What is the point of reduction in science?

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Abstract

The numerous and diverse roles of theory reduction in science have been insufficiently explored in the philosophy literature on reduction. Part of the reason for this has been a lack of attention paid to reduction$_2$ (successional reduction)—although I here argue that this sense of reduction is closer to reduction$_1$ (explanatory reduction) than is commonly recognised, and I use an account of reduction that is neutral between the two. This paper draws attention to the utility—and incredible versatility—of theory reduction. A non-exhaustive list of various applications of reduction in science is presented, some of which are drawn from a particular case-study, being the current search for a new theory of fundamental physics. This case-study is especially interesting because it employs both senses of reduction at once, and because of the huge weight being put on reduction by the different research groups involved; additionally, it presents some unique uses for reduction—revealing, I argue, the fact that reduction can be of specialised and unexpected service in particular scientific cases. The paper makes two other general findings: that the functions of reduction that are typically assumed to characterise the different forms of the relation may instead be understood as secondary consequences of some other roles; and that most of the roles that reduction plays in science can actually also be fulfilled by a weaker relation than (the typical understanding of) reduction.
Keywords: *Diachronic reduction; Synchronic reduction; Limiting reduction; Theory reduction; Inter-theory relations; Correspondence principle.*

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Contents

1 Introduction

Reduction plays numerous and diverse roles in science, but they have been insufficiently explored. Here, I attempt to rectify this, by showcasing the utility—and incredible versatility—of reduction. To do so, I present a non-exhaustive list of various applications of reduction, thus contributing to a more complete picture of the roles of inter-theory relations in science. This paper is motivated by the conviction that a better appreciation of how theory reduction can be, and is, used in science will benefit several different areas of inquiry. Firstly, it can aid our understanding of what counts as a successful reduction, and thus be of assistance in clarifying debates regarding the best account of theory reduction—for example, I argue that this picture suggests the sufficiency of a relatively weak notion of reduction in many particular scientific cases, compared to existing accounts. Secondly, by drawing attention to the various uses of reduction aside from explanation, this project not only opens up new avenues for discussion, but also impacts the question of the relationship between reduction and explanation—for example, I demonstrate (counter to consensus belief) that explanation is not the characteristic function of reduction. Thirdly, it will contribute to our understanding of the nature of scientific theories, and of science itself.

This third motivation is particularly compelling given the current state of fundamental physics, with its indications of moving into a paradigm that relies more heavily on non-empirical means of theory assessment compared to previous eras.[1] Inter-theory relations take on considerable importance in this situation. In fact, many of the examples of roles that I describe here are drawn from the current search for a new theory of fundamental physics. As a case-study, this is especially interesting because of the huge weight being put on reduction by the different research groups involved. Additionally, it presents

[1] Cf. Dawid (2013); Huggett & Wüthrich (2013); Woit (2006).

some unique uses for reduction—revealing, I argue, the fact that reduction can be of specialised and unexpected service in particular scientific cases. In addition, the paper makes two other general findings: one is that the functions of reduction that are typically assumed to characterise the different forms of the relation may instead be understood as secondary consequences of some other roles (which I suggest are better taken as the characteristic functions of these relations). The other main finding is that most of the roles that reduction plays in science can actually also be fulfilled by a weaker relation than (the typical understanding of) reduction.

My methodology involves distinguishing between (a) what reduction is, (b) what it does (i.e., its characteristic function), and (c) what it is useful for (i.e., the secondary roles that reduction plays by virtue of achieving its characteristic function). My aim in this paper is not just to investigate the neglected area (c), but to help shed new light on (a) and (b) by doing so. Of course, however, in order to do this, I need to adopt some preliminary, vague conception of (a) and (b). In fact, I use two such conceptions of each, which are based on scientific-consensus. For (b), I distinguish between a weak notion and a stronger notion of reduction.

The *weak notion* of reduction (WR), obtains when all the successful results of the older and/or less-fundamental theory, O, could be reproduced (i.e., obtained from the theory via methods such as computation, derivation, etc.), in principle, by its more-fundamental/successor theory, N (to within some acceptable degree of error, under appropriate conditions, and within the relevant domain).[2] This notion of reduction is weak because it leaves open the means by which the new theory is thought capable in principle of yielding the relevant results—in particular, we do not need to believe that the (relevant parts of) the older theory are deducible (or derivable) from the new one, for example. In other words, WR is neutral in regards to the relationship between the two theories, other than their approximate shared results in the relevant domain.

By contrast, the *stronger notion* (SR) obtains when all the successful results of O, could be reproduced, in principle, by N (to within some acceptable degree of error, under appropriate conditions, and within the relevant domain) *because* the successful parts of O are derivable in principle from N (under the same qualifications). The two corresponding fuzzy notions of (a) that I start with are then any set of inter-theory relations that achieve WR or SR, respectively.

At the outset, I must make clear that though this paper is concerned with 'theory-reduction' in science, I take this term to not necessarily (or even standardly) refer to entire theories, but *theory fragments*—i.e., any *parts* of scientific theories. Additionally, the theories being referred to are ones that are, were, or will be *accepted* by mainstream science at some point as being correct (i.e., as providing the best descriptions of the phenomena) in their domains of applicability. The reduction, however, need not (and typically will not) hold at a time when both theories involved are considered correct (e.g.,

[2] The vagueness introduced by the "in principle" aspect of this definition is due to our inability in practice of actually going through and obtaining all of the results of the new theory in the old domain (due, e.g., to lack of computing power). Instead, the point is that we obtain enough 'linkages' (which I define below as 'correspondence relations') between the two theories that we believe they approximately share the same results in this domain.

the earlier, or less-fundamental theory may have been demoted to just *approximately* correct, within its domain, at the time when the reduction holds). I ask the reader to please keep these qualifications in mind throughout.

The structure of the paper is as follows. It begins (§2), with a brief elaboration of Nickles' and Wimsatt's distinctions between the two standard conceptions of reduction §2.1, followed by a statement of the two—purposefully very general—conceptions of reduction that I work with in this paper §2.2. §3 compares them with the related notion of *correspondence* (familiar from physics as embodied in the correspondence principle of old quantum theory), and argues that WR is a special case of correspondence. The list of roles of reduction are presented in §4, and some of these (plus two more roles) are then illustrated in the case-study of quantum gravity (§4.1). In §5, I consider the characteristic functions of reduction, before conclusions in §6. Before all this, however, I briefly comment on the issue of neglect.

1.1 Addressing an overlooked aspect of reduction

It is striking that the general philosophy literature on reduction has neglected to thoroughly investigate the myriad uses of reduction. There are two main reasons this seems to have happened. The first is a more-or-less standard assumption that the role of reduction is simply *explanation*: the explanation of 'higher-level' laws (models, theories, theory-parts, etc.) in terms of 'lower-level' ones. This stems, in part, from the deductive-nomological (DN) model of explanation (Hempel & Oppenheim, 1948), which was mirrored by the incredibly influential model of reduction by Nagel (1961). The second reason has to do with a distinction, originally made by Nickles (1973), between two forms of reduction: *reduction$_1$*, being, roughly, the deduction of (corrected) parts of a higher-level theory from (parts of) a lower-level one, under some appropriate conditions, and *reduction$_2$*, being, roughly, various inter-theory relations between (parts of) a new theory and its 'predecessor', under appropriate conditions, that serve heuristic and justificatory roles in theory-succession. This distinction has also been referred to as *explanatory* versus *successional* reduction (Wimsatt, 1976, 2006), and *synchronic* versus *diachronic* reduction (Dizadji-Bahmani et al., 2010; Rosenberg, 2006; van Riel & Van Gulick, 2016).

The general philosophy literature on scientific theory reduction has primarily focused its attention on reduction$_1$, which is exemplified by Nagel-Schaffner reduction. This literature has been largely uninterested in the other type of reduction, which has been relegated to discussions on scientific theory-change, and more-specialised literature.[3] In the rare instances where the literature *on reduction* has explored reduction$_2$, this has mainly been in order to determine its utility for the articulation of reduction$_1$ (i.e., in interpreting Schaffner's (1976) criterion of "strong analogy"). As such, the various important roles that reduction$_2$ plays in science have been largely ignored in this literature. An exception (along with Nickles) is Wimsatt (e.g., 1976; 2006), who elaborates (though also modifies) the distinction, as I discuss below (§2.1).

[3] As evidence of this, consider the *Stanford Encyclopedia of Philosophy* entry on scientific reduction (van Riel & Van Gulick, 2016, §2.1), which briefly mentions, then dismisses this form of reduction as outside its area of interest.

This indifference to reduction$_2$ is unfortunate given its prominence in science, and its versatility beyond supplementing reduction$_1$. There are, indeed, some prominent misconceptions about reduction$_2$. Firstly, although reduction$_2$ plays a large number of roles in science, it has standardly been assumed to be just the demonstration that a new, more-general theory (model, law, etc.) explains the success of the older, special theory (or theories) that it replaces.[4] Secondly, reduction$_2$ is typically thought of as a limiting relation (usually of a characteristic physical constant in the newer, more general theory, going to zero or infinity). Yet, reduction$_2$ is not restricted to this—in different cases, reduction$_2$ may involve several, or no limiting relations, as well as other approximations and inter-theory correspondences. Thirdly, there is some confusion regarding the domains that reduction$_2$ operates on, and the levels that it bridges: what it means to be domain-preserving rather than domain-combining, and whether it is an *inter-level* or *intra-level* relation. I clarify these misconceptions below (throughout the paper, though mostly in §2.1).

Among the findings of this paper is the demonstration that the two forms of reduction may be achieved by the same sorts of relations; indeed, in the case-study considered, the (sought) reduction between the two theories involved is supposed to be *both* reduction$_1$ and reduction$_2$.

2 Two conceptions of reduction

In §2.1 I comment on Nickles' (1973) distinction between reduction$_1$ and reduction$_2$, as well as Wimsatt's (1976; 2006) distinction between explanatory and successional reduction. In §2.2 I present the two conceptions of reduction that I use in this paper, and discuss their relationship with these two other distinctions.

2.1 Existing distinctions

According to Nickles (1973, p. 181), *reduction$_1$* is "the achievement of postulational and ontological economy and *is* obtained chiefly by derivarional reduction as described by Nagel; i.e., reduction$_1$ amounts to the *explanation* of one theory by another". It is, he says, a "domain-combining" relation, in that it may involve the consolidation or elimination of (parts of) the reduced, less-fundamental theory in favour of a more fundamental one. The example of reduction$_1$ that Nickles (1973) presents is the reduction of the theory of optics to that of electromagnetism.

By contrast, Nickles' *reduction$_2$* is a "varied collection of intertheoretic relations" whose great importance lies in its heuristic and justificatory roles in science. As Nickles points out (p. 185), the development of new ideas (theories) is heuristically guided by the requirement that these ideas yield certain established results as a special case, and these ideas are often justified (to a degree) by showing that they bear a certain relation to a predecessor theory. Reduction$_2$ is a "domain-preserving" relation, demonstrating that the

[4]This is in contrast this with reduction$_1$, whose aim is commonly taken as the explanation of higher-level laws (behaviour, theories, fragments of theories, models, etc.) in terms of lower-level ones.

successor theory adequately accounts for the phenomena in the domain inherited from its successful predecessor (p. 185). The terminological convention of reduction$_2$ is reversed from that of reduction$_1$, so that the successor—typically more fundamental—theory is said to reduce to its predecessor.[5] Nickles presents the example of special relativity reducing to Newtonian mechanics. Reduction$_2$ is commonly thought of as just being a limiting relation between theories—indeed, it is often referred to as "limiting reduction". Yet, as Nickles (1973, p. 197) asserts, reduction$_2$ is actually a set of various operations, a number of which may be performed on (parts of) one (or both) of the theories involved (there is no general formula for how this is done). I suggest below that the various relations that may be employed in reduction$_2$ are *correspondence relations* (§3).

It is worth also discussing some of the relationships that Nickles sees between the two conceptions of reduction. Firstly, that reduction$_1$ can be considered a special case of reduction$_2$, where the theories involved are (by necessity) logically compatible—otherwise, the theories related by reduction$_2$ generally need not be logically compatible (Nickles, 1973, p. 186, fn. 6). Secondly, that reduction$_2$ can be of assistance in interpreting reduction$_1$, e.g., by spelling out the relation of "strong analogy" between T_2 (the older, less-fundamental theory) and T_2* (the "corrected version" of this theory) in Schaffner's (1976; 1967) account of reduction$_1$ (Nickles, 1973, p. 195). Finally, note that Nickles (1973, p. 195) emphasises that reduction$_2$ should not be seen as "an imperfect attempt at derivational reduction$_1$, as a failure to achieve reduction$_1$", as, he says, Nagel and others have done—reduction$_2$ is a distinct relation, and need not be viewed as an approximation to the other form of reduction. While Nickles is correct on these points, I nevertheless suggest that reduction$_2$ can still, in some cases, be usefully conceived of as an attempt at derivational reduction. (In fact, below, §5, I argue that the characteristic role of reduction generally is "establishing in principle derivability").

Wimsatt (1976; 2006) elaborates on Nickles' distinction, and relabels it as one between *explanatory* and *successional* reduction. Explanatory reduction, according to Wimsatt, is an *inter-level* relation, relating "levels of organisation" rather than theories.[6] Its aim is to provide a *compositional*, *mechanistic* and *causal* explanation of some large-scale phenomena in terms of shorter-length scale behaviours (Wimsatt, 2006, §4); e.g., explaining the behaviour of gases as clouds of colliding molecules, or the behaviour of genes in terms of the action of DNA (2006, p. 449). Wimsatt is clear that explanatory reduction is no longer best exemplified by Nagel-Schaffner reduction, but that it is richly complex and greatly diverse in its approaches, especially in biology.

Successional reduction, according to Wimsatt, does relate theories, and is supposed to be *intra-level*: holding between newer and older theories, and/or more exact and more approximate theories, and/or more- and less-general theories that apply "at the same compositional level". But this sense of intra-level reduction is supposed to also relate theories that are not level specific, such as in physics (Wimsatt, 2006, p. 450). Wimsatt

[5]This is often referred to as the *physicists' convention*, since it is how the term "reduction" is understood by physicists—the newer, more general, or more fundamental, theory, N *reduces to* the older, more restricted, or less fundamental theory, O. In contrast, the *philosopher's convention* has O reduce to N.

[6]Wimsatt (1976, p. 680) conceives of levels of organisation as "primarily characterized as local maxima of regularity and predictability in the phase space of different models of organization of matter".

agrees that the relations used in successional reduction can sometimes be useful in explanatory reductions, especially in articulating the relation of strong analogy in Schaffner's account of of reduction. However, the aim of successional reduction, Wimsatt says, is to localise and analyse the formal similarities and differences between these theories, and thus aid succession and elaboration of the later theory as well as delimit conditions for safe heuristic use of the earlier theory (1976, p. 677; 2006, p. 449).

Wimsatt (2006, Fig. 2.) argues that in theory-succession the differences as well as the similarities between the old theory, O and the newer one, N, play important roles. The similarities serve to, 1. give prepackaged confirmation of N, by showing that it generates O as a special case (attributed to Nickles, 1973); 2. 'explain away' O, or explain why we were tempted to believe it (attributed to Sklar, 1967); 3. delimit acceptable conditions for heuristic use of O, by determining conditions of approximation (Nickles, 1973). The differences between the theories, serve to, 1. explain facts which were anomalous on O, thus confirming N; 2. suggest new predictive tests of N; 3. suggest reanalysis of data apparently supporting O but not N; 4. suggest new directions for elaboration of$N$. The characteristic role by virtue of which successional reduction aids in theory-succession is thus localising and analysing the similarities and differences between the newer and older theories of the same phenomena.

I argue in this paper that the same relations between theories can play roles in establishing either, or both, explanatory and successional reduction. In the case study I consider (§4.1), both explanatory and successional reduction are sought, since the newer theory (i.e., the theory under-construction) is supposed to not only cover the same domain as its predecessor, but also to underlie it (i.e., be lower-level), and thus to provide a 'mechanistic' explanation of the higher-level phenomena described by its predecessor (although, as we shall see, due to the unusual nature of the particular theory being sought, such explanation may not be accurately described as compositional, mechanistic, nor causal). Indeed, this reductive explanation plays an important role in justifying the new theory, in a way that goes beyond that captured by Wimsatt's notion of successional reduction.

2.2 Conceptions used in this paper

In this paper, I work with two very general conceptions of reduction; I do this not only so as to accommodate the various formal definitions of theory-reduction in the philosophy literature, but because this is how reduction works in science: there is no particular definition, but instead putative reductions are developed and judged on case-by-case bases.[7] To re-state these (from §1):

WR The *weaker conception of reduction* holds when all the successful results of the older and/or less-fundamental theory, O, are obtainable in principle[8] from its more-fundamental/successor theory, N (to within some acceptable degree of error, under appropriate conditions, and within the relevant domain). This conception of re-

[7]This is recognised in the case of reduction$_2$, at least (Nickles, 1973; Wimsatt, 1976).

[8]See fn. 2.

duction remains neutral in regards to any more-specific relationship that may hold between the two theories (see above, §1).

SR The *stronger conception of reduction* holds when all the successful results of the older and/or less-fundamental theory, O, could be reproduced, in principle, by its more-fundamental/successor theory, N (to within some acceptable degree of error, under appropriate conditions, and within the relevant domain) *because* the successful parts of O are derivable (deducible) in principle from N.

Note that, although they are similar, these conceptions of reduction are not to be equated with reduction$_1$ and reduction$_2$, or explanatory versus successional reduction. WR and SR can each be either inter- or intra-level relations. While—as I argue below, §4—SR establishes *relative fundamentality* (i.e., an asymmetric relation of non-causal dependence), this can hold between thermodynamics and statistical mechanics, or between special and general relativity alike. Additionally, although SR establishes relative fundamentality, while WR does not, both notions apply equally well between higher-level/lower-level, older/newer, more-approximate/more-accurate, and special/general theories. On the other hand, as mentioned above, the literature typically conceives of reduction$_1$ as being only between higher-level/lower-level theories, and reduction$_2$ as holding only between older/newer and/or special/general theories.

WR may be deemed too weak for reduction by many philosophers, and yet it seems to be all that scientists practically require in the development of a new theory: many physicists, for instance, would argue that this is the real purpose of reduction, and derivation is just the most efficient and sure means of achieving it. Also, some philosophers (of physics, at least) have used this understanding of reduction. For instance, Rosaler (2017) takes the key feature of reduction as *domain subsumption*: the relation whereby one theory successfully models all real behaviours that are well-modelled on another—i.e., that one description subsumes the domain of applicability of another. (However, below I argue that WR is in fact better conceived of as Correspondence rather than reduction).

Rosaler (2017) also introduces a useful distinction between *conceptions of* reduction and *approaches to* reduction: A conception of reduction is a particular meaning that one assigns to the term 'reduction', while an approach to reduction is a particular strategy for showing that some particular conception of reduction holds. WR and SR—as conceptions of reduction—can be achieved in various ways (they are 'multiply realisable' by different approaches to reduction): for instance, through Nagel-Schaffner reduction, or Kemeny-Oppenheim (1956) reduction, or plausibly any other extant account of reduction on offer in the philosophy literature. However, I resist restriction to a particular account of reduction, and instead go with what seems to be the actual method of scientists: of utilising various *correspondence* relations—including limiting relations, approximations, derivations, etc.—as appropriate the particular scientific case under consideration (these relations are explained in the next section, §3). WR or SR holds when (some assortment of) these relations have been established between the two theories in question, such that scientific consensus holds that WR or SR obtains.

For example, there are numerous and various correspondence relations[9] between quantum and classical mechanics—including mathematical correspondences such faciliated by limiting relations[10]—linking particular laws of quantum theories with classical ones. These are generally believed to be sufficient to establish WR: that quantum mechanics can in principle reproduce all the successful results of classical mechanics (to within some acceptable degree of error, under appropriate conditions, and within the relevant domain).[11] An example of SR is between quantum electrodynamics (QED) and electromagnetism (i.e., Maxwell's theory): numerous and various correspondence relations between the laws of the two theories have been established, including approximations and derivations, such that all successful results of electromagnetism are believed to be derivable in principle from QED, because the successful parts of the theory of electromagnetism are believed to be derivable in principle from QED (to within some acceptable degree of error, under appropriate conditions, and within the relevant domain). So, although particular correspondence relations may be symmetric or asymmetric, they can be used to establish either the asymmetric relation of reduction (both forms), or the symmetric conception of Correspondence, as I now discuss.

(I realise that my fuzzy notions of reduction will admit many cases that would not count according to some particular philosophical definitions, while also missing other cases that may be taken as 'paradigmatic' on particular philosophical accounts. These are, however, unavoidable difficulties faced by any account of reduction—consequences of what Dizadji-Bahmani (2011) calls the "external problem of defining reduction").

3 Correspondence

I distinguish between 'the relation of Correspondence', which I shortly define as the 'generalised correspondence principle', and individual correspondence relations, which, as stated above, may be used to demonstrate Correspondence or reduction—they are, in some sense, the constituent ingredients that may be mixed and matched to obtain the conception of Correspondence (or WR, or SR).

> **A correspondence relation** is any relationship between two theories whose domains of applicabilty (at least partially) overlap, that is employed in theory construction and/or justification. These relations are of most interest when they hold between 'predecessor' and 'successor' theories, i.e., O and N. (Typically, though not invariably, these relations are intended to help demonstrate *Correspondence*—i.e., that the theories approximately share the same results in the overlap regions; see below).

[9] See §3.

[10] Although limiting relations are involved in derivations, they—strictly speaking—can only establish that solutions of the new equations coincide with solutions of the old equations in the limit (Hüttemann & Love, 2016, p. 468).

[11] The various correspondence relations between quantum and classical mechanics are explored, e.g., in Radder (1991); Bokulich (2008); that these are insufficient to establish that classical mechanics is 'contained within' quantum mechanics, is argued in, e.g., Post (1971).

These inter-theory relationships are a motley collection, and there is no general recipe for which and how many to use in order to do the job of establishing Correspondence (or reduction). These relations include—but are not restricted to—mathematical correspondences such as limiting relations and law correspondence. Radder (1991) identifies three types of correspondence relations, and Hartmann (2002) describes seven in a list that is not supposed to be exhaustive—I mention just a selection of these here by way of illustration (please refer to the cited works for more details, including various limitations and disclaimers associated with each of these).

Term correspondence N incorporates certain terms from O (Hartmann, 2002; Post, 1971). As Hartmann explains, this is the weakest form of correspondence, which is, moreover, presupposed by almost all the other forms of correspondence. An example is 'mass', which is carried over from Newtonian mechanics to special relativity, with a shift in meaning (Kuhn, 1962).

Numerical correspondence N and O (approximately) agree on the numerical values of some quantities (Hartmann, 2002; Radder, 1991). An example is the spectrum of hydrogen in the Bohr model and in quantum mechanics; although each of these theories utilise different assumptions in calculating the spectrum, they nevertheless obtain the same numerical values (Da Costa & French, 1993; Scerri, 1993).

Law correspondence Some laws from O also appear (approximately) in N. An example is the kinetic energy in classical mechanics and in the special theory of relativity. For low velocities, $T_{CM} = 1/2mv^2$ and $T_{STR} = m - m_0c^2 = 1/2mv^2 \cdot (1 + 3/4\beta^2 + \mathcal{O}(\beta^4))$ are approximately the same (Hartmann, 2002).

Structure correspondence Some mathematical structures (e.g., symmetries, groups, etc.) used by O bear well-defined relations to some of those in N. An example is the relation between the inhomogeneous Lorentz group, used in special relativity, and the inhomogeneous Galilei group of Newtonian mechanics, which 'correspond' in a precise mathematical sense (Hartmann, 2002; Saunders, 1993).

Model correspondence A model which belongs to O is also used in N. An example is the harmonic oscillator, which is widely used in classical mechanics, and is also applied in quantum mechanics and in quantum field theory (Hartmann, 2002).

Principle correspondence A key principle used by O also features in N (but may be re-interpreted or generalised). An example is the 'principle of relativity', a form of which appears in Newtonian mechanics, and other forms of which are used by the special and general theories of relativity. Another example is the 'principle of background independence', which is seen as a key feature of general relativity, and utilised by various quantum gravity approaches in their attempts to construct a 'successor' to general relativity (as discussed below, §4.1).

Now, to the 'relation of Correspondence'; its precursor (which captures a specific instance of the idea) is familiar from the common understanding of the *correspondence* *principle*, which states, roughly, that quantum mechanics reduce to classical mechanics in the domain where the latter is successful.[12] Thus, Correspondence is often thought of as reduction$_2$, and is usually taken to hold between a newer theory, N, and the older, established theory, O that it replaces. Yet, as discussed below, Correspondence is also used from O to N, as a means of inferring (parts of) N from O (Radder, 1991; Bokulich, 2008). This way of using Correspondence is explicitly heuristic, and does not strictly match definitions such as numerical or law correspondence—rather, it is an exploratory, 'working' Correspondence, used to discern something of the structure of the theory-in-development from current theories.

Post (1971), and subsequent literature, speaks of the "generalised correspondence principle" (GCP). My formulation[13] of the GCP (and thus, the statement (a) of what Correspondence is) is:

> **GCP** Any two theories whose domains of validity (partially or fully) overlap must *Correspond* to one another in those domains.
>
> **Correspondence** between two theories whose domains of validity (partially or fully) overlap, is taken to obtain when sufficient (individual) correspondence relations have been demonstrated between these theories such that we are satisfied that the two theories are *compatible*—i.e., share approximately the same results—within the overlap region(s).[14]

This statement of Correspondence reveals the concept as tautological: two theories that both successfully describe a given domain will necessarily be compatible within that domain, *by definition*. So, correspondence relations are of the most practical interest in the cases where the success of one theory within that domain has not been demonstrated. The relationships are then articulated in order to establish that the new theory does indeed successfully describe that domain, by virtue of standing in these particular relationships to a theory whose success here has been directly and thoroughly established. As such, Correspondence is partly a 'shortcut to results'—one of the main reasons that the relationships between the two theories are articulated is in order to avoid actually having to derive *all* the results from the new theory in the old domain.

Correspondence is taken to hold when the correspondence relationships demonstrated between two theories are sufficient that the two theories are believed to be compatible (sharing approximately the same results) in the relevant domain (this is the primary role (b) of correspondence). This subjectivity in judging successful cases of correspondence is a consequence of its nature as a 'shortcut to results': a full demonstration of compatibility, by comparing all results in the relevant domains, would defeat much of the purpose of the inter-theory relations. But, importantly, even if we did compute all the results of both theories, we would still need correspondence relations between the theories in order to

[12]The correspondence principle was famously proposed by Niels Bohr in the context of old quantum theory, yet the common understanding of the principle is most certainly not what Bohr meant by it (Bokulich, 2014).

[13]Note that this is an original formulation, and thus differs from Post's (1971) GCP.

[14]Note that the theories need not be compatible in any other sense!

interpret and connect these results—to 'match them up' and determine whether they are indeed compatible (this is the 'identificatory role', I.1.c., listed in §4).

Note that WR is a *special case* of Correspondence, that obtains when the overlap in the theories' domains of success is the *entire* domain of the older/less fundamental theory—i.e., domain subsumption. SR can also, in a sense, be thought of as a special case of correspondence, that achieves *in principle compatibility* via the establishment of *in principle derivability*. However, I argue that it is more natural to think of this as reduction, rather than mere correspondence. Correspondence has two features that make it a broader concept than reduction: firstly, it can hold between *any* two theories, provided they have some domain of overlap (i.e., they do not need to stand in a relation of relative fundamentality, general/special, or successor/succeeded); secondly, Correspondence need not be asymmetric. As well as being a broader concept than reduction, Correspondence is also weaker: it establishes compatibility, while reduction (SR) can do this plus establish stronger relations, such as relative fundamentality (as I discuss in §4 and §5). SR is thus more useful than Correspondence, because it can play all the same roles as Correspondence, and more.

4 Some roles of reduction

The numerous roles of Correspondence and reduction can be categorised into three broad types, which roughly conform to three stages of science: theory development, theory acceptance, and theory use. Additionally, though, we need to distinguish the roles (c) that these relations are able to play, according to which characteristic function (b) facilitates their doing so. Firstly, I list some of the roles that these relations are supposedly[15] able to play by virtue of establishing compatibility—these roles are all able to be satisfied equally-well by Correspondence, WR and SR. Following this, I list some of the roles that reduction is supposedly[16] able to play by virtue of establishing in-principle-derivability—these roles are exclusive to SR (and other derivational conceptions of reduction). As the examples in the lists show, in many cases, the roles of Correspondence and reduction are not fulfilled by any individual correspondence relations, but rather the whole idea of Correspondence or reduction once it has been established (i.e., once enough correspondence relations have been articulated between the two theories such that we believe they are compatible in the relevant domains). In §4.1, I illustrate how some of these roles feature in a particular scientific case-study.

I. ROLES PLAYED IN VIRTUE OF ESTABLISHING COMPATIBILITY
These roles are supposedly able to be played by any account of reduction or Correspondence that establishes (or is intended to establish) that the newer/more-fundamental theory, N, is able *in principle* to reproduce all the successful results of the older/less-fundamental theory O to within some acceptable degree of error, under appropriate

[15]Whether the relations can actually achieve any of these roles in any particular real scientific case, depends on many factors, most of which will be case-specific. I do not explore these here.

[16]See fn. 15.

conditions, and within the relevant domain (being the intersection of the domains of applicability of the two theories). Thus, these roles may be achieved by either WR or SR.

1. **Heuristic.** Roles of Correspondence and reduction in theory construction or development of N. As Radder (1991) states, and Bokulich (2008) explores in detail, these roles may be played by correspondence from O to N (rather than the more familiar 'from N to O'). Particular links between aspects of O and aspects of N (the theory in development), can, for instance, serve as:

    (a) **Guiding principles:** Tentative guides, or aspirations, that may or may not feature in the final formulation of N. As an example, consider Bohr, in his development of old quantum theory, using the harmonics of the classical motion of the electron in the initial stationary state as a heuristic 'selection rule' in judging what quantum transitions should be allowed between stationary states (Bokulich, 2008, 2014).

    (b) **Postulates:** Assumed as key features of N. An example is the equivalence principle of general relativity, which took a result of Newtonian mechanics (the equivalence of inertial and gravitational mass) and elevated it to the status of a postulate.

    (c) **'Data':** Since O is successful, it can act analogously to empirical data for the new theory to be built around (this also features strongly in the 'justificatory roles'). An example of this is the use of particular features of general relativity, such as Lorentz invariance and the metric structure of spacetime, as constraints (in the relevant domain) in developing a theory of quantum gravity (see §4.1).

2. **Justificatory.** Roles of Correspondence and reduction in theory acceptance; These roles are about legitimising the new theory, N, by appeal to the established theory, O. The diverse items on this list may be appealed to either, or both, as constraints, or means of confirmation. Constraints are *criteria of theory acceptance* (also referred to as 'criteria of success', or 'definitional' criteria), meaning that a new theory will not be accepted unless it satisfies these. Means of *confirmation*, on the other hand, are non-necessary, but desirable features that serve to increase credence in the theory.

    (a) **Preservation of success:** A constraint on N is that it be at least as successful as O. Compatibility ensures that the successes of O are not lost in the move to N; compatibility guards against (and so correspondence relations are invoked in order to minimise) 'Kuhn-losses', which include losses in the ability to explain certain phenomena whose authenticity continues to be recognised, losses of scientific problems (a narrowing of the field of research), an increased specialisation and increased difficulty in communicating with outsiders.[17] This role will be achieved most effectively by reduction rather than Correspondence more generally, since reduction shows that the relevant overlap in the domains

[17]Definition from Hoyningen-Huene (1993, p. 260).

of success of the two theories is the entire domain of success of O. Both relations may also be used to evaluate 'acceptable losses'.

(b) **Explanation of success:** A constraint on N is that it explain why O is as successful as it is. The old theory, from the perspective of the new theory, is (to some degree) incorrect, yet it is successful by virtue of being compatible with the new theory in the relevant domain. (Here, this justifies the adoption of N, yet this condition is also used to measure scientific progress, and, in a sense, justify the enterprise as a whole; see I.3.b, below).

(c) **Identification:** There is a requirement that N describes all the same systems as N, but at different scales, or otherwise under different conditions (N of course will typically also describe many more systems than O). Correspondence can help us identify that this is actually the case. However, Correspondence does not standardly demonstrate that one theory is more fundamental than another (i.e., that the behaviour described by one depends, in a sense, on that of the other). For this, reduction is required, II.3.i., below.

(d) **Problem-solving explanation:** N may explain, or 'explain away', features of O that are problematic, or otherwise apparently stand in need of explanation. The solution to particular such problems may be set as part of the criteria of acceptance of N (i.e., in the definition of what would count as a successful theory of N), or be unexpected successes of N that serve as additional evidence for its being correct (i.e., as means of confirmation).

An example is the 'measurement problem' of quantum theory, the solution of which is often viewed as a constraint on a successful theory of quantum gravity (i.e., a prospective theory may not be accepted unless it solves this). Another example, mentioned below (§4.1), is the resolution of general relativity singularities by quantum gravity.

(e) **Non-empirical confirmation:** As Dawid (2013) has claimed in his "meta-inductive argument", our credence in a theory N may be increased in virtue of its standing in particular relationships to established theories and frameworks; e.g., by N exemplifying some key features of O; empirical evidence for the other theories can indirectly also support N. (This, along with I.2.b., can also be understood as part of what Friedman (2001) calls "prospective communicative rationality" in a revolutionary transition, pr paradigm-change: framing N as being, in some sense, a "natural continuation" of the older framework).

An example of this is role is string theory's relationship with the framework of quantum field theory—the fact that string theory shares many correspondences with quantum field theory is taken by proponents as strong evidence in its favour (Dawid, 2013). Another example, mentioned below, is an instance of 'principle correspondence' between general relativity and loop quantum gravity (§4.1).

(f) **'Predictions':** Since O is successful, N unexpectedly recovering particular aspects of O in the relevant domain, can be viewed as support for N. In order for these 'predictions' to lend support for N, the recovered features of O should be those that are actually involved in O's success. (Interestingly, however, the justification here can go the other direction, so that, N's recovery of particular features of O may be taken as evidence of these features being the ones that are responsible for O's success—this 'mutual justification' is part of I.3.c., below). An example, mentioned below (§4.1), is the recovery of the correct spacetime dimension (in correspondence with general relativity), by an approach to quantum gravity known as causal dynamical triangulations. This example of 'numerical correspondence' serves to increase credence in the approach as being on the right track.

3. **Efficient** Roles in justifying the continued use of the older theory, O, through its relationship to N; or refining, or correcting O through its relationship to N:

   (a) **Practical:** O may be more efficient to practically apply in a given situation (weighing up factors such as computational time and effort versus accuracy and precision), and such use is legitimated because O is relevantly compatible with the new theory N, and its successes retained (it is interesting to note, comparing with the 'justificatory roles', that N and O are used to mutually justify one another).

   (b) **Retrospective rationality:** N should explain why O is as successful as it is, in order to maintain the impression of scientific progress, and the connection between a theory's being successful and its being approximately correct. Thus, this role is not just about justifying O, but the scientific enterprise itself. O, from the perspective of N, is (to some degree) incorrect, yet it is successful by virtue of being compatible with N in the relevant domain—which, in this case, is the *entire* domain of success of O (note, this is not necessarily the case in I.2.b., which is the justification of N). Thus, this role is achieved by reduction only (not Correspondence). One account of retrospective rationality is Friedman (2001), which describes it as the demonstration that the old paradigm (or, really, theories within it) is contained within the new one, as an approximate special case. For Friedman, this is an important activity that occurs during "revolutionary transitions" of scientific theory-change.

   (c) **Revealing redundancy:** The new theory may reveal unphysical aspects of the older theory, or features that are unnecessary for (or not directly involved in) the success of O—for instance, by not matching-up with (or failing to recover) particular features of O, or by failing to link to O in domains where O has not been directly tested.

   (d) **Correcting:** N may not correspond to O, but a corrected version, O^* (Schaffner, 1976).

   (e) **Further development:** The "inverse correspondence principle" refers to cases where problems in N are used to guide the further development of O (Sánchez-Ron, 1983).

II. ROLES PLAYED IN VIRTUE OF ESTABLISHING DERIVABILITY
These roles are supposedly able to be played by any account of reduction that establishes (or is intended to establish) that the successful aspects of the older/less-fundamental theory, O, are able in principle to be derived from the newer/more-fundamental theory, N, to within some acceptable degree of error, under appropriate conditions, and within the relevant domain. Thus, these roles are are able to be established by SR.

2. **Justificatory.** (See description in above list).

   (i) **Demonstrating relative fundamentality:** If O reduces to N (using the 'philosopher's convention' of terminology, fn. 5), then the laws of O may be said to *depend upon* the physics described by N. The idea is that, if the successful parts of O are (appropriately) able to be derived from N, then O is, in a sense, "embedded within" N (and we can thus say that the physics described by N is entirely responsible for O's success—see item 3(f), below). This may be used to justify the adoption of N.

3. **Efficient.** (See description in above list).

   (f) **Explanation:** 'Higher level' regularities, described by O, can be explained by showing that their descriptions are derivable (to within some acceptable degree of error, under appropriate conditions, and within the relevant domain), from 'lower level' ones, described by N. Note that this requires that N be more fundamental than O, i.e., that the behaviour described by N is in some (non-causal) sense (at least partially) 'responsible for' that described by O (In other words, it is what enables us to label the two theories as 'lower level' and 'higher level'). This demonstration is achieved by the same relation, of reduction (II.2.i., above). Reductive explanation does *not*, however, require strict deducibility, nor that it hold between entire theories, and nor that these be formulated in first-order languages (Sarkar, 2015).

   (g) **Unification:** Two senses can supposedly be achieved by reduction: 1. The demonstration of the unity of science, *à la* Oppenheim & Putnam (1958), for example, and 2. Unification in the physicists' sense, where two distinct concepts, laws, etc., that (typically) feature in different theories O_1 and O_2, are shown to be consequences of some single entity in a more fundamental theory, N. Both of these senses of unification require that N be more fundamental than O.

4.1 Example: quantum gravity

Quantum gravity (QG) is the as-yet-undiscovered theory that describes the phenomena at the intersection of the domains of necessity of both general relativity (GR) and quantum field theory (QFT); i.e., it describes the domains in which both general relativistic and quantum field theoretic effects are supposed to be non-trivial. These domains include, for instance, the Planck scale (10^{-35}m), black holes, and cosmological singularities (such as the 'big bang'). Part of the difficulty with finding a theory is the extreme nature of these regimes, which preclude direct experimental testing (although tests in accessible regimes are not ruled out), and currently there are no unequivocal data that QG is definitely required to explain (although there are potential candidates). This empirical disconnect means that more weight has fallen on other guides to theory construction and evaluation, including those offered by reduction and Correspondence that appear in the lists above.[18]

Although there is currently no theory of QG, there are several different approaches towards a theory (i.e., different research programs), including string theory, loop quantum gravity (LQG), causal set theory, causal dynamical triangulations, and group field theory (to name but a few).[19] The plurality of approaches is due, in part, to the fact that it is unclear what an acceptable theory of QG would look like. Apart from the minimal characterisation just mentioned, there is little agreement as to the criteria of theory success. Significantly, the 'recovery' of GR from QG is perhaps one of *the only* generally-agreed upon constraints on QG. Because it is part of the definition of QG that it subsume the domain of GR, this 'recovery' must mean reduction (either WR or SR, §2.2); meanwhile, its relationship to QFT (and to particular QFTs) may be the weaker one, of Correspondence (Crowther, 2017). Note that the different approaches are all in various stages of development and maturity—none, so far, are complete theories, and thus we cannot expect reduction to hold at this stage. Nevertheless, there are many individual correspondence relations being exhibited, developed, and appealed to across all the approaches, with the ultimate aim of establishing their reduction to GR (on the 'physicists' convention' of terminology, fn. 5).

Illustrating the heuristic roles of Correspondence (I.1.), many of the approaches to QG start from current theories and 'work down'; for example, *canonical* and *covariant* approaches to QG begin by quantising (parts of) GR, using different methods.[20] As well as being useful points of departure, current theories serve as 'empirical anchors' because of their established success (I.1.c.); thus correspondence relations with GR and QFT are also used as means of justification for different approaches to QG (I.2.)—although at this stage, these individual correspondence relations do not establish WR or SR for any of the approaches, they nevertheless lend weight to the approaches which demonstrate them. Some of the key principles of current theories are adopted as guiding principles in the search for QG (I.1.a.); for instance, background independence—which was an appreciable issue in the development of GR, and is exemplified in GR's substantive[21] *general covariance*—is a significant principle in LQG (Rovelli, 2004). Additionally, the fact that the use of this principle in LQG successfully resolved some (otherwise apparently unresolvable) problems in the development of the approach is taken as a means of non-empirical confirmation (I.2.e.), serving to increase credence in LQG as being on the right track.

An example of principle correspondence is the relationship of QG approaches to

[18]This section draws heavily from Crowther (2017), please refer to this for more details.

[19]See, respectively: Polchinski (1998a,b); Rovelli (2004) and Rovelli & Vidotto (2014); Henson (2009); Ambjorn et al. (2012); Oriti (2012).

[20]Accessible introductions to QG include Kiefer (2006); Rickles (2008).

[21]Cf. Norton (2003).

Lorentz invariance: the characteristic symmetry of special relativity, which is well-supported by current spacetime theories, as well as recent experimental tests designed to detect possible Planck-scale violations.[22] Lorentz invariance is thus a strong choice as a guiding principle (I.1.a), postulate (I.1.b), and 'datum' (I.1.c) for QG. It is used in all three of these heuristic roles in causal set theory, where it is achieved through the random 'sprinkling' process by which causal sets are constructed, which ensures there is no preferred frame that results (Dowker et al., 2004). The ability of causal set theory to provide models that are consistent with Lorentz invariance in the relevant limit serves also in the justificatory role of non-empirical confirmation (I.2.e).

As these examples show, successfully using correspondence relations from O to N in the heuristic role of theory-construction can also serve as part of theory-justification, even if such correspondences are "built in" by hand, rather than derived as 'predictions'. However, having correspondences in this way is no guarantee of having them in the 'recovery' direction, from N to O, which is the one more strongly associated with theory-justification. To see this, note that both LQG and causal set theory describe structures that correspond to spacetimes, because these structures are originally *constructed from* spacetimes (i.e., models of GR)—using heuristic, O to N correspondences. And yet both approaches have difficulty recovering spacetime (i.e., models of GR) from the multitudes of other possible structures described by their theories. They each seek a dynamics that naturally 'picks out' the structures in their theory that correspond to spacetimes in the appropriate domains.[23]

The justificatory roles of preservation of success (I.2.a) and explanation of success (I.2.b) are seemingly only demonstrable post-hoc, once we have a close-to-fully-developed theory that is otherwise acceptable as a replacement for GR in the relevant domains. However, string theory provides an example of how (I.2.b) might work, with one of its correspondences: in order for the theory to be well-defined, the background spacetime containing the string must satisfy an equation that has the Einstein field equations (the central equations of GR) as a large-distance limit. If string theory replaces GR, then this correspondence may be appealed to in order to explain the success of GR: even though GR is only 'approximately correct', it works because it features, or approximates, some aspects of string theory. In other words, the idea is that GR is successful partly in virtue of employing the Einstein field equations in the appropriate domain.

The shared importance of these equations is an example of law correspondence (§3). Huggett & Vistarini (2015) argue that the background metric field that features in the theory is composed of stringy excitations and, given that it satisfies the Einstein field equations, is to be identified with the gravitational field. Hence, this correspondence also plays the identificatory role (I.2.c), which justifies the theory through the demonstration that it describes the same systems as GR—as required by any acceptable theory of QG. Additionally, these correspondence relations would be part of establishing that GR *reduces to* string theory (on the 'philosopher's convention' of terminology); if this were achieved, then the correspondence relations would serve in the justificatory role of establishing

[22]See, e.g., Collins et al. (2009); Liberati & Maccione (2011).

[23]This is related also to the issue of *emergence*, cf. Crowther (2016); Wüthrich (2017).

relative fundamentality (II.2.g), and *all* of the efficiency roles (I.3.a.–II.3.f), including the explanation of the higher-level behaviour described by GR—e.g., QG may describe the 'atoms of spacetime', i.e., its constituents[24]; or, if gravity is conceived of as a *force* akin to the other fundamental forces, then QG might provide a 'mechanistic explanation' (loosely speaking) of how the force is constituted, and an explanation of why gravity is so 'weak' compared to the other fundamental forces[25].

An example of the problem-solving explanatory role (I.2.d), is the goal of QG to explain or 'explain away' the problematic singularities in GR (e.g., those of black holes and the big bang). Depending on the approach under consideration, this may be set as a criterion of acceptance of QG (i.e., part of the definition of the theory), or just serve as a means of confirmation (i.e., an added bonus of an otherwise acceptable theory, and additional evidence of its being correct). An example of a correspondence acting as a 'prediction' (I.2.f) in QG is the recovery of the correct spacetime dimension—in accordance with GR, in the appropriate domains—by causal dynamical triangulations (Ambjørn et al., 2004). This numerical correspondence lends support for the approach, given that the dimension of spacetime in GR is likely involved in the theory's success.

There is one more peculiarity of QG that I must mention: the theory is expected by many researchers to not only profoundly alter our current understanding of spacetime (as described by GR), but perhaps to not feature conceptions of space and time at all. (This is why, although there should be *explanatory* reductions (in Wimsatt's sense, §2.1) between QG and GR, these would be difficult to properly interpret as compositional, mechanistic, or causal—all of which seem to involve spatiotemporal notions). This leads to two particular (but related) problems, to which Correspondence and reduction can provide solutions. These are the problems of establishing *local beables* and *empirical coherence* for a theory that does not feature notions of space and time. The idea of local beables comes from John Bell, who meant it to refer to the things that we take to be real, and which are definitely associated with particular spacetime regions. The issue is that a theory without local beables is not only apparently unable to be experimentally verified (since our experiments necessarily only involve local beables), but that it may be empirically incoherent: its means of verification may undermine the reasons for believing it correct. Huggett & Wüthrich (2013), however, show how—for a variety of QG approaches—one can potentially derive local beables, and thus avoid the challenge of empirical incoherence. In some cases, this is done by establishing correspondence relations with GR, but in others it is not necessary to make contact with full GR spacetime in order to find a notion of local beables (and so correspondence is not necessarily required for these two roles).

Thus, illustrating the unexpected utility and versatility of reduction and Correspondence are two additional, specialised roles (which rely just on the establishment of *compatibility*, rather than derivability), are:

I.2. Justificatory.

(g) **Local beables:** For a theory lacking conceptions of space and time (and thus

[24] See, e.g., Dowker (2005); Oriti (2014); Padmanabhan (2016).
[25] This is the case, e.g., in string theory, (Polchinski, 1998a,b)

"local beables" (Bell, 1987)), making contact with established spatiotemporal theories is one means of deriving local beables (Huggett & Wüthrich, 2013). (The possession of local beables is a criterion of acceptance, but note that neither Correspondence nor reduction are necessary means of achieving it).

(h) **Empirical coherence:** Experimental testing is necessarily carried out in space and time, so the means of testing a theory that says there is no space and time may undermine the reasons for believing the theory correct; correspondence with established spatiotemporal theories is one means of avoiding this problem of "empirical incoherence" (Huggett & Wüthrich, 2013). (Empirical coherence is a criterion of acceptance, but note that neither Correspondence nor reduction are necessary means of achieving it).

5 The characteristic function of reduction

The characteristic function of reduction is typically assumed to be *explanation*: either the explanation of higher-level theories in terms of lower-level ones (in the case of reduction$_1$), or the explanation of the success of a replaced theory by the theory that replaces it (in the case of reduction$_2$). As I have argued, both of these are indeed roles of reduction—however, they are not the characteristic functions of the relation. I briefly re-state these roles. The role that is typically thought to characterise reduction$_2$, explanation of success (I.2.b.), has the success of O explained by being in principle *compatible* with N, ECD—i.e., that the two theories share the same results (to within some acceptable degree of error, under appropriate conditions, and within the relevant domain). This role does not depend upon 'in principle derivability' of O from N, but just requires that the new theory, N, is accepted as correct. Thus, this role is able to be played by Correspondence (including WR) as well as SR. It is not exclusive to reduction. (This does not, of course, mean that reduction is not explanatory; and note too that if *all* the successes of O are to be explained in this way, then at least WR is required).

The role usually thought to characterise reduction$_1$ is the idea of deductive explanation. As mentioned in (II.3.f.), the weaker, more realistic sense of this idea does not require strict deduction but rather the establishment (i.e., the scientific-consensus belief) that the older theory be derivable in principle (to within some acceptable degree of error, under appropriate conditions, and within the relevant domain) from the newer one: the less-fundamental behaviours described by O are thereby explained. This explanation requires that the behaviour described by O be dependent on that of N; this demonstration of relative fundamentality is also provided by the correspondence relations that establish in-principle derivability.[26] Thus, the characteristic function of reduction, by virtue of which

[26] Recall that derivability (which may be established using relations of correspondence which need not be asymmetric) is being used to demonstrate the asymmetric relation of reduction: i.e., that all the successful parts of the reduced theory can (approximately and appropriately) be obtained from the reducing theory. The idea of dependence, or relative fundamentality, is that the reduced theory is thus shown to be embedded within the reducing theory (and hence that the physics described by the reduced theory depends on that of the reducing theory).

these roles are achieved, is the establishment of *in principle derivability.* This is part of *SR*, the stronger understanding of reduction.

The characteristic function of reduction is, I argue, the establishment of in principle derivability. It is by virtue of achieving this that reduction is able to play all of the heuristic, justificatory and efficiency roles listed above. (Though note that the heuristic roles utilise the idea that in principle derivability *will be* established, rather than its actual establishment, since these roles pertain to a theory-in-development, rather than holding between two sufficiently-developed theories). The weaker conception of reduction (WR, being a special case of Correspondence), as the establishment that the newer theory can in principle reproduce all the results of the older one is insufficient for achieving the roles in part II of the list above.

6 Conclusion

The numerous, diverse—and in some cases, particular and unexpected—roles of reduction are achieved by the relation establishing that an older and/or less-fundamental theory[27], O is derivable in principle from a newer and/or more-fundamental theory, N, to within some acceptable degree of error, under appropriate conditions, and within the relevant domain. These roles go far beyond the typical function assigned to reduction, of explaining O by reference to N. I have argued that they may be achieved by either reduction$_1$ or reduction$_2$, since either of these conceptions of reduction may be used to establish the *in principle derivability* of O from N (to within some acceptable degree of error, under appropriate conditions, and within the relevant domain). This is using the more accurate, broader conception of reduction$_2$, on which it is not simply a limiting relation (as clarified in §1.1 and §2.1, which address some common misunderstandings associated with this conception of reduction). Thus, here I suggest that reduction$_1$ and reduction$_2$ are not essentially different in respect to the roles they play in science.

Yet, most of the roles of reduction can in fact be achieved by a weaker relation, Correspondence, that establishes just the *in principle compatibility* of the two theories. That is, that N is able in principle to reproduce all the successful results of O to within some acceptable degree of error, under appropriate conditions, and within the relevant domain—which is the overlap of the domains of success of the two theories involved. A special case of Correspondence (which above, I referred to as WR, the weaker notion of reduction) is *domain subsumption*, when the overlap in the domains of success of the two theories is the entirety of the domain of success of O.

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[27]Keeping in mind the qualifications regarding my use of this term, outlined on p.3.

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