

On the Copernican Turn in Semantics*

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Abstract: Alberto Coffa used the phrase ‘the Copernican turn in semantics’ to denote a revolutionary transformation of philosophical views about the connection between the meanings of words and the acceptability of sentences and arguments containing those words. According to the new conception resulting from the Copernican turn, here called ‘the Copernican view’, rules of use are constitutive of the meanings of words. This view has been linked with two doctrines: (A) the instances of meaning-constitutive rules are analytically and *a priori* true or valid; (B) to grasp a meaning is to accept its rules. The pros and cons of different versions of the Copernican view, ascribable to Wittgenstein, Carnap, Gentzen, Dummett, Prawitz, Boghossian and other authors will be weighed. A new version will be proposed, which does neither imply (A) nor (B).

1. The chain and the links.

In honest discussions, where both sides seek truth, we are sometimes forced by an argument to change our views even when we have grown fond of them. What kind of force is constraining us? Constraint is suggested by the Cartesian comparison of deduction to a chain. The chain has many links: ‘we survey the links one after the other, and keep in mind that each link from first to last is attached to its neighbour’.

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The binding force of the deductive chain depends on memory and on the binding force of individual links. For Descartes (1985: 14-5) the force of a link is perceived through ‘mental intuition’: ‘the conception of a clear and attentive mind, which is so easy and distinct that there can be no room for doubt about what we are understanding’.

Many philosophers today reject immediate self-evidence and infallible certainty. They reject Cartesian intuition. But most still think that the metaphor of reasoning as a chain is apt: an argument advanced in a discussion is a *concatenation* of individual steps. Steps may be challenged. Challenges may be countered by a justification. Justification is a further argument. Thus a regress is started, which terminates when the disputants are driven to a point where no further (non-circular) justification seems possible. Usually, however, the disputants do not feel uncomfortable. They do not feel that a justification is needed.

Let us call ‘**epistemic uses of an expression *E***’ the linguistic acts by which we perform or endorse an assertion or inference containing *E*. We can distinguish those epistemic uses which are primitive for a speaker from those that are not.

A use *U* of *E* is a **primitive use of *E* for *S***, *if, and only if*, *S* expects of every competent speaker *P* that

- a*) in using *E*, *P* accepts *U* and neither envisages the possibility of a (non-circular) justification of *U*, nor acknowledges the need to give such a justification;
- b*) *P* is convinced that whoever understands and uses *E* treats *U* in the way specified in (*a*).

The compelling force of an argument depends on the force of primitive uses. Primitive uses for S seem obvious to S, but they are a problem for the philosopher who wonders about the force of arguments. Epistemic uses which are not primitive can be justified, if challenged. Justifications explain why we accept them. But why do we accept primitive uses?

Cartesian intuition is one possible answer: we accept primitive uses because we have some rational insight into a self-evident truth or the self-evident validity of an inference. Bertrand Russell (1912: 111) explains our acceptance of those statements or inferences for which ‘we cannot find any further reason’ by crediting us with intuitive knowledge. Intuitive knowledge is immediate knowledge of self-evident truths or inferences. Examples of the ‘highest degree’ of self-evidence are some general *a priori* principles. These are intuitively known by directly perceiving a relation between universals. According to Russell, we grasp the validity of such principles through our immediate awareness of the fact that a relation obtains between universals. Direct awareness ‘without the intermediary of any process of inference or any knowledge of truths’ is called ‘acquaintance’ by Russell. The general truths of logic or mathematics enjoying the highest degree of self-evidence are based on our *acquaintance with universals*. To be acquainted with a universal is to grasp a concept (52).

2. Concepts first.

Russell’s picture embodies a straightforward view: *our grasp of concepts always comes first; inferences, judgments, rules and our knowledge of their validity*

always come later. As far as language is concerned, meanings come first, linguistic uses come later. I shall call this view *the concepts-first view*. For Russell, we first give meanings to ‘two’ ‘four’ and ‘plus’ by associating those words with the corresponding concepts. On the basis of the way the words are combined, we then see that the statement ‘two plus two is four’ means that a certain fact concerning those concepts obtains. Being acquainted with the asserted fact, we can finally perform the epistemic act of intuitive knowledge: immediately establishing *a priori* the truth of the statement.

Many philosophers have advocated the concepts-first view. Laurence Bonjour (1998: 102) contends that we accept elementary logical inferences and arithmetical sentences on the basis of a ‘rational insight’ which is not very different from Russell’s intuitive knowledge. Bonjour thinks that we come to accept the sentence ‘nothing can be red all over and green all over at the same time’ because the ‘metaphysically independent properties or universals’ (181) ‘redness and greenness are themselves before the mind in a way that allows their natures and mutual incompatibility to be apparent’ (162); thus, ‘the necessity of such a proposition is seen or grasped or apprehended as an act of rational insight or rational intuition’ (102).

The concepts-first view faces serious difficulties. In works written in around 1930, Ludwig Wittgenstein deals with rules whose instances are primitive uses (which we shall henceforth call ‘primitive rules’). Regarding the primitive rule that ‘two negations yield an affirmation’, an advocate of the concepts-first view would

think that such a rule could be discovered by examining ‘what “lies behind” the word “not”’ (1969: §14). Those who favour this idea speak of understanding

as if understanding were an instantaneous grasping of something from which later we only draw consequences which already exist in an ideal sense before they are drawn (§18),

because

“that two negations yield an affirmation must already be contained in the negation that I am using now”,

the rules [...] follow from the nature of negation. So that in a certain sense there is first of all negation, and then the rules of grammar (§15).

Many problems undermine this line of thought. *First:* what kind of entity is the concept or meaning which already subsists before all judgments and rules? *Second:* what is it to grasp a concept independently of accepting judgments or inferences involving it? *Third:* in what sense does a rule of use ‘follow’ from such a concept or meaning? What kind of fact is, for example, the fact that (the concept of) negation ‘contains’ the rule of double negation elimination? *Fourth:* how do we come to know such a fact?

The epistemic act by which one derives a primitive rule from the antecedently given concept is mysterious. The concept (whatever it is) does not yield the rule in the same sense in which ‘carbon and oxygen yield carbonic acid’ (§14). It cannot be merely a causal relation. The main reason is that, according to the concepts-first view, a subject acquires *a priori* knowledge of the truth or validity of all recognizable

instances of the rule. An empirical investigation might perhaps show that subjects who satisfy certain independent empirically specified conditions of concept-possession (e. g. for negation) under suitable circumstances come to accept certain judgments or inferences (e. g. from not-not- A to A). This empirical fact, however, would not settle the issue because the problem would remain: how can the causal process through which concept-possession generates acceptance of a rule make the rule *valid* and turn acceptance into *a priori knowledge* of validity? Advocates of the concepts-first view do not think that a primitive use (e. g. a double negation elimination) is merely accepted. They think that it is *a priori* known to be (certainly) valid. Since causal psychological processes often lead us into errors, we need more than a causal connection between two psychological attitudes in order to have a special faculty of knowledge, leading us to truth. Even if the subject knew of a causal psychological connection, this would be only a piece of empirical knowledge. Empirical knowledge does not explain, nor justify, the subject's treating the connection between concept and primitive use as a necessary, immediate, non-empirical connection which is *essential* to the concept.

On the other hand, the person who comes to accept primitive rules cannot derive them from antecedent meanings *by means of logic*, because the rules in question include precisely the most primitive logical rules. Today, in a metalanguage in which we develop a semantics for a language L we deduce from axioms stating the meanings of the logical constants of L that certain logical laws are valid in L . We can do so by exploiting corresponding logical laws in the metalanguage. As Michael

Dummett (1973: 296-7) argues, this strategy is fully legitimate if our aim is to explain *why* already accepted logical laws are valid and not to persuade a person who does not accept those laws. But we are now dealing with the different point of view of someone who has to persuade himself or herself *that* the basic logical laws are valid. According to the advocates of the concepts-first view, such a subject is at an epistemic stage, so to say, *before* logic. They contend that the subject grasps the concept of negation before accepting the logical rules concerning negation. They cannot credit this subject with the use of logic in the process through which he or she *comes to accept* the basic rules of logic. On the contrary, they believe there to be a non-empirical and non-logical sense in which a primitive logical rule is known to be valid (and thus accepted) by drawing its validity from the meanings of the relevant logical words. Knowledge of the validity of double negation elimination is thought to be non-empirically and non-logically derived from the meaning of negation. Such a non-empirical and non-logical faculty is called ‘insight’ or ‘intuition’. But the name does not make it any less obscure. They can say that we immediately know the validity of the rule because it is evident. ‘Evident’, however, is only a way to hide our inability to explain it.

3. The Copernican turn.

In the 1930s, Wittgenstein proposes an alternative view. He rejects the idea that concepts or meanings subsist independently of primitive rules. The rule of double negation elimination does not follow from an already formed concept of negation, but

‘constitutes negation’ (1969: §14). A general formulation of the idea can be found in *Remarks on the Foundations of Mathematics*:

We can conceive the rules of inference – I want to say – as giving the signs their meaning, because they are rules for the use of these signs.
(Wittgenstein 1956: VII, §30)

This move revolutionizes the relation between primitive epistemic uses and meanings. Alberto Coffa (1991: 267) called it ‘the Copernican turn in semantics’.

The semantic explanatory route does not go from [...] “objects” or meanings to the laws concerning them and then to our reasonable linguistic behaviour, but the other way around, from our behaviour to meanings. The ultimate explanatory level in semantics is not given by reference to [...] objects or meanings, but by reference to the meaning-giving activity of human beings, an activity embodied in their endorsement of rules.

The traditional view is: concepts come first, rules come later. The Copernican view is: rules and concepts come *together*, at the same time. An anticipation of this view can be seen in Kant. In the *Critique of Pure Reason* (A 126), the faculty of understanding is defined as ‘the faculty of concepts’ and as ‘the faculty of rules’. Kant says that if ‘adequately understood’ the two definitions are equivalent even if the second ‘is more fruitful, and approximates more closely to its essential nature’.

A less Kantian formulation of the Copernican turn is in terms of primitive uses. The traditional view is: meanings come first, all epistemic uses come later. The Copernican view is: primitive epistemic uses and meanings come together. The

Copernican view offers an alternative explanation for the force of primitive uses. The reason why we accept primitive uses is not that we have some rational insight to the effect that they *follow* from the already independently fixed meanings of the involved words. We accept primitive uses simply because that *is part* of our understanding and accepting those words. Primitive uses do not follow from meaning. They are part of meaning.

Coffa traces the origin of the Copernican view back to two disputes about the axioms of geometry at around the turn of the nineteenth century. Russell maintained that the meanings of geometric indefinables like ‘straight line’ were given by intuition before the relevant terms were incorporated into the axioms. Poincaré objected that he was ‘thoroughly deprived of this intuition’ (1900: 75) and that there is nothing beyond what the axioms say. His view was that axioms determine the meanings of geometric primitives. A similar controversy arose between Frege and Hilbert after the publication of Hilbert’s *Grundlagen der Geometrie* in 1899. Both Hilbert and Poincaré considered the axioms ‘implicit definitions’ or ‘disguised definitions’ of geometric primitives. Coffa (1991: 134) comments: ‘meanings are constituted roughly in the way in which Kantians used to think that we constitute experience or its objects, through the employment of rules or maxims whose adoption is prior to and the source of the meanings in question’.

Hilbert and Poincaré presented a new conception of geometry, but they did not generalize it to language as a whole. A generalized and mature formulation of the Copernican view can be found only in the first half of the 1930s. The decisive

philosophical step was not only taken by Wittgenstein. According to Coffa, there were two protagonists. Carnap in his *Logical Syntax of Language* was the second promoter of the new idea:

let any postulates and any rules of inference be chosen arbitrarily; then this choice, whatever it may be, will determine what meaning is to be assigned to the fundamental logical symbols. (Carnap 1934: v).

A third name should be added: Gerhard Gentzen. In ‘Untersuchungen über das logische Schliessen’, published in 1934, Gentzen writes that the introduction rules of his natural deduction systems ‘constitute, as it were, the “definitions” of the symbols concerned’ (1934: 189), i.e. of logical connectives and quantifiers. If, for example, we tried to explain how we come to know the validity of Gentzen’s rule of conjunction introduction (according to which it is correct to infer ‘A and B’ from a proof of A and a proof of B) by saying that the rule ‘follows’ from an antecedently given concept of conjunction which is independent of the rule, the same difficulties described in the previous section would arise. Gentzen’s idea is that, on the contrary, the rule of conjunction introduction *constitutes* the meaning of ‘and’.

4. Constitutive rules.

Differences notwithstanding, the approaches of Carnap, Gentzen and Wittgenstein contain a common idea: the rules governing the use of a word *constitute* its meaning. This idea has been variously criticized over the last seventy years. Many

critics rightly complain that the thesis of meaning-constitutivity is not clearly formulated. I propose the following definition:

The rules belonging to a finite set R_W are constitutive of the meaning of a word W in a language L , **if, and only if,**

- 1) to understand W in L is to know (implicitly) the rules belonging to R_W ;
- 2) knowledge of each rule in R_W manifests itself in distinctions made between uses of W (and of related words): some are considered correct with respect to the rule, some incorrect, some neither.
- 3) the acts of a speaker X count as speech acts in which W is used in L only if X accepts the rules in R_W .

That we have to add ‘implicitly’ in (1) is one of the morals we can draw from Quine (1936). If ‘knowledge’ in (1) were meant as knowledge of sentences that explicitly state the rules, from which to deduce conclusions about particular uses, our explanation of what it is to understand a word would be led into a vicious circle or infinite regress.

Clause (2) makes explicit the normative ingredient in the notion of rule, which amounts to the distinction between correct, incorrect and neutral uses. If today is a clear day and Ted says ‘The sky is clear’, his assertion is correct and Kim ought to accept it as a correct description of the sky. If Ted asserts ‘The sky is black’, Kim can rightly criticize the assertion as incorrect and Ted ought to withdraw it. If Ted says ‘The sky is happy’, we cannot blame Kim for not endorsing the assertion, nor Ted for making it.

Clause (3) expresses the idea that rules are *constitutive* of the linguistic activity of performing speech acts in which W is used in L. To use a word W in a language L is not simply to emit sound waves. It is also to treat the sound waves (or ink spots) as a particular word in L, i.e. to associate them with certain rules that give to W the meaning it has in L. The particular way in which this idea is developed will settle the issue whether L must be considered an idiolect or a common language. An instructive paper by Kathrin Glüer and Peter Pagin (1999) shows that one should be very careful when defining constitutivity. If, for example, we took the doctrine of constitutive rules to state that a speaker S performs speech acts involving W in L only if S *acts in accordance* with the rules in R_W , the undesirable consequence would follow that no speech acts in which W is used in L would violate the rules in R_W and hence no speech act would be incorrect. But to *accept* a rule is not the same as to comply with it. Football players, during a game, may violate the rule that forbids them to handle the ball deliberately. Nevertheless they accept the rule, as shown by the fact that they (reluctantly) accept the penalty inflicted by the referee when a violation is discovered. One can say that the rule is constitutive of the game of football, even though it is sometimes violated by acts forming part of the game. A plausible thesis of meaning-constitutivity should conform to the *law of breach*: we can accept (constitutive) rules and violate them at the same time.

5. Knowing a rule.

The most problematic clause is the first: what does the implicit knowledge of rules consist in? The problem is highlighted by Wittgenstein's rule-following considerations. They show that we should abandon a widespread mythological view of rules. A rule corresponds to certain distinctions between particular spatio-temporally located acts: some are correct with respect to the rule, some are incorrect, some are neither. When we judge whether a particular act is correct with respect to a rule, we apply the rule. An *application* of a rule R can thus be thought of as a pair: the first element is a particular act and the second is a judgment (which may remain tacit) to the effect that the act is correct, incorrect or neutral. What is the relation obtaining between knowledge of the rule and its applications? The mythological view offers an answer which can be summarized by the *thesis of complete determination*:

after a finite process of training of a subject X relative to a rule R, the fact that X knows R is completely accomplished in such a way that the correctness (or incorrectness) with respect to such a fact of possible future acts in new unconsidered circumstances is determined *in advance*.

It is essential to the rule whether or not a particular act is correct with respect to it. Therefore, if a relation of determination obtains between an already accomplished fact that X knows rule R and the correctness or incorrectness of future actions with respect to R, it should be a non-contingent, non-empirical, non-causal relation. It should be a 'super-strong connection' (Wittgenstein 1953: §196) like the mechanism

of an ‘ideally rigid machine’ whose movements are ‘in a mysterious sense – already *present*’ (§§193-4). The fact that X knows R in this special sense is a ‘superlative fact’ (§192) which, though it is already fully accomplished, extends its non-physical power to infinitely many possible future acts, since the rule ‘traces the lines along which it is to be followed through the whole of space’ (§219). Is this notion of a superlative fact really intelligible? Starting from Wittgenstein’s considerations, Saul Kripke (1982), Crispin Wright (1980, 2001) and other commentators have substantiated perplexities that lead us to reject the notion of a superlative fact. To reject superlative facts is to reject the thesis of complete determination.

Rejection of complete determination, however, is not rejection of the notion of rule. Rather, the moral one can draw is that we should adopt a different, non-mythological conception of rules. Wittgenstein uses the word ‘mythology’ both in the *Philosophical Grammar* (§15) against the concepts-first view and in the *Philosophical Investigations* (§221) against the thesis of complete determination. The move from the mythology of rules to the non-mythological view resembles the Copernican turn. Indeed, it is a radical version of the Copernican turn. According to complete determination, complete knowledge of the rule comes first and its applications come later. According to the non-mythological view, grasp of the rule and its applications come at the same time. Each application contributes to the rule. In other words, a rule is plastic:

a rule is not an entity which is completely fixed before its applications, but it is continuously moulded and shaped by our treating certain particular acts as correct, incorrect or neutral.

After a finite process of training, a person, say Amy, feels sure that she has understood the rule R and can act according to it. In the most basic cases she does not resort to justifications of her acts. She does not choose, but acts *blindly* (§219). Imagine that *A* is one of these acts that Amy is immediately inclined to perform, feeling sure that she has understood. An empirical investigation can detect causes of *A*. But causes do not make *A* correct with respect to R. Incorrect acts would also have their causes. If we reject complete determination, there is nothing to determine the correctness of *A* in advance. When Amy does *A*, she feels that it is right. But the mere fact that *A* seems right to Amy, taken in isolation, is not enough. The notion of correctness is lost if one cannot distinguish between what seems correct and what really is correct (cf. §258). It is a fact that Amy is inclined to do *A* and believes that *A* is correct. But a crucial step is needed to move from a *de facto* inclination to a *de iure* judgment. That is why applications must be publicly checkable: without superlative facts, possible mutual correction is the only basis for a distinction between seeming correct and being correct. The only court before which *A* can be judged are other persons who are engaged in the same practice of following R. What allows us to speak of incorrectness or correctness concerning *A* is therefore the *possibility* that someone else may object to *A* or endorse *A*. In this sense one cannot obey a rule ‘privately’ (§202).

The conception of rules resulting from these considerations is both plastic and public. Knowing a rule is not a fixed state that determines the correctness of future acts in advance, but an ability to act in a way which is subject to possible public check. If we conjoin the public plasticity of rules and the thesis that rules are constitutive of meaning, we obtain a corresponding conception of meaning as plastic and public in accordance with a requirement formulated by Crispin Wright (2001: 56): ‘the proper interpretation of the normativity of meaning [...] has to be compatible with the capacity of ongoing use to determine meaning’.

6. A methodological principle.

The *concepts-first* view, in spite of its problems, has an obvious plausibility at first sight. It seems obvious that we first grasp the concepts ‘square’, ‘side’, ‘diagonal’ and only later come to accept the sentence ‘the diagonal of a square is incommensurable with its side’ or the rule of inference which allows us to infer ‘the side of x is equal to the diagonal of y ’ from the premiss ‘the area of square x is double the area of square y ’. From the fact that Joe does not accept the sentence or the inference, we do not conclude that Joe does not possess one of the involved concepts or does not understand some words.

Serious consideration of such data on linguistic practice leads us to reject a *holistic* version of the Copernican view: that according to which *all rules* of use in which a word W is somehow involved are constitutive of the meaning of W . The holistic view clearly clashes with our pre-philosophical convictions about meaning

and understanding. Speakers confronted with fellow-speakers take *only some* linguistic acts as evidence that interlocutors understand or do not understand some uttered words. *Other* linguistic acts are treated as irrelevant to linguistic understanding though they may be relevant in other respects. Through such reactions to the use of fellow-speakers, implicit knowledge of language reveals itself. If we are interested in explaining linguistic activity as a conscious activity, we should take such data seriously and accept the following *methodological principle*:

meaning theorists should try to respect pre-theoretical criteria of understanding: the more a theory of meaning approximates to full agreement with pre-theoretical criteria of understanding the more adequate it is.

If we adopt this methodological principle, our investigation of meaning and understanding will be guided by the regulative idea that *meaning-constitutive is what speakers treat as meaning-constitutive*. This idea clashes with the holistic version, but can agree with a moderate version of the Copernican view according to which *only some rules of use are meaning-constitutive*. It is a pre-theoretical datum that some rules for using certain words are such that a speaker's manifest ignorance of them is treated as evidence of his/her failure to understand those words. If we discovered that Joe does not know that acceptance of inferences from 'x is a pentagon' to 'x has five sides' is required for a competent use of 'pentagon', we would say that Joe does not understand 'pentagon'.

7. Circumscription of constitutive rules.

The aforementioned inferential acts are for us primitive epistemic uses of ‘pentagon’ (cf. *supra* §1). We might thus try to circumscribe the area of constitutive rules:

a rule R is constitutive of the meaning of a word for a speaker S if, and only if, all applications of R are primitive uses for S.

To make the idea precise one should also take into account the relations of dependence between words established by primitive uses. The dependence of *W* on *V* for a speaker S manifests itself in another kind of pre-theoretical datum: S treats all primitive uses of *V* as also entering into an understanding of *W*. A pre-theoretical datum could be that the primitive uses described above are for S primitive uses of ‘pentagon’, but not of ‘side’ or ‘five’. This fact establishes a non-symmetrical relation of dependence: ‘pentagon’ depends on ‘five’ and ‘side’ for S, but not vice versa. We can thus say that:

a rule R is *constitutive* of the meaning of *W* for S if, and only if,
either all applications of R are primitive uses of *W* (and in this case R is *immediately* constitutive of *W*)
or R is immediately constitutive of some other word *V* such that *W* depends on *V* for S
(and in this case R is *derivatively* constitutive of *W*).

If we add to this circumscription of constitutive rules the plastic conception of rules, we obtain a plastic conception of meaning which best agrees with pre-theoretical criteria of understanding:

the meaning of an expression X is never completely determined or fixed: it is continuously moulded and shaped by primitive uses of X (and possibly of some other related words). Non-primitive uses are not constitutive of meaning.

8. Analyticity and apriority.

Our initial question was: why are primitive uses accepted? The moderate version of the Copernican view provides an answer which does not resort to the mysterious faculty of rational intuition: primitive uses are accepted by speakers who use certain words because they are constitutive of the meanings of those words. Most of its adherents, however, have seen in the Copernican view more than an explanation for our *acceptance* of certain epistemic uses. Coffa highlights the fact that Wittgenstein's view of grammatical rules and Carnap's theory of logical syntax were 'the first genuine alternatives to Kant's conception of the a priori' (259). According to the more ambitious interpretation which Coffa has in mind, the Copernican view can provide an explanation of *a priori knowledge*, *a priori truth* and *validity* (266). Can there be such thing as *a priori* knowledge, i.e. knowledge independent of experience? Quine gave reasons for a negative answer, but many philosophers today think that Quine's criticism can be countered by reviving the Copernican view. Paul Boghossian (1997: 334), for example, defends an epistemological notion of analyticity, an instance of which would occur, assuming that **S** is a sentence and **T** a speaker,

if mere grasp of S's meaning by T sufficed for T's being justified in holding S true. If **S** were analytic in this sense, then, clearly, its apriority would be

explainable without appeal to a special faculty of intuition: mere grasp of its meaning by **T** would suffice for explaining **T**'s justification for holding **S** true. On this understanding, then, 'analyticity' is an overtly *epistemological* notion: a statement is 'true by virtue of its meaning' provided that grasp of its meaning alone suffices for justified belief in its truth.

Boghossian maintains that one has to admit that there are instances of epistemological analyticity if one endorses the thesis that the meaning of certain words is implicitly given by certain principles involving those words. He calls this semantic thesis 'Implicit definition':

it is by arbitrarily stipulating that certain sentences are to be true, or that certain inferences are to be valid, that we attach a meaning to (some of) the involved words (348).

Boghossian makes it clear that in the most basic cases the 'stipulation' cannot consist in explicitly stating certain rules, otherwise the implicit definition thesis would be vulnerable to Quine's criticism and lead to infinite regress or circularity. Hence 'stipulation' should be taken in a broad sense that includes a subject's following a rule without stating it explicitly.

9. Gödel's objection to Carnap.

Let us say that an instance of a rule is a sentence asserted or an inference performed whose assertion or performance are correct according to the rule. The *thesis of epistemological analyticity* is as follows:

if a rule R is meaning-constitutive for a speaker S, then every instance I of R is a true sentence or a valid inference and mere grasp of the meaning of I by S would suffice for explaining S's a priori justification for accepting I as true or, respectively, as a valid inference.

Boghossian refers to Wittgenstein and Carnap as the fathers of this idea, though he rejects their logical ‘Conventionalism’ and ‘Non-factualism’. However, an objection can be raised against Boghossian’s defence of analyticity which Gödel had already raised against Carnap’s *Logical Syntax of Language*. Gödel’s reasoning concerns Carnap’s views, but one can detect in it a core objection addressed against the general thesis that all instances of a meaning-constitutive rule are analytically valid: to be entitled to claim that meaning-constitutive principles are analytically valid, the advocate of epistemological analyticity ought to know that they ‘do not imply any propositions which can be falsified by observation’ (Gödel 1953/59: 347), which in turn requires knowledge that the principles in question are consistent.

10. Arbitrariness?

From Carnap’s perspective, however, it is *not generally* necessary to prove that a system of meaning-giving rules is consistent, because the choice of rules is fully arbitrary:

both the forms of construction for sentences and the rules of transformation (the latter are usually designated as “postulates” and “rules of inference”) may be chosen quite arbitrarily (1934: xv).

‘Arbitrarily’ means that epistemological justification and rational critique are excluded:

no question of justification arises at all, but only the question of the syntactical consequences to which one or other of the choices leads, including the question of non-contradiction (xv).

According to Carnap, the choice between different systems of rules can be evaluated only with respect to individual aims, on practical grounds ‘according to the preferences of anyone of the points of view represented’ (xv). We can thus have languages in which every sentence is derivable on the basis of inconsistent constitutive rules. In languages of this kind every sentence is analytically true. As Michael Potter (2000: 271) remarks: ‘However bizarre such languages may seem, though, *Logical Syntax of Language* gives us no theoretical justification for ruling them out’.

For Carnap, constitutive rules are explicit rules of formal systems. For Wittgenstein, they are mostly implicit rules of natural language and of informal language-games, which he calls ‘rules of grammar’. In spite of this important difference, Wittgenstein also says that the choice of constitutive rules is arbitrary, in a sense similar to that of Carnap’s *Logical Syntax*. For Wittgenstein, too, constitutive rules are beyond rational justification.

You cannot justify grammar. [...] Is grammar arbitrary? Yes, in the sense just mentioned, that it cannot be justified (Wittgenstein 1980: 49).

Not only justification is excluded, but also criticism, because meaning-constitutive rules ‘cannot be right or wrong’. (1956: VII, §30). For Wittgenstein, constitutive rules cannot be rationally evaluated.

Why does Wittgenstein think that the choice of constitutive rules is arbitrary? His point seems to be the impossibility of external constraints with respect to which constitutive rules can be *rationally* evaluated, according to whether or not they agree with such constraints. There are no independent *meanings* of words (Wittgenstein 1969: §133), there is no neutral description of *reality* with which the rules can be compared (§134), there is no general *aim* of language (§133). There are constraints of other kinds, for example natural, causal constraints, and corresponding senses in which the choice of rules is not arbitrary. The latter, however, are not senses pertaining to a rational evaluation as to the rightness or wrongness of the rules.

The thesis that constitutive rules are arbitrarily chosen has problematic consequences. The choice between our arithmetic and ‘the counting of aborigines whose numbers go up to 5’ (1956: III, §84) is arbitrary. The choice between a consistent language and a language where a contradiction is derivable, and hence every sentence is assertible, is arbitrary. The choice between the new chemical language of Lavoisier and the language of phlogiston is arbitrary. Are such consequences plausible?

The crucial question here is whether language has a general purpose. Dummett (1973: 309) writes that by means of language ‘we are enabled to impose an order on reality as it is presented to us’. We can develop the idea by saying that a general aim

of language is to coordinate social interactions and organize experience. If this is so, the aforementioned choices are not arbitrary. In a given epistemic situation, we can rationally evaluate a set of constitutive rules according to how well it serves the general purpose. On this view, the thesis of arbitrariness is wrong.

11. Two ways of rejecting arbitrariness.

Rejecting the idea that our choice of meaning-constitutive rules is beyond rational evaluation can lead the Copernican philosopher to two different options, corresponding to two different ways of conceiving of the relevant rational evaluation. First option: the rational evaluation determines whether the rules are meaning-constitutive on the basis of a pre-condition *a priori* which must be fulfilled in order that rules be capable of constituting genuine meanings; from such a fixed demarcation of constitutivity it follows that meaning-constitutive rules generate analyticity. Second option: rational evaluation is a multifaceted enterprise in the context of subsequent epistemic situations, performed by speakers who already understand the language because they have already grasped the rules constituting meanings: it does not decide whether rules are meaning-constitutive, but only whether they are provisionally acceptable in the given epistemic situation. Those who favour the first option ought to fix a well-grounded boundary to demarcate the area of rules that are genuinely meaning-constitutive in advance, so as to guarantee their rational acceptability once and for all and thus uphold the thesis of analyticity. Those who favour the second option discard the thesis that meaning-constitutive rules

generate apriority and analyticity, and have no qualms about admitting the possibility that rules may be meaning-constitutive but rationally unacceptable.

12. The restrictive Copernican.

Philosophers who take the first option contend that not all rules actually accepted by speakers can be meaning-constitutive, even though their instances are treated as primitive epistemic uses. Only rules that have a certain (perhaps rather complex) property X that guarantees consistency can really give meaning to the associated words. We have already circumscribed the area of meaning-constitutive rules by rejecting the holistic version of the Copernican view on the basis of the regulative idea that meaning-constitutive is what speakers treat as meaning-constitutive. Appealing to pre-theoretical criteria of understanding, we formulated a methodological condition of adequacy for a philosophical theory of meaning. In a sense, this was a restriction imposed by the linguistic practice of speakers on philosophical theories. It was not a restriction on the meaning-giving activity of speakers. The restriction we are now dealing with, on the other hand, is imposed by the philosopher on the meaning-giving activity of speakers for the philosophical purpose of defending analyticity. It amounts to saying that only part of what speakers treat as if it gave meaning really succeeds in giving meaning: not all primitive epistemic uses are meaning-constitutive, but only those that satisfy a special condition which the meaning-theorist specifies by specifying property X. Bob Hale and Crispin Wright, for example, maintain that a pattern of use is fully intelligible

only if it satisfies ‘appropriate constraints of (at least) Conservativeness (and hence consistency), Generality, and Harmony’ (Hale and Wright 2000: 306). On this view, the original rules for infinitesimal calculus given by its inventors, Leibniz and Newton, since they were inconsistent, failed to give meanings to the relevant words.

A supporter of the Copernican view who adopts this restrictive attitude can try to defend the *thesis of epistemological analyticity* according to which instances of a meaning-constitutive rule are analytically valid. His *restricted Copernican view* is:

R is meaning-constitutive for S and every instance I of R is analytically valid (in the sense of epistemological analyticity) *if, and only if*, R is a primitive rule accepted by S and R has property X.

Epistemological analyticity entails that the mere grasp of the meaning of I by S suffices to explain S's *a priori* justification for accepting I as true or as a valid inference. Is the restrictive Copernican entitled to claim that this is the case?

A possible answer is that in order for S to be *a priori* justified, S (or at least some expert, e. g. the philosopher contending that S is justified) should be able *to prove* that R has property X and that X implies consistency. In short, the *consistency guarantee* X should be proved for R. Empirical proof to the effect that so far a contradiction deriving from R (and the relevant context of other rules) has never been discovered cannot be enough to provide an *a priori* justification. The requisite proof of consistency should be *a priori* and analytic. From Gödel's second incompleteness theorem we can conclude that the proof should exploit stronger rules R^1, R^2, \dots, R^n , which do not belong to the set of meaning-giving rules to which R belongs. Hence

proof of the analyticity and consistency of R^1, R^2, \dots, R^n , will also be necessary, and so on: a fatal infinite regress begins. This line of thought does not seem promising.

Another possible answer is that *no proof* of the consistency guarantee X for R is required. There are two possible ways of developing this answer. The first is to contend that *the mere fact* that R has a property X that guarantees consistency, even if *nobody knows* this fact, is sufficient to make R a constitutive rule whose instances are *a priori* and analytically valid. However, it is difficult to see how an unknown fact could be sufficient to yield justification (let alone *a priori* justification). For the speaker S there is no known epistemic difference between a rule R^* without property X whose instances are wrong and rule R whose instances are supposedly acceptable. Thus no difference between following R and following R^* can have any bearing on the justifications that S or anyone else can resort to in order to defend the former or the latter practice. If we think that justification involves our responding to possible criticism by defending our beliefs or acts, we have to conclude that property X and the difference between R and R^* are irrelevant with respect to justification.

13. The very restrictive Copernican.

A second way of developing the idea that no proof is necessary consists in claiming that the fact that R has property X and that X implies epistemic goodness, and hence consistency, is *known* to S (and to the philosopher) without proof: knowledge of such a fact is immediate. The result is a *very restrictive Copernican view*:

R is meaning-constitutive for S and every instance I of R is analytically valid *if, and only if,*

- a) R is a primitive rule accepted by S;
- b) it is immediately known to S that R has property X and that X guarantees that R is epistemically good (and consistent).

Such a strategy requires that property X be very simple and that its epistemic goodness be self-evident. Very simple is the property of being an introduction rule for a logical constant k in Gentzen's sense i. e., roughly speaking, a rule which allows inference to conclusions having k as the principal sign from premises which do not contain k . Gentzen's suggestion that introduction rules constitute the meanings of logical constants and the further idea that these rules are therefore self-justifying have been a constant source of inspiration for Dag Prawitz:

the introduction rules [...] are understood as conferring a meaning on the logical constants by stating what forms proofs of different sentences are to have (and [...the introduction rules] are therefore self-justifying).(Prawitz 1985: 159-160).

These two ideas and the principle which Prawitz (1965: 33) christened 'inversion principle' ('an elimination rule is, in a sense, the inverse of the corresponding introduction rule') may be considered the core of what is now called *proof-theoretic semantics*. Prawitz's proof-theoretic semantics takes the form of a recursive definition of validity for arguments (1973). All the elimination rules of Gentzen's system for first order intuitionistic logic can be proved to be valid (i.e. to

preserve validity) with respect to this notion of validity. Thus, a justification of intuitionistic logic hinging on the meaning-constitutive introduction rules is provided.

Such a justification is philosophically intelligible in the framework of a verificationist theory of meaning in Michael Dummett's sense. Dummett, too, maintains that introduction rules are self-justifying. He explains a necessary feature of 'self-justification' as follows:

Something may be called "self-justifying" only if no proof is needed that it is in order; if, say, a proof of consistency is required, it cannot be self-justifying. [...] It is therefore essential to develop a characterisation that will allow us to recognise a set of logical laws as self justifying by their very form (1991: 251).

The first problem for the very restrictive Copernican is the epistemological status of self-justification. Once the easily recognizable formal property X of the 'self-justifying' rules has been specified, what can the philosopher say about the special kind of immediate knowledge to the effect that inferences of form X do not lead to inconsistency? We have already noted that our treating certain rules as meaning-constitutive does not guarantee that they are consistent. On the other hand, the capacity to immediately recognise that a rule is self-justifying is dangerously similar to intuition. Since this concerns a formal property, it must be some kind of formal intuition. Its nature is not clear and it is not clear how it can escape the objections raised against more traditional notions of intuition. Simple rules may seem obviously reliable. But history teaches us that our feeling of certainty may be misleading. A restrictive Copernican might propose as a necessary and sufficient

requirement of meaning-constitutive primitive rules that they should be introduction and elimination rules in harmony with each other, i.e. rules for which the inversion principle holds true. For example, the rule of conjunction elimination, according to which the conclusion A (or B) can be correctly inferred from the premise ‘A and B’, is in harmony with the rule of conjunction introduction, according to which the conclusion ‘A and B’ can be inferred from the two premises A and B. This is a very easily recognizable formal characterization and, at first sight, seems to guarantee consistency. The advocate of the latter proposal can boast that the harmony requirement succeeds in excluding the rules for the connective ‘tonk’ devised by Arthur Prior (1960) precisely in order to show that the Copernican view leads to inconsistency. However, the harmony requirement is insufficient: Prawitz (1965: 94) gives very simple introduction and elimination rules for naïve set-theoretical membership which are in obvious harmony with each other, but which lead to Russell’s paradox. The necessary conditions that Dummett and Prawitz propose for meaning-constitutive introduction rules banish the naïve set-theoretical introduction rules. Nevertheless, the fact that we do not know of a counterexample to Dummett’s and Prawitz’s proposal does not eliminate the problem: given a characterisation that seems to allow us to recognise rules as self-justifying by their very form, *how* do we know that the rules in question do not lead to inconsistency? Can we really know this *immediately*?

Another epistemological problem concerns rules that are not self-justifying. If, following Gentzen and Prawitz, we take all self-justifying rules to be introduction

rules, there will be rules that are neither self-justifying nor derivable in the usual proof-theoretical sense. The reason is that the conclusion of an introduction which can be legitimately considered self-justifying according to Dummett and Prawitz is always of *higher* complexity than the premises. It follows that a non-self-justifying elimination rule R whose conclusion is of *lower* complexity than the major premiss cannot be justified by deriving it from self-justifying rules, i.e. by constructing a concatenation of applications of introduction rules such that the initial assumptions are the premises of R and the final conclusion is the conclusion of R. Hence for Prawitz (1973: 234) and Dummett (1991: 250), each non-self-justifying non-derivable rule (each elimination rule) should be justified in a different way, by supplying a special justifying operation. The prototypes of Prawitz's justifying operations are the reductions by means of which he (1965) proved the normalization theorems. For example, the rule of *modus ponens*, i.e. conditional-elimination, can be justified by resorting to a conditional-reduction. The existence of such a reduction shows that there is harmony between the rule of conditional-introduction and the rule of conditional-elimination (cf. Dummett 1991: 250). The mere *existence* of a reduction, however, cannot justify anything. From this point of view, the elimination rule can be justified for a person S only if S *knows* that the reduction exists. The rule of *modus ponens* was applied already in the third century before Christ when the Stoics formulated it explicitly. Were the Stoics justified in performing instances of *modus ponens*? Most of us would answer in the affirmative. But can we credit the

Stoics with (implicit) knowledge of the reduction for implication introduced by Prawitz in 1965? In what can such (implicit) knowledge consist?

The problem arises because, following Gentzen and Prawitz, we have considered *modus ponens* a rule which is not meaning-constitutive for the conditional sign, but which should be justified in terms of the corresponding meaning-constitutive introduction rule. Recognizable instances of *modus ponens*, however, are mostly treated as primitive uses that should be accepted by whoever understands and uses the conditional sign. Pre-theoretical criteria encourage us to consider *modus ponens* meaning-constitutive as well (not only conditional-introduction). Thus there is a conflict between Prawitz's very restrictive Copernican view and the regulative idea that meaning-constitutive is what speakers treat as meaning-constitutive.

The conflict does not only concern *modus ponens*, but is clearly much more extensive. Roughly speaking, for Dummett (1991: 258) and Prawitz (1987: 157-161) the property X of meaning-constitutive rules should be the property of being an introduction rule which can be specified in terms of sentences (premises, discharged assumptions) of lower complexity than the conclusion. Many rules lack this property but are treated by speakers as meaning-constitutive: some are elimination rules that can be justified in Prawitz's sense (like *modus ponens*); some are rules of inference that cannot be justified (e. g. classic double negation elimination, which is not intuitionistically valid); some are axioms in mathematical theories, or basic laws of empirical theories, which play the role of implicit definitions of the involved concepts; finally, some principles treated as meaning-constitutive turn out to be

wrong and even inconsistent, like the non-restricted comprehension principle which was constitutive of the meaning of 'set' in Cantor's theory.

The discrepancy between the very restrictive Copernican view and pre-theoretical criteria of understanding is the main objection to this view. Dummett and Prawitz are well aware of the objection but they do not seem to take the problem very seriously. We think it is a serious problem. So we ask: is there a better alternative to the very restrictive Copernican view?

14. Copernican without analyticity.

There is an alternative. Rejecting the view that a choice of meaning-constitutive rules is arbitrary does not necessarily lead to a restrictive Copernican view. It can lead to an admission that rules are meaning-constitutive if, and only if, they are treated as meaning-constitutive but that, nevertheless, they can be criticized and possibly rejected as rationally unacceptable. On this view, the critique of rules does not determine whether the rules are meaning-constitutive. It determines whether the meanings already shaped by the rules are good meanings. This decision is not made in advance once and for all. It is made in the context of given epistemic situations where certain concepts, arguments and conclusions are already accepted, certain problems are considered variously important, some sensory evidence presents itself etc. Epistemic situations change and so the decision as to the acceptability of certain meanings (or concepts) may change. The object of the rational evaluation is not sentences or arguments within a language, but the language itself, or rather a fragment of language, the corresponding meanings and the rules constituting those

meanings. We can call this the *critique of language* (cf. Cozzo 1994: Ch. V, 2002: §22). The critique of language is guided by the general aim of coordinating social interactions and organizing experience. It is multifaceted: simplicity, fruitfulness, consistency are different competing criteria involved in the evaluation. Sensory experience also plays a role. It can be more or less resistant to our attempts to impose linguistic order upon it and its resistance affects our evaluation of the language in question. It affected, for example, the considerations that led scientists to abandon the phlogiston-language in the eighteenth century. Hence we cannot legitimately say that, if a critical evaluation of constitutive rules in a given epistemic situation results in acceptance of certain meanings (or concepts), such a result is independent of experience. We cannot say that our choice of those rules is *a priori*. Nor can we say that instances of those rules are not subject to revision based on sensory experience which changes the epistemic situation significantly. *A priori* principles are not necessarily indefeasible, but they ought to be at least *empirically* indefeasible. We shall thus discard the view that instances of constitutive rules are *a priori* and analytic. Though meaning-constitutive, ‘Phlogiston is present in all flammable bodies’ is not analytically and *a priori* true, simply because, as it turns out, it is not true at all. The Copernican view without analyticity can be summarized as follows:

- i) a rule R is constitutive of the meaning of some word for S if, and only if, all the applications of R are primitive uses for S;
- ii) a set of constitutive rules can be rationally criticized and rejected according to how well it fulfils the function of organizing

common experience in a given epistemic situation (hence rules do not generate analytic validity).

15. The distinction between knowing a rule and accepting a rule.

Admitting a critique of constitutive rules may seem to contradict their being constitutive and to clash with the whole Copernican view. Some formulations of the Copernican view are indeed incompatible with the idea of a critique of constitutive rules. A critique of rules for the correct use of a word, in so far as it is a rational activity, requires the rules and the word to be understood before the rules are rejected, and there is no reason to think that they cease to be understood when they are rejected. So they are both understood and rejected. If the Copernican maintains that understanding consists in *accepting* constitutive rules, he or she cannot avoid the conclusion that a critique of constitutive rules is a contradictory notion, which implies that rules are accepted and rejected at the same time. According to this implausible version of the Copernican view, since understanding entails the acceptance of constitutive rules, those who reject constitutive rules do not understand the relevant words.

This implausible consequence afflicts a formulation of the Copernican view advanced by Boghossian (2003: 239):

inferring according to [a deductive pattern of inference] P is a precondition for having one of the concepts ingredient in it.

In particular, Boghossian thinks that inferring according to *modus ponens* is a precondition for possessing the concept expressed by the English word ‘if’. Timothy Williamson (2003: 251-2) has objected:

Is making inferences by modus ponens [...] a precondition for having the concept *if*? Van MacGee [...] has published purported counterexamples to modus ponens. [...] any particular inference by modus ponens might be rejected by a student who, having just read MagGee’s article, was sufficiently impressed to try to put it into practice. [...] Has the student ceased to understand the word “if”. He may still use it competently. [...]

The student still has the concept *if*.

Williamson’s point is that being willing to infer according to *modus ponens* is not a necessary condition for understanding ‘if’. It is possible to understand ‘if’ without willingness to perform or endorse instances of *modus ponens*. His point is of course general: not only about ‘if’ and *modus ponens*.

Does Williamson refute the Copernican view in general? He does not, but he shows that the Copernican meaning theorist should: *i*) emphasize the distinction between knowing a rule and accepting it; *ii*) explain understanding in terms of knowledge of rules and not in terms of acceptance.

If *W* is a meaningful word, according to the conception of constitutive rules sketched in §4, a speaker *S* understands *W* if, and only if, *S* *knows* the constitutive rules, i.e. knows that *W* should be used in a certain way, e.g. in accordance with a pattern of inference *P*. Suppose that *S* understands *W* in this sense. It does not follow that *S* will use *W* or will accept uses of *W*. It follows only that *S* has the *ability* to use *W* according to *P*. The Wittgensteinian analogy between languages and games is

enlightening in this respect. We can know the rules of a game without being willing to play it. Knowing a rule R amounts to knowing what one should do in order to comply with R. If we agree on the public plasticity of rules, such knowledge is an ability to act in a way which is subject to possible public check. But we can have an ability without being willing to exercise it. Rejecting a recognized instance of a constitutive rule, therefore, does not necessarily show that one does not understand the relevant word. It may show that one has a critical attitude towards the meaning shaped by the rule and rejects the very use of the word, since it has that meaning. In this case one is deliberately abandoning a fragment of language. Similarly one can deliberately violate the rule that a pawn cannot move backwards, thereby showing that one does not want to play chess. The Copernican philosopher should endorse not only the aforementioned ‘law of breach’ (cf. *supra* §4), but also the *law of reluctance*: we can know the constitutive rules without being willing to accept them.

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