Corcoran, J. 2006. “George Boole”. *Encyclopedia of Philosophy*. 2nd edition. Detroit: Macmillan Reference USA, 2006

BOOLE, GEORGE (1815-1864), English mathematician and logician, is regarded by many logicians as the founder of mathematical logic. He could be called the Galileo of logic in that he definitively established the mathematical nature of logic – assuming that it was Galileo (1564-1642) who did this for physics, not, say, Archimedes (287-212 BCE). He is considered to be among the five greatest logicians, the others being the Greek philosopher Aristotle (384-322 BCE), the German mathematician Gottlob Frege (1848-1925), the Austrian mathematician Kurt Gödel (1906-1977) and the Polish mathematician Alfred Tarski (1901-1983). Like Aristotle, he never had the opportunity to take a course in logic. His parents’ economic circumstances precluded the usual formal education. He never took a college course and, thus, never received a bachelor’s degree. Nevertheless, he taught many college courses as Professor of Mathematics and he received honorary doctoral degrees from such distinguished institutions as Trinity College Dublin and Oxford University. These are among the many surprises, ironies, and paradoxes surrounding Boole’s life and work.

His ambition, energy, originality and dedication were evident even when he was a boy. By the age of 26 he had published the first of many articles in mathematics journals. By 29, for his 1844 article “On a General Method in Analysis”, he had won the Royal Society’s gold medal first prize recognizing “the most significant contribution to mathematics” submitted between 1840 and 1844. At 34, he was appointed Professor of Mathematics at Queen’s University. In 1864, when he died tragically just before the age of 50, he was one of the most celebrated figures on the British intellectual scene. In his lifetime he was known almost exclusively for his work in mathematical analysis, a specialty that includes traditional algebra, differential equations, the calculus of finite differences, and, of course, differential and integral calculus. In this field he wrote several articles and two books, both still in print: in 1859 *Treatise on Differential Equations* and a year later *Treatise on the Calculus of Finite Differences*. During his lifetime, few knew his logic at all, and of them few appreciated it. Today his work in mathematical analysis is largely unknown; his fame rests entirely on his logic. Boolean Algebra, the branch of modern mathematics named in his honor, derives from Boole's logic, not from his other mathematics. His work in logic still retains a vigor and freshness; it continues to be read and enjoyed by many people including professional mathematicians and logicians. In 2003, Prometheus Press brought out a new reprint edition of his most mature and most influential book, *An Investigation of the Laws of Thought on which are Founded the Mathematical Theories of Logic and Probabilities* (London, 1854) – known by its shortened title *Laws of Thought* – originally published at his own expense. The non-mathematical passages in this book are lucid and unusually well-written – a testament to Boole's humanistic learning, to his confidence in his own theories, and to his desire to contribute to the advancement of knowledge. In addition to the logic, Boole's 1854 book applies logic to probability theory.

Unlike other revolutionary logical innovators, Boole's greatness as a logician was recognized almost immediately. In 1865, hardly a decade after his 1854 *Laws of Thought* and not even a year after his death, his logic was the subject of a Harvard University lecture "Boole's Calculus of Logic" by Charles Sanders Peirce (1839-1914), America's most creative native logician. Peirce opened his lecture with these prophetic words:

“Perhaps the most extraordinary view of logic which has ever been developed with success is that of the late Professor Boole. His book ... *Laws of Thought* ... is destined to mark a great epoch in logic; it contains a conception which in point of fruitfulness will rival that of Aristotle’s *Organon*”.(See LOGIC, HISTORY OF, section on Boole).

Even though Boole is thought of today as the initiator of a radical revolution that conclusively and irrevocably overthrew the “Aristotelian” paradigm then reigning in the domain of logic, he never thought of himself as opposing Aristotle. He admired Aristotle’s logic – as far as it went. He never criticized any of the *positive* features that Aristotle instituted; he accepted as valid absolutely every argument that was valid according to Aristotle – including those with “existential import”, deducing existential conclusions from universal premises. On the contrary, Boole’s goals included revealing the mathematical nature of Aristotle’s logic, something that he felt Aristotle had failed to clarify, broadening Aristotle’s logic by showing that it could be made to do much more than was envisaged by Aristotle’s followers, and deepening it by penetrating beyond Aristotle’s analysis to the “ultimate” fine structure of the reasoning process – thereby providing it with what he called a “mathematical foundation”, showing that it had much more in common with mathematics than had previously been thought, and thus justifying it. From Boole’s point of view, Aristotle’s faults were all faults of omission, not of commission. Ironically, Boole’s unquestioning acceptance of certain details of Aristotle’s system, e.g., “existential import”, may have been one of the things which led to Boole’s unfortunate mistaken implementation of his own sound ideas.

In the process of extending and deepening Aristotle’s logic, Boole brought many radical ideas into logic. Where Aristotle had represented propositions by a kind of formalized phonetic Greek, Boole represented them by purely ideographic algebraic equations – giving rise to the first successful formalized language in the modern sense. Where Aristotle's propositions were limited to exactly two basic *non-logical* elements, one being the "subject" and one the "predicate", Boole's propositions had no limitation of that kind – they could involve any finite number of basic elements, which Boole represented with the letters familiar from algebra: x, y, z, and so on. In fact, by introducing for the first time in history the two *logical* elements – 1 for "everything" or the universe of discourse and 0 for "nothing" or the empty class – he was able to express propositions of pure logic, propositions devoid of non-logical elements, another historical first. It was Boole who coined the expression "universe of discourse", which is ubiquitous in modern logic, and it was Boole who first suggested the possibility of reinterpreting a formal language by changing the universe of discourse and the meanings of the non-logical symbols.

Where for Aristotle the elements were represented by the Greek words having fixed meanings – for "human", "animal", and other substantives, Boole's letters were reinterpretable. Each of Aristotle's formal sentences expressed exactly one proposition whether true or false, but for Boole any single formal sentence was capable of expressing indefinitely many propositions not necessarily all true (as $x (1 – x) = 0$)or all false (as $x (1 – x) = 1$). Those that expressed only truths he said were "true in virtue of form", perhaps coining this expression also. This innovation was eventually to play a crucial role in modern logic.

For example, with the multiplication sign or juxtaposition representing "logical term-conjunction" (the Boolean "and"), with x for "human", and y for "animal", Boole thought he had expressed Aristotle's "Every human is an animal" by $xy = x$. These innovations opened the way to Boole's most radical, totally unexpected and unprecedented insight: that a fully interpreted equation expressing a proposition, whether true or false, could be considered as an equation with one element regarded as an "unknown" to be solved for in terms of the others. Where Aristotle's focus in formal logic had been exclusively with determining logical validity and invalidity of premise-conclusion arguments, i. e., with what has been called *formal epistemology*, Boole's broader focus included, besides a much expanded formal epistemology, several new concerns, two of which were his wholly new theory of logical equation-solving and his *formal ontology* concerned with axiomatizing logical truths – which he called by the expression *laws of thought*. Boole explicitly recognized, as Aristotle had not, that "class logic", even in its expanded form, could not treat the arguments now dealt with in truth-functional proposition logic. To meet this deficiency he proposed an ingenious reinterpretation of his "class logic" that, in his view, transformed it into a propositional logic. In the process he discovered the key ideas now incorporated into laws of modern truth-function logic, establishing himself as the first modern figure in any history of propositional logic.

Before Boole, logic had been thought of as an *organon* or general instrument necessarily presupposed by any axiomatic science, *not* as an axiomatic science; Boole proposed regarding logic itself as subject to axiomatic treatment. Boole believed that his logic transcended, included, explained, and thus replaced Aristotle's in much the way that Newton's mechanics transcended, included, explained, and thus replaced Kepler's.

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Bibliography

There is an excellent biography of Boole by Desmond MacHale, *George Boole: His Life and Work* (Dublin, 1985), which includes a complete bibliography of Boole's publications. A more intimate portrait is by his wife, M. E. Boole, "The Home Side of a Scientific Mind," in *The University Magazine*, n.s., Vol. 1 (1878), reprinted in her *Collected Works*, Vol. 1 (London, 1931), pp. 1 —53. Boole's only publications in logic are: *The Mathematical Analysis of Logic* (Cambridge, 1847), "The Calculus of Logic", *Cambridge and Dublin Mathematical Journal* Vol.3 (1848), pp.183-198 and *Laws of Thought* (London, 1854), reprinted with introduction by J. Corcoran (Buffalo, 2003). The most important of Boole's unpublished manuscripts on logic-related topics are available in two volumes: one edited by R. Rhees, *Studies in Logic and Probability* (La Salle, IL, 1952), one edited by I. Grattan-Guinness and G. Bornet, *George Boole: Selected Manuscripts on Logic and its Philosophy* (Basel, 1997). In 1976 (revised and enlarged 1986) the mathematician T. Hailperin proposed an interpretation of Boole's logic and probability theory which makes Boole an even more important pioneer in modern mathematics than previous interpreters: *Boole's Logic and Probability* (Amsterdam, 1976 and 1986). A comprehensive collection of critical and expository studies has been compiled by J. Gasser, *A Boole Anthology: Recent and Classical Studies in the Logic of George Boole* (Dordrecht, 2000). An extensive, but incomplete, point-by-point comparison of Boole's logic with the Aristotelian system it was intended to perfect is found in Corcoran, J. "Aristotle's *Prior Analytics* and Boole's *Laws of Thought. History and Philosophy of Logic* 24 (2003), pp. 261-288.

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