Hilbert by Constance Reid
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future. A prime example of such a paper is that of M. A. Garstens entitled "Remarks on Statistical Mechanics and Theoretical Biology." Garstens thinks that the salvation of biology lies in the study of statistical mechanics, but leaves the reader with little more than a statement of his conviction. He certainly does not give us what is in any sense a "draft." A similar sort of paper is that by Waddington on morphogenesis. Waddington talks about the different levels of theorizing needed in developmental biology, but concludes his discussions with statements like "This is certainly a definite and identifiable phenomenon which demands a theory." Hardly a conclusion one would expect in a "draft" of a new biology.

The second group of papers is much more valuable, since this contains the papers where the authors really try to get on with the job of doing some biology. The group includes two papers on genetic nets by Stuart Kauffman, a paper by R. Levins on "whether we can develop a fairly general and widely applicable theory on the structure and dynamics of complex systems, which would be applicable to work in biology at the level of the population, the cell, development and perhaps be of some use in the analysis of other complex systems of a social kind," and a paper by R. C. Lewontin arguing for a view of theoretical population genetics which considers as the basic units chromosomes rather than genes. The last paper, in particular, seems to me to be one of the very few which really attempts to grapple with the supposed aims of the symposia namely to try to improve biology in some way.

The third and final group of papers are those which are in some vague sense "philosophical" —at least, I suppose they are philosophical because they brought on an overwhelming sensation of *déjà vu*. This group contains the papers whose authors are concerned to argue that molecular biologists (and all others of a reductionist bent) are on the wrong track because they overlook biological complexity, biological order, and so on, and so on. Papers by W. M. Elsasser on "The role of individuality in biological theory" and C. Longuet-Higgins on "The seat of the soul" are typical representatives of this group. The latter paper is written in the form of a dialogue between a physicist and a biologist, and the following extract from their conversation will give the reader an idea of what I mean when I say that it all seems so familiar.

> P: All right, so we have to try and define 'organization'. But is it an exclusively biological idea? Don't we physicists use it when we are thinking about perfect crystals, for instance?
> B: Do you really? When I hear physicists talking about such things, they usually seem to use the word 'order', not 'organization'.
> P: All right, but is there a real difference?
> B: Yes, and a very important one in my opinion . . .

Perhaps there is a difference between order and organization. Perhaps the difference is important. It's just that somehow one has seen so many defences of the autonomy of biology along these lines, the whole topic gets too boring to attack or defend.

In conclusion, let me state the obvious, namely that one reviewer is not overly impressed by the results of the symposia towards a theoretical biology. I do not deny that a theoretical biology may be needed, but I do not see that, despite some good contributions, these symposia provide it. Rather, my impression is that most of the contributors are bound together by a shared antipathy to molecular biology or any other kind of biological reductionism, and therefore they would like to found a rival discipline to claim the attention of active biologists. I do not think that so far they have been very successful, and my feeling is that success will continue to elude them. *Michael Ruse, The University of Guelph.*

CONSTANCE REID. ***Hilbert.*** New York Heidelberg Berlin: Springer Verlag, 1970. xi + 290 pp. $8.50.

This intimate, "life and times" biography of perhaps the most influential mathematician since Euclid was written with assistance from many of Hilbert's students and colleagues. Bernays, Courant, Weyl, Landé, Polya, van der Waerden, Tarski, Gödel, and many others are acknowledged in the preface. Hilbert's life is sketched chronologically in twenty-five short chapters spanning 220 pages. The remaining seventy pages include a shortened version of Weyl's important appreciative obituary "David Hilbert and his Mathematical Work" together with twenty-nine interesting and beautifully reproduced photographs of Hilbert, his family and colleagues and his towns of Königsberg and Göttingen. The philosophical interest of the book is considerably enhanced by frequent and long quotations from Hilbert's addresses on foundational issues presented in a setting of the important events of his life. The whole of one chapter is an abridged version of Hilbert's famous address "Mathematical Problems" which shaped the future of mathematics.

Although this fascinating work is not a serious study of Hilbert's personality nor is it an intellectual biography, one can still gather from it some idea of the forces and events which helped to shape Hilbert's views. Except for short stays elsewhere Hilbert spent his first thirty-three years in Königsberg which was still alive with discussion of the philosophy of its most famous citizen. Indeed, Hilbert spent five years in the very *gymnasium* that Kant had attended and, more significantly, for his doctoral examination Hilbert chose to defend Kant's view of the a priori nature of arithmetic judgments. Hilbert never lost respect for Kant; in 1930, almost at the end of his productive life, Hilbert was to reaffirm the synthetic a priori foundation of mathematical knowledge while allowing that Kant had greatly overestimated its role. Hilbert's attachment to a priori nonconstructivist thought could not have been weakened by two other circumstances: first, many of his dramatic achievements rest on existence proofs by contradiction; second, he had sometimes intense personal distaste for the two main proponents of constructivism, Kronecker and Brouwer, who are portrayed in this book as brilliant but dogmatic, unpleasant, and unjust.

In order to focus on the development of Hilbert's thought it is convenient to neglect all but the two central issues in foundations: the "ontological" question of the meanings of mathematical expressions and the epistemological question of how mathematical knowledge is achieved. There are three views which may serve as landmarks in any survey of Hilbert. The "platonist" (neo-Platonic or neo-Kantian) view accepts a correspondence theory of meaning and truth (numerals name numbers, etc.) which is based on belief in objective existence of mathematical entities. Mathematical knowledge, on the platonist view, is achieved by a combination of *non-constructive* insight and strict logical reasoning. The "formalist," on the other hand, believes that mathematical expressions are devoid of meaning and that what counts as "mathematical knowledge" are expressions which are either arbitrarily adopted as starting places or else de-derived from such by application of arbitrary rules. The core of the third "intuitionistic" view is that mathematical expressions indicate "constructions" and that *all* mathematical knowlege is achieved by performing constructions, some initial and some predicated *on* the initial constructions. A "positive" sentence P means that a certain construction has actually been carried out and a "negative" sentence not-P means (not that no such construction has been carried out but rather) that supposition of a construction corresponding to P leads to an absurdity. Clearly "excluded middle" fails here along with the traditional distinction between truth and (potential) knowledge. (The so-called "logicist" view is not of interest here since Hilbert felt that the need for an axiom of infinity made it untenable). It is of crucial importance to note that all of the above views are compatible with "methodological formalism in metamathematics" which is simply adoption of the methodological constraint of eschewing reference to meaning in description of syntactical aspects of axiomatic theories. One sufficient motivation for adopting such a constraint is to be found in the desire to avoid begging questions concerning the relation of syntax to meaning. That Hilbert was a methodological formalist has never been seriously questioned.

In order to understand Hilbert's thought on foundations as developed within this book (and in his writings, it might be added) one must be willing to consider two complications: first, the possibility that Hilbert vacillated; second, the fact that the above views can be consistently combined. On the basis of quotations and commentary in this book it is possible to conclude that at the core of Hilbert's views were three beliefs (all vague and open to subsequent modification): (1) "consistency guarantees existence," (2) soundness of existence proofs by contradiction, (3) objective validity of "classical mathematics" sometimes including much of Cantorian set theory. A fourth idea (which cannot be called a view) that seemed legitimate and impressive to Hilbert was that of postulation of ideal entities in order to fill "gaps" in a universe of investigation. Negative numbers can be regarded "added by postulation" to the natural numbers in order to permit subtraction. To the integers the rationals may be "added by postulation" to permit division. The "imaginary numbers" were postulated in order to provide solutions to polynominals and, most impressively, Kummers' ideal numbers were postulated to permit prime factorization of imaginary numbers. Other possible examples of "ideal postulation" are: points at infinity in geometry, classes in set theory (to permit definition by abstraction) and the null string theory (to simplify certain axioms). Clearly, it is possible to be platonist in regard to one domain of entities while being intuitionist or formalist in regard to "ideal extensions" of that domain. Again, one could be intuitionist in regard to one domain and formalist in regard to an ideal extension.

"Consistency guarantees existence," which plays a large role in this book was reiterated by Hilbert in 1900 and in 1904 . . . and then roundly criticized by Frege and others. Clearly Hilbert should have meant "consistency guarantees satisfiability" and he probably did. In any case, in these early days Hilbert was holding an extreme version of the platonism shared by most mathematicians of his time. Over the next couple of decades Hilbert's views seem to shift in the direction of a compromise between intuitionism and formalism. He was led to distinguish between "contentual" and "ideal" mathematics so that intuitionist principles could be affirmed of the contentual part while the ideal part was to be "justified" (not guaranteed existential import) by means of formal consistency proofs. Hilbert's continuing emphasis on the importance of consistency proofs for justifying classical mathematics led him to conceive of proof theory as having a dual purpose: first, to provide an exact formal description of the syntactical part of *actual* mathematics (formal strings being "images of thought"); second, to provide a consistency proof given the description. Thus methodological formalism is a natural accompaniment to the compromise whereas formalism itself would have been a total capitulation. Although Hilbert weakened his "consistency guarantees existence" view he never really abandoned it as a mistake and, indeed, he moves back toward platonism in one of his last papers where he lists "the question of whether consistency and existence are equivalent for mathematical objects" as an open problem in foundations. Readers not versed in subsequent foundational research may be interested to know that Gödel's results imply the equivalence of consistency and satisfiability in first order logic but (as has been pointed out by Beth, Tarski, and others) their nonequivalence in higher orders. [Consistent, unsatisfiable theories are easy to construct: add to the set of arithmetic inequalities the negated (second order) existential closure of any (first order) "axiom of infinity."] Perhaps in conflict with these results, the author has concluded that "Hilbert's liberating conception of mathematical existence as freedom from contradiction has unquestionably triumphed over the shackling constructive ideas of his opponents."

Because there are no theorems in logic which go by Hilbert's name there is a tendency among younger logicians to underestimate his real achievements in this area and to suppose that his main contribution was in lending his enormous prestige to this new and suspect field. Reid's book should undercut such tendencies; e.g. she points out that a few years before Gödel's solutions Hilbert had already published statements of both the problem of completeness of predicate logic and the problem of incompleteness of arithmetic. (It is ironic that the solution to the latter led directly to the demise of the second part of the Hilbert's Program.)

Because the book is clearly *not* intended as a scholarly biography, some readers might desire more technical accuracy. The following comments are intended as aids to such readers, not as criticism. On p. 34 "consistency guarantees existence" is incorrectly linked to the soundness of nonconstructive existence proofs. On p. 58 proving an axiom as a theorem and proving that its negation leads to a contradiction are counted as *two* of three methods for dealing with doubtful axioms. It is further added that the latter represents an appearance of the concept of consistency. On p. 59 it is stated that since the early noneuclidean geometers did not find the expected contradictions they, therefore, discovered the consistency of noneuclidean geometry. Further down on the same page an "abstract" geometry is described as "a calculus of relations between variables." On p. 60 Hilbert's geometry is made to look "modern" by reporting that he did not regard geometry as a body of truths about the physical universe without adding that he *did* regard it as expressing fundamental facts of our intuition of space. On p. 63, as a stylish way of defining "completeness," Hilbert is said to have wanted his axioms to be *complete* "so that all the theorems could be derived from them." It is also implied that, at that time, Hilbert was concerned with completeness of axiomatic theories in the modern sense whereas in fact his concern was with completeness of the models (a model of a set of axioms is complete if no new elements can be added without falsifying an axiom).

Even as a "life and times," nonscholarly biography the book might receive criticism for containing too many insignificant vignettes and sentimental reminiscences. Some readers may also regret both the sometimes oppressive supply of sterile praise quoted from famous scientists as well as the inclusion of a few unjustifiable and almost cruel anecdotes—one involving Norbert Wiener is especially offensive. The book's shortcomings, however, are overwhelmingly outweighed by its merits and no one with an interest in logic and foundations will want to miss it. *John Corcoran, State University of New York at Buffalo.*