C. I. Lewis

History and Philosophy of Logic

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C. I. Lewis (1883–1964) was the first major figure in history and philosophy of logic—a field that has come to be recognized as a separate specialty after years of work by Ivor Grattan-Guinness and others (*Dawson 2003*, 257). Lewis was among the earliest to accept the challenges offered by this field; he was the first who had the philosophical and mathematical talent, the philosophical, logical, and historical background, and the patience and dedication to objectivity needed to excel. He was blessed with many fortunate circumstances, not least of which was entering the field when mathematical logic, after only six decades of toil, had just reaped one of its most important harvests with publication of the monumental *Principia Mathematica*. It was a time of joyful optimism which demanded an historical account and a sober philosophical critique.

Lewis was one of the first to apply to mathematical logic the Aristotelian dictum that we do not understand a living institution until we see it growing from its birth. He will always be regarded as a unique authority on the founding of mathematical logic because he began his meticulous and broadly informed study of its history when it had reached a certain maturity as a field, but before its later explosive development had a chance to cloud our understanding of its founding and formative years. Lewis's judgment that Boole was the founder of mathematical logic, the person whose work began the continuous development of the subject, stands as a massive obstacle to revisionists whose philosophical or nationalistic commitments render this fact inconvenient. Lewis's articles and books form an essential part not only of history and philosophy of logic, but of logic itself. His criticism of lapses in rigor in *Principia Mathematica* served to notify generations of logicians that proof was not to be identified with formalistic manipulation of esoteric formulas.

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The welcome and long-awaited publication of Murray Murphey's masterful intellectual biography *C. I. Lewis: The Last Great Pragmatist* (Albany: SUNY Press, 2005) is occasion to reexamine Lewis's contribution to this field. Thankfully, Murphey saw fit to include ample discussion of the logical aspect of Lewis's wide-ranging thought—which in its full scope goes far beyond history and philosophy of logic. As Murphey indicates, logic was a small part of the Lewis legacy. He is regarded as a towering figure by many who have little or no appreciation of his great achievements in history and philosophy of logic. I cannot speculate on whether future philosophers will continue to give high marks to Lewis as an epistemologist, a metaphysician, or a value theorist, but I am confident that no historian of logic will be able to ignore his work as a historian of logic, and I strongly suspect and hope that his stature as a philosopher of logic will only increase. Of course, even had he never written a word of history of logic or of philosophy of logic, as the founder of modern modal logic his status as a logician would have been secure. He has a permanent place in history of logic.

This commentary does not begin to do justice to Murphey's excellent treatment of Lewis's logic and contribution to history and philosophy of logic. Rather it aims to complement Murphey's book with comments interrelating Lewis's ideas and connecting them with those of others. It is a pleasure to write this commentary, not only as a tribute to Lewis's contribution to the field but also because I feel a deep affinity with him as a philosopher of logic. I think that Lewis had excellent logical instincts and that he was basically right on important and controversial issues that other "more sophisticated" philosophers of logic were impatient with. Other great philosophers of logic were sometimes dogmatic, cavalier, elitist, arrogant, scientistic, evasive, arbitrary, self-righteous, or dismissive of legitimate criticism; Lewis was humanistic, serious, patient, responsible, sensitive and open-minded. He always gives the impression that he would listen to any informed objection.

Many of my views are developments, refinements, or variants of views Lewis held or would have held. I should say at the outset, however, that—contrary to Lewis—I was never attracted to the Frege-Russell logicism that held that all of mathematics is tautological. I could not accept the first step —the idea that number theory is composed exclusively of laws of logic. I could see that ambiguities would make it possible for an intelligent person to construe the equation "$5 + 7 = 12$" as tautological; but I could not see how such a thing could be said for an inequation such as "$5 + 7 \neq 13$". The view that inequations are tautological is one even Russell himself came to retract, but which Lewis never doubted (*Lewis-Langford 1932*, 211f, *Murphey*, 82).

Logic Includes Formal Epistemology

Before Boole, logic was focused on two central problems of logic as formal epistemology: how to show that a given conclusion follows from given premises that formally imply it, and how to show that a given conclusion does not follow from given premises that do not formally imply it. Aristotle wanted a decisive test or criterion for determining if the conclusion follows, and a decisive test or criterion for determining if the conclusion does not follow. Using other equally traditional terminology, the two central problems were how to establish validity and how to establish invalidity of an arbitrary argument, no matter how many premises or how complicated its propositions.

Aristotle did not solve the problem of formal epistemology in its full generality, nor did he claim to—contrary to what a few later authors seemed to have thought (*Cohen-Nagel 1934*, 110). Although he believed or even knew that he had completed "the logic of categorical propositions", he never thought that he had completed logic. In the opinion of many logicians today, perhaps not a majority, the full problem has still not been solved (despite occasional enthusiastic statements that it has). Aristotle would never have written what Lewis wrote (*1932*, 72): "Given premises *and* conclusion, logic can determine whether this conclusion follows".

This is one of many places where Lewis implicitly aligns himself with Aristotle and Boole by endorsing the traditional view that logic includes formal epistemology, even though at the same time he over-estimates modern achievements. Elsewhere Lewis says that logic can be thought of as an "organon" of demonstrative knowledge or as a "canon of deductive inference" (*1932*, 235 and *Murphey*, Ch. 3). As we will see, Lewis does not limit logic to formal epistemology, to the concern with determining validity or invalidity of premise-conclusion arguments. In this regard he agrees with Boole and disagrees with Aristotle. Nevertheless, he never wavers in his conviction that logic includes formal epistemology at least as a part. In 1957, near the end of his career, he seems to revisit his view of logic as including formal epistemology when he writes: "Logic is concerned only with what is deducible from what . . ." (*Murphey*, 319).

Boole's Thesis: Logic is Mathematical

Perhaps the first Boolean Revolution was announced in his 1847 book *The Mathematical Analysis of Logic* (Cambridge: Macmillan). It was more boldly reiterated on the last page of his 1848 article "The Calculus of Logic" (*Cambridge and Dublin Mathematical Journal* 3:183–198):

> The view which these enquiries present of the nature of language . . . exhibit it not as a mere collection of signs, but as a system of expression, the elements of which are subject to the laws of the thought which they represent. That those laws are as rigorously mathematical as are the laws which govern the purely quantitative conceptions of space and time, of number and magnitude, is a conclusion which I do not hesitate to submit to the exactest scrutiny.

With even more confidence he returned to this theme on page 12 of his 1854 masterpiece *Laws of Thought*: "it is certain that [logic's] ultimate forms and processes are mathematical". He again emphasizes his thesis in the last paragraph of the book. Perhaps Boole's most revolutionary achievement was to persuade the world's logicians that logical reality is mathematical, or at least that logic should be pursued mathematically.

Lewis was one of the first philosophically trained logicians to be fully persuaded of what may be called in retrospect "Boole's Thesis": logic is mathematical. This view is a kind of converse of the equally revolutionary "Logistic Thesis" in the form "mathematics is logical", also accepted by Lewis (*Murphey,* Ch. 1). Despite the fact that both theses tended to attract superficial or scientistic thinkers who, for example, denigrated the more humanistic or value-oriented side of philosophy, Lewis never lost touch with his deep concern with the human condition, nor was he ever tempted to exaggerate the importance of logic at the cost of other fields of philosophy (*Murphey,* Chs. 7 and 8). He never called logic "the essence of philosophy".

Logic Includes Formal Ontology

A second Boolean Revolution was incipient in his 1847 and 1848 works, but not fully announced until he heralded it by titling his 1854 book *An Investigation of the Laws of Thought,* bringing to the forefront of logic tautologies and traditional logical laws that have no immediately evident connection to deduction: e.g., identity of "indiscernibles", the law of contradiction, and the law of excluded middle for properties. Boole did not see the laws of thought as part of metaphysics as had Aristotle. He and many other logicians felt that this revolution restored to logic part of the domain previously but illegitimately controlled by metaphysics. In the eyes of some—Frege, Lukasiewicz and Quine to name three, though not Lewis and not Boole—this revolution relegated testing of premise-conclusion arguments to the background.

The characterization of logic as formal ontology continues to be problematic. Lewis spoke of *logistic* as the science of types of order (*1918,* 3) and of "laws of logic true of every subject-matter" (*1932,* 21). However, being sensitive to excesses and errors of exaggerated claims about formal ontology, he explicitly distanced himself from "those who take logical truth to state some . . . miraculous property of reality" giving rise to "mystic wonderment about nothing"(*1932,* 312). Other logicians had their own characterizations of logic as formal ontology. Frege spoke of "pure logic" composed of laws "upon which all knowledge rests", laws that disregard "particular characteristics of objects" (*1879,* Preface). Tarski, after acknowledging the role of logic as an organon or canon of "deductive sciences", said that logic "analyzes the meaning of the concepts common to all sciences, and establishes the general laws governing these concepts" (*Tarski 1941/1994,* xii, *1986,* 145).

For Aristotle, syllogistic deduction was an instrumental analytic activity—as is perhaps signaled by his using the word *Analytics* for what was later called *logic* and by the fact that his logical works came to be known as the *Organon* (instrument). Deduction, deducing conclusions from premises, stood on its own; it did not require principles previously judged to be true. In fact, deduction was an activity used instrumentally in the process of judging. In a way, deduction was prior to judgment in that it was applicable to premises regardless of whether they had previously been judged to be true, and indeed without regard to whether they were true or false. Even the codification of laws of thought or tautologies related to deduction, such as the *dictum* or a propositional cognate of *modus ponens*, brought into logic by Frege, were beyond the Aristotelian limits of logic, and for that matter beyond the Boolean limits—neither Aristotle nor Boole could even state in their formal languages propositional analogues of standard rules of deduction.

The two conceptions of logic, as formal epistemology and as formal ontology, are vaguely similar respectively to the traditional two conceptions of logic as an instrument and as a science. Cf., for example, Lewis's discussion (*1932*, 235).

Principles of Deduction: Peano's Double-use Conception

Frege and Lewis went far beyond Boole, and a fortiori Aristotle, in widening the scope of logic as formal ontology—including not only a richer complement of logical principles unconnected to deduction *per se*, but also principles that Lewis said have a "double use": as laws and as rules of deduction (*1918*, 324; *Murphey*, 96). It may have been the double-use view that induced Lewis in his comparison of logic conceived as formal epistemology with it as formal ontology to say that the latter "is much more inclusive"(*1932*, 235).

Murphey fails to say anything about how Lewis may have arrived at the bizarre double-use view, or how successful he was in persuading his readers to accept it. As a result of research by two Italian scholars, we can be confident that this view originated with Peano (*Borga and Palladino 1992*). They establish by persuasive quotations from Peano's writings that he subscribed to what Lewis called the "double-use" conception of logical principles (*1932*, 235 and cf. *1918*, 351ff), the view that logical principles not only serve as premises *from which* inferences are made but also justify inferences by serving as rules *according to which* inferences are made. The double-use conception, so alien to contemporary thinking and (except for Borga and Palladino) totally ignored by contemporary philosophers and historians, seems to have been transmitted by Peano to Russell, Whitehead, Lewis and others (*Corcoran and Nambiar 1994*).

Implication is Intensional

Every proposition carries a *message* about the elements in its universe of discourse; it has an *information* content—to adapt the terminology common among English logicians who succeeded Boole and pursued the mathematical paradigm he established. In order for a premise-conclusion argument to be valid it is necessary and sufficient for the information of the conclusion to be contained in, be part or all of, the information of the premises. The word 'information' was used repeatedly in this sense by Boole, De Morgan, Jevons, Venn and others, as documented in Section 2 of my 1998 article "Information-theoretic logic". Lewis was familiar with all of these logicians (*Lewis 1918*, Bibliography, *1932*, 1–15, *Murphey*, Chs. 1 and 3). In 1918 he wrote that inference depends on "meaning, logical import, intension", not extension (*1918*, 328f; *Murphey*, 96). He uses various words taking the place of *information*: *logical force, import, meaning, content,* and *logical significance* are all used in the space of two pages (*Lewis-Langford 1932*, 88–89). Lewis is explicit in his view that it is by reference to the information contents of premise and conclusion—not truth-values—that we can know that the one implies the other (*1932*, 261).

The information content of a proposition is its intension; the "reality" that it is about is its extension. Lewis never wavers in his endorsement of the view that logical implication is an intensional relation (*Lewis 1912*, 526; *1932*, 120; *Murphey*, 117).

Implies and Infers

The slogan "Propositions imply; people infer" is a useful over-simplification. But it is a good opening to make the point that one risks one's reputation for taste, perhaps also for sanity, by saying that a proposition infers. Whatever inferring is, it is an act performed by a person in forming a belief called an inference. It is performed in a specific interval of time on beliefs held at the time, and it results in a further belief. Once, a philosopher told me that my belief in other minds "was [arrived at by] an inference, not an observation".

Cohen and Nagel (*1934*, 7f) have this straight when they write: "*inference* . . . is a temporal process . . . *implication* . . . is an objective relation between propositions. An implication may hold even if we do not know how to infer one proposition from another." Of course, both of these words have more than one sense relevant to formal logic. We can hope for the day when authors define these two words before using them.

Deduction is Prior to, not Based on, the Principles of Deduction

As mentioned above, for Aristotle and Boole, the process of deducing does not require principles of deduction previously judged to be true. Boole made this picture more complicated when he noted that a chain of step-by-step deduction—while cogent, gapless, completely sound and logical—could nevertheless use as "premises" logical laws not among the original premises of the argument. Thus, logical judgments in some cases were used as "logical premises" in the process of deduction. Boole must have grasped something that became explicit in logic only much later, namely that tautologies are devoid of information and thus do not add information to the premises, something that would vitiate the deduction. Later, logic came to focus on the fallacy (charged to Euclid for one) called *premise-smuggling*: interpolating into an alleged step-by-step deduction information not contained in the stated premises. Boole implicitly realized that deductive use of logical laws was sound deductive practice, and not a case of premise-smuggling. Of course, Frege, Russell, and Lewis held the same view.

In Aristotle's now famous syllogistic model of deduction there was literally no place for what later came to be known as principles of syllogistic deduction such as *Dictum de omni et nullo* and laws of conversion. The propositions he chose to include in his model were all *categorical*—each was a subject-copula-predicate proposition of one of the four standard forms; there were no conditionals. An Aristotelian deduction containing "No square is a circle" as a premise or intermediate conclusion could be deductively augmented by addition of the converse "No circle is a square", but no such Aristotelian deduction could ever literally contain—as a premise, intermediate conclusion or final conclusion—the principle of deduction "If no square is a circle, then no circle is a square". For Aristotle conversion was done by a *rule*, not by use of a *principle*. Likewise for Boole, in no step-by-step deduction from an *equation* to its converse could a principle of conversion occur—because such a principle is a conditional, not an equation. Boole's language contained nothing but equations.

In sharp, almost diametrical opposition to Aristotle and Boole, in 1913 Lewis published an apodictic theory, a theory of proof, which required—for proof of a given theorem in a given system—a prior proof of a "principle of deduction", a logical or "strict" implication whose consequent was the theorem and whose antecedent was a postulate or previous theorem (*1913*, 429). For Lewis, *material modus ponens* [from p and p materially implies q deduce q] was inadequate for genuine proof—what he said was needed is what may be called *apodictic modus ponens* [from a known p and a known p strictly implies q infer q].

This astounding view is the basis of one of Lewis's main criticisms of *Principia Mathematica* (*Lewis 1914*, 598; *Murphey*, 78). Moreover, as Lewis well knew, this put him in disagreement with almost every modern logician. He apparently did not realize that he was also contradicting Aristotle. Curiously, Murphey spells out this criticism, which Lewis regarded as crucial, without telling us how it was received in the logic community or by Whitehead and Russell. I think that it is safe to say that no major logician ever accepted Lewis's apodictic theory.

To be explicit, since Aristotle first articulated it, there has been essentially no disagreement with the view that in order to prove a conclusion using a certain proposition as the premise it is necessary for the conclusion to be strictly implied by the premise. The issue concerns whether a cogent proof must actually contain as one of its lines or steps a proposition to that effect, i.e. a "strict-implicational" proposition that the conclusion is strictly implied by the premise. To accept this would be to reject all logics before Lewis or to claim that, contrary to what their authors seem to be saying, these logics really contained a means of expressing strict implications. If Lewis had read Carroll's "What the Tortoise Said to Achilles" (*1895*, 278–280), he might have changed his mind.

Inference, Deduction, and Derivation

Frege suggests using the word 'infers' for the epistemic activity of judging a proposition to be true by deducing it as a consequence from propositions known to be true (e.g., *1974*, 3; *1984*, 402). On the third page of his famous piece on negation (*1952*, 199), he said: "Of course we cannot infer anything from a false thought". Even though no other logician I know of has adopted Frege's suggestion—certainly not Lewis and not Cohen or Nagel—I have come to see the wisdom of it. I find it not at all restrictive, as one might at first think. It has been said that one of Aristotle's greatest discoveries was that the same process of *deduction* used to draw a conclusion from premises known to be true is also used to draw conclusions from propositions whose truth or falsity is not known, or even from premises known to be false. There is no doubt that Aristotle demonstrated his grasp of this point in the first few pages of *Prior Analytics* where he distinguishes demonstrative from non-demonstrative syllogisms. There Aristotle wrote (*Prior Analytics*, 24a7, Gasser *1991*, 232): "Every proof is a deduction but not every deduction is a proof". Later, he teaches that whether the conclusion of a premise-conclusion argument follows from the premises depends on the form of the argument, not on the truth or falsity of the premises.

Inference, which produces knowledge of the truth of its conclusion, and deduction, which produces knowledge that its conclusion is strictly implied by its premise, must both be distinguished from derivation, which consists in arriving at a string of characters by means of rule-governed manipulations starting with a given string of characters. Derivation by itself does not and cannot produce knowledge. No proposition may be *inferred* from a contradiction because no contradiction is known to be true, as Lewis taught. Every proposition may be *deduced* from a contradiction, as Lewis taught. And what can be *derived* from what depends upon which character-manipulating rules are allowed, as Lewis taught. Lewis's robust yet nuanced common sense continues to expose the confused pretensions of many of his successors, as Murphey ably demonstrates in chapter after chapter.

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