



What Is Mathematical Logic?

Review Author[s]:
John Corcoran; Stewart Shapiro

Philosophy of Science, Vol. 43, No. 2 (Jun., 1976), 301-302.

Stable URL:
<http://links.jstor.org/sici?sici=0031-8248%28197606%2943%3A2%3C301%3AWIML%3E2.0.CO%3B2-A>

Philosophy of Science is currently published by The University of Chicago Press.

Your use of the JSTOR archive indicates your acceptance of JSTOR's Terms and Conditions of Use, available at <http://www.jstor.org/about/terms.html>. JSTOR's Terms and Conditions of Use provides, in part, that unless you have obtained prior permission, you may not download an entire issue of a journal or multiple copies of articles, and you may use content in the JSTOR archive only for your personal, non-commercial use.

Please contact the publisher regarding any further use of this work. Publisher contact information may be obtained at <http://www.jstor.org/journals/ucpress.html>.

Each copy of any part of a JSTOR transmission must contain the same copyright notice that appears on the screen or printed page of such transmission.

JSTOR is an independent not-for-profit organization dedicated to creating and preserving a digital archive of scholarly journals. For more information regarding JSTOR, please contact support@jstor.org.

centaurs—an illustration credited to R. J. Richman). Further difficulties, concerning types and tokens and kinds of ambiguities that Scheffler himself introduces, require additional adjustments. These are all, it seems, controlled by Goodman's own fundamental suggestion—roughly, "that the difference in meaning between two words is a matter of either their own difference in extension or that of any of their parallel compounds" (p. 259). So that, for instance, although "there are neither centaurs nor unicorns, there certainly are centaur-pictures and unicorn-pictures and, moreover, they are different" (p. 259). In fact, Goodman apparently holds that "'description' constitutes a suffix capable of yielding all the wanted distinctions for every pair of words P and Q ," applied to any actual inscription of the form "a P that is not a Q " or "a Q that is not a P " (p. 259). The question arises whether ambiguities can be detected in some interesting sense by such an addition that cannot be detected by the primary expressions themselves. Scheffler is particularly careful to demonstrate, against Rudner for instance, that Goodman's claim that "no two words [types] have the same meaning" (p. 258) need not, under the pressure of the kinds of ambiguity that need to be resolved, lead to "the stronger conclusion that no two tokens have the same meaning under any circumstances" (p. 260). The essay repays close attention. *Joseph Margolis, Temple University.*

JOHN N. CROSSLEY *et al.* *What is Mathematical Logic?* London, Oxford and New York: Oxford University Press, 1972. iii + 82 pp. \$1.95.

The authors of this little book attempt to "introduce the very important ideas in modern mathematical logic without the detailed mathematical work which is required of those with a professional interest in logic." The intended audience includes "those who have no mathematical training." It consists in six short chapters: Historical Survey, Completeness of Predicate Calculus, Model Theory, Turing Machines and Recursive Functions, Gödel's Incompleteness Theorems, Set Theory. As indicated by several reviews in English, French and German, an expert in logic can read the book through and be left with the impression that the authors have succeeded in their aim.

The book, together with its reception in the logic community is a remarkable, and probably rare, event. Its main author, John Crossley, is a logician of high reputation and its publishing house is perhaps the most distinguished of the world. It has been reviewed in exceptionally favorable terms by several logicians including Largeault, deRijk, Lightstone, Kneebone, A. K. Austin, Kilmister and Tichy. In these reviews only four misprints are noted and one review says that the book is remarkably free from printing errors.

But a restrictive criterion for counting printing errors turns up at least twenty-five, not counting several places where definitions are given without italicizing the defined term, not counting several places where terms are used without definition (or before their definitions appear) and not counting fluctuating terminology. Moreover, every chapter of the book reveals mistakes which can be accounted for by assuming that the authors were not thinking through what they were saying. In Chapter 1 there is a strangely undecipherable flow chart of the history of logic which includes the names Boole, Bolzano and Löwenheim while omitting Peano, Zermelo, Hilbert, Post, Gentzen, Tarski and Church. Boole's work is said to have been "extended by Frege" and the first-order predicate calculus is said to have been used "for making logical deductions" up until 1920 without the knowledge of its completeness (when in fact it had not been formulated as a separate system until two years before Gödel proved its completeness). In Chapter 2 an example is given which is claimed to include three sentences true in a certain interpretation; but one is not a sentence and one is false. Moreover, a completeness proof is sketched for a logic which is not complete (even after a typographical error resulting in an invalid axiom scheme is corrected). [The mathematical mistake can be seen as the result of a pedagogical mistake, *viz.* using one set of primitives and giving the axioms in another.]

A similar pedagogical mistake in Chapter 3, *viz.* writing the axioms of discrete

linear order using prefix notation, ' $P(x, y)$ ' instead of the usual infix notation ' $x < y$ ', lead the authors to another mathematical mistake—the last two axioms are wrong. The discussion of the "Skolem paradox" is garbled; some of the discussion only makes sense if one is talking about "Henkin models," some of it makes sense only if one is talking about countable elementary submodels of the "intended interpretation" of set theory and some of it does not seem to make sense at all. Chapter 4, "Turing Machines and Recursive Functions," does not mention by name Church or Church's Thesis although, of course, Church's Thesis is appealed to throughout. In Chapter 5, the discussion of the proof of the incompleteness theorem is so garbled that it is virtually useless. For example, where the authors want to consider the formula which says that x is the Gödel number of a proof of the result of substituting the numeral for y in the formula whose Gödel number is y [viz. $Pf+(x, y, y)$], what they do say is that they want to consider the formula which says "that x is the Gödel number of a proof of the formula that you get when you put for its free variable the numeral for its own Gödel number." In the same paragraph, the letter G is defined to denote a certain formula having y free and then it is used as if it denoted the result of replacing the occurrences of y by the Gödel number of G . In Chapter 6, the set theory presented is inconsistent. One contradiction slips in through the statement of the comprehension scheme which omits the restriction on the variables allowed free in the formula for the separating property. Of course, this is excuseable—but in stating the scheme the authors add "... there may be other free variables as well but they do not bother us." The discussion of the axiom of infinity seems to imply that that axiom is needed to prove the existence of the null set—which can be got using the comprehension scheme (presented earlier). The discussion of the axiom of choice tells the reader that the axiom of choice is doubted because the other axioms "give us concrete constructions for sets." (Does the comprehension scheme give concrete constructions? How about the power-set axiom?)

Aside from the charge of twenty-five printing errors, all of the above charges can be easily verified by reading the relevant chapters. The reviewers also have criticisms of a more subjective nature, viz. that the book is poorly written and that its intended audience could not begin to follow it. The project of introducing the material of this book in less than 100 pages to readers with no mathematical training is impossible and the authors have understandably failed. But the above criticisms indicate that they failed in a way not attributable to the nature of their project—there is not a mistake in the book that the authors could not have noticed themselves. It is equally true that the previous reviewers could have noticed them.

Incompetent performance by competent people and its subsequent whitewashing by other competent people has become common in other areas of our culture. Are we witnessing the growth of the infection into logic? *John Corcoran and Stewart Shapiro, SUNY at Buffalo.*

R. C. LEWONTIN. *The genetic basis of evolutionary change*. New York: Columbia University Press, 1974. 346 pp. \$5.75.

This is a fascinating book. Although written by a biologist for biologists, I hope I can give some idea of why I, a philosopher, found it exciting, and thus persuade other philosophers of science that it ought to be on their reading lists. Because my interest is philosophical rather than scientific, my review will be a bit one-sided; but I hope I can give a fair flavor of the book nevertheless.

From the moment that it first appeared in the *Origin of Species*, evolutionary theory through natural selection has been at the center of controversy. Admittedly many of Darwin's critics based their attacks, with some good reason, on religion—a source which, for better or for worse, most of us no longer find very worrisome. But some critics, most notably the Cambridge mathematician and geologist William Hopkins, struck a vein which is still being mined today. They argued that judged by the canons of the best kind of science, in particular judged by Newtonian astronomy, evolutionary