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► JOHN CORCORAN AND GERALD RISING, *Expressing set-size equality*.

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The set of points on a sphere *is* the set of points one radius from its center and it *equals* the set of points on any one of its radii, where ‘equals’ means “is in one-one correspondence with”. Each angle of an equilateral triangle *equals* each of the other two but it *isn’t* either of them. Many fields routinely distinguish identity and equality. Nothing *is* something else, but many things *equal* something else [this BULLETIN, vol. 19 (2013), pp. 255–256].

Identity and equality are routinely expressed by singular relational verbs—for identity: ‘is’, ‘is the same [...] as’, ‘is identical to’; for equality: ‘equals’, ‘is equal to’. Separation is clear: in fact ‘equals’ often connotes “not identical”—given availability of ‘is’.

Plural predicates such as ‘are equal’ [this BULLETIN, vol. 19 (2013), pp. 254–5] often express equality. Plural predicates can also express identity, but less smoothly: zee and zed are the same letter; zee and zed are one letter—the plurality of ‘are’ clashes with the singularity of its grammatical complement. Moreover, two-name conjunctions (as ‘zee and zed’) often connote that the names denote distinct entities.

Clarity is sadly diminished when numbers or sets are discussed.

We routinely read, e.g., that two plus three *equals* five, even in contexts emphasizing that two plus three actually *is* five.

Moreover, despite agreement that *two* sets in one-one correspondence *are equal* [in size, or cardinality], authors often avoid writing that two such sets are equal—without qualification such as ‘numerically’ or ‘in size’: perhaps because ‘equals’ is often enough misleadingly used for ‘is the same set as’.

Avoiding the natural verb ‘equals’ requires artificial substitutes: ‘equipollent’ [Suppes], ‘equinumerous’ [Boole], ‘similar’ [Dedekind], ‘equivalent’ [Tarski], ‘can be put in one-one correspondence with’ [Huntington], etc.

We review established substitutes, their rationales, and reasons stated for avoiding ‘equals’.