(T&F) THPL100387

HISTORY AND PHILOSOPHY OF LOGIC, **24** (2003), 261–288

Taylor & Francis
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Aristotle's *Prior Analytics* and Boole's *Laws of Thought*

JOHN CORCORAN
Philosophy, University of Buffalo, Buffalo, NY 14214, USA

Received 2 May 2003 Accepted 12 May 2003

Prior Analytics by the Greek philosopher Aristotle (384 – 322 BCE) and *Laws of Thought* by the English mathematician George Boole (1815 – 1864) are the two most important surviving original logical works from before the advent of modern logic. This article has a single goal: to compare Aristotle's system with the system that Boole constructed over twenty-two centuries later intending to extend and perfect what Aristotle had started. This comparison merits an article itself. Accordingly, this article does not discuss many other historically and philosophically important aspects of Boole's book, e.g. his confused attempt to apply differential calculus to logic, his misguided effort to make his system of 'class logic' serve as a kind of 'truth-functional logic', his now almost forgotten foray into probability theory, or his blindness to the fact that a truth-functional combination of equations that follows from a given truth-functional combination of equations need not follow truth-functionally. One of the main conclusions is that Boole's contribution widened logic and changed its nature to such an extent that he fully deserves to share with Aristotle the status of being a founding figure in logic. By setting forth in clear and systematic fashion the basic methods for establishing validity and for establishing invalidity, Aristotle became the founder of logic as formal epistemology. By making the first unmistakable steps toward opening logic to the study of 'laws of thought'—tautologies and laws such as excluded middle and non-contradiction—Boole became the founder of logic as formal ontology.

> ... using mathematical methods ... has led to more knowledge about logic in one century than had been obtained from the death of Aristotle up to ... when Boole's masterpiece was published.
>
> *Paul Rosenbloom 1950*

1. Introduction

In *Prior Analytics* Aristotle presented the world's first extant logical system. His system, which could be called a logic today, involves three parts: first, a limited domain of propositions expressed in a formalised canonical notation; second, a method of deduction for establishing validity of arguments having unlimited numbers of premises and, third, an equally general method of countermodels for establishing invalidity. In *Laws of Thought* Boole presented the world's first mathematical treatment of logic. His system, which does not fully merit being called a logic in the modern sense, involves a limited domain of propositions expressed in a formalised language as did Aristotle's. In fact, Boole intended the class of propositions expressible in his formalised language not only to include but also to be far more comprehensive than that expressible in Aristotle's. However, Boole was not entirely successful in this. Moreover, where Aristotle had a method of deduction that satisfies the highest modern standards of soundness and completeness, Boole has a semi-formal method of derivation that is neither sound nor complete. More importantly, Aristotle's discussions of his goals and his conscientious persistence in their pursuit make of both soundness and completeness properties that a reader could hope, if not expect, to find Aristotle's logic to have. In contrast, Boole makes it clear that his primary goal was to generate or derive solutions to sets of equations regarded as conditions on unknowns. The goal of gaplessly deducing conclusions from sets of propositions regarded as premises is mentioned, but not pursued. Contrary to

History and Philosophy of Logic ISSN 0144-5340 print/ISSN 1464-5149 online © 2003 Taylor & Francis Ltd
http://www.tandf.co.uk/journals
DOI: 10.1080/01445340310001604707

Aristotle, Boole shows little interest in noting each law and each rule he uses in each step of each derivation. Accordingly, the deductive part of Boole's equation-solving method is far from complete: associative laws are missing for his so-called logical addition and multiplication, to cite especially transparent but typical omissions. As for a possible third part of Boole's logic, a method of establishing invalidity, there is nothing answering to this in the realm of equation-solving. Perhaps accordingly, there is essentially no discussion in Boole's writings concerning independence proofs for demonstrating that a given conclusion is not a consequence of given premises, and there is certainly nothing like a method of countermodels anywhere to be seen.

In *Prior Analytics* Aristotle addressed the two central problems of logic as formal epistemology: how to show that a given conclusion follows from given premises that formally imply it and how to show that a given conclusion does not follow from given premises that do not formally imply it. Using other equally traditional terminology, Aristotle's problems were how to establish validity and how to establish invalidity of an arbitrary argument,[1] no matter how many premises or how complicated its propositions. Aristotle could not have failed to notice that his problems are much more general than those he solved in detail (*Rose 1968*, 11, *Aristotle Sophistical Refutations*, ch. 34). For his initial partial solution he presented the world's first extant logical system. His system, which is somewhat similar to a modern logic, involves three parts:[2] a limited domain of propositions expressed in a formalised language, a formal method of deduction for establishing validity of arguments having an unlimited number of premises and an equally general method of countermodel or counterargument for establishing invalidity. The underlying principles for both methods continue to be accepted even today (*Corcoran 1973*, 25–30, *1992*, p. 374). Aristotle achieved logical results that were recognised and fully accepted by subsequent logicians including George Boole. The suggestion that Boole rejected Aristotle's logical theory as incorrect is without merit or ground despite the fact that Boole's system may seem to be in conflict with Aristotle's. Interpretations of Aristotle's *Prior Analytics* established the paradigm within which Boole's predecessors worked, a paradigm which was unchallenged until the last quarter of the 1800s after Boole's revolutionary insights had taken hold. The origin of logic is better marked than that of perhaps any other field of study—*Prior Analytics* marks the origin of logic (*Smith 1989*, p. vii and *Aristotle Sophistical Refutations*; ch. 34).[3]

The writing in *Prior Analytics* is dense, elliptical, succinct, unpolished, convoluted, and technical, unnecessarily so in the opinion of many.[4] Its system of logic is presented almost entirely in the space of about fifteen pages in a recent translation, chapters 1, 2, and 4 through 6 of Book A, and it is discussed throughout the rest of Book A,

1 An argument or, more fully, a premise-conclusion argument is a two-part system composed of a set of propositions called the premises and a single proposition called the conclusion. By definition, an argument is valid if the conclusion is logically implied by the premise set, and it is invalid otherwise, i.e. if the conclusion supplements the premise set or contains information beyond that in the premise set. For further discussion of logical terminology see *Corcoran 1989*.

2 The tripartite character of modern logics is so well established that it is more often presupposed than specifically noted or discussed. But see *Corcoran 1973* (pp. 27–30), *1974* (pp. 86–87) and *Shapiro 2001*. Aristotle's text does not mention the tripartite nature of his own system nor does it formulate its aims in the way stated here. For a discussion of aims a logical theory may have, see *Corcoran 1969*.

3 David Hitchcock (pers. comm.), commenting on this paragraph, wrote: 'Now *Sophistical Refutations* 34 does claim originality, but the claim is not for the *Prior Analytics*, as is commonly thought, but for the material which precedes that concluding chapter, namely the *Topics* and *Sophistical Refutations*'. Either way the fact remains that the origin of logic is remarkably well marked.

4 It is my opinion that Aristotle's prose in *Prior Analytics* is perversely 'reader unfriendly'. I have heard this expressed

especially chapters 7, 23–30, 42 and 45. It presupposes no previous logic on the part of the reader. There was none available to the audience for which it was written[5]—even for today's reader a month of beginning logic would be more than enough. However, it does require knowledge of basic plane geometry, including ability and experience in deducing non-evident theorems from intuitively evident premises such as those taken as axioms and postulates a generation or so later by Euclid (fl. 300 BCE). Especially important is familiarity with *reductio ad absurdum* or indirect deduction. Aristotle repeatedly refers to geometrical proofs, both direct and indirect.[6] It also requires the readers to ask themselves what is demonstrative knowledge, how do humans acquire it, what is a proof, and how is a proof made?

The publication of *Laws of Thought* in 1854 launched mathematical logic. Tarski (*1941/1946*, p. 19) notes that the continuous development of mathematical logic began about this time and he says that *Laws of Thought* is Boole's principal work. Lewis and Langford (*1932/1959*, p. 9) are even more specific: they write that Boole's work 'is the basis of the whole development [of mathematical logic] . . .'. If, as Aristotle tells us, we do not understand a thing until we see it growing from its beginning, then those who want to understand logic should study *Prior Analytics* and those who want to understand mathematical logic should study *Laws of Thought*. There are many wonderful things about *Laws of Thought* besides its historical importance. For one thing, the reader does not need to know any mathematical logic. There was none available to the audience for which it was written—even today a little basic algebra and some beginning logic is all that is required. For another thing, the book is exciting reading. C. D. Broad (*1917*, p. 81) wrote: '. . . this book is one of the most fascinating that I have ever read. . .it is a delight from beginning to end . . .'. It still retains a kind of freshness and, even after all these years, it still evokes new thought. The reader comes to feel through Boole's intense, serious and sometimes labored writing that the birth of something very important is being witnessed. Of all of the foundational writings concerning mathematical logic, this is the most accessible[7].

It is true that Boole had written on logic before, but his earlier work did not attract much attention until after his reputation as a logician was established. When he wrote this book he was already a celebrated mathematician specialising in the branch is known as analysis. Today he is known for his logic. The earlier works are read almost exclusively by people who have read *Laws of Thought* and are curious concerning Boole's earlier thinking on logic. In 1848 he published a short paper 'The Calculus of Logic' (*1848*) and in 1847 his pamphlet 'The Mathematical Analysis of Logic' was

by others, but I do not recall seeing it in print often. Ross is on the right track when he says that Aristotle's terminology is 'in some respects confusing' (*1923/1959*, p. 37). Rose (*1968*, pp. 10–11) and Smith (*1989*, p. vii) are unusually insightful and frank.

5 This statement may have to be qualified or even retracted. In his recent book *Aristotle's Earlier Logic*, John Woods *2001* presents further evidence that Aristotle addressed these issues in earlier works. In fact, previous scholars including Bochenski (*1956/1961*, p. 43) have used the expression 'Aristotle's second logic' in connection with the system now under discussion (*Corcoran 1974*, 88). Moreover it can not be ruled out that even earlier works on logic available to Aristotle or his students have been lost without a trace. In keeping with suggestions made by Hitchcock (pers. comm.), we should bear in mind not only that there are important logical works that we know have been lost such as those of Chysippus (280–207 BCE), but also that there may have been important contributions that we have not heard of, e.g. from one of the other ancient civilisations.

6 W. D. Ross (*1923/1959*, p. 47) points out that 'there were already in Aristotle's time *Elements of Geometry*'.

7 The secondary literature on Boole is lively and growing, as can be seen from an excellent recent anthology (*Gasser 2000*) and a nearly complete bibliography that is now available (*Nambiar 2003*). Boole's manuscripts on logic and philosophy, once nearly inaccessible, are now in print (*Grattan-Guinness and Bornet 1997*).

printed at his own expense.[8] By the expression 'mathematical analysis of logic' Boole did not mean to suggest that he was analyzing logic mathematically or using mathematics to analyze logic. Rather his meaning was that he had found logic to be a new form of mathematics, not a form of philosophy as had been thought previously. More specifically, his point was that he had found logic to be a form of the branch of mathematics known as mathematical analysis, which includes algebra and calculus.[9]

Bertrand Russell (*1903*, p. 10) recognised the pivotal nature of this book when he wrote: 'Since the publication of Boole's *Laws of Thought* (1854), the subject [mathematical logic] has been pursued with a certain vigour, and has attained to a very considerable technical development'. Ivor Grattan-Guinness (*2004*) notes that Boole's system received its definitive form in this book.[10] Although it is this work by Boole that begins mathematical logic, it does not begin logical theory. The construction of logical theory began, of course, with Aristotle, whose logical writings were known and admired by Boole. In fact, Boole (*1854*, p. 241) explicitly accepted Aristotle's logic as 'a collection of scientific truths' and he regarded himself as following in Aristotle's footsteps. He thought that he was supplying a unifying foundation for Aristotle's logic and that he was at the same time expanding the ranges of propositions and of deductions that were formally treatable in logic. Boole (*1854*, p. 241) thought that Aristotle's logic was 'not a science but a collection of scientific truths, too incomplete to form a system of themselves, and not sufficiently fundamental to serve as the foundation upon which a perfect system may rest'. Boole was one of the many readers of *Prior Analytics* who failed to discern the intricate and fully developed logical system that Aristotle had devised. What Kretzmann (*1974*, p. 4) said of Aristotle's *On Interpretation* applies with equal force to *Prior Analytics*: 'In the long history of this text even what is obvious has often been overlooked'. Boole was not the first or the last in a long series of scholars who wrote about Aristotle's logical works without mentioning even the presence of Aristotle's references to geometrical demonstrations. It was not until the early 1970s that philosophically and mathematically informed logicians finally discovered Aristotle's system (*Corcoran 1972* and *Smiley 1973*). This new understanding of Aristotle's logic is fully reflected in the 1989 translation of *Prior Analytics* by Robin Smith.[11]

As has been pointed out by Grattan-Guinness (*2003* and *Grattan-Guinness and Bornet 1997*), in 1854 Boole was less impressed with Aristotle's achievement than he was earlier in 1847. In 'The Mathematical Analysis of Logic' (*Boole 1847*) Aristotle's logic plays the leading role, but in *Laws of Thought* (*1854*) it occupies only one chapter of the fifteen on logic. Even though Boole's view of Aristotle's achievement waned as

8 Without even mentioning the 1848 paper, Boolos (*1998*, p. 244) in the course of crediting Boole with key ideas relating to the disjunctive normal form and the truth-table method, notes that the 1847 work is much less well-known than the *Laws of Thought*. Kneale and Kneale (*1962/1988*, 406) say that Boole and a friend paid for the cost of publishing *Laws of Thought*.

9 Books on this subject typically have the word 'analysis' or the words 'real analysis' in the title. For a short description of this branch of mathematics, see the article 'Mathematical Analysis' in the 1999 *Cambridge Dictionary of Philosophy* (*Audi 1999*, pp. 540–1). A recent, and in many respects revealing, axiomatic formalisation of analysis can be found in *Boolos et al.* (*2002*, pp. 312–18). However, this book does not discuss the question of why its axiomatic analysis deserves the name.

10 It remains a puzzle to this day that despite significant changes Boole says in the preface to *Laws of Thought* that it 'begins by establishing the same system' as was presented in 'The Mathematical Analysis of Logic'.

11 Smith's scholarship combines knowledge of modern mathematical logic with an appreciation of Aristotle's thought gained through reading Aristotle's writings in the ancient Greek language. For a useful discussion of some of the linguistic and interpretational problems that Smith confronted see the critically appreciative essay-review by James Gasser (*1991*).

Boole's own achievement evolved, Boole never found fault with anything that Aristotle produced in logic, with Aristotle's positive doctrine. Boole's criticisms were all directed at what Aristotle did not produce, with what Aristotle omitted. Interestingly, Aristotle was already fully aware that later logicians would criticize his omissions; unfortunately he did not reveal what he thought those omissions might be (*Aristotle, Sophistical Refutations*, ch. 34).

It is likewise true, of course, that Boole does not begin the practical application of logical reasoning or deduction in mathematics. Early use of deduction in mathematics began long before Aristotle. It has been traced by Kant (1724–1804) as far back as Thales (625?–547? BCE), who is said to have deduced by logical reasoning from intuitively evident propositions the conclusion that every two triangles, no matter how different in size and/or shape, nevertheless have the same angle-sum. This point is made in the preface to *The Critique of Pure Reason* (*Kant 1781/1887*, B, pp. x–xi). Thales' result, reported over two centuries later as Theorem I.32 in Euclid, was strikingly important at the time and is still fundamental in geometry and trigonometry. Today unfortunately, it is often taken for granted without thought to how stunning it once was, to what might have led up to it, to how it might have been discovered, to how it might have been proved to be true, or even to whether there might have been one or more alleged proofs that were found to be fallacious before a genuine proof was discovered. This is one of Aristotle's favorite examples of the power of logical deduction[12] (*Aristotle, Prior Analytics*, pp. 48a33–37, 66a14, 67a13–30, *Posterior Analytics*, pp. 85b5, 85b11, 85b38, 99a19; *Smith 1989*, p. 164).

Likewise, philosophical concern with problems of understanding the nature of logical reasoning also predates Aristotle's time. In a way, concern with understanding the nature of logical reasoning is brought to a climax by Socrates (469? –399 BCE), who challenged people to devise a criterion, or test, for recognising proofs, a method for determining of a given alleged proof whether it indeed is a proof, i.e. whether it proves to its intended audience that its conclusion is true, or whether, to the contrary, it is fallacious despite any persuasiveness the audience might find it to have (*Plato, Phaedo*, pp. 90b–90e).[13]

Perhaps the identification of logic as a potential field of study, or as a possible branch of learning, should be taken as the time when humans, having discovered the existence of logical deduction, were able to perceive a difference between objective proof and subjective persuasion. For more on this see *Corcoran 1994*.

2. Colloquial and formalised languages

The key insight for unlocking the intricacies of logic was the same for Boole as it was for Aristotle. It required a combination of two closely-related points: first, distinguishing the grammatical form of a sentence used to express a proposition from

12 More on the role of deduction in early mathematics can be found in any history of the subject, e.g. *History of Greek Mathematics* (*Heath 1921/1981*), which goes somewhat beyond Kant in its statement that 'geometry first becomes a deductive science' with Thales' contributions (Vol. I, p. 128).

13 As in other similar cases, we can wonder whether the credit should go to the historic Socrates, i.e. to Socrates himself, or to the Platonic Socrates, i.e. to Plato. Hitchcock (pers. comm.) suggests that the available evidence in this case favors crediting Plato not Socrates. This point, which may seem academic to some, may take on added importance as logic-related disciplines such as critical thinking come to have separate identities and require their own histories. Rose (*1968*, p. vi) says that Plato set the stage for Aristotle's logic and he speculates that Plato may have had even more involvement.

the logical form of the proposition expressed by the sentence; and second, recognising that the grammatical form of a sentence used to express a proposition does not necessarily correspond to the logical form of the proposition expressed. As a first approximation, we can think of a sentence as a series of inter-related written words and we can think of a proposition as a series of inter-connected meanings or concepts that may or may not have been expressed in words. Aristotle had already said that although sentences are not the same for all humans what they express is the same for all (*Kretzmann 1974*, p. 4); *Aristotle, On Interpretation*, ch. 1, pp. 16a3 – 18).[14] The fact that Boole distinguishes sentences from the propositions they express or are used to express, noted by Broad (*1917*, p. 83), becomes increasingly evident as his exposition unfolds. See *Boole 1854/2003* (p. 25) and below. Wood addresses this issue explicitly (*1976*, pp. 3 – 7).

Given two sentences expressing one and the same proposition, often one corresponds more closely to the logical form of the proposition than the other. Often one reveals more of the logical structure of the proposition or contains fewer logically irrelevant constituents. Some of the easiest examples of the grammatical–logical discrepancy are found in the so-called elliptical sentences that have been shortened for convenience or in the so-called expletive sentences that have been redundantly lengthened for emphasis or for some other rhetorical purpose. The sentence 'Zero is even and one is not even' seems neither elliptical nor expletive. But, the sentence 'Zero is even and one is not' is elliptical—the second occurrence of 'even' has been deleted. And the sentence 'Truly, so-called zero is genuinely even and, in fact, strictly speaking, one really is not actually even' is expletive—the added expletives, 'truly', 'so-called', 'genuinely', 'in fact', 'strictly speaking', 'really' and 'actually', contribute nothing to the information content conveyed.

For Boole, the grammatical form of the eleven-word sentence 'Some triangles are acute, some obtuse and, of course, some neither' corresponds less closely to the logical form of the proposition it expresses than does the grammatical form of the twenty-one-word sentence 'Some triangles are acute triangles and some triangles are obtuse triangles, and some triangles are neither acute triangles nor obtuse triangles'. The two sentences—the original sentence, which is both elliptical and expletive, and its complete, unabridged and unannotated logical paraphrase or translation—both express the same proposition.[15] For Boole the logical form of the proposition expressed corresponds more closely to the grammatical form of the longer logical paraphrase than to the grammatical form of the original shorter sentence. From a logical point of view the expletive is mere annotation or decoration. It can be compared to the wrapping paper on a candy. In the case just considered, the logical paraphrase adds ten words not in the original and it drops two of the original words, namely the two words 'of' and 'course' that are in the original but make no logical contribution.[16]

14 This passage, one of the most important in the history of semantics, should probably not be construed as involving anticipations of modern abstract, as opposed to mental, conceptions of meaning, which began to emerge about the time of Boole.

15 Many issues concerning logical form remain to be settled. For example, it is not yet settled whether both occurrences of 'and' in the above logical paraphrase are needed. Perhaps the second alone is sufficient. It may well be that 'and' is not a binary connective that combines exactly two propositions in each application but that it is instead a multinary connective that combines two or more propositions, different numbers in different applications, three in this case. Cf. *Lewis and Langford 1932* (pp. 310, 341n.) and *Corcoran 1973* (p. 35).

16 The distinction between the grammatical form of a sentence expressing a certain proposition and the logical form of the proposition expressed is treated in the article 'Logical form' in the 1999 *Cambridge Dictionary of Philosophy Audi 1999* (pp. 511 – 512) and in the article of the same name in the 1996 *Oxford Dictionary of Philosophy Blackburn 1996,* (pp. 222 – 223). The distinction between sentences and propositions is found in many historically informed logic

The grammatical – logical discrepancy came to the attention of a wide audience through its roles in Russell's *Principles of Mathematics* (*1903*, pp. 48 – 58) and in his theory of descriptions (*1905*, pp. 95 – 97). This theory holds that a large class of sentences of widely varying grammatical forms all express, perhaps contrary to appearances, existential propositions that are also expressible by sentences beginning with the expression 'there exists a' which is characteristic of existentials. This theory implies in particular that the sentence 'Two is the number that is even and prime', which is grammatically an equational sentence, does not express an equational proposition but rather it expresses the existential proposition also expressed by the sentence 'There exists a number which is even and prime and which is every even prime number, and which two is'. To be more faithful to Russell's exact words we would have to use something like the following: 'There exists a number *x* which is even and prime and every number which is even and prime is *x*, and two is *x'*.

The usual convention is to use single quotes for making names of sentences and other expressions, for example names of words, phrases, symbols, etc. Thus 'One plus two is three' is a five-word English sentence and 'square' is a six-letter English word, both of which were used by Boole, but neither of which would have been recognised by Aristotle. Following Bertrand Russell (*1903*, pp. 53ff., *1905*, p. 99) and others,[17] double quotes are used in naming propositions and other meanings. Thus, "One plus two is three" is a true proposition known both to Boole and to Aristotle and "square" is a concept also well known to both. In familiar cases, expressions express meanings or senses and they name entities or things. Thus, the sentence 'One plus two is three' expresses the proposition "One plus two is three" and the number-word 'three' names the number three.

With the grammatical – logical distinction in place, one task of the logician is to devise a system of canonical notation so that sentences of ordinary language can be translated into canonical sentences or logical paraphrases that correspond better to the logical forms of the propositions. One modern mathematical logician noted the necessity for purposes of logic 'to employ a specially devised language . . . which shall reverse the tendency of the natural languages and shall follow or reproduce the logical form' (*Church, 1956*, p. 2). Once a system of logical paraphrase has been adopted the logician devises a set of transformations corresponding to logical inferences in order to be able to derive from a set of premise-sentences the sentences expressing the conclusions that logically follow from the propositions expressed by the premise-sentences. In this connection it had been common to use the expression 'calculus' or even 'calculus ratiocinator', but now that the distinction between deducing and calculating has been clarified the term 'calculus' is found less often in reference to formalization of deduction.

In regard to canonical notation, the ultimate goal is to devise a logically perfect language in which each sentence expresses exactly one proposition and the grammatical form of each sentence corresponds exactly to the logical form of the proposition it expresses, at least in the sense that two sentences in the same grammatical form express propositions in the same logical form. In a logically perfect language there are no elliptical sentences, no expletives, no rhetorical flourishes, no ambiguities, etc. The particular font, the particular alphabet of characters, the

books, e.g. *Cohen and Nagel 1962/1993* (pp. xxii – xxiii), *Church 1956* (pp. 23 – 26). For a deeper, more thorough, and more contemporary discussion of the range of issues related to logical form see the book *Logical Forms* (*Chateaubriand 2001*). For a contrasting approach to these issues and for alternative ways of using the expressions 'logical form' and 'grammatical form' read the chapter 'Logical form and grammatical form' in the book *Using Language* (*Kearns 1984*).

17 For example, *Cohen and Nagel 1962/1993* (p. xxii) and *Atlas 1989* (pp. xiv – xv).

particular system of orthography and the like are all insignificant. Translation from one logically perfect language to another suitable for the same discourse would be little more than a one-to-one substitution. Thus, there is essentially only one logically perfect language for a given discourse.[18] The expressions 'logically perfect language' or 'formalised language' have almost completely replaced the older and less appropriate expression 'lingua characteristica' (character language) since it is irrelevant whether each simple concept is expressed by a single character as opposed to a string of two or more characters and besides this older expression misleadingly omits connotation of logical form, which is the relevant issue, cf. *Wood 1976* (pp. 45–53, esp. 66), *Cohen and Nagel 1993* (pp. 112).

Boole and Aristotle seem to be in agreement on the above theoretical points, which in various forms are still widely accepted by logicians today (*Corcoran 1992*). Boole also agrees with Aristotle's view that every simple (or 'primary') proposition was composed of three immediate constituents: the subject term, the predicate term, and the connector, which is sometimes called the copula. For example, for Aristotle the proposition "Every square is a rectangle" would have as its subject "square", as its predicate "rectangle", and as its connector something expressed by the discontinuous remainder of the sentence, 'Every ... is a ___', which was considered by Aristotle to express a unitary meaning, just as today the discontinuous fragments 'if ... then___', 'either ... or___', and the like are widely considered to express single unitary constituents. Aristotle would translate the sentence into 'Rectangle belongs-to-every square' to emphasise that the terms are in some sense the terminals, extremities or ends of the proposition and that the connector is a unitary device connecting the predicate to the subject with the predicate in some sense coming first (*Aristotle, Prior Analytics*, pp. 25a1–26; *Rose 1968*, pp. 10, 14). His pattern was *P-c-S*, Predicate–connector–Subject. For Aristotle there were four connectors. Besides the so-called universal affirmative connector already mentioned, i.e. "belongs-to-every", there is the universal negative "belongs-to-no", the existential affirmative "belongs-to-some", and the existential negative represented awkwardly 'does-not-belong-to-every' or 'non-belongs-to-some'. 'Some quadrangle is not a square' translates to 'Square does-not-belongs-to-every quadrangle'. Aristotle used upper-case letters for the terms and when it was clear what the connector was he would represent the proposition by a juxtaposition of two letters as AB, BC, RS, etc. *Aristotle, Prior Analytics*, pp. 25a, 26a, 28a).[19] Views similar to the subject–connector–predicate view of simple propositions are no longer accepted as characterising a wide class of propositions (*Corcoran 2001*, pp. 61–75). In fact, Boole (*1854*, pp. 52–3) was one of the last logicians to think that it characterises all propositions treated in logic. But even aside from this almost certainly intended limitation of Aristotle's formal language, it is difficult to overlook the fact that it does not express co-extensionality propositions such as "The quadrangles are the quadrilaterals", "Being an equilateral triangle is being an

18 As has been noticed by Tarski and others, in certain cases there are significant differences between languages suitable for different discourses. For example a language for arithmetic or number theory typically has proper names for each of the entities in its universe, i.e. for each of the numbers, because it is customary to refer to individual numbers by name. In contrast a language for geometry typically has no proper names because it is not customary to refer to an individual geometrical entity by name, e.g. to a point, a line or a square. For more on logically perfect languages, which are also sometimes called formalised languages, see e.g. *Church 1956* (pp. 1–68, esp. 2, 47, 55). Another early substantive use of the concept 'logically perfect language' is in *Frege 1892* (p. 86), which also contains the earliest use of the expression.

19 For more information on modern discussions of Aristotle's logic see *Corcoran 1974b*, *Smiley 1973* and *Smith 1989*.

equiangular triangle'' or ''Being a square is being an equilateral rectangle'', a point that should have been of great importance to Boole but which he does not seem to notice.

It is important to note that for Aristotle the subject of ''Every square is a rectangle'' is ''square'' and not ''Every square''. Likewise the predicate is ''rectangle'' and not ''is a rectangle''. This is not the so-called subject-predicate conception of propositions which takes ''Every square is a rectangle'' to have two immediate constituents instead of three. As W. D. Ross (*1923/1959*, p. 32) puts it, 'the copula appears ... completely disengaged from the predicate'. Apparently, Aristotle's interest in classification into species and his interest in the species-genus relationship influenced his choice of the simple propositions with which to start his study of consequence and independence (*Wood 1976*, p. 7). We can speculate that more than good fortune was operative in Aristotle's choice of a class of propositions in which each subject is also a predicate and vice versa, in which each affirmative has a uniquely correlated negative opposite and vice versa, and whose connectors include the simplest logical relations among species, namely inclusion and exclusion.

Boole agreed with Aristotle that the two terms of a proposition were substantives, expressed by common nouns or noun phrases and not just adjectives or verbs. See, e.g. *Boole 1847* (p. 20), *1854* (pp. 27, 42, 52, 60–1). For both whatever is a predicate of propositions is also a subject of propositions, and conversely (*Ross 1923/1959*, p. 32). That is why above 'Some triangle is acute' got translated to 'Some triangle is an acute triangle'. In Boole's (*1854*, p. 52) own words: To say that 'snow is white' is for the ends of logic equivalent to saying that 'snow is a white thing'. He had almost made the same point twenty-five pages earlier (*1854*, p. 27), as noted by Wood (*1976*, p. 5). An adjective complement such as 'acute' was considered to be elliptical for a noun-phrase complement such as 'acute triangle' in Boole's logical paraphrase, as in Aristotle's. For both Aristotle and Boole, each of these sentences expresses a relation between two substantives—but not the same relation and not the same two substantives, as we will see below. Both would say that the logical form of ''Every square is equilateral'' is better expressed by 'Every square is an equilateral' or by 'Every square is an equilateral square' in which the adjective is replaced by a noun or a noun phrase.

Aristotle did not say anything about how to treat complex subjects and predicates as in the sentences 'Every rectangle that is equilateral is a square', 'Every square is a quadrangle that is equiangular', or 'Every equiangular polygon that is acute-angled is a triangle that is equilateral'. By default, he has been construed as regarding each term as unitary so that the above three sentences would be translated respectively 'Square belongs-to-every rectangle-that-is-equilateral', 'Quadrangle-that-is-equiangular belongs-to-every square' and 'Triangle-that-is-equilateral belongs-to-every equiangular-polygon-that-is-acute-angled'.

Consequently, Aristotle's theory did not and could not treat inferences that required recognition of complex terms. Starting with simple cases such as deducing ''Every male human is mortal'' from ''Every human is mortal'', or deducing ''Some human is mortal'' from ''Some male human is mortal'', we could go on to inferences involving multiple premises each of which has multiple simple terms such as deducing ''Every equilateral triangle is equiangular'' from ''Every equilateral triangle is an acute-angled polygon that is equiangular''. Aristotle could not have been unaware of these omissions. It is very likely, perhaps almost certain, that such propositions and such inferences were simply beyond the scope of the limited project that he had set for himself.

3. Boole's view of his relation to Aristotle

As indicated above, Boole was an accomplished mathematical analyst who thought he knew Aristotle's logic thoroughly and who found Aristotle's logic to be flawless as far as it went. However, as we will soon see in detail, Boole did not think that Aristotle's logic went deep enough or wide enough. One of the goals of Boole's work was to preserve the results that Aristotle had achieved while at the same time contributing in two contrasting ways to the further development of the project that Aristotle had begun. Boole wanted to simplify Aristotle's system in one respect while making it more complicated in other respects. Boole wanted, on the one hand, to unify Aristotle's logic and to provide it with an algebraic-mathematical foundation. Early in his logical work he said that logic should not be associated with philosophy but with mathematics (*Boole 1847*, p. 13). In *Laws of Thought* he expressed this conviction about logic: 'it is . . . certain that its ultimate forms and processes are mathematical' (*1854*, p. 12). On the other hand, Boole wanted to broaden Aristotle's logic by expanding the range of propositions whose forms could be adequately represented and by expanding the basic inferential transformations so that the derivations familiar to Boole from mathematics, such as substitution of equals for equals and applying the same operation to both sides of an equation, could be carried over to ordinary syllogistic argumentation.

Boole may have seen the relation of his mathematical logic to Aristotle's syllogistic logic somewhat as Einstein was to see the relation of his relativistic mechanics to Newton's classical mechanics. In both cases, roughly speaking, the older theory provided a paradigm and a class of accepted results for the new, and the older results were to become either approximations or limiting cases for results in the newer theory. In both cases, the later theorist accepted what he took to be the goals of the earlier theorist, but then went on to produce a new theory that he took to better fulfill those goals. Demopoulos (pers. comm.) suggests that Boole may have thought the relation of his logic to Aristotle's to be like the relation of Newton's theory of gravity to the Keplerian theory of planetary motion based on Kepler's three laws. In each of the two analogies the newer theory was thought of as a broadening of the older. Accepting the Newton/Kepler analogy would suggest that Boole's theory explains or gives the reasons for Aristotle's logical results while excluding the possibility that Boole may have thought that Aristotle had been deficient and wrong, not just partial or incomplete. From Newton's viewpoint Kepler's theory gave correct descriptions of the motions of the planets but without an explanation; Newton explained why Kepler was right.

We can be almost certain that Boole accepted this analogy to some extent and that he felt his relation to Aristotle comparable in some way to Newton's relation to Kepler (*Boole 1854*, p. 5). However, it is not certain what he thought that way was, what basis of comparison Boole had in mind, nor is it clear how far Boole would take it. Other possible analogies come to mind: Newton/Galileo, Newton/Archimedes, Russell/Frege, Gödel/ Peano and Veblen/ Euclid. The description in *Van Evra 2000* of the development of the relevant area of logic up through the end of Boole's career in logic, showing how the whole field was in flux, may be useful in determining which analogy is more appropriate.

Boole's relationship to Aristotle's logic was also in some ways analogous to the relationship that a family might have to a beautiful old house that they had acquired and that they wished to preserve as they had found it. But, at the same time that they regarded the old house as in many ways something to be preserved untouched, they also perceived that it needed a new foundation based on new technology and they saw that it was not large enough to accommodate all the members of the family, so it needed to be made larger. After the renovations the original house, though in some sense still there, could hardly be recognised in the renovated one except by members of the family that persisted in the belief that they had really left the original intact.

4. Boole's reconstruction of Aristotle's logic

Boole's first step of simplification and unification was to try to show how Aristotle's four connectors could be reduced to one and how each of the four connectors that Aristotle had treated as unitary were in fact colloquial, or idiomatic and not in correspondence with any actual part of the logical structure of the propositions in which they seem to occur. This project was similar to the one mentioned above under taken by Russell *1905* almost 50 years later involving the so-called definite descriptions. For Boole, the logical form of a proposition such as "Every square is a polygon", treated by Aristotle as "Polygon belongs-to-every square", is really an equation, two terms connected by equality, an equation in which nothing corresponds to "belongs-to-every". Here, equality is the strictest mathematic equality, "is-one-and-the-same-as", also called numerical identity. Roughly speaking, as incredible as this may seem, Boole would have said that "Every square is a polygon" is better thought of as "Every-square is [one-and-the-same-as] a-polygon", or better as "All-squares is [one-and-the-same-as] some-polygons", taking the so-called quantity indication to be parts of the subjects and predicates and not as part of the connector, as Aristotle did (*Boole 1847*, pp. 21–25, *1854*, pp. 28, 59–61). This is not as if what Aristotle saw as a duck Boole saw as a rabbit (*Audi 1999*, pp. 310–311). No duck is a rabbit. Where Aristotle saw predications Boole saw equations. Boole realised that his theory of logical form was in radical opposition to Aristotle's, but he seems to have thought that Aristotle had just not gone deep enough, not that Aristotle was fundamentally mistaken. Boole's pattern was S–*is*–P, Subject–is–Predicate, or $S = P$, Subject equals Predicate. For Aristotle 'every' and 'some' contributed to the expression of the connectors; for Boole they contributed to the expressions of the terms.[20] There are two points here for Boole: first that these propositions were really equations between class-terms, not predications of common substantives to common substantives, and second that the so-called quantified-subject and quantified-predicate expressions were used as class-names. The first of these two ideas was to prove very fruitful for Boole. The second would get him into trouble (*Gasser 2000*, p. 116). Which class could "Some polygons" correspond to? Could 'all squares' as in 'All squares are polygons' be a name of the class of squares, as Boole (*1854*, p. 28) thinks? Boole is forcing propositions into forms that cannot contain them and he is misreading English. But, he is not the last logician to commit what have been called segmentation fallacies. 'All humans' does not name a class as Russell (*1905*, p. 95) finally disclosed to us in his famous "On denoting". It is not a defect of English that 'the class of humans' is singular and 'all humans' plural.

For Boole, the only connector was the one represented by the equality sign, as in the following: $(1 + 2) = 3$. $(1 + 2) = (2 + 1)$. $((1 + 2) + 3) = (1 + (2 + 3))$. This is

20 Some contemporary philosophers of logic that take a Kuhnian view of historically given theories might conclude that Aristotle's theory is incommensurable with Boole's and that neither are commensurable with modern theories of logic. My view is that all theories of logical form are to some extent comparable. Aristotle's deliberately simplified view has much more truth to it, so to speak, than Boole's overly ambitious theory which strikes me as very wrong and wrong-headed. Aristotle's theory was at least a good first step grounded in sound logical intuition, whereas Boole's theory of logical form of primary propositions is a grotesque distortion, perhaps even an example of the Segmentation Fallacy (like deducing "Dogs are cats" from the premises "Dogs are animals" and "Cats are animals").

the so-called is-of-identity, which is one of the meanings expressed in normal English by the two-letter word 'is' as in ''One plus two is three'' or ''Twain is Clemens'' (*1854*, pp. 27, 35). The propositions involving the is-of-identity are often expressed by expletive sentences to emphasize the nature of the connector. For example, ''Mark Twain is Samuel Clemens'' is sometimes expressed by the expanded expletive sentence 'Mark Twain is the same person as Samuel Clemens'. The is-of-identity is not to be confused with the is-of-predication as in 'Mark Twain is clever', which is used to say that Mark Twain has the property of being clever, and not to assert the absurdity that Mark Twain is identical to the property of being clever.

Since Boole has only one connector where Aristotle had four, Boole has to redefine the Aristotelian terminology for classifying propositions based on which of the four connectors they involved: universal affirmative, universal negative, particular affirmative, and particular negative. In his words, 'it will be convenient to apply ... the epithets of logical quantity, 'universal' and 'particular', and of quality, 'affirmative' and 'negative', to the terms of propositions, and not to the propositions themselves' (*1854*, p. 228). This was not the only Aristotelian terminology that Boole was to ''reinterpret'' (see section 6 below).

By itself, it seems implausible for Boole to suggest replacing all four Aristotelian connectors with one new connector. But Boole had another innovation that would make the replacement seem plausible to many logicians. Boole wanted to treat as logically complex those terms which are expressed by grammatically complex expressions. As mentioned above, such complex terms were not treated by Aristotle in *Prior Analytics* or were treated by Aristotle as simple, as unitary. In order to show what Boole was doing, brackets can be used to make the transition from English to Boole's equations. For Boole, the three expressions 'rectangle that is equilateral', 'rectangle or circle' and 'rectangle that is not square' would be thought of, respectively, as '[[rectangle] and [equilateral]]', '[[rectangle] or [circle]]' and '[[rectangle] not [square]]'. And these in turn would be represented by terms made up of letters as in mathematical analysis or algebra. The multiplication-dot is used here for 'and' where Boole (*1854*, p. 27) used the times sign, the plus sign is used for 'or', and the minus sign for 'not' in this sense. The simple terms were represented by letters. Thus the above three expressions would ultimately translate to '(r·e)', '(r + c)' and '(r – s)'. Since Boole has only one connector where Aristotle had four, Boole cannot use the Aristotelian terminology for classifying propositions based on which of the four connectors they involved: universal affirmative, universal negative, particular affirmative and particular negative. On his view these distinctions can only relate to grammatical form, not to logical form. But instead of abandoning it as mistaken, or relegating it to the realm of grammar, he redefines the expressions. For Boole the expressions 'all squares', 'all things not squares', 'some squares' and 'some things not squares' are names of terms, which are respectively universal affirmative, universal negative, particular affirmative, and particular negative, in his new terminology. Using old terminology to express new ideas is a frequent occurrence in the history of logic.

Boole's innovations did not stop there. In his logic, Aristotle did not recognise the universal term 'entity' or 'thing' nor did he recognize the null term 'nonentity' as in 'Being an entity is being either a square or a non-square' and 'Being a nonentity is being a square that is not a square'. These two omissions were probably deliberate and based on theoretical considerations (*Corcoran 1974*, p. 104).[21] In any case the two omissions take on special significance in modern mathematical studies of Aristotle's

21 Aristotle's system, as it stands unchanged since Aristotle made the final touches, has an architectonic beauty and

system (*Lukasiewicz 1951*, *Corcoran 1972* and *Smiley 1973*). However Boole did not merely recognise both the universal term and the null term, he accorded them central theoretical status. Seeing an analogy between universality and unity, Boole took the digit '1' to express the universal term. Seeing an analogy between non-being and zero, he took the digit '0' to express the null term (*1854*, pp. 47, 411).

Now we are ready to see the first of the ways that Boole (*1847*, p. 21) treated the universal affirmative proposition as an equation. He may have first noticed that universal affirmative propositions are logically equivalent to propositions that resemble equations in several respects. For example, "Every square is a polygon" is logically equivalent to the co-extensionality proposition "Being a square is being both a square and a polygon". Once this is seen the following reduction is suggested. [Square] = [[square] and [polygon]]. Using Boole's notation this corresponds to: s = (s·p). The universal negative is even simpler. "No square is a circle" is logically equivalent to "Being a nonentity is being a square that is a circle". This translates to the following. [Nonentity] = [[square] and [circle]]. And in Boole's notation this translates to the equation: 0 = (s·c). Alternative translations for these two and Boole's treatment of the other two are given on the same page (*1854*, 228). Boole has gone beyond Aristotle by formally recognizing complex terms containing two simple terms each and at the same time reducing the four simple kinds of propositions to equational form. But he introduced another innovation that must be explained before we can proceed.

Boole's (*1854*, pp. 42, 47) use of the symbol '1' found in *Laws of Thought*, and by translation also the word 'entity', marks a milestone in logic. In his earlier work (*Boole 1847*, p. 15), '1' expressed "the universe", where the universe is taken to be the most comprehensive class, the uniquely determined class of which every other class is a subclass. This corresponds to using the word 'entity' in its broadest, invariable sense. Thus in the earlier *1847* work, '1' is a constant, or label-word, like 'square', 'triangle', 'seven', or 'Aristotle', an expression whose referent is considered to remain fixed regardless of the context in which it is used. Label-words remain, so to speak, attached to what they label. Label-words contrast with indexicals or pointer-words such as 'you', 'me', 'here', 'there', 'this', 'now', 'yesterday' and the like, as well as richer words such as 'father', 'mother', 'school' and 'home' whose referents are not fixed until they are used in a particular context at a particular time and place by a particular person for a particular audience. What a pointer points to depends on where it is, what its orientation is and what happens to be in its way, so to speak. For example, the sentence 'Yesterday you told me that Mother was home' can be used to express many different propositions. On a given day it may be used to express a large number of propositions each distinct from any proposition it could be used to express on any other given day. One given person could use the sentence to express to a second given person a proposition that no third person could use it to express to a fourth person.

simplicity that Aristotle may have been reluctant to interfere with even if he had good reason to. Besides, complicating changes, especially those of a potentially controversial nature may diminish the philosophical or pedagogical usefulness of the system (*Lear 1980, Corcoran and Scanlan 1982*). William of Ockham (c. 1285 – c. 1349), one of the most systematic and rigorous of the medieval logicians, retained Aristotle's formalised language and system of deductions unchanged while explicitly including propositions involving null terms treated as non-logical (*Corcoran 1981*). However, it is very likely that 'entity' and 'nonentity' would be regarded as logical concepts. In contexts like Aristotle's logic or Boole's the extensions, the universe of discourse and the null class, are logical notions in Tarski's sense (*Tarski 1986*). Accordingly, propositions having 'entity' or 'nonentity' as subject or predicate do not have the same logical forms as propositions having nonlogical concepts as terms and thus special rules of inference must be added for them.

Label-words are also called tags; pointer-words are also called deictics, egocentrics, demonstratives, token-reflexives or even here-nows (*Lewis and Langford 1932*, p. 312; *Copi and Gould 1967*, p. 128). Of course, 'mother' and 'home' are both ambiguous and in some senses they are used as label-words, e.g. when preceded by an article as in 'Every mother should have a home'.

In sharp contrast, in Boole's book '1' becomes an indexical and by translation so does 'entity', 'being', 'thing', 'individual', etc. In *Laws of Thought*, '1' indicates not "the universe" but the limited subject matter of the particular discourse in which it is used, what Boole calls 'the universe of [the] discourse' which varies from one discourse-context to another (*Boole 1854*, 42).

This is the first time in the history of the English language that the expression 'universe of discourse' was ever used. In this way Boole opens up room for one and the same language to be used in many different interpretations. In arithmetic discourses the universe of discourse is often the class of natural numbers, in geometric discourses the universe of discourse is a class of geometrical figures, and in set theory the universe of discourse is a class of sets.[22] It is important to notice that Boole seems to regard the context sensitivity or context relativity of the class-name '1' to pertain only to its extension (reference or denotation) and not to its intension (meaning, sense or connotation). Thus, as a class name, '1' has a fixed meaning something like "the universe of discourse", whereas the extension of "the universe of discourse" is different in different contexts. This is remarkably similar to the functioning of personal egocentric words such as, e.g. 'I', which has the same sense whenever it is used but its reference is context-sensitive. When I say it, the reference is me; but when you say it, the reference is not me but you.

The idea of multiple universes of discourse is one of the key ideas of modern logic (*Sagüillo 1999*). Modern logic is almost inconceivable without the concept of universe of discourse. In each modern mathematical theory the special universe of discourse contains all of the entities that are subjects of the propositions of the theory. For example, in number theory there are propositions about zero, propositions about one, propositions about zero and one, and so on. Boole already made this point in 1854, 'this universe of discourse is in the strictest sense the ultimate subject of the discourse' (*1854*, p. 42). However he went far beyond the view just attributed to modern mathematical logic. For Boole, not only was each proposition about some entity or entities in the universe of discourse, but also it was about the universe of discourse itself. Moreover, both of the terms of an equation contained the concept of the universe of discourse and, indeed, each expression of a term of an equation contains at least one occurrence of the character '1' referring to the universe of discourse (*1847*, pp. 15, 1854, p. 44). For example, Aristotle's "Every square is a rectangle", considered as a proposition of a general theory of geometry, would be treated by Boole as "Being an entity that is square is being an entity that is rectangular that is square" where the word 'entity' names the universe of geometrical discourse. One Boolean equation for this would be '$(s\cdot 1) = (s\cdot(r\cdot 1))$'. This aspect of Boole's semantics or theory of propositions – called the principle of holistic reference—does not seem to have been

22 For more on universe of discourse, see any good philosophical dictionary. For example, see 'Universe of discourse' in the 1999 *Cambridge Dictionary of Philosophy* (*Audi 1999*, p. 941) or the article of the same name in the 1996 *Oxford Dictionary of Philosophy* (*Blackburn 1996*, p. 387). The principle of universe relativity—that one and the same sentence can be used to express different propositions according to which universe the discourse of its use has—was resisted by certain schools of logic for many years. As late as 1903 a major logician, Bertrand Russell (*1903*), could write a 500-page book without mentioning the concept of universe of discourse or even using the expression.

explicitly noted by Boole scholars. To be explicit, Boole's principle of holistic reference is his view that each and every equational proposition refers to the universe of discourse as such.

This strange and fascinating view that, in his words *Boole 1854* (p. 42), 'the universe of discourse is the ultimate subject' of every proposition is foreshadowed in Boole's earlier work (*1847*, pp. 15, 16), where he explicitly says that when an "elective symbol" such as 'x' occurs by itself it is to be taken as elliptical for the class-name 'x(1)'. Thus in that work, where the universe is the universe of discourse '1' occurs in every term expression except some involving '0', e.g. '0' itself and '(x·0)'. But in *Laws of Thought* this view is in a way more explicit and in a way less explicit (*1854*, pp. 42, 43). Boole's view is based on his insightful epistemic principle that the mental process of formulating a propositional thought begins with the act of conceiving of the universe of discourse. Any subsequent specialisation of the subject of the proposition is construed as a concept based on the concept of the universe of discourse in addition to whatever else it involves. To be clear, Boole's view is that the meaning of, e.g. the common noun 'human' in a particular discourse—the common noun concept expressed by 'human'—has a logical form corresponding to 'entity that is a human' or 'entity that is human' (*1854*, p. 42). Boole *1854* (p. 27) writes: it is the same thing to say "Water is a fluid thing" as to say "Water is a fluid". This means that for Boole the sentences not using the word 'entity' are elliptical for corresponding sentences that do use the word 'entity', or a synonym such as 'thing' or 'being'. For example, 'Being a square is being both a square and a polygon' is elliptical for something like 'Being an entity that is square is being both an entity that is square and an entity that is polygonal'. This in turn means that the equations not using the digit '1' are elliptical for equations that do use '1' (*1847*, p. 15). For example, the equation 's = (s·p)' is elliptical for something like '(s·1) = (s·(p·1))'. Likewise, 'Being a nonentity is being a square that is a circle' is elliptical for something like 'Being a nonentity is being an entity that is square that is a circle'. Thus, the equation '0 = (s·c)' is elliptical for something using '1' like '0 = ((s·1)·c)'. In Boole's language no fully expressed equation is composed entirely of a connector, letters and operation symbols without a digit. Every such equation contains at least two occurrences of digits, one on each side.

Boole is not the only important logician to accept something similar to the principle of holistic reference. Chateaubriand (*2001*, p. 53) says '... Frege held that statements ... could not refer to isolated aspects of reality ... that their connection to reality must be ... [total]'. So it looks like Frege accepted something like the principle of holistic reference even though he differed with Boole on many other points. Chateaubriand's own view is nearly diametrically opposed to the principle of holistic reference (*2001*, ch. 2). But he does not go so far as the principle of mentioned reference, i.e. that in a logically perfect language not every sentence mentions the universe of discourse and each sentence refers only to the referents of the non-logical expressions occurring in the sentence.

Although Aristotle was silent on complex terms, Boole analyzed even Aristotle's simple-term propositions as equations involving complex terms composed of two simple terms. Remember Aristotle's "Every square is a rectangle", is seen by Boole as "Being an entity that is square is being an entity that is rectangular that is square". Where Aristotle saw simplicity Boole found complexity.

Once complex terms containing two simple terms are available, complex terms involving any number of simple terms are also available: equilateral polygon, equilateral polygon that is equiangular, equilateral polygon that is both equiangular and six-sided and so on. Once these are available, propositions involving three or more simple terms are also available: "Every equilateral triangle is equiangular", "Every equilateral quadrangle that is equiangular is a square", "Some equilateral polygon that is equiangular is neither a triangle nor a square". Boole's logic is starting to look a lot more extensive than Aristotle's logic of two-term propositions.

5. Systems of deduction

Once equational propositions are available chains of reasoning can be constructed that involve, besides analogues of the syllogistic inferences taken over from Aristotle's system, equational inferences taken over from mathematical analysis including algebra and arithmetic. Now Boole's deductions—his chains of inferences—look radically different from Aristotle's, but there are even more subtle differences. It was very important to Aristotle that his system of deductions—his system of chains of deductive inferences—was based on epistemically immediate[23] inferences, i.e. on primitive inferences that are logically evident in themselves and which cannot be further explained without logical redundancy. If a chain of reasoning is compared to a word, then an epistemically immediate (elementary, basic, or primitive) inference would correspond to the process of adding a letter. There is no simpler way to make a word than to add letters one by one. Aristotle called the four most prominent of his immediate inferences perfect syllogisms suggesting that they have a kind of ultimate gaplessness. However, in Boole's system each of Aristotle's 'immediate' inferences was seen to involve long and intricate chains of equational steps. What was immediate for Aristotle required mediation for Boole. Again, Aristotelian simplicity becomes Boolean complexity. For example, Aristotle would go from the two premises "Every square is a rectangle" and "Every rectangle is a polygon" immediately—in one step—to the conclusion "Every square is a polygon". Boole broke this down into eight tediously meticulous equational steps. The first step is going from the second premise, "Every rectangle is a polygon" to an intermediate conclusion gotten by something analogous to multiplying equals by equals, namely "Every square that is a rectangle is a square that is a polygon", "multiplying" both sides of the equation by "square". The entire gapless eight-step deduction has been written out by Susan Wood[24] (*Corcoran and Wood 1980*, p. 619; *Gasser 2000*, p. 111).

Thus, Boole's logic may give the appearance of being much more logically precise, detailed and refined than Aristotle's. Boole thinks that he is filling gaps in Aristotle's reasoning. In *Laws of Thought*, he (*1854*, p. 10) says that the inferences that Aristotle took to be immediate 'are not the ultimate processes of Logic'. The claim to have discovered the most fundamental forms of reasoning—often taken to be implicit in Aristotle's writings, or made on his behalf by his admirers—seems to be roundly refuted by Boole.

Perhaps most importantly, Boole was convinced that he had discovered the logically perfect language. He says, 'Let us imagine any ... language freed from

23 This use of 'immediate inference' agrees with that of Church (*1956*, p. 49n) in the case where the rules of inference are in suitable correspondence with deductive practice. As with Church's usage, an immediate inference may but need not be limited to a single premise. Thus, this sense of 'immediate' has no connection with the number of premises ,which may in fact be as few as zero or as many as three, or even more. It seems incongruous to define an immediate inference as one based on one premise, especially in the standard situations where any finite number of premises can be conjoined into one and thus where every inference without exception is very similar to an inference that would be properly said to be immediate in that sense. Cf. *Lewis and Langford 1932* (p. 341) and *Cohen and Nagel 1962* (p. 8).

24 This article, which includes several results achieved in Wood's University of Buffalo doctoral dissertation (*Wood 1976*), contains a much more detailed analysis of Boole's system of deductions.

idioms and divested of superfluity ... transition from such a language to the notation of analysis [used here] would consist of no more than the substitution of one set of signs for another, without change ... of form ...' (*Boole 1854*, p. 174).

To a modern reader one of the most surprising things to notice in the comparison of Aristotle's treatment of deductive processes to Boole's is that Aristotle was by far the more scientific and mathematical. Aristotle took his own deductive system as an object of scientific interest and he proves mathematically that two of his four two-premise rules were eliminable. Aristotle shows that every conclusion deducible from a given set of premises using any or all of his rules including the four two-premise rules is also deducible from the same premise set without using either of the two so-called particular two-premise rules, called Darii and Ferio by medieval logicians. There is nothing in Boole's writings remotely comparable to this. Perhaps this limitation of Boole's thinking was a by-product of his lack of interest in determining a complete system of gapless deductions. In the course of seeking such a system, where such exists, it is almost inevitable that the logician will overshoot the mark, so to speak, and list more rules than are necessary, thus creating a system admitting of an eliminability result. In fact, in many cases, if not all, each set of rules that a logician finds epistemically immediate for a given formalised language contains one or more rules that are eliminable in the above sense. Thus even in cases where no complete system exists, an attempt at epistemic comprehensiveness may produce a system admitting of eliminability.

For Boole as for Aristotle there was never a question of limiting the number of premises a deduction might have or of limiting the number of intermediate conclusions that might be needed before the final conclusion could be reached. Each implicitly acknowledged the existence of deductions based on a single premise, on two premises, three premises and so on. However it was Aristotle not Boole who considered and rejected the possibility of a deduction with an infinite premise set (*Scanlan 1983*). Moreover it was Aristotle not Boole who did preliminary calculations concerning the maximum number of intermediate conclusions needed to arrive at a conclusion from a given number of premises (*Corcoran 1974*). Since Aristotle's formal language had no tautologies, there was no possibility for him of a zero-premise immediate inference (i.e. a logical axiom), a zero-premise valid argument or a zero-premise deduction. It was Boole who clearly recognised the role of tautologies in deduction of non-tautologies from non-tautologies, a milestone in the history of logic (*1847*, p. 18; *Gasser 2000*, pp. 106, 108; *Rhees 1952*, p. 224). Although Boole was clear before perhaps anyone else of the cogency of the introduction into a deduction of an obvious tautology, i.e. that such use of tautologies was not fallacious premise-smuggling, nevertheless he still did not notice the existence of valid zero-premise arguments.

Aristotle's two-premise eliminability result is only one of what today are called proof-theoretic results in *Prior Analytics*. A definitive survey is found in Smith's (*1984*) appropriately titled work 'Aristotle as proof theorist'. The branch of logic that studies deductive texts as purely syntactic objects apart from any deductive, apodictic or other epistemic significance they may have has been misleadingly called proof theory since it was so-named by David Hilbert (1862–1943). Deduction theory would be a better name, but even this expression carries unwarranted connotations not shared by a more abstract designation such as derivation theory. But Aristotle may have had metalogical results that transcend proof theory. It is certain that he raised the metalogical question of the deductive completeness of his own system: whether every consequence of given propositions is deducible from them by a deduction constructed in accord with his system (*Corcoran 1972*, p. 701, *1974*, §5, pp. 119–122).

Aristotle may have tried to prove the following: given any finite set of propositions expressible in his formalised language, if one follows logically from the rest then it is possible to deduce the former from the latter by a deduction of his system. This is the thesis of the article aptly titled 'Aristotle's completeness proof' by Timothy Smiley (*1994*, pp. 25–38).

6. Laws of thought

Boole also thought that he had unearthed the true analytic form of the laws of thought. For laws[25] of non-contradiction, e.g. "Being a nonentity is being both a square and an entity that is not a square", he had '0 = (s·(1 – s))' (*1854*, p. 49). After quoting Aristotle in the original Greek language using Greek characters, Boole says that he has proved that 'the principle of contradiction', formerly thought to be an axiom of metaphysics, is really a law of logic, a law of thought, to use his expression. A fact not often noticed is that laws of non-contradiction play no role in Aristotle's syllogistic logic; his language lacks the capacity to even state them (*Corcoran 1972*). Boole should have been clear that Aristotle's law of non-contradiction is a single proposition about all qualities, as is evident from Boole's own translation of Aristotle: 'It is impossible that the same quality should belong and not belong to the same thing' (*1854*, p. 49).[26] For present purposes Boole's translation is close enough to that of recent scholars. For example Anton (*1972*, p. 44) has 'The same attribute cannot at the same time belong and not belong to the same subject in the same respect'.

Boole was fully aware that he was breaking new ground with his symbolic statement of laws of non-contradiction in a logical system. He opened the door of logic to what is now called formal ontology (*Cohen and Nagel 1962*, p. xli, ch. IX and *Corcoran 1994*, pp. 9, 18, 19), i.e. the discipline that, in Tarski's words, establishes general laws governing the concepts common to all sciences (*Tarski 1941/1994*, pp. xii, *1986*, p. 145). This initiated a paradigm shift in logic so that many years later Russell (*1919*, p. 169) could say that 'logic is concerned with the real world just as truly as zoology, though with its more abstract and general features'. Before Boole logic was the study of formal validity and invalidity, what has been called formal epistemology (*Corcoran 1994*, pp. 9, 18, 19). After Boole logic came to include formal ontology, which became the almost exclusive concern of Frege and then of the so-called logicists who virtually ignored the role of logic in deducing the consequences of propositions not yet known to be true, or even known to be false (*Corcoran 1969*).

For laws of commutativity, e.g. "Being an equilateral and an equiangular is being an equiangular and an equilateral", Boole had "(e·a) = (a·e)" (*1854*, p. 31). One of his proudest discoveries was what he called the index laws, e.g., for "Being both a circle and

25 The plural must be used here. Since every proposition in the same logical form as 'Being a nonentity is being both a square and an entity that is not a square' is also a theorem of Boole's system, e.g. 'Being a nonentity is being both a sphere and an entity that is not a sphere', it is clear that whatever Boole has here he has many, not just one. This is an observation that Boole did not make. Moreover, he was never articulate about the differences, if any, among the senses he attached to the words 'law', 'principle' and 'axiom'. John Anton pointed out the need for this note.

26 Boole is totally oblivious of the obvious fact that he has not come near stating Aristotle's law in his formal language of equations. Quite aside from Boole's lack of modality for 'impossible', he has no variables and no way to universally quantify an equation. Boole has indefinitely many individual propositions in the same form as '0 = (m·(1 – m)', but he has no universalization having them all as instances, say, 'For every class X, 0 = (X . (1 – X)'. This criticism of Boole also applies to his other laws: he has individual instances '(a·b) = (b·a)' but no universal 'For every class X, for every class Y, (X·Y) = (Y·X)'

a circle is being a circle'' he had ''(c.c) = c'' (*1854*, p. 31). Boole realised very clearly that the realm of logical laws—including many which are now called tautologies—is infinitely more extensive than ever imagined before,[27] a point which he develops later himself in an unpublished manuscript written after 1854 (*Rhees 1952*, pp. 211–29). In the same unpublished manuscript Boole writes that his 'laws of thought' are 'propositions true in consequence of their form alone' (*Rhees 1952*, p. 215), which is a locution that never appears in *Laws of Thought* or in earlier published writings.

In order to be clear about how sweeping Boole's innovations in this aspect of logic were it is necessary to juxtapose two striking facts. The first is that Boole was fully aware that his formal language could express infinitely many tautologies or laws of thought all having different logical forms. Just as the laws of non-contradiction and the laws of excluded middle each involve one substantive term, Boole recognised analogous laws involving two substantive terms, three substantive terms, and so on (*Rhees*, *1952*, p. 223). The second is that Aristotle's formal language could not express even one tautology or law of thought. It is widely believed that Aristotle's formal language could express trivial tautologies such as ''Every square is a square'' and ''Some square is a square'', but this is not the case. Even Lukasiewicz (*1951*, p. 20) admits that 'Neither of these laws was explicitly stated by Aristotle . . .'. Aristotle took strictly the principle that in every proposition involving predication one thing is predicated of something else (*Corcoran*, *1974*, p. 99; *M. Mulhern 1974*, pp. 144–5; *Smith 1989*, p. xix). Even when Aristotle's language is extended to include one-termed propositions and thus also tautologies, as was apparently done by later Peripatetics (*Lukasiewicz 1951*, p. 20), there is still a huge chasm to bridge before the wealth of 'laws of thought' available in Boole's theory would be reached.

7. Boole's inadequacies

If this essay were to stop here the reader would be left with the impression that Boole had mounted a sweeping and devastating, if unintended, victory over Aristotle—a victory that would erase Aristotle's place of leadership in logic and install Boole as the new leader. But, unfortunately or fortunately, things are not so simple.

In the first place, Boole's work is marred by what appear to be confusions, incoherencies, fallacies, and glaring omissions. Michael Dummett (*1959*, p. 203 and Gasser *2000*, p. 79) wrote that readers of Boole's logical writings will be unpleasantly surprised to discover 'how ill-constructed his theory actually was and how confused his explanations of it.'

Boole appears to have been trying to do several things at once without realising it. Some of his goals were incompatible, or at least incompatible given the methods that he had chosen. I will mention a few of Boole's projects. Boole was developing a new non-quantitative and non-geometric branch of mathematics, the algebra of classes. He was describing the logical forms of certain propositions expressed in English using compound noun phrases and he was analyzing deduction involving such propositions. He was constructing something like what we now know as truth-functional logic. He was reconstructing Aristotle's syllogistic logic. He was developing a distinctive

27 For more on laws of thought and the distinction now made between laws of thought and tautologies see any good philosophical dictionary. For example, see 'Laws of thought' and 'Tautology' in the 1999 *Cambridge Dictionary of Philosophy Audi* (*1999*, pp. 489, 902–3) or the articles of the same name in the 1996 *Oxford Dictionary of Philosophy Blackburn* (*1996*, pp. 214, 373). Also see Corcoran *2001*.

epistemology of logic. He was making the first steps toward logic in Tarski's sense, i.e. logic as formal ontology.

In the course of executing such goals mistakes are inevitable. I will mention only two main flaws. Others are documented in the literature (*Nambiar 2003*; *Corcoran and Wood 1980*; *Dummett 1959*). And the reader will doubtlessly find others that have not yet been mentioned in print.

In the first place, for all of the power and intricacies of Boole's system, he somehow managed to overlook indirect reasoning, or *reductio ad absurdum*, one of the most important and productive forms of inference. Indirect reasoning, also called reasoning by contradiction, is one of the most common forms of reasoning found in Euclid's *Elements* and in general humanistic and scientific reasoning (*Hardy 1940/1992*, p. 94). In Aristotle's system there are two parallel types of deductions, direct and indirect. For direct deduction, after assuming the premises and identifying the ultimate conclusion to be reached, the reasoner derives intermediate conclusions one-after-the-other by epistemically immediate inferences until the ultimate conclusion is achieved. In Boole's system the deductions are all direct. Readers should confirm this astounding point for themselves (*Wood 1976*, p. 25).

For indirect reasoning, as demonstrated repeatedly in Aristotle's work, a conclusion is deduced from given premises by showing that from the given premises augmented by the denial of the conclusion a contradiction can be derived. After the premises have been set and the conclusion to be reached has been identified, the next step is to assume for purposes of reasoning the denial of the conclusion. Then intermediate conclusions are entered step-by-step until a contradiction has been reached, which means that the last intermediate conclusion in the chain of intermediate conclusions is the contradictory opposite of one of the previously deduced conclusions or of one of the assumptions. In other words, to show indirectly that a conclusion follows from given premises, one shows that the denial of the conclusion contradicts the premises, i.e. that if the premises were true it would be logically impossible for the conclusion not to be true. Indirect reasoning, though commonplace in mathematical analysis, is absent from Boole's system (*Corcoran and Wood 1980*, p. 27). The avoidance of indirect reasoning sometimes requires painfully circuitous and unnatural paths of reasoning. The fact that Boole omitted even the mention of indirect deduction, or *reductio* reasoning, is astounding. It is not that he disapproved of it on some puristic principle; he did not even mention it. How can this glaring omission be explained?

The second flaw that will be mentioned here is a fallacy that is found in many of the most crucial of Boole's deductions. This fallacy has become known as the Solutions Fallacy because it involves confusing solutions to an equation with its implications, or consequences. A solution to an equation is not necessarily logically deducible from, or implied by, the equation. The numerical equation '$x = (x \cdot y)$' does not imply the solution '$x = 0$' or '$y = 0$'. Note that x and y may both be 1 and still the equation would be satisfied, of course. However, '$x = 0$' does imply '$x = (x \cdot y)$'. But in trying to derive a consequence, Boole thought it was cogent 'to replace ... equations expressive of universal propositions by their solutions ... in all instances ...' (*1847*, p. 42). For example, when Boole does what had been known as conversion-by-limitation,[28] deducing "Some polygon is a square" from "Every square is a polygon"

28 Full treatment of conversion and the Solutions Fallacy requires discussion of Boole's unsuccessful attempt to translate the existential, or particular, propositions into his formal language (*Broad 1917*, p. 90). Boole's notorious vee symbol, analogous to the Hilbert epsilon, would have to be discussed (*Corcoran and Wood 1980*, §2.5).

and deducing "Some polygon is not a square" from "No square is a polygon", his only "justification" is that the equations representing the respective conclusions are solutions to those representing the premises (*1847*, pp. 27, 28). In *Laws of Thought*, Boole never repudiates the fallacious reasoning; but, as a result of by a few innovations it is much more difficult to detect. Nevertheless the Solutions Fallacy is still there to be detected by those who know what they are looking for (*1854*, p. 229).

Boole did not focus on the analogy or similarity between, on one hand, syllogistic reasoning as found in Aristotle and, on the other, ordinary algebraic reasoning as found in analysis. He did not conceive of two similar or identical, but parallel styles of deduction. Instead, he compared syllogistic reasoning with the process of solving equations. He conceived of two equation-solving contexts: one in mathematical analysis involving numbers or quantities, the other in logic involving substantives or classes. After producing a few equational analogues of Aristotelian deductions, Boole makes the following revealing statement (*1854*, p. 230).

> From these instances it is seen that conversion is a particular application of a much more general process in Logic, of which many examples have been given in this work. That process has for its object the determination of any element in any proposition, however complex, as a logical function of the remaining elements.

Incredibly, Boole is thinking of conversion, not as a form of inference, but as a form of equation-solving. This is very likely part of why he missed indirect deduction (or *reductio* reasoning). There is no such thing as indirect equation-solving, of course. And in fact there is no such thing in Boole's equational language as the denial of an equation; Boole has no inequality sign and no way to apply 'It-is-not-the-case-that' to an equation. The negation of a given proposition is another proposition that contains the entire given proposition as a proper part and which has in addition a certain negating concept enclosing the given proposition as an envelope encloses a letter. "Not every number is even" is the negation of "Every number is even". But the negation of "Not every number is even" cannot be expressed in English by the sentence 'Not not every number is even' because that is not grammatical. Rather we must use 'It is not the case that not every number is even', or something to that effect. In a logically perfect language the negation of the proposition expressed by a given sentence is the result of attaching a certain affix to the sentence. Thus it would be moving in the direction of a logically perfect language if we were to accept the convention to avoid all negative expressions except 'it is not the case that' and to agree that it always be used with parentheses unless we have some other way to indicate to what it is being applied. Thus we would avoid writing 'It is not the case that Abe is able and Ben is brave', but instead we would write 'It is not the case that (Abe is able and Ben is brave)' or '(It is not the case that (Abe is able) and Ben is brave)' depending on which proposition was intended. It is characteristic of negation that each proposition has exactly one negation, that the negation of the negation of a given proposition is a different proposition which is nevertheless logically equivalent to the given proposition, and that no proposition is materially equivalent, or equivalent in truth-value, to its own negation.

The absence of negation in Boole's formal language may have resulted from following Aristotle too closely, as Aristotle's formal language also lacks negation. However, Aristotle was able too express the denial, or the contradictory opposite of any of his propositions. For him an indirect deduction of, say "Rectangle belongs-to-every square" takes as its reductio assumption "Rectangle does-not-belong-to-every square" and vice versa, and an indirect deduction of, say "Rectangle belongs-to-no circle" takes as its reductio assumption "Rectangle belongs-to-some circle" and vice versa. Aristotle takes the *reductio* assumption to be the contradictory opposite of the conclusion, not the negation. The contradictory opposite of the contradictory opposite of a given proposition is that given proposition. Of course the contradictory opposite of a given proposition is logically equivalent to its negation, a point that Aristotle could not have made without the concept of negation which he did not seem to have. The concept of negation may have been discovered in Boole's lifetime.

Another strange thing that may help to explain Boole's blindness to the Solutions Fallacy is his perverse practice of using the letters 'x', 'y' and 'z' both for propositional terms (instead of 'a', 'b' and 'c') and also for metalanguage variables (instead of 'S', 'M' and 'P'). As any analyst or algebra student knows, x, y and z are 'unknowns' and when we see unknowns in an equation, we want to find a solution. No term in the proposition 'No square is a circle' is an 'unknown'; there is nothing to solve. But if '0 = (x.y)' is used to express a universal negative proposition, as Boole did, it may seem that there is something to solve after all.

In the preface to *Laws of Thought* Boole (*1854*, p. ix) indicates what logic and what algebra is needed to read his book; he does not say that the reader needs to be able to deduce equational conclusions from equational premises, but rather he says that the reader needs to know 'the solution of simple equations'.

Boole's Solutions Fallacy is not like the fallacies found in Euclid, or in routine contemporary mathematics, which are mistakes that can be corrected leaving the results intact. The latter are mistaken ways of doing things that nevertheless could have and should have been done correctly. Rather, once Boole's fallaciously derived intermediate conclusions are deleted, there is no cogent way to fill the resulting gaps in the deduction; his ultimate conclusions can not be reached by cogent reasoning from the respective premises. Boole did not, and could not, cogently deduce the equational analogues of some of Aristotle's immediate inferences. Boole's alleged reduction of Aristotle's syllogistic logic to equational logic does not work. It is as if an Olympic champion were made to return his medal a century after the race because of something historians unearthed. The technical details of this result are available in several places (*Corcoran and Wood 1980*, §§2.4, 2.5, 2.6; *Gasser 2000*, pp. 115–21; *Nambiar 2003*, ch. 5).

Boole is one of the most misunderstood of the major philosophers of logic. He gets criticised for things he did not do, or did not do wrong. He never confused logic with psychology. He gets off without blame for errors and omissions he should have corrected himself. He gets credit for things he did not do, or did not do right. He did not write the first book on pure mathematics, he did not present a decision procedure, and he did not devise 'Boolean algebra'. And perhaps worst of all he fails to get credit for subtle logical insights and for discoveries that must have been difficult. Even where there is no question of blame or praise, his ideas are often misdescribed or, worse, ignored. As will be seen below, he never used the plus sign for exclusive-or, contrary to many logicians and historians including, e.g. Bochenski (*1956/1961*, pp. 298, 302), Boolos (*1998*, p. 244) and Lewis and Langford (*1932/1959*, p. 10). Boole's three 'signs of operation' (*1854*, p. 27) do not denote what are known today as Boolean operations. He never subscribed to the so-called Boolean interpretation; for example he never expressed the slightest doubt that "Every triangle is a polygon" implies "Some triangle is a polygon". If he had he would never have been tempted by the Solutions Fallacy. The syllogisms rejected by the misnamed Boolean interpretation, precisely those that cannot be derived without the Solutions Fallacy, are fully accepted by Boole (*1847*, pp. 35–42). And he fails to receive credit for many significant contributions that he did make.

Besides Boole's actual mistakes, he made some conventions that have been found to be so cumbersome as to invite expressions of displeasure and disapproval, two of which concern his 'logical' definitions of the plus sign and the minus sign. For Boole, when the classes indicated by 'x' and 'y' have no elements in common, then, as is to be expected, the expression '(x + y)' indicates the pooling of the elements, what is now called the union.[29] But, if the two classes share even one element then, '(x + y)' has no referent and in such a case '(x + y) = (y + x)' does not express a true proposition, and it does not express a false proposition either. Similarly, '(x – y)' has no referent unless the class indicated by 'y' is totally included in the class indicated by 'x' (*1864*, p. 33). Someone like Boole coming from mathematical analysis would be familiar with terms that do not name and with sentences that do not express propositions. For example, in ordinary algebra, the expression '(x/y)' names the indicated quotient if 'x' names any number and 'y' names any number other than zero. However, if 'y' names zero, then '(x/y)' is without a referent, it is 'undefined': division by zero is impossible and, for example, '(1/0)'does not name anything. Thus if 'y' names zero, then the equation '((x/y)/(z/y)) = (x/z)' does not express a true proposition, but it does not express a false one either, even though it expresses one that is true if 'y' names any other number.

Just as Boole criticised Aristotle's omissions, modern logicians criticize Boole's omissions. Perhaps the most often cited is Boole's complete lack of relational logic. Nothing Boole wrote bears on the logical connections among propositions involving relations such as numerical preceding ('is less than') and numerical following ('is greater than') (*Cohen and Nagel 1993*, pp. 113–6). What is in a way even more surprising is how little equational logic Boole discovered. It is clear that he did not grasp the problem of formalizing the equational logic that he used (*Corcoran and Wood 1980*; *Gasser 2000*). Even though he proudly states his laws of commutativity, e.g. '(x·y) = (x·y)', he is oblivious of laws of associativity, e.g. '(x·(y·z)) = ((x·y)·z)'. Thus he had nothing to say on such elementary points of equational logic as whether associativity implies or is implied by commutativity. What many logicians do not realise is that Boole failed to do for equations what Aristotle did for the four categorical predications.

Boole was more interested in the algebra of logic than he was in the logic of algebra. In particular, he was more interested in solving equations arising from his algebraic representation of Aristotle's logic than he was in the details of the deductive processes presupposed in algebra. This equation-solving focus, fostered perhaps by the expression 'algebra of logic', was to persist for many years. Indeed, in Skolem's (*1928*, pp. 512–24) well-known expository article 'On mathematical logic', the discussion of Boolean logic is entirely concentrated on solving equations as opposed to performing deductions. This is especially significant because his discussion of other logics is concentrated on performing deductions and not on solving equations.

29 It may seem strange, perhaps perverse, that Boole decided to have his 'logical sum' of overlapping classes 'undefined', but Cantor did the same thing with his 'union' (*1895–7*/*1915*, p. 91). Besides, what is two stones and two stones when the two groups of stones overlap? Boole was thinking of analysis as involved with combining quantities not numbers. There is nothing perverse here until this ' + ' is taken as the meaning of the English word 'and' as in 'Mathematicians and philosophers admire Boole as much as Aristotle' (*1854*, p. 32).

There are probably more logically independent falsehoods said about Boole, his logic and his philosophy of logic than about the accomplishments and opinions of any other major logical philosopher or philosophical logician. Rosenbloom anachronistically says that Boole was the first to study Boolean algebra[30] and he does not even get Boole's nationality right (*1950*, p. 5). Boole has a way of exciting interest in his work. He was probably as widely read during the last century as Frege, Russell, Lewis, Quine, Tarski, Carnap or Church.

8. Conclusion

Despite his faults, George Boole is one of the greatest logicians of all time and he ranks even higher as a philosopher of logic than as a logician. His 1854 *Laws of Thought* was his only mature book on logic. It has been read by generations of logicians and by students of logic. And each new generation finds new things to admire and new things to criticise. It is interesting that of all the giants of philosophy of logic it is Boole that people feel most free to criticise. Aristotle, Ockham, Frege, Russell, Gödel, Church, Quine and even Tarski made statements that go counter to the most deeply held doctrines of our period, but their 'mistakes' either go unmentioned or are glossed over in a respectful way.

Aristotle has no rival to the title of founder of logic, even though his achievement would be unthinkable without the emphasis on deductive reasoning in geometry that he found in Plato's Academy, or without the deep and critical awareness of the power of proof and the danger of fallacy fostered by Socrates.[31] Boole can be, and often is, regarded as the founder of mathematical logic. For example, Lewis and Langford wrote (*1932/1959*, p. 89): 'almost all developments of symbolic logic ... [have] been built up gradually on the foundation laid by Boole ...'. The only person besides Boole who is ever mentioned as the founder of mathematical logic is the great German logician Frege, whose works are often forbiddingly difficult. But even Frege's most ardent supporters do not fail to accord founder status to Boole, although one of them does try to make Boole share honors with others: 'Boole, De Morgan, and Jevons are regarded as the initiators of modern [mathematical] logic, and rightly so' *van Heijenoort* (*1967*, p. vi). Copi and Gould (*1967*, p. 75) agree with many other logicians who say that Frege is regarded as the second founder of modern symbolic logic after Boole.

I.M. Bochenski, the foremost historian of logic of the last century, agrees with much of what has been said above. In his section on Boole he refers to Boole as '... the first to outline clearly the program of mathematical logic [and] the first to achieve a partial execution'. He concludes: 'Boole resembles Aristotle both in

30 For more on Boolean algebra, a branch of modern algebra, read Tarski's 1935 classic paper, Rosenbloom's idiosyncratic but logically impeccable presentation (*1950*, ch. 1) or Hailperin's 1981 article 'Boole's Algebra isn't Boolean Algebra' reprinted in the Boole anthology (*Gasser 2000*, pp. 61–78). On the deceptively misnamed Boolean interpretation, which is a 'reinterpretation' of the universal sentences according to which the universal does not imply the corresponding existential, contrary to both Aristotle and Boole, read the passages on existential import in *Parry 1965*, *Corcoran and Scanlan* (*1982*, pp. 76–78) and *Cohen and Nagel* (*1993*, pp. 41, 58, 62, 68, 91). Incidentally, the ideas in *Parry 1965* in combination with those in *Corcoran 1972* show how Aristotle's logic can be smoothly and faithfully modeled in a modern many-sorted logic with identity, thus in the eyes of many logicians vindicating Aristotle's and Boole's doctrine that the 'two' universal propositions imply the respective corresponding existentials.

31 The relevance of geometry to the aims of Aristotle's *Analytics* is emphasized by Ross (*1923/1959*, pp. 47), who points out that Euclid was only a generation later than Aristotle, and, as quoted above, that 'there were already in Aristotle's time *Elements of Geometry* which Euclid simply augmented and recast'.

point of originality and fruitfulness; indeed it is hard to name another logician, besides Frege, who has possessed these qualities to the same degree ...' (*1956/1961*, p. 298).

In 1974, after having concentrated on Aristotle's logic for several years working with Lynn Rose, John and Mary Mulhern, Robin Smith and others, but before beginning the intense collaboration on Boole with Susan Wood, Calvin Jongsma and Sriram Nambiar, I believed and wrote that Aristotle and Boole were comparable in comprehensiveness and accuracy given their distinct goals and their distinct logical inheritance (*1974*, p. 123). After studying Boole along with recent logicians such as Tarski for some years, my estimation of Aristotle has increased and my estimation of Boole has decreased (*Corcoran 1980*, pp. 638–9, *2000*, pp. 130–1). Boole's many sound and imaginative contributions to logical thought, which have not been adequately described in this article or in any other work of which I know, must be accorded the greatest respect. Nevertheless, it must also be said that the gulf between modern logic and Boole is much greater than that between modern logic and Aristotle. In several respects, including their respective philosophical conceptions of the nature of logic and its role in intellectual life, their comprehensiveness, their level of rigor, and their metalogical awareness, Aristotle seems superior to Boole and closer to contemporary thinking. My guess would be that Aristotle would have less trouble understanding Gödel's results than Boole.

When the history of is finally written, many people might be surprised at how many of its central concepts are to be found in some form in the writings of Aristotle and Boole. Perhaps the most obvious point to make in this connection is that, as mentioned above, the method of countermodels for independence proofs (that demonstrate the absence of logical consequence), a cornerstone of model theory, is prominent in *Prior Analytics*, but sadly absent from *Laws of Thought*. Ironically, the device of reinterpreting language that is such a vivid feature of the modern form of the method of countermodels is prominently and unmistakably present in *Laws of Thought*, but totally absent from Aristotle's writings. In this regard it should be noted that the device of reinterpreting non-logical constants is not yet present in Tarski's 1936 'On the concept of logical consequence' (*1956/1983*, pp. 409–16), which is widely regarded as one of the defining papers of model theory. Here, rather than reinterpreting a non-logical constant, Tarski replaces it by a variable which then can be assigned suitable values.[32] The same awkward avoidance of reinterpreting the object-language persists until as late as the mid-1980s. Church (*1956*, p. 324) continues the Tarskian practice of replacing '... each of the undefined terms [by] a corresponding variable of the same type'. Even as late as 1986, Quine (*1970/1986*, ch. 4) avoids reinterpreting non-logical constants, which he calls lexical elements. As mentioned already, even the expression 'universe of discourse', today ubiquitous in model theory, is of Boolean coinage as is his special usage of the word 'interpretation'. But the term 'universe of discourse' is not yet in the 1936 Tarski paper nor is the idea of changing the universe of discourse there, cf. *Scanlan and Shapiro 1999*, (pp. 149–51). Boole probably also deserves to be mentioned in any history of proof theory even

32 Tarski continues this use of variables in all editions of his classic textbook *Introduction to Logic and to the Methodology of Deductive Sciences* (*1941/1994*, §38) long after abandoning it in his technical publications in model theory e.g. *Tarski et al. 1953* (p. 8). Typical of contemporary treatments is that found in *Boolos et al. 2002*, (pp. 102–105, 114–119, 137).

though Aristotle was better at it. Boole's idea of reinterpreting a formal language opened the way for thinking of a language without any interpretation and this is one of the basic ideas of proof theory. Besides, Boole expressed awareness of the mechanical, algorithmic potential in his 'Calculus of Deductive Reasoning' (*1847*, pp. 2, 11), a very nearly oxymoronic expression which continues to reverberate with proof-theoretic overtones.

Aristotle and Boole both used variables, just as they both used truth-functional connectives, definite descriptions, etc. but neither shows any awareness of the peculiar logical features of variables such as ranges of variation, equivalence of alphabetic variants, free versus bound occurrences, and so on. Neither contributed to the understanding of the nature of variables and neither deserves more than a footnote in the history of the variable. Of course neither uses the word 'variable' in this sense which had become common by the time of Russell (*1903*, pp. 5, 6) but which is not found in Boole's writings, or in Frege's for that matter. It is historically important to be clear about the fact that neither Aristotle nor Boole had variables in their formalised object languages, contrary to what has been written, e.g. *Lukasiewicz 1951* (pp. 7–10), *Smith 1989* (p. xix).

If we divide logic into formal epistemology and formal ontology as has been done above, then we can give credit where due by saying that Aristotle was the founder of logic as formal epistemology and that Boole was the founder of logic as formal ontology. Aristotle laid down the groundwork for a science of determining validity and invalidity of arguments. Boole laid down the groundwork for a science of formal laws of being, in Tarski's words 'general laws governing the concepts common to all sciences' or 'the most general laws of thinkables' (*1941/1994*, pp. xii), to use the words that Kneale and Kneale applied to what Boole called 'laws of thought' (*1962/1988*, p. 407).

It has been said that Galileo's greatest achievement was to persuade the world's scientists that physical reality is mathematical, or at least that science should be pursued mathematically. In his words, 'The Book of Nature is written in mathematical characters'. In a strikingly similar spirit, Boole (*1854*, p. 12) stated 'it is certain that [logic's] ultimate forms and processes are mathematical'. Perhaps Boole's greatest achievement was to persuade the world's logicians that logical reality is mathematical, or at least that logic should be pursued mathematically.

Acknowledgements

For bringing errors and omissions to my attention, for useful suggestions, and for other help, I gladly acknowledge the following scholars: O. Chateaubriand (Brazil), D. Novotny (Czech Republic and USA), B. Smith (Germany and USA), J. Sagüillo (Spain), J. Gasser (Switzerland), I. Grattan-Guinness (UK), S. Burris, D. Hitchcock, G. Reyes and J. Van Evra (Canada), J. Anton, G. Boger, M. Brown, J. Burgess, W. Corcoran, J. Dawson, B. Decker, L. Jacuzzo, J. Kearns, W. Lawvere, K. Lucey, S. Mitchell, S. Nambiar, P. Penner, M. Scanlan, S. Wood, J. Yu and J. Zeccardi (USA). Earlier versions were presented at the Buffalo Logic Colloquium in March 2003 and the University of South Florida in April 2003. This work is gratefully dedicated to Professor Ivor Grattan-Guinness, founding Editor of the international journal *History and Philosophy of Logic*, who has done more than any other person toward establishing the combined field of history and philosophy of logic as a rigorous, productive, and recognized field of scholarship.

References

Anton, J. 1972. On Aristotle's Principle of Contradiction and its Platonic Antecedents, *Philosophia* **2**, 266–80. Reprinted *Anton 1996*.

Anton, J. 1996. *Categories and Experience*. Oakvale, NY: Dowling College Press.

Aristotle. *On Interpretation*.

Aristotle. *Posterior Analytics*.

Aristotle. *Prior Analytics* (Smith 1989).

Aristotle. *Sophistical Refutations*.

Atlas, J. 1989. *Philosophy without Ambiguity*. Oxford: Oxford University Press.

Audi, R., ed. 1999. *Cambridge Dictionary of Philosophy*. Cambridge: Cambridge University Press.

Blackburn, S. 1996. *Oxford Dictionary of Philosophy*. Oxford: Oxford University Press.

Bochenski, I. 1956/1961 *History of Formal Logic*. Trans. and ed. by I. Thomas. Notre Dame, IN: Notre Dame University Press.

Boole, G. 1847. *The Mathematical Analysis of Logic*. Cambridge: Macmillan.

Boole, G. 1848. The Calculus of Logic. *Cambridge and Dublin Mathematical Journal* **3**, 183–98. Reprinted in Rhees 1952, pp. 125–40.

Boole, G. 1854/2003. *Laws of Thought*. Cambridge: Macmillan. Reprinted with introduction by J. Corcoran. Buffalo: Prometheus Books.

Boolos, G. 1998. *Logic, Logic, and Logic*. Cambridge MA: Harvard University Press.

Boolos, G., Burgess, J. and Jeffrey, R. 2002. *Computability and logic*. Cambridge: Cambridge University Press.

Broad, C. D. 1917. Review of *Boole 1854*, *Mind* **26**, 81–99.

Bunge, M. 1973. *Methodological Unity of Science*. Dordrecht: Kluwer.

Cantor, G. 1895–7/1915. *Contributions to the Founding of the Theory of Transfinite Numbers*. Trans. by P. Jourdain. New York: Dover.

Chateaubriand, O. 2001. *Logical Forms*. Campinas: CLE Unicamp.

Church, A. 1956. *Introduction to Mathematical Logic*. Princeton: Princeton University Press.

Cohen, M., and Nagel, E. 1993. *Introduction to Logic*. Indianapolis: Hackett.

Copi, I. M., and Gould, J. A. 1967. *Contemporary Readings in Logical Theory*. NY: Macmillan.

Corcoran, J. 1969. Three Logical Theories, *Philosophy of Science* **36**, 153–77.

Corcoran, J. 1972. Completeness of an Ancient Logic, *Journal of Symbolic Logic* **37**, 696–702.

Corcoran, J. 1973. Gaps Between Logical Theory and Mathematical practice. In Bunge 1973, pp. 23–50.

Corcoran, J., ed. 1974a. *Ancient Logic and its Modern Interpretations*. Dordrecht: Kluwer.

Corcoran, J. 1974b. Aristotle's Natural Deduction System. In Corcoran 1974, pp. 85–131.

Corcoran, J. 1981. Ockham's Syllogistic Semantics, *Journal of Symbolic Logic* **57**, 197–8.

Corcoran, J. 1989. Argumentations and Logic. *Argumentation* **3**, 17–43.

Corcoran, J. 1992. Logical Methodology: Aristotle and Tarski. *Journal of Symbolic Logic* **46**, 374.

Corcoran, J. 1994. The Founding of Logic. *Ancient Philosophy* **14**, 9–24.

Corcoran, J. 2001. Second-Order Logic. In Zeleny and Anderson 1998.

Corcoran, J. and Jongsma, C. 1983. Review of Hailperin 1981. *Mathematical Reviews* **83e**, 01038.

Corcoran, J. and Scanlan, M. 1982. Review of Lear 1980. *Philosophical Quarterly* **32**, 76–86.

Corcoran, J., and Wood, S. 1980. Boole's Criteria of Validity and Invalidity, *Notre Dame Journal of Formal Logic* **21**, 609–39. Reprinted in *Gasser 2000*.

Dummett, M. 1959. Review of Rhees 1952, *Journal of Symbolic Logic* **24**, 203–9. Reprinted in *Gasser 2000*.

Frege, G. 1892. On sense and nominatum. Trans. by H. Feigl. Reprinted in Copi and Gould 1967, pp. 75–92.

Gasser, J. 1991. Aristotle's Logic for the Modern Reader, *History and Philosophy of Logic* **12**, 235–40.

Gasser, J., ed. 2000. *Boole Anthology*. Dordrecht: Kluwer.

Grattan-Guinness, I. and Bornet, G. eds. 1997. *George Boole: Selected Manuscripts on Logic and its Philosophy*. Basel: Birkhauser.

Grattan-Guinness, I., ed. 2004. *Landmarks in Western Mathematics 1640–1940*. Amsterdam: Elsevier (forthcoming).

Hailperin, T. 1981. Boole's Algebra Isn't Boolean Algebra, Gasser 2000, pp. 61–78 (reviewed in *Corcoran and Jongsma 1983*).

Hardy, G. H. 1940/1992. *A Mathematician's Apology*. Cambridge: Cambridge University Press.

Heath, T. 1921/1981. *History of Greek Mathematics*. NY: Dover.

Kant, I. 1781/1887. *Critique of Pure Reason*. Trans. with introd. by N. K. Smith. NY: Modern Library.

Kearns, J. 1984 *Using Language*. Albany: SUNY Press.

Kneale, W. and Kneale, M. 1962/1988. *Development of Logic*. Oxford: Oxford University Press.

Kretzmann, N. 1974. Aristotle on Spoken Sound Significant by Convention. In Corcoran 1974, pp. 1–21.

Lear, J. 1980. *Aristotle and Logical Theory*. Cambridge: Cambridge University Press.

Lewis, C. I. and Langford, C. H. 1932/1959. *Symbolic Logic*. NY: Century. Reprinted NY: Dover.

Lukasiewicz, J. 1951. *Aristotle's Syllogistic*. Oxford: Oxford University Press.

Mulhern, J. 1974. Modern Notations and Ancient Logic. In Corcoran 1974, pp. 71–82.
Mulhern, M. 1974. Corcoran on Aristotle's Logical Theory. In Corcoran 1974, pp. 133–48.
Nambiar, S. 2000. The Influence of Aristotelian Logic on Boole's Philosophy of Logic: the Reduction of Hypotheticals to Categoricals. In Gasser 2000, pp. 217–40.
Nambiar, S. 2003. *Origin of Mathematical Logic: Boole's Assimilation of Traditional Logic to Mathematical Analysis*. Dordrecht: Kluwer.
Parry, W. 1965. Quantification of the Predicate and Many-sorted Logic, *Philosophy and Phenomenological Research* **26**, 342–60.
Plato. *Phaedo*.
Quine, W. 1970/1986. *Philosophy of Logic*, Cambridge, MA: Harvard UP.
Rhees, R., ed. 1952. *Studies in Logic and Probability by George Boole*. LaSalle, IL: Open Court.
Rose, L. 1968. *Aristotle's Syllogistic*. Springfield, IL: Thomas.
Rosenbloom, P. 1950. *Elements of Mathematical Logic*. NY: Dover.
Ross, W. D. 1923/1959. *Aristotle*. NY: Meridian Books.
Russell, B. 1903. *The Principles of Mathematics*. Cambridge: Cambridge University Press.
Russell, B. 1905. On denoting. *Mind*, n. s. 14, 479–93. Reprinted in Copi and Gould 1967, pp. 93–104.
Russell, B. 1919. *Introduction to Mathematical Philosophy*. London: George Allen and Unwin.
Sagüillo, J. 1999. Domains of Sciences, Universe of Discourse, and Omega Arguments, *History and Philosophy of Logic* **20**, 267–80.
Scanlan, M. 1983. On Finding Compactness in Aristotle, *History and Philosophy of Logic* **4**, 1–8.
Scanlan, M. and Shapiro, S. 1999. The Work of John Corcoran, *History and Philosophy of Logic* **20**, 149–58.
Shapiro, S. 2001. Modeling and Normativity: How Much Revisionism Can We Tolerate?, *Agora* **20**, 159–73.
Skolem, Th. 1928. On Mathematical Logic, *Norsk Matematisk Tidsskrift* **10**, 129–42. Reprinted in translation in van Heijenoort 1967.
Smiley, T. 1973. What is a Syllogism?, *Journal of Philosophical Logic* **2**, 136–54.
Smiley, T. 1994 Aristotle's Completeness Proof, *Ancient Philosophy* **14**, 25–38.
Smith, R. 1984. Aristotle as Proof Theorist, *Philosophia Naturalis* **21**, 590–7.
Smith, R., trans. 1989. *Aristotle's Prior Analytics*. Indianapolis: Hackett.
Tarski, A. 1935. On the Foundations of Boolean algebra. In Tarski 1956/1983, pp. 320–41.
Tarski, A. 1941/1946. *Introduction to Logic and to the Methodology of Deductive Sciences*. Trans. O. Helmer. New York: Oxford University Press.
Tarski, A. 1941/1994. *Introduction to Logic and to the Methodology of Deductive Sciences*. Trans., ed. with preface and biographical sketch of the author by J. Tarski. New York: Oxford University Press.
Tarski, A. 1956/1983. *Logic, Semantics, Metamathematics*. Trans. J. H. Woodger. Indianapolis: Hackett.
Tarski, A. 1986. What are Logical Notions?, *History and Philosophy of Logic* **7**, 143–54.
Tarski, A., Mostowski, A. and Robinson, R. 1953. *Undecidable Theories*. Amsterdam: North-Holland.
Van Evra, J. 2000. The Development of Logic ... 1600–1900, *History and Philosophy of Logic* **21**, 115–34.
van Heijenoort, J. 1967. *From Frege to Gödel*. Cambridge MA: Harvard University Press.
Wood, S. 1976. George Boole's Theory of Propositional Forms. PhD dissertation, University of Buffalo.
Woods, J. 2001. *Aristotle's Earlier Logic*. Oxford: Hermes Science Publishing.
Zeleny, M. and Anderson, C. A. 2001. Logic, Meaning, and Computation: Essays in Memory of Alonzo Church. Dordrecht: Kluwer.