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This largely expository paper explores analogues of ω-completeness and ω-inconsistency, starting with "finite cases". For each positive integer *n*, let L*n* be a first-order language having exactly *n* [individual] constants, say the digits '0', '1', etc. of a base-*n* arithmetic notation. An *instance of a universal sentence* is the result of deleting the quantifier and replacing every occurrence of the freed-up variable by one and the same constant. The *complete induction sentence* [for L*n*] is the sentence expressing "for every object *x, x* is 0 or *x* is 1 or etc.", for L3, in symbols, A*x*(*x* = 0 V *x* = 1 V *x* = 2). A set of sentences is *n-complete* iff it [deductively] yields every universal sentence each of whose instances it yields. Theorem COM: In order for a set to be *n*-complete it is necessary and sufficient for it to yield complete induction. A set of sentences is *n-inconsistent* iff it [deductively] yields the negation of some universal sentence each of whose instances it yields. Theorem INC: in order for a set to be *n*-inconsistent it is necessary and sufficient for it to yield the negation of complete induction. In that both of these theorems reduce a condition concerning infinitely many deductions to the deducibility of a single sentence, they are syntactic, or proof-theoretic, results analogous to semantic, or model-theoretic, results about ω-completeness and ω-inconsistency in second-order languages announced in my abstract "Semantic omega properties and mathematical induction", *Bulletin of Symbolic Logic* 3 (1997) 280. When we leave either the "finite" case or the second-order case to consider intermediate cases such as sublanguages (involving constants for zero and successor) of the usual first-order languages of number theory, we find that they do not admit of ω-completeness or ω-inconsistency being reducible to the deducibility of a single sentence.