *Syllogistik des Aristoteles*, vol. 2. Tuebingen: H. Lamp, 1896–1900. Neudruck, Leipzig: Koehler.

Malink, M. 2008. 'Tōi vs Tōn in *Prior Analytics* 1.1–22.' *Classical Quarterly* 58, 519–536.

—. 2009. 'A Non-extensional notion of conversion in the Organon.' *Oxford Studies in Ancient Philosophy* 37, 105–141.

—. 2013. *Aristotle's Modal Syllogistic*. Cambridge, MA: Harvard University Press.

—. 2020. '*Demonstration by reductio ad impossibile* in *Posterior Analytics* 1. 26.' *Oxford Studies in Ancient Philosophy* 58, 91–155.

Marion, M. and Rückert, H. 2016. 'Aristotle on Universal Quantification: A Study from the Perspective of Game Semantics.' *History and Philosophy of Logic* 37, 201–229.
Mignucci, M. 1965. *La teoria aristotelica della scienza* (Pubblicazioni della Facoltà di Magistero dell'Università di Padova, VII). Florence: Sansoni.
—. 1996. 'Aristotle's Theory of Predication.' In A. Ignacio and C. María (eds.), *Studies on the History of Logic. Proceedings of the III. Symposium on the History of Logic*. Berlin: Walter de Gruyter, 1–20.
—. 2000. 'Parts, Quantification and Aristotelian Predication.' *Monist* 83, 3–21.
Morison, B. 2008. 'Book Notes.' *Phronesis* 53, 209–222.
—. 2015. 'What is a Perfect Syllogism?' *Oxford Studies in Ancient Philosophy* 48, 107–166.
Patterson, R. 1993. 'Aristotle's Perfect Syllogisms: Predication and the *dictum de omni*.' *Synthese* 96, 359–378.
—. 1995. *Aristotle's Modal Logic: Essence and Entailment in the Organon*. Cambridge: Cambridge University Press.
Priest, G. 1995. 'Etchemendy and Logical Consequence.' *Canadian Journal of Philosophy* 25, 283–292.
Rose, L. 1968. *Aristotle's Syllogistic*. Springfield: Thomas.
Sanford, D. 2003. *If P, Then Q: Conditionals and the Foundations of Reasoning*, 2nd ed. London: Routledge.
Simons, P. 1987. *Parts: A Study in Ontology*. Oxford: Oxford University Press.
Smiley, T. 1959. 'Entailment and Deducibility.' *Proceedings of the Aristotelian Society* 59, 233–254.
Smith, R. 1982. 'What is Aristotelian *Ecthesis*?' *History and Philosophy of Logic* 3, 113–127.
—. 1989. *Aristotle's Prior Analytics*. Indianapolis: Hackett.
Tweedale, M. 1987. 'Aristotle's Universals.' *Australasian Journal of Philosophy* 65, 412–426.
Varzi, A. 2016. 'Mereology.' In E. Zalta (ed.), *The Stanford Encyclopedia of Philosophy* (Winter 2016 Edition). https://plato.stanford.edu/archives/win2016/entries/mereology.
Vlasits, J. 2019. 'Mereology in Aristotle's Assertoric Syllogistic.' *History and Philosophy of Logic* 40, 1–11.