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The Founding of Logic

Modern Interpretations of Aristotle's Logic

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Introduction

Since the time of Aristotle's students, interpreters have considered *Prior Analytics* to be a treatise about deductive reasoning—more generally, about methods of determining the validity and invalidity of premise-conclusion arguments. People studied *Prior Analytics* in order to learn more about deductive reasoning and to improve their own reasoning skills. Some people naively and irresponsibly thought that the deductive reasoning described by Aristotle was entirely adequate for derivation of the theorems of geometry from the basic premises. Moreover, even such perceptive people as Boole who were insightful about the gross lack of comprehensiveness in the rules of deduction attributed to Aristotle nevertheless understood Aristotle to be treating the process of deducing conclusions implied by *given* premise-sets. These interpreters understood Aristotle to be focusing on two epistemic processes: First, the process of establishing knowledge that a conclusion follows necessarily from a set of premises (that is, on the epistemic process of extracting information implicit in explicitly given information) and, second, the process of establishing knowledge that a conclusion does not follow. Despite the overwhelming tendency to interpret the syllogistic as *formal epistemology*, it was not until the early 1970s that it occurred to anyone to think that Aristotle may have developed a theory of deductive reasoning with a well worked-out system of deductions comparable in rigor and precision with systems such as propositional logic or equational logic familiar from mathematical logic.

When modern logicians in the 1920s and 1930s first turned their attention to the problem of understanding Aristotle's contribution to logic in modern terms, they were guided both by the Frege-Russell conception of logic as *formal ontology* and at the same time by a desire to protect Aristotle from possible charges of psychologism. They thought they saw Aristotle applying the *informal* axiomatic method to formal ontology, not as making the first steps into formal epistemology. They did not notice Aristotle's description of deductive reasoning. Ironically, the *formal* axiomatic method (in which one explicitly presents not merely the substantive axioms but also the deductive processes used to derive theorems from the axioms) is incipient in Aristotle's presentation.

Partly in opposition to the axiomatic, ontically-oriented approach to Aristotle's logic and partly as a result of attempting to increase the degree of fit between interpretation and text, logicians in the 1970s working independently came to remarkably similar conclusions to the effect that Aristotle indeed had produced the first system of formal deductions. They concluded that Aristotle had analyzed the process of deduction and that his achievement included a system of natural deductions including both direct and indirect deductions which, though simple and rudimentary, was semantically complete.

Where the interpretations of the 1920s and 1930s attribute to Aristotle a system of *propositions* organized deductively, the interpretations of the 1970s attribute to Aristotle a system of *deductions*, extended deductive discourses, concatenations of propositions, organized epistemically. The logicians of the 1920s and 1930s take Aristotle to be deducing laws of logic from axiomatic origins; the logicians of the 1970s take Aristotle to be describing the process of deduction and in particular to be describing deductions themselves, both those deductions that are proofs based on axiomatic premises and those deductions that, though deductively cogent, do not establish the truth of the conclusion but only that the conclusion is implied by the premise-set.

Thus, two very different and opposed interpretations had emerged, interestingly both products of modern logicians equipped with the theoretical apparatus of mathematical logic. The issue at stake between these two interpretations is the historical question of Aristotle's place in the history of logic and of his orientation in philosophy of logic. This paper affirms Aristotle's place as the founder of logic taken as formal epistemology, including the study of deductive reasoning. A by-product of this study of Aristotle's accomplishments in logic is a clarification of a distinction implicit in discourses among logicians—that between logic as formal ontology and logic as formal epistemology.

Aristotle's Logic: New Goals, New Results

Our understanding of Aristotle's logic has increased enormously in the last sixty years. It is gratifying to review the cascade of progress beginning with the independently achieved but remarkably similar advances reported in 1929 by Jan Łukasiewicz and in 1938 by James Wilkinson Miller. Penetrating examination and critical evaluation of the Łukasiewicz-Miller viewpoint in the 1950s and 1960s set the stage for work in the early 1970s by Timothy Smiley and myself. Subsequent work in the late 1970s and early 1980s by various people including Timothy Smiley, Robin Smith, Michael Scanlan and myself can be seen as culminating, at least for the moment, in the 1989 translation and commentary on *Prior Analytics* by Robin Smith.

Since the early 1970s the progress has been dialectical rather than linearly cumulative, and the spirit of objectivity and sympathetic criticism currently pervading the field makes it likely that the dialectical paradigm will continue.

Each succeeding contribution not only benefitted from and built on previous efforts but it also found itself refining, revising, correcting, and refuting various features of previous efforts, often previous efforts by the same author. There is no such thing as a unique fixed contemporary doctrine on Aristotle's logic. Indeed, as far as I know, none of the current investigators has yet to become committed to a definitive viewpoint although there is wide agreement on loose, general principles.

In some instances, e.g. Smith's 1982 work on ecthetic deduction, progress was made by widening the range of Aristotelian processes amenable to treatment by modern methods of symbolic logic. In some cases, e.g. Scanlan's 1983 work on issues of compactness in the *Analytics*, progress consisted largely in clarifying issues and correcting errors of previous works. In some cases, e.g. the independent work by Smiley and by myself on syllogistic deduction, progress involved radically increasing the degree of fit required between modern interpretative reconstruction and the Aristotelian text. Neither Łukasiewicz nor Miller attempted to reconstruct a logical system originally presented by Aristotle. Cf. Corcoran and Scanlan 1982, 78–79. The repayment for these two independent attempts at increasing the precision of interpretation was a sweeping revision of our understanding of Aristotle's most fundamental concepts, methods, and goals. In retrospect it is difficult to imagine two types of interpretative theories more thoroughly different than those proposed before World War II by Łukasiewicz and by Miller, on one hand, and those proposed twenty-five years after the war by Smiley and by myself, on the other.

The pre-war Łukasiewicz-Miller view takes Aristotle's main concerns in *Prior Analytics* to be ontic, to the exclusion of epistemic concerns. Łukasiewicz goes so far as to say that the logic of *Prior Analytics per se* has nothing to do with thinking, that it concerns thought *per se* no more than does a mathematical theory. It is true of course that this nonepistemic approach was motivated in part in order to defend Aristotle from false charges of psychologism. The post-war Smiley-Corcoran view takes Aristotle's main focus to be on epistemic concerns, as opposed to but not excluding ontic concerns. According to this view, among other things Aristotle was concerned with how human beings deduce a conclusion from premises that logically imply it. More generally, Aristotle was concerned with the methodology of deduction and counterargumentation: with how we know that *this* conclusion follows logically from *these* premises and with how we know that *this* conclusion does not follow from *these* premises. These questions, according to the Smiley-Corcoran view, were motivated largely by Aristotle's concern to understand the axiomatic method that he treats in the *Posterior Analytics* and that was practiced in the Academy. *Prior Analytics* studies an aspect of the axiomatic method; it does not *use* the axiomatic method.

Thus the Łukasiewicz-Miller view attributes to Aristotle's *Prior Analytics* a goal drastically different from the goal attributed by the Smiley-Corcoran view. One should expect then that the methods attributed to Aristotle would likewise differ significantly and indeed this is the case, with some curious qualifications and exceptions. The main exception has to do with the already mentioned fact that neither Łukasiewicz nor Miller claims to have reconstructed a system actually to be found in *Prior Analytics*, or even an approximation thereof. It is a bizarre irony that defenders of the Łukasiewicz-Miller view are often found to be claiming for the view merits that its originators disclaimed.

Instead of having as a goal understanding the processes of determining whether or not a given conclusion follows from a given premise-set, Łukasiewicz and Miller think that Aristotle's goal is the establishing as true or as false, as the case may be, of certain universally quantified conditional propositions, viz. those that 'correspond' to what have been called valid and invalid categorical arguments with arbitrarily large premise sets. For example, the following is to be established as true: given any three terms A, B, and C, if A belongs to every B and C belongs to no A, then B belongs to no C. And the following is to be established as false: given any three terms A, B, and C, if A belongs to every B and B belongs to every C, then C belongs to every A. In contrast, according to Smiley and Corcoran, Aristotle's goal rather was to develop and apply a method of deductive reasoning to deduce a proposition 'B belongs to no C' from propositions 'A belongs to every B' and 'C belongs to no A', and in regard to the second example, the Smiley-Corcoran view is that Aristotle's goal was to exhibit a method for establishing that a proposition 'C belongs to every A' is not a necessary consequence of propositions 'A belongs to every B' and 'B belongs to every C'. According to the Smiley-Corcoran viewpoint, examples of the first sort, i.e., establishing that a given conclusion follows from given premises that actually imply it, are handled by Aristotle's method of deduction which involves both direct (ostensive) deductions and indirect (per impossible) deductions. Aristotle's method of deduction is treated in more detail below. Here I want to indicate that because Łukasiewicz does not realize what Aristotle is doing with indirect deductions, Łukasiewicz thinks that Aristotle does not understand *per impossible* reasoning. In fact, he thinks that Aristotle's misunderstanding of indirect reasoning was so defective that Aristotle commits a fallacy not just here and there but each and every time it is used in the syllogistic. See Corcoran 1974. According to the Smiley-Corcoran viewpoint, examples of the second sort, i.e. establishing that a given conclusion does not follow from given premises actually not implying it, are handled by Aristotle's method of counterarguments which consist in exhibiting another argument in the same form having premises known to be true and conclusion known to be false. Łukasiewicz also thinks that this method is fundamentally flawed. See Łukasiewicz 1957, 72. It is worth noting that the two interpretations, the pre-war as well as the post-war, are in complete agreement that Aristotle's syllogisms are *not* abstract forms (whether forms of propositions, forms of arguments or forms of deductions). In the case of the Łukasiewicz-Miller view, the 'terms' of the syllogism (indicated by letters A, B, C above) are actually object-language variables, not place-holders in forms. In the case of the Smiley-Corcoran view, the 'terms' are concrete substantives such as 'human,' 'animal,' 'plant,' etc. There is no evidence whatever to support the widespread belief that Aristotle postulated abstract forms over and above concrete propositions, concrete arguments, or concrete deductions. Indeed, such postulation would have been characteristically platonistic and nonaristotelian.

The Background of Aristotle's Logic

In order to appreciate what is accomplished in the *Organon*, more specifically, in the *Analytics*, it is necessary to have some appreciation of the state of development of geometry and arithmetic in the Academy during the period immediately preceding and during Aristotle's writing of these works. It is also helpful to be aware of Plato's attitude toward the mathematical knowledge of his time and to understand Plato's views on the role of mathematical knowledge in education and in intellectual life. Some graduates of modern universities do not achieve knowledge of mathematics comparable to that required by Plato for admission as a beginner in the Academy. Plato believed that mathematical knowledge serves as a paradigm of knowledge itself, a paradigm through which people may come to grasp the nature of knowledge and to realize the criteria that must be applied to distinguish genuine knowledge from mere belief. Cf., e.g. *Republic*, Book 7, 525a–527c and Vlastos 1988, 138.

Especially important for appreciating Aristotle's logic is acquaintance with dialectic, with the Socratic method of hypotheses and with the method of analysis as applied in geometry. Cf. Hintikka and Remes 1974 and Corcoran 1979. Perhaps the most important fact to bear in mind is that an axiomatically organized geometry text was in use in the Academy when Aristotle was a student. Cf. Heath 1926, i, 116. This geometry text probably resembled the book by Euclid that came to replace it. Cf. Knorr 1975, 7, 22.

Two points of resemblance are especially relevant. Firstly, two kinds of proof were used. On the one hand there were direct proofs which erased doubt and established knowledge of their conclusions by so-to-speak building up the conclusion from material already established. On the other hand there were indirect proofs which erased doubt and established their conclusions by so-to-speak first inviting the doubt to be openly embraced for purposes of reasoning and then showing that such embrace was in conflict with already established results.

The second point of resemblance is the orientation toward the ontic as opposed to the epistemic; this is not to impose a dichotomy or to suggest that in actual practice one orientation excludes attending to the other. Rather one should say that in a typical axiomatization of a science there is an orientation toward the ontology of the science, toward the class of objects comprising the subject-matter or genus of the science, rather than on the process or processes of knowledge being employed. Euclid, e.g., sets forth the basic axioms and definitions first and then, without saying anything about the processes of deduction to be used, he proceeds to the extended elaboration of one chain of reasoning after the other.

This pattern of articulating the ontic while leaving the epistemic in an unarticulated and tacit state is repeated in axiomatization after axiomatization even up to the present day. For more on this point, including a remark by Alonzo Church, see Corcoran 1973, especially pages 24–29. People are sometimes surprised to learn that Hilbert's famous 1899 axiomatization of geometry, though carefully stating the axioms in clear natural language (not in a symbolic language), proceeds to deduce consequence after consequence without any discussion of the methods used. In other words, the geometrical axioms are presented but the rules of deduction from the underlying logic are left tacit.

Church 1956, 27 speaks of *the informal axiomatic method* when the underlying system of deductions is left tacit as in Euclid's *Elements* and Hilbert's 1899 *Grundlagen*; he contrasts it with what he calls the *formal axiomatic method* wherein the deductions themselves are formally analyzed and made explicit. Using Church's terminology one can say that Łukasiewicz and Miller are on firm ground when they imply that Aristotle did not use the *formal* axiomatic method. See e.g. Łukasiewicz 1929, 106. They think that Aristotle's syllogistic is an application of the *informal* axiomatic method and thus, in particular, that Aristotle does not articulate the rules of deduction of his syllogistic system. According to the Smiley-Corcoran view, Aristotle did not employ the formal axiomatic method in the syllogistic because he did not employ any axiomatic method there. However, in *Prior Analytics* Aristotle does indeed present a fully explicit and self-contained system of deductions with meticulously described rules of deduction and with what amounts to a definition of a complete set of formal deductions including direct deductions and indirect deductions. Thus, according to the Smiley-Corcoran view, although Aristotle does not apply any axiomatic method (whether formal or informal) in *Prior Analytics* he does make the essential first step beyond the informal axiomatic method toward the formal axiomatic method by clearly indicating the possibility of articulating the means of deduction.

Intuition and Deduction: Two Epistemic Processes

In an axiomatization of a science the information is *concentrated* in the propositions explicitly stated as axioms and this information is extracted from the axiom set and amplified, often in surprising ways, in the course of the development of the series of theorems. The theorems are all contained in the axiom set, as is shown by the chains of reasoning that extract them. The information processing techniques, which are typically not explicit, are not regarded as part of the axiomatization but rather as rules of deduction in the underlying logic presupposed by the axiomatization in question *and* presupposed by other axiomatizations of the same science or of different sciences. The axiom set is proper to the particular science under study but the underlying logic is topical neutral.

In order for a proposition to be an axiom it is not sufficient for it to be true; it must be known to be true by the scientist. In order for an information processing technique to be a rule of deduction it is not sufficient for it to be logically sound; it must be cogent or logically epistemic for the persons who use the underlying logic. Compare Quine 1986, pages 49, 83, and 98. To say that an information processing technique, or information transformation, is logically sound is to say that the result of applying the process has no information not already in the information that it is applied to, i.e., that the information or the resultant or conclusion is contained in its raw material of premises. Put another way, a logically sound process produces from a given set of premises conclusions that are logically implied by that set. A conclusion is logically implied by a given premise-set if it is logically impossible for the conclusion to be false were the premises all true, i.e. if the negation of the conclusion contradicts the premise-set. But not every logically sound information process is a rule of deduction of an underlying logic; a rule of deduction has the appropriate evidentiality or obviousness, it must be usable by persons to produce knowledge that a conclusion is implied by given premises. For more on this point see Weaver 1988.

The epistemically effective information processing relevant to the axiomatic method, and to the hypothetical method used by Socrates and to the analytic method used in geometry, serves to extract information implicit in the propositions to which it is applied, in other words, to produce logical consequences from premises, and to do it in a way that makes evident to those using the processes that the conclusions indeed follow. These epistemic processes have come to be known collectively as *deduction*. The process of establishing the axioms to begin with has been called *intuition* recently; an earlier term still in use this way is *induction* See Hintikka 1980 and Corcoran 1982. All three words are ambiguous.

In the hypothetical method, deduction is used to determine consequences of a hypothesis which is rejected as false when one of its consequences is determined to be false. The process of deduction, being part of the underlying logic, merely determines implied consequences of propositions. Deduction applies to information content *per se*; it is not limited merely to true propositions and certainly not merely to propositions already known to be true.

To say that deduction is truth-preserving is an understatement at best; it is usually an insensitive and misleading half-truth; and it often betrays ignorance of logical insights already achieved by Aristotle. See Myhill 1960, 461–463. Deduction is not merely truth-preserving, it is information-conservative, i.e. consequence-conservative in the sense that every consequence of a proposition deduced from a given set of propositions is already a consequence of the given set; there is no information in the deduced proposition not already in the set of propositions from which it was deduced. Not every truth-preserving transformation is consequence-conservative, but, of course, every consequence-conservative transformation is truth-preserving. For example, the rule of mathematical induction is truth-preserving but not consequence-conservative. See the Appendix below.

The epistemic status of knowledge of axioms contrasts sharply with that of knowledge of deduction. The former is propositional knowledge (or 'know-that') whereas the latter is operational knowledge (or 'know-how'); it is an epistemic skill. To avoid confusion we should note that intuition and deduction are both operational knowledge; the one productive of propositional knowledge, the other productive of implicational knowledge, i.e. knowledge that a given proposition is logically implied by a given set of propositions.

The initial-versus-derivative distinction applies both to propositional knowledge in the narrow sense and to implicational knowledge, i.e. to results of pure deduction. There are clear senses in which in an axiomatic science the axioms are *initial* and the theorems are *derivative*. When we turn to the processes by which the theorems are deduced from the axioms we see the step-by-step reasoning reported in what are called *deductions*; a deduction is a discourse or argumentation composed of a premise-set, a conclusion, and a chain of reasoning that makes evident that the conclusion follows logically from the premise-set. These chains of reasoning also break down into *initial* and *derivative*. The initial chains of reasoning are those which are composed of a single link, so to speak: these are the chains in which the conclusion is deduced immediately from the premise-set. The derivative chains are constructed by concatenating the initial chains. Just as axioms are epistemically fundamental in the realm of propositional knowledge, the initial chains of reasoning are fundamental in the realm of deduction. The proximate product of the deductive process is the chain of reasoning that forms the core of the discourse or argumentation that we call a deduction. There are many interesting analogies between the realm of propositional knowledge and other realms of knowledge. Here we have mentioned only one: an axiom is to a theorem as an immediate (one-link) chain of reasoning is to a mediated (multi-link) chain of reasoning. For further discussion see Corcoran 1989. Aristotle was fully aware of both of the above applications of the initial-versus-derivative distinction: that in the realm of known propositions and that in the realm of epistemically effective chains of reasoning.

The initial-versus-derivative distinction in the realm of known propositions is implicit in Aristotle's view that every proposition known to be true is *either* known by 'induction' *or* deduced by a chain of immediate inferences whose ultimate premises are known by induction. This of course is closely related to Aristotle's *truth-and-consequence conception of proof*, viz. that demonstrating the truth of a proposition is accomplished by showing that it is a *logical consequence* of propositions already known to be *true*. The initial-versus-derivative distinction in the realm of cogent argumentations is exemplified in Aristotle's theory of the completing of syllogism: an incomplete (or 'imperfect') syllogism is completed ('perfected') by chaining together simple syllogisms that are already complete in themselves ('perfect').

It should be noticed that the initial-versus-derivative distinction applies in many other places as well; it would be a mistake to think that it is limited to axiomatic sciences and systems of deductions. The ordinary recursive grammars used by modern symbolic logicians apply this distinction to formalized languages: the 'atomic' formulas are initial and the 'molecular' formulas are derivative. In fact, the initial-versus-derivative distinction is applied in several ways in the context of a formalized language. In Corcoran 1976, several kinds of recursive (or generative) grammars are discussed: term grammars that generate derivative terms from initial terms, sentential grammars that generate derivative sentences from initial sentences, deduction grammars that generate derivative deductive discourses (designed to express chains of reasoning) from initial deductive discourses, and 'theorem grammars' (or axiomatizations) that generate propositions (theorems) deductively known from propositions (axioms) known initially without use of deduction.

Every axiomatic system involves an initial-versus-derivative structure. But, as is almost too obvious to mention, not every system involving an initial-versus-derivative structure is axiomatic. Thus detection of an initial-versus-derivative structure in a certain text is no sign that the text contains an axiomatic system. In particular, the fact that the Aristotelian syllogistic system has an initial-versus-derivative structure with the 'perfect' syllogisms as initial does not imply that the syllogistic is an axiomatic system. Never once does Aristotle apply to a perfect syllogism one of the terms that he characteristically uses elsewhere for axiom *per se*.

There are several ways of misconstruing Aristotle's syllogistic as an axiomatic system; Łukasiewicz 1951 is representative of one type of misconstrual while Parry and Hacker 1991 represents an entirely different type of misconstrual. The 'perfect' syllogisms *do* have a certain priority, they *are* fundamental, they *are* basic, the 'imperfect' syllogisms *are* known through the 'perfect' ones. Nevertheless, the 'perfect' syllogisms are *not* axioms for Aristotle and the syllogistic is not an axiomatic system.

Logic as Formal Ontology

There are several different conceptions of the nature of logic. Here I want to contrast an ontic conception with an epistemic conception. On one ontic conception logic investigates certain general aspects of 'reality', of 'being as such', in itself and without regard to how (or even whether) it may be known by thinking agents: in this connection logic has been called *formal ontology*. On one epistemic conception, logic amounts to an investigation of deductive reasoning *per se* without regard to what it is reasoning about; it investigates what has been called *formal reasoning*. On this view, logic is part of epistemology, viz. the part that studies the operational knowledge known as deduction. It has been said that one of the main goals of epistemically-oriented logic is to explicate the expression 'by logical reasoning' as it occurs in sentences such as:

a deduction shows how its conclusion can be obtained by logical reasoning from its premise-set.

Relevant to the axiomatic method there would be two branches of epistemology: one to account for knowledge of the axioms and one to account for how knowledge of the theorems is obtained from knowledge of the axioms, in other words, one investigating induction and one investigating deduction. The latter is logic according to the epistemic conception.

On the ontic view of logic, on the other hand, logic is an attempt to gain knowledge of the truth of propositions expressible using only generic nouns (*individual*, *property*, *relation*, etc.) and other 'logical' expressions. In the framework of *Principia Mathematica* those are propositions expressible using only variables and logical constants. *Principia Mathematica* is an excellent example of an axiomatic presentation of logic as formal ontology. Below are some typical laws of formal ontology.

Excluded middle: Given any individual and any property either the property belongs to the individual or the property does not belong to the individual.

Noncontradiction: Given any individual and any property it is not the case that the property both belongs to the individual and does not belong to the individual.

Identity: Given any individual and any property, if the property belongs to the individual then the individual has the property.

Dictum de omni: Every property A belonging to everything having a given property B which in turn belongs to everything having another property C likewise belongs to everything having that other property C.

Dictum de nullo: Every property A belonging to nothing having a given property B which in turn belongs to everything having another property C likewise belongs to nothing having that other property C.

Commutation of Complementation with Conversion: Given any relation R the complement of the converse of R is the converse of the complement of R.

From this sample of logic as ontic science we can see how the focus is on ontology, or, as has been said by others, on the most general features of reality itself and not on methods of gaining knowledge. According to Russell 1919, 169, 'logic is concerned with the real world just as truly as zoology, though with its more abstract and general features.' These six laws are purely ontic in that they involve no concepts concerning a knowing agent or concerning an epistemic faculty such as perception, judgement, or deduction. This is not to deny that there is an epistemic dimension to logic as ontic science but only to affirm that the focus if ontic. Every science in so far as it is science has an epistemic dimension. The epistemic differs from the ontic more as size differs from shape than as, say, animal differs from plant.

Logic as ontic science was referred to above as *formal ontology*. Logic as epistemic metascience may in like manner be called *formal epistemology*. It is important and interesting to note that both are called *formal logic* but for very different reasons. Some formal onticists justify the adjective *formal* by reference to the fact that its propositions are expressed exclusively in general logical terms without the use of names denoting particular objects, particular properties, etc. cf. Russell 1919, 197. Some formal epistemicists justify the adjective *formal* by reference to the fact that the cogency of an argumentation is subject to a principle of form and in particular to the following principles: (1) every two argumentations in the same form are either both cogent or both noncogent, (2) every argumentation in the same form as a deduction is itself a deduction. In fact, some formal epistemicists such as Boole claimed, with some justification, that they were dealing with the forms of thought, i.e. with the forms of cogent argumentations. For more on cogency of argumentations and the principles of form see Corcoran 1989.

Formal onticists are often easy to recognize because of their tendency to emphasize the fact that formal ontology does not study reasoning *per se*. In fact, the formal onticists often think that the study of reasoning belongs to psychology and not to logic. For example, Łukasiewicz in his famous book on Aristotle's syllogistic makes the following two revealing remarks. Łukasiewicz 1957 pages 12 and 73, respectively. 'Logic has no more to do with thinking than mathematics.' '[Aristotle's] system is not a theory of the forms of thought nor is it dependent on psychology; it is similar to a mathematical theory...'

There are significant differences among formal onticists. For example, even among those that emphasize the truth-preserving character of deduction some accept the view that it is consequences-conservative as well and some reject this view. For example, Łukasiewicz 1929, 16 explicitly rejects the view that deduction is a process of information extraction. He says that in deductive inference '...we may obtain quite new results, not contained in the premises'.

Conclusion

The tendency of interpreters to find an epistemically-oriented theory in Aristotle has been overwhelming. With the exception of James Wilkinson Miller's 1938 book and the writings of Jan Łukasiewicz and those directly influenced by these two, few interpreters have found a theory of formal ontology in Aristotle's *Prior Analytics*. Down through the ages, with these exceptions, interpreters have agreed that *Prior Analytics* is about methods of determining validity and invalidity of arguments. People studied *Prior Analytics* in order to learn more about deductive reasoning and in order to improve their own reasoning skills.

Despite the overwhelming tendency to interpret the syllogistic epistemically it wasn't until the early 1970s that it occurred to anyone to wonder whether

Aristotle had a developed theory of deductive reasoning with a well worked-out system of deductions comparable in rigor and precision with the systems then familiar from mathematical logic. Of the logicians that studied *Prior Analytics* from this point of view, two of them published articles in same twelve-month period with remarkably similar systems affirming in clear and unequivocal terms the epistemic nature of *Prior Analytics*: Corcoran 1972 and Smiley 1973.

The simpler of the two articles holds that Aristotle's theory of deductions recognizes two kinds of extended deductions of conclusions from arbitrarily large premise sets: direct deductions and indirect deductions. A direct deduction of a conclusion from given premises begins with the premises and proceeds by chaining together simple one-premise and two-premise inferences until the conclusion is reached. An indirect deduction of a given conclusion from given premises is in effect a direct deduction of a pair of contradictory opposites from the premises augmented by the contradictory opposite of the conclusion. This view is spelled out in more detail in the introduction to Smith's 1989 translation of Aristotle's *Prior Analytics*.

According to the ontic interpretation the syllogistic is a system of true propositions about inclusional relations among classes. It is a system which is organized deductively, axioms followed by deduced theorems, by employment of an underlying logic never explicitly mentioned by Aristotle. It is a system whose place in the *Organon*, in Greek philosophy, and in the history of philosophy raises many problems. When we turn to the epistemic interpretation the changes are dramatic. From the epistemic perspective the syllogistic is a system of deductions or chains-of-reasoning. It is organized according to an initial-versus-derivative structure with the derivative components as chainings of initial components. It is a system which can be seen to explain epistemic processes of deduction presupposed by the Socratic hypothetical method, by the so-called method of analysis, by the axiomatic method and even by dialectic itself. According to the epistemic interpretation, the focus of the syllogistic is on methods as opposed to results; it concerns the process of deduction rather than conclusions *per se*. One might say that it concerns how to think rather than what to think. And it is a step toward understanding the nature of proof as opposed to persuasion and toward fulfilling the demand made by Socrates in the *Phaedo* for a *technê logikê*. This step made by Aristotle was so firm, so detailed, and so well-developed that it warrants the title of THE FOUNDING OF LOGIC.

Appendix

The word *conservative* is used in connection with political thinking in several senses, two of which are relevant here. On the one hand, a person's thinking is called conservative to the extent that he aims at preserving and protecting what he takes to be traditional principles and values. On the other hand, a person's thinking is called conservative to the extent that he aims at preventing new principles and values from being realized. The word *conservative* in the expressions *information-conservative* and *consequence-conservative* is used in analogy with the second, negative or privative sense. In an application of an information-conservative or consequence-conservative process the resultant contains no information not in the premises processed; such a process 'prevents' new information from intruding.

Uses of hyphenated, adjectival expressions ending with 'conserving', 'conservative', 'preserving', and 'preservative' seem to be intrinsically confusing. Use of such expressions without explanation and warning is misleading. For example, 'information-conserving' suggests the bizarre condition that the resultant contain all of the information contained in the datum, exactly the reverse of what is wanted. 'Every rectangle is a parallelogram and every square is a rectangle' contains all of the information contained in its consequence, 'Every square is a parallelogram', *not* the other way around.

We can say that a given process is *contained in* or *constrained by* a given relation if in every application the premises or raw material is in the given relation to the resultant. A truth-preserving process is constrained by material implication but not by logical implication, whereas a consequence-conservative process is constrained by logical implication (and thus also *a fortiori* by material implication).

To see that not every truth-preserving process is consequence-conservative it is sufficient to consider the rule of mathematical induction which, for example, when applied to the two propositions 'zero is even' and 'every natural number which is the successor of an even natural number is even' results in 'every natural number is even.' This resultant is materially implied by the given premise-set since the second premise is false and, of course, every proposition is materially implied by every set of propositions having a false member. On the other hand, the resultant is not logically implied by the given premise-set. To see this use the method of counterarguments: 'zero is integral' and 'every real number which is the successor of an integral real number is integral' are both true whereas 'every real number is integral' is false, of course; one-half is not integral, for example.

The rule of mathematical induction, which is applicable only to propositions about the natural numbers (or, what amounts to the same thing, to sentences interpreted appropriately in the universe of natural numbers), is a rule of *inference* in Frege's sense but, as we have seen, it is not a rule of *deduction* in the information-extracting sense usual in logic. There are rules of *derivation*, let us say, which are truth-preserving but which are not even rules of inference. One extreme case is what I call *the rule of truth-deriving*: from an arbitrary set of propositions derive an arbitrary true proposition. This example shows that the property of being truth-preserving could not merit the emphasis that it has gotten.

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