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Aristotle on the Individuation of Syllogisms

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Discussion of the Aristotelian syllogism over the last sixty years has arguably *centered* on the question whether syllogisms are inferences or implications.[1] But the *significance* of this debate at times has been taken to concern whether the syllogistic is a logic or a theory, and how it ought to be represented by modern systems.[2]

Largely missing from this discussion has been a study of the few passages in the *Prior Analytics* where Aristotle provides explicit guidance on how to *individuate* syllogisms. Aristotle asserts that syllogisms in the traditional moods of Camestres and Cesare are the same syllogism at 27a14, that the same syllogism can have distinct conclusions at 50a5 and that a syllogism can have several conclusions at 53a5 and 53a10. Scholarly reaction to these passages has been predominantly deflationary. Some hold that Aristotle is discussing the individuation not of any syllogism whatsoever but only of an argument with several sub-conclusions, which loosely may be called a syllogism (e.g., Ross 1949 and Smith 1982). Others hold that Aristotle is not discussing the individuation of syllogisms at all but merely claiming that the premises of distinct syllogisms can have the same conclusion (e.g., Striker 2009). I will argue that these deflationary readings are unsatisfactory. Taken together, the passages yield compelling evidence that Aristotle individuates syllogisms by unordered premise pairs. A syllogism then does not have the form of one of the traditional moods, since a mood is a pattern with a fixed premise order and a single conclusion. I develop an interpretation of syllogisms sensitive to this textual evidence.

Before proceeding, let me make a point of clarification. I have characterized the scholarly reaction to these passages as sparse and predominantly deflationary. Predominantly, not wholly. Duerlinger 1968a, 1968b, 1969 offers article-length discussions of some of this textual evidence. Since the view I am putting forward

[1] For the view that instances of syllogisms are implications, see, e.g., Łukasiewicz 1957, Patzig 1968. For the view that syllogisms are inferences, see, e.g., Austin 1952, Rose 1968, Smiley 1973, and Corcoran 1972.

[2] Proposals over the years have included viewing the syllogistic as a diagrammatic reasoning system (Euler 1768), a semantic tableau (Carroll 1887, Beth 1955), an axiomatic theory (Łukasiewicz 1957, Patzig 1968), a logic of multiple sorted quantification (Smiley 1962), a natural deduction system (Corcoran 1972, Smiley 1973), a connexive logic (McCall 1967), a fragment of a generalized quantifier theory (van Benthem 1984, van Eijck 1985, Westerståhl 1989), an inductive construction (Martin 1987, 1997), a sequent calculus (Crabbé 2003, Tennant 2014), a relevant logic (Irvine and Woods 2004, 65), a natural logic (van Benthem 2008), a metatheory (Pelletier and Hazen 2012), a linear logic (Englebretsen 1991, Pagnan 2013), and a dialogical logic (Dutilh Novaes 2015).

might be confused with Duerlinger's, it may be worth emphasizing from the start some of the similarities and differences. Duerlinger identifies syllogisms with concludent premise pairs. Duerlinger's thesis and the view I offer share a consequence, namely, that syllogisms do not necessarily have a single conclusion and so differ structurally from the traditional moods. But to *individuate* syllogisms by unordered premise pairs is not to *identify* syllogisms with such pairs, and although I will argue that syllogisms differ from moods, I will also offer an interpretation according to which syllogisms have one or more conclusions. I will return to these points, when I discuss in more detail the view that syllogisms are concludent premise pairs.

I begin by reminding readers of the broad outlines of the syllogistic. Here I will also briefly rehearse the evidence for taking syllogisms to be inferences (§1). I then argue that syllogisms are individuated by unordered premise pairs (§§2-4). Together these observations suggest that syllogisms may be fruitfully thought of as inferences from unordered premise pairs to some conclusion or other, and I conclude by briefly discussing a consequence of this thesis for the representation of the syllogistic (§5).

I

Aristotle characterizes syllogisms at *APr* i 1.24b18-22 as follows:[3]

> **T1**: a syllogism is a discourse in which, certain things having been supposed, something different from the things supposed results of necessity because they are so. By 'because these things are so' I mean 'resulting through them', and by 'resulting through them' I mean 'needing no further term from outside in order for the necessity to come about'.[4]

This characterization suggests that a syllogism is an ordered pair consisting of a set of suppositions and a set of results. Let me use the lower case letters 'p' and 's', with or without a subscript, as variables ranging over propositions. And recall the standard use of angle brackets for ordered n-tuples and curly brackets for unordered n-tuples.Then we can say that a syllogism would seem to have the form $<\{p_1, \ldots, p_n\}, \{s_1, \ldots, s_m\}>$ such that $\{p_1, \ldots, p_n\}$ and $\{s_1, \ldots, s_m\}$ stand in a certain relation of acceptability. (I will tweak this claim later.) The definition in **T1** places certain further constraints on what can count as a syllogism. The plural characterization of the suppositions in 'certain things having been supposed' suggests that, for the n-tuple of suppositions, $n > 1$. The characterization of the result as 'something different from the things supposed' suggests that for the n-tuple of suppositions and the m-tuple of results, and with $1 \leq i \leq n$ and $1 \leq j \leq m$, $p_i \neq s_j$. Aristotle characterizes the acceptability relation as obtaining when $\{s_1, \ldots, s_m\}$ results of necessity through $\{p_1, \ldots, p_n\}$. The nature of the acceptability relation

[3] I follow Smith's 1989 translation of the *Prior Analytics* with occasional alterations.

[4] συλλογισμὸς δέ ἐστι λόγος ἐν ᾧ τεθέντων τινῶν ἕτερόν τι τῶν κειμένων ἐξ ἀνάγκης συμβαίνει τῶι ταῦτα εἶναι. λέγω δὲ τῶι ταῦτα εἶναι τὸ διὰ ταῦτα συμβαίνειν, τὸ δὲ διὰ ταῦτα συμβαίνειν τὸ μηδενὸς ἔξωθεν ὅρου προσδεῖν πρὸς τὸ γενέσθαι τὸ ἀναγκαῖον.

is historically controversial, and I will discuss the nature of this relation further below. Finally, it is standardly assumed that a syllogism has a single result and so m=1. A goal is to question this assumption. I will turn to this issue in §2. But before beginning this task, let me continue to sketch the broad outlines of the syllogistic. Much of this sketch may be familiar to the reader, but having the details before us will prove fruitful in what follows, especially when in §5 I discuss the representation of the syllogistic.

We have seen that Aristotle characterizes the syllogism in **T1** (24b18-22). What Aristotle actually proceeds to do in *APr* i 4-6 is to classify tertiary ordered sequences of categorical propositions. Call a *mood* the form of an ordered sequence $<p_1, p_2, s>$ where each member is a categorical proposition. I will assume a set of term variables A, B, C, …. The assertoric categorical propositions have the forms:

BaA: pronounced 'B belongs to every A'
BeA: B belongs to no A
BiA: B belongs to some A
BoA: B belongs to not every A.

The moods are classified into three figures, which have the following canonical format. (I will note in the next section that Aristotle appears occasionally to vary from this format.) The first two members of the sequence contain the two terms of the third member respectively and a common or middle term: in the first figure, the middle term is in the predicate position of the first member and in the subject position of the second member; in the second and third figures, the middle is the predicate or the subject, respectively, of both of the first two members. So, for example, one of the moods of the first figure, called by its medieval mnemonic, 'Barbara', is the pattern:

A belongs to every B.
B belongs to every C.
So A belongs to every C.

I will use the traditional names for the moods and, as I did above, I will occasionally express a mood as an ordered sequence. So for example, Barbara may be represented as <AaB, BaC, AaC>.

The assertoric syllogistic is in part a two-stage classification of moods. Aristotle also presents at length the apodeictic and problematic syllogistic, classifications of moods with modal operators. In the interests of keeping our discussion from ballooning, I will restrict discussion to the assertoric syllogistic. In *APr* i 4-6, Aristotle considers various combinations for the three figures and shows which are acceptable and which unacceptable. The acceptable moods of the first figure are taken to be evidentially acceptable: immediately following the syllogism definition in **T1**, Aristotle (24b22-23) characterizes these moods as standing 'in need of nothing else besides the things taken in order for the necessity to be evident'. The acceptability of the acceptable moods of the second and third figures is established by showing that these moods stand in a certain relation to one of the moods of the first figure—often, that of convertibility. That is to say, Aristo-

tle takes such moods as (one of the first figure moods) Celarent:

<AeB, BaC, AeC>

as obviously acceptable. He then establishes the acceptability of such moods as Cesare

<MeN, MaO, NeO>

by converting the first member to

NeM

by means of the conversion rule *e-conversion* and then appealing to Celarent, so to derive NeO from NeM and MaO.

Another method to establish the acceptability of the acceptable sequences is indirect proof. For example, the indirect proof of Baroco, from *APr* i 5.27a36-b1, is:

> **T2**: Again, if M belongs to every N but does not belong to some X, it is necessary for N not to belong to some X. (For if it belongs to every X and M is also predicated of every N, then it is necessary for M to belong to every X; but it was assumed not to belong to some.)[5]

It is controversial how to describe what happens in Aristotle's indirect proofs. But according to one plausible reading, the above passage assumes the premises of Baroco and shows that its conclusion follows by assuming the negation of one of its premises and using Barbara to derive a contradiction.[6]

I have presented the syllogistic in interpretatively neutral terms of the acceptability of sequences. The historical interpretation and representation of these sequences, their acceptability and the resulting structure of the syllogistic has arguably reflected the logical concerns of the interpreter's time. In the 50's and 60's, Łukasiewicz 1957 and Patzig 1968 took syllogistic forms to be true generalized conditionals and so instances of these forms, implications.[7] In the early 70's, by contrast, Corcoran 1974 and Smiley 1973 independently argued that syllogistic forms are valid inference rules and instances of these forms, deductions.

It is now fairly standardly held that syllogisms are inferences. And indeed, the arguments for the reading of syllogisms as implications are unpersuasive.[8] I will

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[5] πάλιν εἰ τῶι μὲν N παντὶ τὸ M, τῶι δὲ Ξ τινὶ μὴ ὑπάρχει, ἀνάγκη τὸ N τινὶ τῶι Ξ μὴ ὑπάρχειν· εἰ γὰρ παντὶ ὑπάρχει, κατηγορεῖται δὲ καὶ τὸ M παντὸς τοῦ N, ἀνάγκη τὸ M παντὶ τῶι Ξ ὑπάρχειν· ὑπέκειτο δὲ τινὶ μὴ ὑπάρχειν.

[6] A third method to establish the acceptability of second and third figure moods is exposition. Take the first two members of Darapti, A belongs to every B; and C belongs to every B. Now set out some particular B, say b. Then we may infer from the first member A belongs to b and from the second member C belongs to b. So it follows that A belongs of something to which C also belongs; hence A belongs to some C. For discussion, see Smith 1982. Finally, the unacceptability of the unacceptable sequences is typically established by counter-instance.

[7] If p and q are open sentences and Q a string of universal quantifiers, one for each free variable in $(p \supset q)$, then $Q(p \supset q)$ is a universalized conditional. So the syllogistic form of Barbara, on this interpretation, looks like this: For all A, B, C: if B holds of every A and C holds of every B, then C holds of every A.

[8] Łukasiewicz 1957, 1-3, 20-30 and Patzig 1968, 3-4, e.g., defend their view that syllogisms are

assume that syllogisms are inferences, and so I will take the acceptability relation—the relation between a premise set $\{p_1, \ldots, p_n\}$ and a conclusion set $\{s_1, \ldots, s_m\}$ in a syllogism—to be inferential. It may seem trivial to note this assumption, given its broad acceptance today, but it will prove fruitful to flag now that syllogisms are inferences, since it will allow us later to distinguish our proposal o from a view that might on the surface otherwise resemble it. The reader might view the next sections as the argument that, even if syllogisms are inferences, they ought not to be interpreted as moods.

II

In this section, I examine the textual support for the thesis that syllogisms are individuated by unordered premise pairs. The result, along with the considerations from the preceding section, is that syllogisms are inferences from a given unordered premise pair to some conclusion or other. In going through the evidence for the claim, I make six ports of call.

I begin with an uncontroversial observation. Aristotle occasionally states moods with non-canonical premise order. For example, Aristotle typically states the major premise first but at *APr* i 6.28a26-29 Aristotle states the major premise of Felapton <AeB, CaB, AoC> after the minor premise. This phenomenon is well recognized. Examples of this observation in the secondary literature include Łukasiewicz 1957, 34, Rose 1968, ch. 10, Thom 1979, 753, and Morison 2015, 111. Aristotle's lax approach to premise order is not sloppiness. For it is not merely that Aristotle can mention moods with alternative premise order; he also uses moods within reductio proofs in a way that suggests a tacit permutation rule. For example, the proof of Felapton at *APr* i 6.28a26-28 requires the non-canonical premise order. The medieval mnemonic encodes this step with an 'm' and Thom 1979, 753 formalizes this rule as follows: 'Let *p* be a protasis and Q, R be sequences of protases that do not differ otherwise than in the ordering of their elements. Then, if Q/p is a thesis so is R/p.'

The authors in the secondary literature who comment on Aristotle's use of moods with non-canonical premise order, however, do not claim that the syllogisms are individuated by unordered premise pairs but rather endorse some weaker claim. For example, Thom's permutation rule allows only that the validity of the mood $<p_1, p_2, s>$ entails that there is a valid mood $<p_2, p_1, s>$. Thom perhaps views Aristotle as advocating a stronger thesis. Thom 1979, 753 says: 'Aristotle sometimes tacitly permutes theses, stating their premisses at one time

not inferences but implications in part by noting that Aristotle generally presents syllogisms in conditional form. For example, Barbara is stated at 25b37-39 as: 'if A is said of every B and B of every C, then it is necessary for A to be predicated of every C'. But it would be natural in some contexts to express inferences as conditionals where, if the premises hold, then the conclusion follows. Austin 1952, Rose 1968, 25 and Corcoran 1972, 278 all make this observation. Alexander *in An Pr*. 373, 29-35 claims that '"if A, then B" means the same as "B follows from A"'. Morison 2011 holds that Aristotle uses conditionals to assert not the syllogism but the conditions under which a syllogism results—namely, the premises in the antecedent—and the conclusion that can be drawn when those conditions obtain.

in one order, at another in another.' This suggests that the validity of the mood $<p_1, p_2, s>$ entails that there is a valid mood $<p_2, p_1, s>$ that is the same mood as $<p_1, p_2, s>$. To give a second example from the secondary literature, Morison 2015, 111 draws from this evidence the conclusion that 'Aristotle did not think the order of premises was important', which may also suggest that moods with inverted premise order are the same syllogism. Of course, these views are still weaker than my proposal that moods with different *conclusions* can be the same syllogism. So let us turn to the next port of call, Aristotle's discussion of the subaltern moods.

The five traditional subaltern moods are (in the first figure) Barbari <PaM, MaS, PiS> and Celaront <PeM, MaS, PoS>, (in the second figure) Cesaro <MeP, MaS, PoS> and Camestrop <MaP, MeS, PoS> and (in the fourth figure, discussed below) Camenop <MaP, SeM, PoS>. These are all derivable from moods in one of the traditional four figures by the additional application of a rule of subalternation to the conclusion of the original mood. So for example, Barbari can be shown to be a valid mood by applying a-i subalternation, the inference from PaS to PiS, to the conclusion of Barbara. Aristotle is committed to the validity of these subaltern syllogisms. Indeed, it would be remarkable, given all that he says, were he to have failed to recognize these syllogisms.[9] A charitable reading of their omission is not that Aristotle missed this class of syllogism, but that he did not believe these to be a class distinct from syllogisms he does discuss. On this reading, since they share the same premise pair, Barbara and Barbari are not treated as distinct items in the classification of assertoric moods in *APr* i 4-6. Notice that the absence of subaltern moods establishes only that, if perhaps distinct syllogisms share the same premise pair, they are not treated as distinct moods in this classification.

This general line of interpretation is venerable. Arnauld and Nicole 1662, 142 write that

> people have been satisfied with classifying syllogisms only in terms of the nobler conclusion, which is general. Accordingly they have not counted as a separate syllogism the one in which only a particular conclusion is drawn when a general conclusion is warranted.

By 'nobler' Arnauld and Nicole appear to mean the stronger conclusion. Citing Arnauld and Nicole, Parsons 2014, 16 takes a similar interpretation: 'Aristotle has only 19 moods because he is examining which combinations of premises can yield a valid conclusion.' These authors appear to be making a weaker claim about the *classification* of syllogisms, and not the stronger claim I aim to defend about their *individuation*. There is certainly an emphasis on the premise pairs in the classification of moods in *APr* i 4-22. Aristotle typically notes, for a given premise pair, that a conclusion necessarily follows or that no conclusion necessarily follows. In the former case, he then gives a proof for a specific example of

[9] E.g., Aristotle does not explicitly state a-i subalternation, but *Top* ii 1.109a1-6 suggests that AaB entails AiB.

such a conclusion. The classification then is on the basis of the quantity and quality of the premises. Aristotle does not *classify* the moods according to what conclusions are entailed by some set of premises or other. To show that, for a given arrangement of premises, some conclusion or other follows, is sufficient for Aristotle's aims.

But I wish to go further than the Arnauld and Nicole interpretation and show that sharing the same premises is not a peculiarity of classification but a criterion for counting syllogisms as the same or distinct. There is more direct evidence that syllogisms are individuated by their premises. Aristotle occasionally characterizes two or more moods as the same syllogism or as different syllogisms. Our third port of call is Aristotle's discussion of the so-called fourth figure moods. In the canonical format for the fourth figure, the middle term is in the predicate position of the second premise and in the subject position of the first premise. So the fourth figure moods have the form <MxP, SyM, PzS>. Although Aristotle does not include the fourth figure moods among those discussed in the classification of *APr* i 4-6, he does explicitly recognize their validity. Aristotle appears to view some fourth figure moods as not distinct syllogisms from those already discussed in the classification of *APr* i 4-6. In *APr* ii 1.53a3-14 Aristotle recognizes Bramantip <MaP, SaM, PiS>, Camenes <MaP, SeM, PeS> and Dimaris <MiS, PaM, PiS>. Here is the Smith 1989, 65 translation:

> **T3**: Now, seeing that some deductions are universal and others are particular, all the universals always deduce several results; among particular deductions, positive deductions deduce several things, but negatives only deduce their conclusions. For, although the privative <particular> premise does not convert, the other premises convert; and the conclusion is one thing predicated about another, so that the other deductions deduces several things. For example, if A has been proved to belong to every B or to some, then it is also necessary for B to belong to some A; and if A has been proved to belong to no B, then neither does B belong to any A (and this conclusion is different from the previous one). However, if A does not belong to some B, it is not also necessary for B not to belong to some A, since it is possible for it to belong to every.[10]

Smith translates *sullogismos* and its cognates with 'deduction' and its cognates. This passage provides some evidence that Aristotle not only views the items classified in *APr* i 4-6 as individuated by premise pairs, as Arnauld and Nicole seem to hold, but that syllogisms themselves are so individuated.

[10] ἐπεὶ δ' οἱ μὲν καθόλου τῶν συλλογισμῶν εἰσὶν οἱ δὲ κατὰ μέρος, οἱ μὲν καθόλου πάντες αἰεὶ πλείω συλλογίζονται, τῶν δ' ἐν μέρει οἱ μὲν κατηγορικοὶ πλείω, οἱ δ' ἀποφατικοὶ τὸ συμπέρασμα μόνον. αἱ μὲν γὰρ ἄλλαι προτάσεις ἀντιστρέφουσιν, ἡ δὲ στερητικὴ οὐκ ἀντιστρέφει. τὸ δὲ συμπέρασμα τί κατά τινός ἐστιν, ὥσθ' οἱ μὲν ἄλλοι συλλογισμοὶ πλείω συλλογίζονται, οἷον εἰ τὸ Α δέδεικται παντὶ τῶι Β ἢ τινί, καὶ τὸ Β τινὶ τῶι Α ἀναγκαῖον ὑπάρχειν, καὶ εἰ μηδενὶ τῶι Β τὸ Α, οὐδὲ τὸ Β οὐδενὶ τῶι Α, τοῦτο δ' ἕτερον τοῦ ἔμπροσθεν· εἰ δὲ τινὶ μὴ ὑπάρχει, οὐκ ἀνάγκη καὶ τὸ Β τινὶ τῶι Α μὴ ὑπάρχειν· ἐνδέχεται γὰρ παντὶ ὑπάρχειν.

I have two reasons for believing this. First, although he clearly holds that Bramantip is derivable from Barbara by inverting the premises and converting the conclusion, Aristotle does not appear to view Bramantip as itself an argument that includes the conversion rule as an additional premise. Aristotle holds that a syllogism strictly has just two premises. I will defend this claim in §4, and so will not put any weight on this first consideration here.

But here is a second reason for taking this third port of call to support my thesis. Aristotle uses at 53a5 and 53a9 in **T3** (53a3-14) the somewhat peculiar locution πλείω συλλογίζονται. Liddell and Scott take one meaning of the deponent verb συλλογίζομαι to be 'to infer by way of syllogism'. With the adverb πλείω, the expression πλείω συλλογίζονται at 53a5 and 53a10 means literally the same as 'to infer severally by way of syllogism'. The object of συλλογίζομαι is the conclusion so inferred, and Smith 1989, 65 rightly translates the qualified expression as 'deduce several results' or 'deduce several things'. But the subject of the verb is not the premises of the inferences, but the deduction itself. And so Aristotle gives the appearance of claiming that one and the same syllogism can have several distinct conclusions, and not that the premises of one syllogism can also be the premises of another syllogism with a distinct conclusion from the first. For these reasons, Aristotle does not seem to view Bramantip, Camenes, and Dimaris as distinct syllogisms from Barbara, Celarent, and Darii, respectively.

This reading of *APr* ii 1 has precedent. A broadly similar reading plays an important role in the Duerlinger 1968a, 497-498 and 1968b, 231 argument that syllogisms are concludent premise pairs. The line of interpretation is controversial, however, and I will consider alternative readings in §4, and clarify my points of agreement and disagreement with Duerlinger in §5.

But let me lay out all of the textual evidence before discussing objections. Aristotle also recognizes the two remaining fourth figure moods, Fesapo <MeP, SaM, PoS> and Fresison <MeP, SiM, PoS>. Neither of these is equivalent to a first figure mood with inverted premises and a converted conclusion. And Aristotle treats Fesapo and Fresison differently from the fourth figure moods discussed in *APr* ii 1. The relevant passage is *APr* i 7.29a19-27. Here then is our fourth port of call, again in the Smith 1989 translation:

> **T4**: It is also clear that in all the figures, whenever a deduction does not come about, then when both the terms are positive or privative no necessary result comes about at all; but when one term is positive and the other privative, then when the privative is taken as universal, a deduction of the minor extreme in relation to the major always comes about. For example, if A belongs to every or to some B and B to no C: if the premises are converted, it is necessary for C not to belong to some A. And similarly also in the case of the other figures, for a deduction always comes about through conversion.[11]

[11] Δῆλον δὲ καὶ ὅτι ἐν ἅπασι τοῖς σχήμασιν, ὅταν μὴ γίνηται συλλογισμός, κατηγορικῶν μὲν ἢ στερητικῶν ἀμφοτέρων ὄντων τῶν ὅρων οὐδὲν ὅλως γίνεται ἀναγκαῖον, κατηγορικοῦ δὲ καὶ

Fesapo and Fresison are derivable from Ferio by a process that includes converting the premises. However, Aristotle does not claim that for this reason the original premises 'syllogize severally'. Were syllogisms individuated by ordered premises and a conclusion, we would expect all of the fourth figure syllogisms—Fesapo and Fresison, along with Bramantip, Camenes, and Dimaris—to be treated uniformly. But where the premises of Bramantip, Camenes, and Dimaris are merely inverted from their correlated first figure moods, converting the premises of Fesapo and Fresison are also converted from their correlated first figure mood. This conversion of premises suffices to yield a distinct syllogism (provided of course that some conclusion or other follows from the converted premises).

Aristotle is nonetheless willing to call Fesapo and Fresison syllogisms. He thus appears to concede that these fourth figure moods, unlike Bramantip, Camenes, and Dimaris, fall outside of the classification of *APr* i 4-6. And so if a fourth figure mood shares unordered premise pairs with a first figure mood, then Aristotle identifies the first and fourth figure moods; but if a fourth figure mood does not share unordered premise pairs with a first figure mood, then Aristotle treats that fourth figure mood as a syllogism distinct from the moods classified in *APr* i 4-6.

I turn to our fifth port of call. At *APr* i 42.50a5-7 Aristotle writes the following, with some variation from the Smith translation: **T5**: 'Let us not fail to notice that not all the conclusions in the same syllogism are from a single figure, but rather one is through this figure and one is through another.'[12] Here in **T5** Aristotle explicitly discusses the individuation of syllogisms and under what conditions apparently distinct arguments are the same syllogism. He appears to claim that one and the same syllogism can have distinct conclusions. Moreover, one and the same syllogism can be in distinct figures. A natural suggestion is that Aristotle is referencing the fourth figure moods he recognizes in *APr* ii 1, Bramantip, Camenes, and Dimaris. Admittedly, this would be a controversial reading and I will consider an alternative interpretation in §4.

Our sixth and final port of call. So far, we have seen that Aristotle identifies moods classified within *APr* i 4-6 that share unordered premise pairs. Camestres <MaN, MeX, NeX> and Cesare <MeN, MaX, NeX> are two second figure moods with inverted premises. Recall, Aristotle typically introduces a mood by noting, for a given arrangement of premise forms, that a conclusion necessarily follows. In *APr* i 5 Aristotle introduces Camestres and Cesare together by noting first that there is a deduction in the second figure, where the middle term is the predicate of both premises, provided one term is an a-proposition and the other an e-proposition. Here is the passage in full, again with some variation from the

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στερητικοῦ, καθόλου ληφθέντος τοῦ στερητικοῦ ἀεὶ γίνεται συλλογισμὸς τοῦ ἐλάττονος ἄκρου πρὸς τὸ μεῖζον, οἷον εἰ τὸ μὲν Α παντὶ τῶι Β ἢ τινί, τὸ δὲ Β μηδενὶ τῶι Γ· ἀντιστρεφομένων γὰρ τῶν προτάσεων ἀνάγκη τὸ Γ τινὶ τῶι Α μὴ ὑπάρχειν. ὁμοίως δὲ κἀπὶ τῶν ἑτέρων σχημάτων· ἀεὶ γὰρ γίνεται διὰ τῆς ἀντιστροφῆς συλλογισμός.

[12] Μὴ λανθανέτω δ᾽ἡμᾶς ὅτι ἐν τῶι αὐτῶι συλλογισμῶι οὐχ ἅπαντα τὰ συμπεράσματα δι᾽ ἑνὸς σχήματός ἐστιν, ἀλλὰ τὸ μὲν διὰ τούτου τὸ δὲ δι᾽ ἄλλου.

Smith translation.

> **T6**: When the terms are universal, there will be a syllogism when the middle belongs to all of one term and none of the other, no matter which one the privative is in relation to, but otherwise in no way. For let M be predicated of no N but of every X. Then, since the privative converts, N will belong to no M. But M was assumed to belong to every X, so that N belongs to no X (for this has been proved earlier). Next, if M belongs to every N but to no X, then neither will N belong to any X. For if M belongs to no X, neither does X belong to any M; but M belonged to every N; therefore, X will belong to no N (for the first figure has again come about). And since the privative converts, neither will N belong to any X, so that there will be the same syllogism (ὁ αὐτὸς συλλογισμός). (It is also possible to prove these results by leading to an impossibility.) It is evident, then, that a syllogism comes about when the terms are related in this way, but not a perfect syllogism.[13] (27a3-16)

Here Aristotle asserts that when the premises share a predicate and one is an a-proposition and the other an e-proposition, there is a syllogism since at least one conclusion follows necessarily. He goes on to show that this is the case by deriving both Cesare and Camestres. He calls Camestres the same syllogism (ὁ αὐτὸς συλλογισμός) as Cesare at 27ª14. Like the case of Bramantip and Barbara, Aristotle clearly identifies Camestres and Cesare. As in our previous port of call, this is a controversial reading, and I will consider an alternative in §4.

But it may be helpful to the reader first to pause and sum up the evidence considered to this point. I have drawn on Aristotle's claim that some syllogisms syllogize several results at 53a5 and 53a10 in **T3** (53a3-14), that distinct conclusions can belong to the same syllogism at 50a5 in **T5** (50a5-7), and that Camestres and Cesare are the same syllogism at 27ª14 in **T6** (27a3-16), as evidence that syllogisms are individuated by unordered premise pairs. These passages are among the very few of which I am aware where Aristotle gives explicit guidance on how to individuate syllogisms: 27a14 and 53a18 are the only occurrences in Aristotle of ὁ αὐτὸς συλλογισμός, with τῷ αὐτῷ συλλογισμῷ at 50a5; and 53a5 and 53a10 are the only occurrences of πλείω συλλογίζονται.[14]

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[13] καθόλου μὲν οὖν ὄντων ἔσται συλλογισμὸς ὅταν τὸ μέσον τῶι μὲν παντὶ τῶι δὲ μηδενὶ ὑπάρχηι, ἂν πρὸς ὁποτερωιοῦν ᾖι τὸ στερητικόν· ἄλλως δ' οὐδαμῶς. κατηγορείσθω γὰρ τὸ Μ τοῦ μὲν Ν μηδενός, τοῦ δὲ Ξ παντός. ἐπεὶ οὖν ἀντιστρέφει τὸ στερητικόν, οὐδενὶ τῶι Μ ὑπάρξει τὸ Ν· τὸ δέ γε Μ παντὶ τῶι Ξ ὑπέκειτο· ὥστε τὸ Ν οὐδενὶ τῶι Ξ· τοῦτο γὰρ δέδεικται πρότερον. πάλιν εἰ τὸ Μ τῶι μὲν Ν παντὶ τῶι δὲ Ξ μηδενί, οὐδὲ τὸ Ξ τῶι Ν οὐδενὶ ὑπάρξει (εἰ γὰρ τὸ Μ οὐδενὶ τῶι Ξ, οὐδὲ τὸ Ξ οὐδενὶ τῶι Μ· τὸ δέ γε Μ παντὶ τῶι Ν ὑπῆρχεν· τὸ ἄρα Ξ οὐδενὶ τῶι Ν ὑπάρξει· γεγένηται γὰρ πάλιν τὸ πρῶτον σχῆμα)· ἐπεὶ δὲ ἀντιστρέφει τὸ στερητικόν, οὐδὲ τὸ Ν οὐδενὶ τῶι Ξ ὑπάρξει, ὥστ' ἔσται ὁ αὐτὸς συλλογισμός. ἔστι δὲ δεικνύναι ταῦτα καὶ εἰς τὸ ἀδύνατον ἄγοντας. ὅτι μὲν οὖν γίνεται συλλογισμὸς οὕτως ἐχόντων τῶν ὅρων, φανερόν, ἀλλ' οὐ τέλειος·

[14] Aristotle uses τὸν αὐτὸν…συλλογγισμόν at 29a28-29. The passage in which the expression occurs immediately follows **T4**, at *APr* i 7.29a27-29: 'It is also clear that putting an indeterminate

III

The interpretation that syllogisms are individuated by unordered premise pairs may be surprising, given the long historical association of syllogisms with the traditional moods. In section 4, I will consider several objections. But in this section I first note that the view is consistent with several other features of the *Prior Analytics*. First, notice that the view is consonant with the characterization of a syllogism in **T1** (24b18-22), where Aristotle characterizes a syllogism as 'a discourse in which, certain things having been supposed, something (τι) different from the things supposed results of necessity because these things are so'. The second occurrence of the indefinite pronoun *tis* in this characterization (here declined as *ti*) could be read as referring to an *unspecified* conclusion. If so, the characterization only requires that the premises of a syllogism entail some conclusion or other. The reader might object that, if the same syllogism can have multiple conclusions, then Aristotle should speak of several results, and **T1** should have the neutral plural of *tis*, *tina*. But the thesis that syllogisms are individuated by unordered premise pairs entails only that a syllogism has some conclusion or other, even if there are multiple conclusions available. Nothing we have seen requires that a statement of a syllogism should explicitly draw all available conclusions. I return to this point in §5.[15]

The interpretation is also consistent with the various signposts and metalogical observations scattered throughout the *Prior Analytics*. The two books of this work include a formal study of the assertoric and modal moods, but are largely concerned with two other topics: heuristics, the choice of appropriate premises for desired conclusions, and analytics, the reformulation of given arguments into syllogistic form. Aristotle marks the transitions between these sections with signposts that give good evidence of how he views the goals of these sections.

Aristotle never states an explicit aim of *specifying* the syllogisms—laying out the premises and conclusions of each syllogism. His avowed aim is rather to show what pairs of propositions are the premise pairs of a syllogism. For example, he begins his formal study of the moods with the following exhortation at *APr* i 4.25b26-27, **T7**: 'Let us say now through what premises, when, and how every syllogism comes about'.[16] Aristotle appears to state three aims here—to state through what premises every syllogism comes about, when every syllogism comes about, and how every syllogism comes about. How ought we to take these

premise in place of a positive particular will produce the same deduction in every figure' (δῆλον δὲ καὶ ὅτι τὸ ἀδιόριστον ἀντὶ τοῦ κατηγορικοῦ τοῦ ἐν μέρει τιθέμενον τὸν αὐτὸν ποιήσει συλλογισμὸν ἐν ἅπασι τοῖς σχήμασιν). Indeterminate propositions lack explicit quantity but we might take them to have hidden universal or particular quantity that must be supplied in context. For example, they seem to be equivalent to particular propositions in this passage but, as Striker 2009, 77 notes, the examples given at 24a21-22, 'the science of contraries is the same' and 'pleasure is not a good', seem to be equivalent to universal propositions. Regardless, 29a27-29 is neutral on the question whether syllogisms are individuated by unordered premise pairs.

[15] Moreover, **T1** might be merely a sufficient condition for being a syllogism, and having at least one conclusion (along with meeting the other criteria) would suffice.

[16] Διωρισμένων δὲ τούτων λέγωμεν ἤδη διὰ τίνων καὶ πότε καὶ πῶς γίνεται πᾶς συλλογισμός·

aims? Aristotle has calls to **T7** (25b26-27) at *APr* i 22.40b12-14, 27.43a16-19, and 32.46b38-40. Let us take a closer look at the second of these.

> **T8**: From what has been said, then, it is clear how every syllogism comes about, both through how many terms and premises and what relationships they are in to one another, and furthermore what sort of problem (πρόβλημα) is proved in each figure, and what sort in more and what in fewer figures.[17] (43a16-19)

Again, Aristotle emphasizes his aim of showing what pairs of propositions are the premise pairs of a syllogism. The goal of laying out the relationships among these propositions may correspond to Aristotle's stated aim in **T7** at 25b27 to explain 'how' (πῶς) syllogisms come about. But it is not entirely clear how the three clauses in **T7** correspond to achievements claimed in **T8**. And so it is unclear what the differences are among the three clauses in **T7**. I doubt that distinct roles for each clause—through what premises every syllogism comes about, when every syllogism comes about and how every syllogism comes about—can be assigned. Indeed, these clauses may well be pleonastic. The most that can confidently be said is that Aristotle aims to show what premise pairs are the premise pairs of a syllogism. This is consistent with taking syllogisms to have multiple available conclusions. Aristotle does show a concern in **T8** with the conclusions that can be drawn from concludent premise pairs. As Smith 1989, 114 notes, the expression πρόβλημα invariably in the *Prior Analytics* has the sense of 'types of categorical sentences…found as conclusions'. But Aristotle takes himself only to have shown what conclusion follows in each figure, that is, what moods, with a strict premise order and a single conclusion, correspond to syllogisms. And again, this is consistent with taking syllogisms to have multiple available conclusions.

Aristotle goes on to describe how one might seek syllogisms for a given conclusion. A complete specification of syllogisms arguably would serve Aristotle's purposes here better. For example, if I wanted to derive AiC, I could look for the premises AaB and BaC, an heuristic strategy obscured by the omission of the subaltern mood Barbari from the discussion of *APr* i 4-6. So it seems that I must view Aristotle's discussion of heuristics in *APr* i 23-27 as an impoverished method. However, any interpretation of syllogisms faces this difficulty, whether Barbari is regarded as a neglected mood or as a mood corresponding to the same syllogism to which Barbara corresponds. Moreover, Aristotle does not purport to offer an exhaustive heuristic method. Aristotle lays out his aims in the next passage that, immediately following **T8**, is *APr* i 27.43a19-24.

> **T9**: Now it is time to explain how we may ourselves always be supplied with syllogisms about what is set up, and the route by which we may obtain the principles concerning any particular

[17] Πῶς μὲν οὖν γίνεται πᾶς συλλογισμὸς καὶ διὰ πόσων ὅρων καὶ προτάσεων, καὶ πῶς ἐχουσῶν πρὸς ἀλλήλας, ἔτι δὲ ποῖον πρόβλημα ἐν ἑκάστωι σχήματι καὶ ποῖον ἐν πλείοσι καὶ ποῖον ἐν ἐλάττοσι δείκνυται, δῆλον ἐκ τῶν εἰρημένων.

> subject. For surely one ought not only study the origin of deductions, but also have the power to produce them.[18]

Judging from **T9**, Aristotle only aims to supply syllogisms and have the power to produce them. He does not aim to have an exhaustive supply of valid inferences from a given premise pair to whatever conclusion follows from those premises. Nor does Aristotle aim to give the reader an unlimited or maximal power of the production of syllogisms. And yet again, this is consistent with taking syllogisms to have multiple available conclusions.

Similar comments can be made on Aristotle's discussion of analysis, or the leading of syllogisms back into one of the moods. Aristotle's explicit characterization of his aims in this section seems to be to take a given informal argument and present it as one of the moods. Consider his discussion at i 32.46b40-47a5:

> **T10**: We must explain how we can lead syllogisms back into the figures stated previously, for this part of our inquiry still remains. For if we should study the origin of syllogisms, and also should have the power of finding them, and if, moreover, we could resolve those which have already been produced into the figures previously stated, then our initial project would have reached its goal.[19]

Not unlike in the case of heuristics discussed above, it may seem that this is an impoverished method: would it not be better to have more valid inferences from premise pairs to conclusions with which to work, when aiming formally to represent an informal argument? My response to this objection is similar to my response to the objection in the heuristics case. Any interpretation of syllogisms faces difficulties of this sort. For Aristotle's professed aim in *APr* i 4-6, to classify all valid moods, is unfulfilled.

Yet, this raises an interesting question. On the interpretation I have pushed for, there are striking differences between moods and syllogisms. Aristotle's explicit aim is to provide an exhaustive list of syllogisms. Yet he proceeds in *APr* i 4-6 by classifying assertoric moods. Why does Aristotle proceed in this way? I return to this question in §5.

IV

Let me respond now to a few objections. I have looked at Aristotle's claims that some syllogisms syllogize several results at 53a5 and 53a10 in **T3** (53a3-14), that distinct conclusions can belong to the same syllogism at 50a5 in **T5** (50a5-7), and that Camestres and Cesare are the same syllogism at 27a14 in **T6** (27a3-

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[18] Πῶς δ᾽ εὐπορήσομεν αὐτοὶ πρὸς τὸ τιθέμενον ἀεὶ συλλογισμῶν, καὶ διὰ ποίας ὁδοῦ ληψόμεθα τὰς περὶ ἕκαστον ἀρχάς, νῦν ἤδη λεκτέον· οὐ γὰρ μόνον ἴσως δεῖ τὴν γένεσιν θεωρεῖν τῶν συλλογισμῶν, ἀλλὰ καὶ τὴν δύναμιν ἔχειν τοῦ ποιεῖν.

[19] πῶς δ᾽ ἀνάξομεν τοὺς συλλογισμοὺς εἰς τὰ προειρημένα σχήματα, λεκτέον ἂν εἴη μετὰ ταῦτα· λοιπὸν γὰρ ἔτι τοῦτο τῆς σκέψεως. εἰ γὰρ τήν τε γένεσιν τῶν συλλογισμῶν θεωροῖμεν καὶ τοῦ εὑρίσκειν ἔχοιμεν δύναμιν, ἔτι δὲ τοὺς γεγενημένους ἀναλύοιμεν εἰς τὰ προειρημένα σχήματα, τέλος ἂν ἔχοι ἡ ἐξ ἀρχῆς πρόθεσις.

16). One might take a deflationary view of the evidence in **T3** (53a3-14), **T5** (50a5-7), and **T6** (27a3-16), and hold that, in one way or another, Aristotle means something other than an ordinary syllogism in these passages. For example, one might take Aristotle to intend in some of these passages an *extended* sense of 'syllogism' under which a syllogism is an argument that may contain a string of moods or a mood along with other premises. Some commentators take this reading of the claim that some syllogisms syllogize several results at 53a5 and 53a9-10 in **T3**. Ross 1949, 425 takes the question here to be 'what conclusions, besides the primary conclusion, a syllogism can be held to prove implicitly'. Smith 1989, 183 appears to agree, holding that Aristotle 'shows that an additional conclusion can be derived from many deductive forms through conversion'. And many commentators hold that Aristotle is speaking in **T5** of a string of moods. For example, Ross 1949, 414 notes that Aristotle means 'an extended argument in which more than one syllogism occurs'. Smith 1989, 174 agrees, noting that Aristotle 'means an extended deduction which may contain subsidiary deductions in several figures'. Striker 2009, 235 concurs.

Aristotle allows for this extended sense of 'syllogism'. He considers in *APr* i 25.42b1-26 and *APr* ii 18 arguments that consist of a series of syllogisms. Aristotle is willing to call such arguments syllogisms, and so they are exceptions to the thesis that syllogisms have no more than three terms and two premises. But he appears to view these arguments as reducible to, and equivalent with, two-premise moods. So in this sense even these arguments are two premise syllogisms. Aristotle repeats that a syllogism has only three terms at *APr* i 28.44b6, 30.46a6 and *APo* i 9.81b10. And generally Aristotle uses an extended sense of 'syllogism' in contexts where it is clear that he is talking about strings of strict syllogisms. For example, in *APr* i 25 Aristotle's intention is to show that, since strings of strict syllogisms are equivalent to syllogisms in one of the figures, they do not fall entirely outside of the classification of moods in *APr* i 4-6. In passages where the context is not restricted in this way, then surely the strict sense of 'syllogism' is the interpretative default. I will assume that strict syllogisms are the intended sense unless it proves impossible to understand Aristotle's intention otherwise. Aristotle does not flag that he means syllogism in its extended sense in **T3** and **T5**. And it would be surprising if the extended sense of syllogism is intended in **T3** or **T5**. For these passages do not concern extended argument forms. Rather, Aristotle's interest is with the relation between fourth figure moods and moods of one the figures classified in *APr* i 4-6. So it is unlikely that this deflationary strategy can provide an attractive reading of these passages. Similarly, such a gambit cannot be used to dismiss the evidence in **T6**. Here too, Aristotle does not seem to be discussing extended arguments; rather, he is comparing the two moods Camestres and Cesare. So the postulation of an extended sense of syllogism is not a general strategy to object to the thesis that syllogisms are individuated by unordered premise pairs.

Another deflationary strategy would be to take 'syllogism' occasionally to be used synecdochically to mean some part of a syllogism. For example, one might

take Aristotle's intention when he claims that certain syllogisms 'syllogize severally' is to assert that certain pairs of *premises* deduces several conclusions, and so the same unordered premises can be contained in *distinct* syllogisms. However, Aristotle has the resources to say this, since he can distinguish the premises from the syllogism with the *protasis* terminology, used throughout the *Prior Analytics*. Alternatively, one might take 'syllogism' to refer occasionally to the *conclusion* of a syllogism. Such a move would deflate Aristotle's characterization of Camestres and Cesare as the same syllogism. Striker 2009, 101, for example, takes 'the same syllogism' to mean 'a [distinct] syllogism with the same conclusion. Here as in many other places Aristotle uses the word "syllogism" both for an entire argument and for its conclusion. His special term for conclusion, [*sumperasma*], appears only from chapter 8 onwards.' On this reading, Camestres and Cesare share a conclusion but are distinct syllogisms. And the view has the added support that Aristotle might lack the terminological resources when he is writing **T6** (27a3-16) to distinguish syllogisms from their conclusions. However, the reading at best only handles our sixth port of call, Aristotle's claim that Camestres and Cesare are the same syllogism at 27a14 in **T6**. The reading does not handle well our third and fifth ports of call, Aristotle's claims respectively that some syllogisms syllogize several results at 53a5 and 53a10 in **T3** (53a3-14), and that distinct conclusions can belong to the same syllogism at 50a5 in **T5**. The suggestion that Aristotle lacks the terminology to distinguish a syllogism from its conclusion when writing these passages is highly implausible. Aristotle uses συμπέρασμα as the conclusion of a syllogism at 53a19-20, in a passage immediately following **T3**.

What then motivates these deflationary readings of Aristotle's claim that Camestres and Cesare are the same syllogism at 27a14? It is perhaps the belief that, since Camestres and Cesare are distinct moods in the classification of *APr* i 4-6, they must be distinct syllogisms. Such a belief might rest on the assumption that the classification of moods is intended to provide a list of syllogisms that is both exhaustive and mutually exclusive. Were this assumption correct, then the suggestion that Camestres and Cesare are really the same syllogism would run counter to Aristotle's intentions in classifying the moods. So let me now discuss this assumption. Aristotle gives the impression that the classification in *APr* i 4-6 is intended to be exhaustive. For example, as we have seen, he announces his intention to classify all syllogisms (πᾶς συλλογισμός) at *APr* i 4.25b27, b31 in **T7** (25b26-27) and **T8** (43a16-19) and elsewhere. By *APr* i 23, Aristotle has shown that the syllogisms in the three figures can be completed by Barbara and Celarent; he (40b20-23) writes that 'it will now be evident that this holds for every syllogism without qualification, when every one has been proved to come about through some one of these figures'. These and other metatheorems require that Aristotle have an exhaustive list of valid syllogisms.

As we have seen, Aristotle recognizes that the method of classification of moods in *APr* i 4-6 *underdetermines* valid syllogisms. For Aristotle acknowledges in *APr* i 7 that Fesapo and Fresison are syllogisms left unclassified in *APr* i

4-6. It may be that Aristotle holds that the classification, when supplemented by these two outliers, is exhaustive. If so, he would have been clearer to have flagged the inclusion of these two moods in such discussions as *APr* i 23. Regardless, it appears to be Aristotle's intention to provide a list of syllogisms that is exhaustive. However, I know of no passage that suggests that the classification in *APr* i 4-6 is intended to be mutually *exclusive*. Aristotle seeks a method systematically to go through every permutation of terms in given pairs of premises, and show which pairs of premises entail some conclusion or other. Nothing in this way of proceeding requires that the resulting list be mutually exclusive. Repetition of a syllogism would not undermine Aristotle's intentions; it would be merely inefficient. And since I believe that there is good evidence that syllogisms are individuated by unordered premise pairs, and so certain pairs of moods, such as Camestres and Cesare, are not distinct syllogisms, I am inclined to believe that the classification of moods in *APr* i 4-6 *overdetermines* the valid syllogisms.

I bring this section to a close. In §2, I examined evidence that syllogisms are individuated by unordered premise pairs. This evidence includes Aristotle's claim that some syllogisms syllogize several results at 53a5 and 53a10 in **T3** (53a3-14), that distinct conclusions can belong to the same syllogism at 50a5 in **T5** (50a5-7), and that Camestres and Cesare are the same syllogism at 27a14 in **T6** (27a3-16). In this section, I have considered several deflationary readings of these passages. The deflationary readings are not well motivated, and so fail to undermine the evidential support the passages provide.

V

Sections §§2-4 are the main line of my argument, and this section will not add any further support to the thesis that syllogisms are individuated by unordered premise pairs. Instead, I discuss a few consequences of this thesis. The discussion will be too brief to be fully satisfying. But I hope to indicate in broad strokes how the considerations raised interact with questions about the representation of the syllogistic.

The textual evidence examined above might suggest to the reader one of two potentially misleading interpretations of syllogisms, and I discuss each in turn. First, one might view syllogisms as concludent premise pairs. On this view, syllogisms are not themselves inferences but are conjunctive propositions that license drawing one or more conclusions. As I noted in §2, Duerlinger 1968a, 1968b appeals to **T3** to defend this line of interpretation. The thesis I am putting forward, the view that syllogisms are individuated by unordered premise pairs, is of course a weaker claim, since syllogisms can be individuated by premise pairs without being identified with such pairs. Although I have argued that **T3**, **T5**, and **T6** offer good support for the weaker thesis, they offer no support for the stronger thesis. The weaker claim, moreover, is consistent with viewing syllogisms as inferences. And although I merely sketched some of the considerations in favour of this inferential reading in §1, it is now standardly held, I believe it to be cor-

rect, and there is no reason to saddle the thesis that syllogisms are individuated by unordered premise pairs with the denial that syllogisms are inferences. The reader might view one of my goals, then, as to revisit some of the textual evidence examined by Duerlinger, along with other textual evidence, in light of more recent interpretative developments.

I turn to the second of the two misleading interpretations of syllogisms. The discussion to this point might suggest to the reader that syllogisms are multiple-succedent sequents. Single-succedent sequents are expressions such as

$p_1, \ldots, p_n \vdash s$

where the antecedent $p_1, \ldots, p_n$, the premises together, entail the conclusion or succedent, s.[20] A multiple-succedent sequent such as

$p_1, \ldots, p_n \vdash s_1, \ldots, s_n$

is a generalization of this notion of a sequent. To view syllogisms as such sequents requires an unusual interpretation. The standard informal understanding of sequents takes a conjunctive reading of the antecedent and a disjunctive reading of the succedent, so that the above multiple-succedent sequent has the same interpretation as

if every p_i is true, then at least one s_i is true.

Were syllogisms multiple-succedent sequents, they would require a conjunctive reading of the succedent, and so have the same interpretation as

if every p_i is true, then every s_i is true.

That is to say, the members of the antecedent together entail all of the members of the succedent.

The suggestions that Aristotle recognizes a multiple conclusion logic and that he understands multiple conclusions conjunctively both have precedent in Malink's innovative interpretation of Aristotle's discussion in *APr* ii 5-7 of circular proof. Malink 2013, 220 presents the general structure of a circular proof as: 'P_1 is proved from P_2, P_2 is proved from P_3, . . ., P_{n-1} is proved from P_n, and P_n is proved from P_1 ($n \geq 1$).' Malink persuasively argues that the P_i must be pluralities of propositions in order for Aristotle to give a syllogistic analysis of this argument form. Malink 2013, 246 concludes that 'a plurality of propositions can be deduced from another plurality'. Malink notes that the multiple conclusions must be read conjunctively and holds that such a reading 'is arguably more natural'.[21] My results perhaps may be seen as a complement to Malink's thesis: Aristotle's recognition of something like a multiple conclusion logic flows not just from the demands of his discussion of circular proof but from his view of the syllogism itself.

However, there would be something potentially misleading in this reading of syllogisms as *themselves* multiple-succedent sequents. The considerations raised here support only the thesis that syllogisms are individuated by unordered premise pairs. As we have just seen, since syllogisms are rightly viewed to be

[20] The turnstile is an object language expression here, and not as it is more commonly used, a metalanguage expression.

[21] On this latter point, compare Tennant 1997.

inferences, they are not concludent premise pairs. But neither do all available conclusions need to be explicitly drawn in a syllogistic inference. Rather, syllogisms are inferences from unordered premise pairs to some conclusion or other from among a set of available conclusions. It is in this sense that, for example, Barbara and Barbari are the same syllogism, despite having different conclusions. For this reason, it might be clearer to say that a syllogism *corresponds* to a multiple-succedent sequence. In the terminology introduced in §1, two moods $<p_1, p_2, s_1>$ and $<p_3, p_4, s_2>$ are the same syllogism only if there is a multiple-succedent sequent $<\{p_1, p_2\}, \{s_1, \ldots, s_m\}>$ such that (with '=' here denoting identity) either $p_3=p_1$ and $p_4=p_2$ or $p_3=p_2$ and $p_4=p_1$, and for some i such that $1 \leq i \leq m$, $s_2=s_i$, and for some j such that $1 \leq j \leq m$, $s_2=s_j$.[22] So, for example, Barbara <AaB, BaC, AaC> and Barbari <PaM, MaS, PiS> are the same syllogism because they both correspond to <{AaB, BaC}, {AaC, AiC, …}>.

The proposal that syllogisms are related in this way to multiple-succedent sequents might further suggest to the reader that the syllogistic can be fruitfully thought of as a sequent calculus. But the reasoning Aristotle uses to show the acceptability of the second and third figure moods is recalcitrant to representation as a sequent calculus.[23] The method of indirect proof (such as that which Aristotle presents in **T2** (27a36-b1)) and his proofs for the conversion rules both rely on a reductio rule.[24] A characteristic mark of natural deduction systems, in contradistinction to sequent calculi, is the facility to make, track, and subsequently discharge arbitrary assumptions.[25] This feature is key to Gentzen's original presentation and is the sense in which such a deduction system is 'natural': its employment reflects this aspect of actual reasoning.[26] As the work of Smiley 1973, Smith 1982 and others show, it is attractive to represent the argumentation that Aristotle himself *employs* in *APr* i 4-6 by a natural deduction system.

Notice that there results a point of some interest for the interpretation of the syllogistic and its representation by modern systems. Corcoran 1974, 280 takes the interpretation of syllogisms as implications to entail that the syllogistic is not a natural deduction system and therefore not a logic. But there since has been recognition that a wide range of systems can provide a logical interpretation of the syllogistic. The diversity of representations I footnoted in the article's opening paragraph is a testament to the flexibility of Aristotle's achievement. To give

[22] 'Only if': there are other constraints, as mentioned in §1, such as the requirement that no conclusion is the same proposition as any of the premises.

[23] I do not claim that the valid syllogisms cannot be systematically presented within a sequent calculus. Crabbé 2003 and Tennant 2014 independently represent the syllogistic in this way, although neither aims to follow Aristotle's own presentation.

[24] Aristotle proves e-conversion at 25a5-17 by employing a reductio principle and the square of opposition (or, at least, the contradictory opposition between e- and i-propositions). He goes on to establish the other conversion rules by reductio proofs that employ the established e-conversion.

[25] Pelletier 1999 argues that this feature is the most characteristic mark of natural deduction systems.

[26] Gentzen's 1934, 74 professed aim is 'to set up a formalism that reflects as accurately as possible the actual logical reasoning involved in mathematical proofs'.

just two relatively recent examples, Martin 1997 interprets the moods as inferences but represents the syllogistic as a construction, with the perfect moods as basic elements. On this reading, the syllogistic has a structure redolent of an axiomatic theory. Pelletier and Hazen 2012, 52 also reject the assimilation of logics with natural deduction systems while presenting the syllogistic as a metatheory. I have argued that syllogisms are related to multiple-succedent sequents. As we have seen, although syllogisms correspond to multiple-succedent sequents, the reasoning used in the syllogistic resembles natural deductions. There is no tension here. For the syllogistic directly derives not syllogisms but moods. The question of how to represent the syllogistic is sensitive to the interpretation of moods, but it is an issue that is one step removed from the interpretation of syllogisms.

We now can return to the question I raised at the end of §3. Why does Aristotle attempt to classify all of the syllogisms by going through the moods? Let me sketch an incomplete answer. Aristotle's aim is not just to *enumerate* the moods but to *show* what moods are valid. Aristotle takes the first figure moods, along with a reductio rule and the conversion rules, as intuitively valid inferences. It is relatively easy to see that the first figure moods are valid. For example, it is arguably easier to appreciate that the ordered premise pair <AaB, BaC> entails AaC than to see that {AaB, BaC} entails both AaC and AiC. Moreover, moods can be employed in proofs more straightforwardly than multiple-succedent sequents. The reasoning used by conversion and reductio proofs are facilitated by using rules that allow us to derive from a pair of lines in the proof a determinate, unique conclusion. The use of moods, and not syllogisms, has these advantages. But to show, for example, that the mood Cesare <MeN, MaX, NeX> is valid makes it clear that there is a syllogism with unordered premise pairs {MeN, MaX} and with NeX *among* its conclusions. In this way, Aristotle *proves* that a mood is valid, and thereby *establishes* a correlated syllogism.[27]

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BIBLIOGRAPHY

Arnauld, A. and P. Nicole. 1662. *La logique ou l'art de penser*. Translated in J. Buroker 1996. *Logic or the Art of Thinking*. Cambridge: Cambridge University Press.

van Benthem, J. 1984. 'Questions About Quantifiers' *Journal of Symbolic Logic* 49: 443-466.

van Benthem, J. 2008. 'A Brief History of natural logic' Technical Report PP-2008-05, Institute for Logic, Language & Computation. url: http://www.illc.uva.nl/Publications/ResearchReports/PP-2008-05.text.pdf.

Beth, E. 1955. 'Semantic Entailment and Formal Derivability' *Mededelingen van de Koninklijke Nederlandse Akademie van Wetenschappen, Afdeling Letterkunde*. N.R. vol 18, no 13, 309-342. Reprinted in J. Hintikka ed. 1969. *The Philosophy of Mathematics*. Oxford University Press.

Carroll. L. 1887. *The Game of Logic*. London: MacMillan.

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Corcoran, J. 1972. ‘Completeness of an Ancient Logic’ *Journal of Symbolic Logic* 37: 696-702.
Corcoran, J. 1974. ‘Aristotle’s Natural Deduction System’ 85-131 in J. Corcoran ed. *Ancient Logic and Its Modern Interpretations*. Dordrecht: Reidel.
Corcoran, J. 1994. ‘The Founding of Logic’ *Ancient Philosophy* 14: 9-24.
Crabbé, M. 2003. ‘The formal theory of syllogisms’ *Review of Modern Logic* 9: 29-52.
Duerlinger, J. 1968a. ‘Aristotle’s Conception of Syllogism’ *Mind* 77: 480-499.
Duerlinger, J. 1968b. ‘Drawing Conclusions from Aristotelian Syllogisms’ *Monist* 52: 229-236.
Duerlinger, J. 1969. ‘*Sullogismos* and *sullogizesthai* in Aristotle’s Organon’ *American Journal of Philology* 90: 320-328.
Dutilh Novaes, C. 2015. ‘The Formal and the Formalized: the Cases of Syllogistic and Supposition Theory’ *Kriterion* 131: 253-270.
van Eijck, J. 1985. ‘Generalized Quantifiers and Traditional Logic’ 1-21 in J. van Benthem and L. ter Menlen edd. *Generalized Quantifiers in Natural Language*. Grass. vol. 4. Dordrecht: Foris.
Englebretsen, G. 1981. *Three logicians. Aristotle, Leibniz, and Sommers and the syllogistic.* Assen: Van Gorcum.
Euler, L. 1768. Lettres à une Princesse d’Allemagne sur Divers Sujets de Physique et de Philosophie, Saint-Petersbourg: De l’Academie des Sciences, 1768. (H. Hunter trans. 1997. Letters of Euler to a German Princess on Different Subjects in Physics and Philosophy. Thoemmes Press.)
Gentzen, G. 1934. ‘Untersuchungen über das logische Schließen’ *Mathematische Zeitschrift* 39: 176-210. (‘Investigations into Logical Deduction’ in M. Szabo ed. 1969. *The Collected Works of Gerhard Gentzen*. North-Holland.)
Lear, J. 1980. *Aristotle and Logical Theory*. Cambridge: Cambridge University Press.
Lemon, O. and I. Pratt 1998. ‘On the Insufficiency of Diagrams for Syllogisms’ *Notre Dame Journal of Formal Logic* 39: 573-580.
Łukasiewicz, J. 1957. *Aristotle’s Syllogistic*. 2nd edn. 1st edn. 1951. Oxford: Oxford University Press.
Maier, H. 1936. *Syllogistik des Aristoteles*. 3 vols. Tuebingen: H. Lamp, 1896-1900. Neudruck, Leipzig: Koehler, 1936.
Malink, M. 2009. ‘A Non-Extensional Notion of Conversion in the Organon’ *Oxford Studies in Ancient Philosophy* 37: 105-141.
Malink, M. 2013. ‘Aristotle on Circular Proof’ *Phronesis* 58: 215-248.
Martin, J. 1987. *Elements of Formal Semantics*. Orlando FL: Academic Press.
Martin, J. 1997. ‘Aristotle’s Natural Deduction Reconsidered’ *History and Philosophy of Logic* 18: 1-15.
McCall, S. 1967. ‘Connexive Implication and the Syllogism’ *Mind* 76: 346-356.
Morison, B. 2011. ‘What Was Aristotle’s Concept of Logical Form?’ 172-188 in B. Morison and K. Ierodiakonou edd. *Episteme, Etc*. Oxford: Oxford University Press.
Morison, B. 2015. ‘What Is a Perfect Syllogism?’ *Oxford Studies in Ancient Philosophy* 48: 107-166.
Pagnan, R. 2013. ‘Syllogisms in Rudimentary Linear Logic, Diagrammatically’ *Journal of Logic, Language and Information* 22: 71-131.
Parsons, T. 2014. *Articulating Medieval Logic*. Oxford: Oxford University Press.
Patzig, G. 1968. *Aristotle’s Theory of the Syllogism*: *a logico-philological study of Book A of the Prior Analytics*. Dordrecht: Reidel.
Pelletier F.J. 1999. ‘A Brief History of Natural Deduction’ *History and Philosophy of Logic* 20: 1-31.
Pelletier F.J. and A. Hazen. 2012. ‘A History of Natural Deduction’ 341-414 in D. Gabbay, F.J. Pelletier and E. Woods edd. *Handbook of the History of Logic.* vol 11. North-Holland: Elsevier.
Rose, L. 1968. *Aristotle’s Syllogistic*. Springfield IL: C.C. Thomas.
Ross, W.D. ed. 1949. *Aristotle’s Prior and Posterior Analytics.* Oxford: Clarendon Press.
Smiley, T. 1962. ‘Syllogism and Quantification’ *Journal of Symbolic Logic* 27: 58-72.
Smiley, T. 1973. ‘What Is a Syllogism?’ *Journal of Philosophical Logic* 2: 136-154.
Smith, R. 1982. ‘What Is Aristotelian *Ecthesis*?’ *History and Philosophy of Logic* 3: 113-127.
Smith, R. 1989. *Aristotle’s Prior Analytics*. Indianapolis: Hackett.
Striker, G. 1998. ‘Aristotle and the Uses of Logic’ 209-226 in J. Gentzler ed. *Method in Ancient Phi-*

losophy. Oxford University Press.
Striker, G. 2009. *Aristotle's* Prior Analytics *Book I Translated with An Introduction and Commentary*. Oxford: Clarendon Press.
Tennant, N. 1997. *The Taming of the True*. Oxford: Oxford University Press.
Tennant, N. 2014. 'Aristotle's Syllogistic and Core Logic' *History and Philosophy of Logic* 35: 120-147.
Thom, P. 1979. 'Aristotle's Syllogistic' *Notre Dame Journal of Formal Logic* 20: 751-759.
Westerståhl, D. 1989. 'Aristotelian Syllogisms and Generalized Quantifiers' *Studia Logica* 48: 577-585.