

MR0450016 (56 #8315) 02A05**Hambourger, Robert****A difficulty with the Frege-Russell definition of number.***J. Philos.* **74** (1977), no. 7, 409–414.

It is widely agreed by philosophers that the so-called “Frege-Russell definition of natural number” is actually an assertion concerning the nature of the numbers and that it cannot be regarded as a definition in the ordinary mathematical sense. On the basis of the reasoning in this paper it is clear that the Frege-Russell definition contradicts the following three principles (taken together): (1) each number is the same entity in each possible world, (2) each number exists in each possible world, (3) some entities existing in the actual world do not exist in every possible world. Since these principles seem to be true, the paper is a refutation of the Frege-Russell definition. The paper does more. It shows that the contradictory of the Frege-Russell definition follows even when principles 2 and 3 are replaced by one considerably weaker principle.

The ideas contained in the paper are related to two earlier objections to the definition. The first, sometimes attributed to the mathematician, C. S. Keyser, is that existence of the numbers as defined implies the existence of infinitely many particulars in each possible world. The second is, in effect, an idea which is said to have led Whitehead to reject the definition of number to which he had subscribed in *Principia Mathematica* [A. N. Whitehead and B. Russell, *Principia mathematica*, Vol. I, Cambridge Univ. Press, Cambridge, 1910; Jbuch. **41**, 83; Vol. II, 1912; Jbuch **43**, 93; Vol. III, 1913; Jbuch **44**, 68]. Whitehead is supposed to have said that he could not believe that the number two changes every “time twins are born”. The mathematician H. Jeffreys expressed similar ideas [*Philos. of Sci.* **5** (1938), 434–451].

One of the merits of the author’s work is that it refutes the Frege-Russell definition without the need to take sides on controversial points presupposed by the Keyser and Whitehead objections. The objections made by the author are therefore not to be identified with the Keyser and Whitehead objections. Even if the author’s work is to be regarded as a refinement and integration of previous ideas, it is nevertheless a contribution—not only because the basic points are well worth repeating but also because the refinements are logically significant improvements and because the author has stated them clearly and concisely in the idiom of contemporary philosophy.

Reviewed by *J. Corcoran*