Corcoran, J. 2010. Four entries for a Spanish-language dictionary of logic: “counterarguments and counterexamples”(137–42), “deduction and deducibility” ”(168–70), “logical form and formalization” ”(257–59), “truth-values”(627–63). Luis Vega, Ed. *Compendio de Lógica, Argumentación, y Retórica*. Madrid: Trotta.

Deduction and Deducibility

Deduction is the process of determining that a conclusion follows from premises, is an implication of premises, or is a logical consequence of premises—to use three of many synonymous expressions. Euclid is thought to have used deduction alone in extracting his theorems from his basic premises: his axioms, postulates, and definitions—propositions which he had previously established to be evident by intuition, induction, or some other non-deductive process. But the evidential nature of the premises, or raw material, or data, to which deduction is applied is neither necessary for nor presupposed by the evidential nature of deduction itself. The same process of deduction used to deduce true theorems from axioms known to be true is also used to deduce consequences from propositions not known to be true—and even to deduce false consequences from propositions, which might thereby be known to be false. After all, we often determine that a proposition is false by deducing from it a consequence already known to be false. By deducing a certain conclusion from given premises, a person comes to know that the conclusion is a logical consequence of those premises. However, by deducing a certain conclusion from given premises, a person does not *thereby* come to know that the conclusion is true unless those premises are known to be true.

Deduction makes evident that the conclusion is a consequence of the premises; *in itself* it does not make the truth of the conclusion evident—although if the truth of the premises is evident, then deduction is part of a richer process called *demonstration* or *proof* that does make evident the truth of the conclusion. As Aristotle said, demonstration presupposes deduction, but deduction does not presuppose demonstration. In fact, as implied above, deduction can be used when demonstration is impossible. People who accept deduction while rejecting demonstration are often called *deductivists* or *formalists*. However, deduction is not the only logical process; but it is one of the two central processes used repeatedly in logic: each relates to its own special problem type.

The two related problem types central to logic are consequence problems and independence problems. *Consequence problems* have the form: to make evident that a given conclusion is a consequence of a given premise set—if it is. *Independence problems* have the form: to make evident that a given conclusion is not a consequence of a given premise set—if it is not. Traditionally, a proposition *not* a consequence of a set of propositions is said to be *independent* of the latter.

A lengthy *deduction* that Andrew Wiles discovered shows the Fermat conjecture to be a consequence of established mathematical axioms. Consequence problems were solved by *deduction*: deducing the conclusion from the premises using a series of steps conforming to “rules of deduction”. The known rules of deduction were discovered by watching the process of deduction take place or by reviewing reports of deductive activity, concrete applications of deduction.

Reinterpreting ‘number’, ‘zero’, and ‘successor’ so as to produce true propositions from the other two axioms and a false proposition from the Mathematical Induction axiom shows the latter to be independent of the other two axioms of Gödel’s 1931 axiomatization of arithmetic. Independence problems were solved by *reinterpretation*: reinterpreting non-logical constants so as to produce true premises and false conclusion.

A proposition that is a consequence of (or is independent of) a given premise set is said to be a *hidden* consequence (or a *hidden* independence) if it is not obviously such. Without hidden consequence, deduction would be pointless. Without hidden independence, independence proof would be pointless. Hidden consequence and hidden independence are basic for justifying the study of logic and, indeed, for justifying the existence of logic as a field. This point is rarely made. In fact, the existence of hidden consequence and independence has been denied.

Deduction is a human process which has been applied for many centuries: it has been used to establish many consequence relations of conclusions to premise sets. But it has been used only finitely many times. As time goes on, deduction establishes more and more consequence relations. Its potential is infinite. The ideal limit of the potential for future applications of deduction is called *deducibility*. This raises the question of the comparison of *consequentiality*, the totality of consequence relations to deducibility, the ideal totality consequence relations which potentially can be made evident to human beings. Can every consequence relation be made evident? Or are there consequences of premises sets that can never be known to be consequences of those respective premise set? Does deducibility coincide with consequence? Is deducibility co-extensive with consequentiality? Or is deducibility less extensive? Is the extension of deducibility a proper subset of the extension of consequentiality? If the mathematical models studied by Kurt Gödel in the early 1930s are reliable representations of the human situation discussed above, the answer is no: there are consequences of the axioms of arithmetic that cannot be deduced from those axioms.