MR862448 (88j:01013) 01A50 01A75 03-03 51-03
Saccheri, Girolamo [Saccheri, Giovanni Girolamo]

★**Euclides vindicatus.**
Second edition.
Translated from the Latin and edited by George Bruce Halsted.
With notes by Paul Stäckel and Friedrich Engel translated from the German by F. Steinhardt.
Chelsea Publishing Co., New York, 1986. xxx+255 *pp.* $16.95. *ISBN* 0-8284-0289-2

Girolamo Saccheri (1667–1733) earned a permanent place in the history of mathematics by discovering and rigorously deducing an elaborate chain of consequences of an axiom-set for what is now known as hyperbolic (or Lobachevskian) plane geometry. Such an axiom-set can be constructed from a suitable axiom-set for Euclidean geometry by replacing the parallel postulate with a proposition to the effect that through a point outside of a given line there is more than one line parallel to the given line (although this is not how Saccheri did it). Saccheri's development of hyperbolic geometry is found in Book I of his work *Euclides vindicatus* (*Euclid vindicated*) published in the year of his death. The avowed purpose of Book I was not, however, advancement of non-Euclidean geometry but rather, on the contrary, proof of the parallel postulate from the other basic premises of Euclid's geometry, a goal attempted by many mathematicians both before and after. It was not until more than a century later, about 1870, that such a proof was to be known to be impossible. As implausible as this may seem, Saccheri's remarkable but ultimately unsuccessful "proof" was apparently the first to involve indirect reasoning as its main strategy [see, e.g., R. Torretti, *Philosophy of geometry from Riemann to Poincaré*, see p. 45, Reidel, Dordrecht, 1978; MR0525381 (80h:01021)]. It was only after a long chain of carefully deduced consequences that Saccheri fallaciously arrived at a contradiction. It may have been the failure of this extensive, imaginative, and largely cogent effort that led, or helped to lead, future generations of mathematicians to conjecture the impossibility of its goal, i.e., to conjecture the independence of the parallel postulate. It is certain that Saccheri's work influenced subsequent mathematicians, including Lambert, Gauss, Bolyai and Lobachevskiĭ [see, e.g., Torretti, op. cit., Chapter 2; A. M. Dou, Notre Dame J. Formal Logic **11** (1970), 385–415; see especially p. 396; MR0297503 (45 #6557); errata; MR0349320 (50 #1814)]. Even apart from its fruitful influence, *Euclides vindicatus* is a storehouse of evidence for the history of mathematics, the history of logical rigor and the history of mathematical philosophy.

The present volume contains a Latin edition of Book I of *Euclides vindicatus* together with an English translation on facing pages by the American mathematician G. B. Halsted. The English translation originally appeared in segmented form [Amer. Math. Monthly **1** (1894), 70–72; 112–115, 149–152, 188–191, 222–223, 259–260, 301–303, 345–346, 378–379, 421–423; ibid. **2** (1895), 10, 42–43, 67–69, 108–109, 144–146, 181, 214, 256–257, 309–313, 346–348; ibid. **3** (1896), 13–14, 35–36, 67–69, 109, 132–133; ibid. **4** (1897), 10, 77–79, 101–102, 170–171, 200, 247–249, 269–270, 307–308; ibid. **5** (1898), 1–2, 67–68, 127–128, 290–291] and was later published in book form [Open Court, Chicago, Ill., 1920; Jbuch **47**, 35]. The latter was reprinted in 1971 by the current publisher. The 1920 first edition contains a rather enthusiastic "Introduction" and a series of notes by the translator. The 1986 second edition adds a new preface signed "A. G." and more than fifty new notes originally written in German by the distinguished scholars P. Stäckel and F. Engel for their own work [*Die Theorie der Parallellinien von Euklid bis auf Gauss*, Teubner, Leipzig, 1895; Jbuch **26**, 57] and translated for the present volume by F. Steinhardt. The subject index was apparently never completed; it consists of only thirty-two items, eighteen of which list but one page location each and one of which lists no page location at all. The English translation is often difficult, occasionally impossible, to follow. In a few cases, the new notes include alternative translations which are much superior to the original translation.

{Reviewer's remarks: (1) On two pages of this book Saccheri refers to his previous and equally original book *Logica demonstrativa* (Turin, 1697) to which 14 of the 16 pages of the editor's "Introduction" are devoted. At the time of the first edition, 1920, the editor was apparently not acquainted with the secondary literature on *Logica demonstrativa* which continued to grow in the period preceding the second edition [see D. J. Struik, in *Dictionary of scientific biography, Vol. 12*, 55–57, Scribner's, New York, 1975]. Of special interest in this connection is a series of three articles by A. F. Emch [Scripta Math. **3** (1935), 51–60; Zbl **10**, 386; ibid. **3** (1935), 143–152; Zbl **11**, 193; ibid. **3** (1935), 221–333; Zbl **12**, 98]. (2) It seems curious that modern writers believe that demonstration of the "nondeducibility" of the parallel postulate vindicates Euclid whereas at first Saccheri seems to have thought that demonstration of its "deducibility" is what would vindicate Euclid. Saccheri is perfectly clear in his commitment to the ancient (and now discredited) view that it is wrong to take as an "axiom" a proposition which is not a "primal verity", which is not "known through itself". So it would seem that Saccheri should think that he was convicting Euclid of error by deducing the parallel postulate. The resolution of this confusion is that Saccheri thought that he had proved, not merely that the parallel postulate was true, but that it was a "primal verity" and, thus, that Euclid was correct in taking it as an "axiom". As implausible as this claim about Saccheri may seem, the passage on p. 237, lines 3–15, seems to admit of no other interpretation. Indeed, Emch [op. cit.; Zbl **12**, 98, see pp. 232–233] takes it this way. (3) As has been noted by many others, Saccheri was fascinated, if not obsessed, by what may be called "reflexive indirect deductions", indirect deductions which show that a conclusion follows from given premises by a chain of reasoning beginning with the given premises augmented by the denial of the desired conclusion and ending with the conclusion itself. It is obvious, of course, that this is simply a species of ordinary indirect deduction; a conclusion follows from given premises if a contradiction is deducible from those given premises augmented by the denial of the conclusion—and it is immaterial whether the contradiction involves one of the premises, the denial of the conclusion, or even, as often happens, intermediate propositions distinct from the given premises and the denial of the conclusion. Saccheri seemed to think that a proposition proved in this way was deduced from its own denial and, thus, that its denial was self-contradictory (p. 207). Inference from this mistake to the idea that propositions proved in this way are "primal verities" would involve yet another confusion.}

The reviewer gratefully acknowledges extensive communication with J. Gasser and M. Scanlan. *J. Corcoran*