Chapter 6
Inferentialism and Connexivity

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Abstract This paper investigates the relationships between two claims about conditionals that are often discussed separately. One is the claim that conditionals express inferences, in the sense that a conditional holds when its consequent can be inferred from its antecedent. The other is the claim that conditionals intuitively obey the characteristic principles of connexive logic. Following a line of thought that goes back to Chrysippus, we suggest that these two claims may coherently be understood as distinct manifestations of a single and more basic idea, namely, that a conditional holds when its antecedent is incompatible with the negation of its consequent. The account of conditionals we propose is based precisely on this idea.

Keywords: Inferentialism, Connexivity, Evidential conditional, Incompatibility, Chrysippus.

6.1 Overview

The two claims investigated in this paper have received growing attention in the most recent literature on conditionals, although they have been discussed in quite different circles and from different angles. The first claim, which we call *inferentialism*, is that a conditional $p \rightarrow q$ holds when q can be inferred from p, or equivalently when p provides a reason for accepting q.[1] As long as the term 'valid' is understood in a sufficiently broad way, which includes both conclusive and defeasible reasoning, this claim may be phrased as follows: a conditional holds when the inference from its antecedent to its consequent is valid. The idea that true conditionals amount to valid arguments goes back to the Stoics. Sextus Empiricus reports the Stoic view as follows:

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[1] We take the term 'inferentialism' from Douven and colleagues, see [21] and [12].

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> The conclusive argument is sound, then, when after we conjoin the premises and create a conditional that begins with the conjunction of the premises and finishes with the conclusion, this conditional is itself found to be true.[2]

Iacona uses the term 'Stoic Thesis' to denote the view that an argument is valid if and only if the corresponding conditional is true.[3] Since inferentialists regard the left-hand side of this biconditional as an analysis of its right-hand side, the terms 'inferentialism' and 'Stoic Thesis' may be used as synonymous.

Inferentialism is an intriguing view because it accounts for some central aspects of the ordinary use of conditionals. Consider the following examples, which concern a series of tosses of a fair coin and Real Madrid's football season:

(1) If the first 5 tosses are all heads, there will be at least 5 heads in the first 1.000 tosses.
(2) If Real Madrid loses the first 10 matches, its coach will be fired.
(3) If Real Madrid loses the first 10 matches, there will be at least 5 heads in the first 1.000 tosses.
(4) If Real Madrid loses the first 10 matches, 5+5=10.

Intuitively, (1)-(4) are not all equally compelling: while (1) and (2) seem perfectly reasonable, (3) sounds definitely odd, and (4) may produce some puzzlement at least. These intuitive differences cause explanatory troubles to most extant account of conditionals. On the material account, where $p \to q$ is read truth-functionally as $p \supset q$, (1)-(4) are all true as long as their antecedent is false, which is very likely. On Adams' probabilistic account, according to which the acceptability of $p \to q$ is measured by $P(q|p)$, the conditional probability of q given p, (1)-(4) are all highly acceptable, due to the high probability of their consequent given their antecedent.[4] On the Stalnaker-Lewis modal account, according to which $p \to q$ is true just in case q is true in the closest world, or worlds, in which p is true, (1)-(4) are all true, given that their consequent is true in the closest worlds in which their antecedent is true.[5] A similar result is obtained on the belief revision account due to Gärdenfors and others, according to which $p \to q$ is acceptable just in case q belongs to the belief state obtained by adding p to one's set of beliefs.[6] All the accounts just considered seem to miss one crucial fact, namely, that in (1) and (2) the inference from the antecedent to the consequent seems justified, while the same does not hold for (3) and (4). This is precisely the kind of intuition that inferentialism intends to capture.

The second claim, which we call *connexivity*, is that an intuitively adequate theory of conditionals should vindicate some central principles of connexive logic. Here we will focus on a set of principles which we take to be prominent, without

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[2] Sextus Empiricus, *Against the Logicians*, II, 417, in [37], p. 171. Scholars also agree that the Stoic notion of a valid argument was not restricted to formal validity, and might easily have included inferences that are now broadly classified as inductive rather than deductive. See [2], p. 123.

[3] [19].

[4] [1].

[5] [38], [25].

[6] [15].

any intention to suggest that they are equally important, or that they provide an exhaustive characterization of connexive systems as distinct from other systems of conditional logic. Let us start with the following four principles, which we will call *basic connexive principles*:

P1 $\neg(\neg p \rightarrow p)$
P2 $\neg(p \rightarrow \neg p)$
P3 $\neg((p \rightarrow q) \wedge (p \rightarrow \neg q))$
P4 $\neg((p \rightarrow q) \wedge (\neg p \rightarrow q))$

P1 and P2 are alternative formulations of *Aristotle's Thesis*.[7] P3 is *Abelard's First Principle*, sometimes phrased in conditional form as *Weak Boethius Thesis*: $(p \rightarrow q) \supset \neg(p \rightarrow \neg q)$.[8] P4 is *Aristotle's Second Thesis*.[9] To these four principles one may add the following, which we will call *embedded connexive principles*:

P5 $(p \rightarrow q) \rightarrow \neg(p \rightarrow \neg q)$
P6 $(p \rightarrow \neg q) \rightarrow \neg(p \rightarrow q)$
P7 $(p \rightarrow q) \rightarrow \neg(\neg p \rightarrow q)$
P8 $(\neg p \rightarrow q) \rightarrow \neg(p \rightarrow q)$

P5-P8 are more complex than P1-P4 in that they contain embedded occurrences of $\rightarrow$. P5 and P6 are alternative formulations of *Boethius Thesis*, which is stronger than Weak Boethius Thesis on the assumption that $\rightarrow$ is stronger than $\supset$.[10] Finally, P7 and P8 may be called *Boethius Left Thesis*.[11] These two principles bear to P4 the same relation that P5 and P6 bear to P3: if one replaces the main connective in P7 and P8 with $\supset$, one obtains a weaker claim which is equivalent to P4.

The eight principles just listed also have restricted versions, which may be spelled out as follows:

P9 $\Diamond\neg p \supset \neg(\neg p \rightarrow p)$
P10 $\Diamond p \supset \neg(p \rightarrow \neg p)$
P11 $\Diamond p \supset \neg((p \rightarrow q) \wedge (p \rightarrow \neg q))$
P12 $\Diamond\neg q \supset \neg((p \rightarrow q) \wedge (\neg p \rightarrow q))$
P13 $\Diamond p \supset ((p \rightarrow q) \rightarrow \neg(p \rightarrow \neg q))$
P14 $\Diamond p \supset ((p \rightarrow \neg q) \rightarrow \neg(p \rightarrow q))$
P15 $\Diamond\neg q \supset ((p \rightarrow q) \rightarrow \neg(\neg p \rightarrow q))$
P16 $\Diamond\neg q \supset ((\neg p \rightarrow q) \rightarrow \neg(p \rightarrow q))$

P9-P12 may be called *restricted basic connexive principles*, while P13-P16 may be called *restricted embedded connexive principles*. One main reason for regarding

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[7] The reference is to Aristotle, *Prior Analytics* 57b14, where a statement of the form 'If not-p, p' is rejected as impossible.

[8] See [41]. The reference is to Boethius, *De Syllogismo Hypothetico* 843D. This principle is not mentioned in McCall's characterization of connexivity, see [27], p. 435. In [1], P3 is called 'principle of subjunctive contrariety'. Another label is 'Strawson's Law', see [13].

[9] This label is used in [18]. In [31], P4 is called 'Secondary Boethius Thesis'.

[10] See [41].

[11] In [14], P7 and P8 are called 'Boethius $\neg l$ Thesis', where l stands for 'left'.

P9-P16 as possible alternatives to P1-P8 is that arguably P1-P8 draw their intuitive appeal from sentences that involve ordinary contingent antecedents and consequents, that is, sentences which are also vindicated by P9-P16. For example, the following sentence instantiates P1, so it is true on the assumption that P1 is valid:

(5) It is not the case that, if it is not snowing, it is snowing

But the same holds on the assumption that P9 is valid, given that the antecedent of P9 is satisfied: it is possible that it is not snowing. Similar examples can easily be found for P2-P8 and P10-P16 respectively. In other words, even though P1-P8 are stronger than P9-P16, it seems that P9-P16 provide a sufficiently strong measure of intuitive adequacy.

In this paper, the claim that conditionals intuitively obey the characteristic principles of connexive logic will be understood as follows: an intuitively adequate theory of conditionals must validate at least some version of the basic connexive principles, that is, either P1-P4 or P9-P12. Note that none of the accounts of conditionals considered above is connexive in this sense, in that they validate at most a proper subset of the basic connexive principles.

Inferentialism and connexivity are distinct claims motivated by different kinds of considerations, so they may easily be regarded as independent from each other. There is no obvious answer to the following question: if a theory of conditionals were explicitly designed to be inferentialist, would it also be connexive? Similarly, there is no obvious answer to the following question: if a theory of conditionals were explicitly designed to be connexive, would it also be inferentialist? In fact the two issues, by and large, are discussed separately.

This paper instead suggests that inferentialism and connexivity, at least on one coherent understanding of them, are closely related. Sections 6.2-6.4 discuss three accounts of conditionals that qualify as both inferentialist and connexive to some extent, and point out some of their limits. Section 6.5 introduces the central idea of the paper, namely, that a conditional holds when its antecedent is incompatible with the negation of its consequent. Section 6.6 outlines our own account, the evidential account, which rests on this idea. Finally 6.7 explains how the evidential account holds together inferentialism and connexivity.

6.2 The strict account

The first account to be discussed, the *strict account*, says that $p \rightarrow q$ is true just in case it is impossible that p is true and q is false, that is, $p \rightarrow q$ is read as $\Box(p \supset q)$. A strict conditional is true when the corresponding material conditional is necessarily true.

The strict account is a classical alternative to the material account. Historically, the contrast between these two accounts goes back to the divergence between Philo and Diodorus as described by Sextus Empiricus:

> Philo says that a sound conditional is one which does not begin with a truth and end in a falsity (e.g. when it is day and I am conversing, ‘If it is day, I am conversing’). Diodorus says that it is one which neither could nor can begin with a truth and end in a falsity. According to him, the conditional just stated seems to be false, since if it is day but I shall be silent, it will begin with a truth but end in a falsity. But ‘If it is not the case that there are indivisible elements of existing things, there are indivisible elements of existing things’ is true — for it will always begin from something false, viz. ‘It is not the case that there are indivisible elements of existing things’, and — according to him — end in something true, viz. ‘There are indivisible elements of existing things’.[12]

The two examples mentioned by Sextus Empiricus in this passage can be phrased as follows, replacing ‘indivisible elements of existing things’ with ‘atoms’:

(6) If it is day, I am conversing
(7) If there are no atoms, there are atoms

While both (6) and (7) may be true according to the material account, only (7) is true according to the strict account, for it is possible that the antecedent of (6) is true but its consequent is false.

The strict account is inferentialist in a straightforward sense: as long as validity is understood as necessary truth preservation, in accordance with the classical deductive definition, this account implies that a conditional is true just in case the corresponding argument is valid. Some logicians of the past endorsed inferentialism so construed. For example, the equation between true conditionals and deductively valid arguments underlies the treatment of strict implication suggested by C. I. Lewis:

> These are the vital distinctions of the ordinary meaning of “implies” — for which ‘p implies q’ is equivalent to ‘q can validly be inferred from p’ — from that implication which figures in the algebra.[13]

The strict account is connexive as well, as it validates P9-P12, given some widely accepted assumptions about the underlying modal logic. In what follows we will take for granted that $\Box$ obeys S5, which we regard as a natural choice in this context. Consider P9. If $\neg p$ is true in some world, $\Box(\neg p \supset p)$ is false, so $\neg(\Box(\neg p \supset p))$ is true. P10 holds for a similar reason. Consider P11. If p is true in some world, either q or $\neg q$ is false in that world. It follows that $\Box(p \supset q) \wedge \Box(p \supset \neg q)$ is false, hence $\neg((\Box(p \supset q) \wedge \Box(p \supset \neg q))$ is true. Consider P12. If q is false in some world, either p or $\neg p$ is true in that world. It follows that $\Box(p \supset q) \wedge \Box(\neg p \supset q)$ is false, hence $\neg((\Box(p \supset q) \wedge \Box(\neg p \supset q))$ is true.

Note that the strict account also validates P13-P16. P13 and P14, like P11, hold in virtue of the falsity of $\Box(p \supset q) \wedge \Box(p \supset \neg q)$, provided that p is true in some world. Similarly, P15 and P16 hold in virtue of the falsity of $\Box(p \supset q) \wedge \Box(\neg p \supset q)$, provided that q is false in some world. The strict account does not validate P1-P8, though, because P1-P8 have counterexamples when the antecedent condition of P9-P16 is not satisfied. However, as noted above, this is not really a problem as long

[12] Sextus Empiricus, *Outlines of Scepticism*, 159, in [36], pp. 95-96.
[13] [24], p. 529.

as it is granted that the intuitive core of the connexive principles boils down to implications that P1-P8 share with P9-P16.[14]

The strict account has some virtues, as it validates some logical principles that are definitely plausible in an inferentialist perspective, while it invalidates other logical principles that are definitely not plausible in an inferentialist perspective. An example of the first kind is *AND*, the principle according to which $p \rightarrow q$ and $p \rightarrow r$ entail $p \rightarrow (q \wedge r)$. This principle arguably holds for reasons in general: if p is a reason for q, and p is a reason for r, it seems to follow that p is a reason for $q \wedge r$. A similar example is *OR*, the principle according to which $p \rightarrow r$ and $q \rightarrow r$ entail $(p \vee q) \rightarrow r$. This principle is also intuitive: if each of p and q is a reason for r, it seems to follow that $p \vee q$ is a reason for r. An example of the second kind is *Conjunctive Sufficiency*, the principle according to which $p \wedge q$ entails $p \rightarrow q$. Clearly, it may happen that p and q both hold but are totally unrelated, so that p is not a reason for q.

However, the strict account is not fully satisfactory as an inferentialist theory of conditionals. Its main shortcomings stem from the fact that it adopts a deductive criterion, necessary truth preservation, as a condition for the truth of conditionals. At least three crucial implications of this fact deserve attention, as they may easily be perceived as undesirable results in an inferentialist perspective.

First, the strict account validates *Monotonicity*, the principle according to which $p \rightarrow q$ entails $(p \wedge r) \rightarrow q$ for any r. This principle is at odds with the very idea that $\rightarrow$ represents defeasible inference: on a widespread understanding of defeasibility, to say that p is a defeasible reason for q is to say that, for some r, $p \wedge r$ is not a reason for q. As long as Monotonicity is granted, it is hard to see how the theory can explain the difference between intuitively good and intuitively bad defeasible inferences. Consider the sentences (2) and (3) in our initial list of examples provided in section 6.1. On the strict account, (2) and (3) turn out to be both false, given that the truth of their antecedent does not rule out the falsity of their consequent.[15]

Second, the strict account validates *Right Weakening*, the principle according to which $p \rightarrow q$ entails $p \rightarrow r$ whenever $q \models r$, where $\models$ is classical logical consequence. Although Right Weakening may be perfectly reasonable when one restricts consideration to conclusive reasons, it becomes more problematic as a rule for reasons in general. Arguably, it may be the case that p is a reason for q without thereby being a reason for r, in spite of the fact that $q \models r$, because by weakening the conclusion the positive relevance of the premise can decrease or get lost. So it might be contended that an inferentialist theory of conditionals should *not* validate Right Weakening.[16]

Third, the strict account validates *Necessary Consequent*, the principle according to which $\Box q$ entails $p \rightarrow q$ for any p, and *Impossible Antecedent*, the principle

[14] [18] argues that the strict account preserves the intuitive content of the connexive principles.

[15] As [17] suggests, an advocate of the strict account could explain the apparent failure of Monotonicity in ordinary language by adopting some moderately revisionary hypotheses about the formal representation of conditionals in a modal language. However, such hypotheses are not generally accepted as part of the strict account.

[16] [35] and [7] provide some arguments against Right Weakening.

according to which $\neg\Diamond p$ entails $p \rightarrow q$ for any q.[17] It might be argued that these two principles are at odds with the pretheoretical understanding of reasons. Consider the sentence (4) in the list of examples provided in section 6.1. This sentence may cause some puzzlement, as observed in section 6.1, given that there is no clear intuition to the effect that its antecedent provides a reason for accepting its consequent. A similar example for Impossible Antecedent is obtained by contraposing (4). Of course, much here depends on one's inclinations towards classical logic. But at least it is an open question whether an inferentialist theory of conditionals should validate Necessary Consequent and Impossible Antecedent.

6.3 Douven's threshold/increment account

The second account, which we call the *threshold/increment account*, has been suggested by Douven. On this account, $p \rightarrow q$ is acceptable just in case two conditions are satisfied: (i) $P(q|p)$ is high enough (relative to a threshold greater than 0.5), (ii) p gives some evidential support to q in the sense that $P(q|p)$ is higher than $P(q)$, the *un*conditional probability of q itself. The second condition, which marks a key difference with respect to Adams' probabilistic account, is intended to capture the intuition that p must be relevant to q. In Douven's framework, a logic for $\rightarrow$ is developed from the idea of acceptability preservation for all probability distributions and all thresholds.[18]

The threshold/increment account is inferentialist in that it assumes that q can be inferred from p when (i) and (ii) are satisfied. In other words, acceptable conditionals are taken to correspond to valid arguments, in a sense of 'valid' that is not purely deductive and covers defeasible inferences.

As far as connexivity is concerned, it must be noted that the threshold/increment account, due to expressive limitations of its language, cannot validate the embedded connexive principles. Nonetheless it might be classified as connexive, as it validates some version of the basic connexive principles. More precisely, it validates P1, P2, P11, and P4. Consider P1. If $0 < P(p) < 1$, then $P(p|\neg p) = 0$, so $\neg p \rightarrow p$ is not acceptable because (i) is not satisfied. If $P(p) = 1$ or $P(p) = 0$ (ii) cannot be satisfied. In both cases, $\neg(\neg p \rightarrow p)$ is acceptable. A similar reasoning shows that P2 holds. Consider P11. If $P(p) > 0$, either $P(q|p)$ or $P(\neg q|p)$ is lower than or equal to the 0.5 threshold, so either $p \rightarrow q$ or $p \rightarrow \neg q$ is unacceptable, which makes acceptable the negation of their conjunction. Note that we don't get the same result on the supposition that $P(p) = 0$, for Douven, following Adams, assumes that $p \rightarrow q$ and $p \rightarrow \neg q$ are both acceptable in that case, provided that $0 < P(q) < 1$. This is why P3 does not hold.[19] Now consider P4. If $P(\neg q) = 0$, then $P(q) = 1$, so neither $P(q|p)$ nor $P(q|\neg p)$ can be greater than $P(q)$, against (ii). If $P(\neg q) > 0$,

[17] These two principles have been widely discussed at least since the middle ages, under the label *Necessarium a quodlibet* and *Ex impossibili quodlibet*.

[18] See [11].

[19] [11], pp. 179-180.

then $P(q) < 1$. If $P(p) > 0$ and $P(\neg p) > 0$, then either $P(q|p)$ or $P(q|\neg p)$ is lower than or equal to $P(q)$, again against (ii). If $P(p) = 0$, then $P(q|\neg p) = P(q)$, and if $P(\neg p) = 0$, then $P(q|p) = P(q)$. So $p \to q$ and $\neg p \to q$ are never jointly acceptable, which makes the negation of their conjunction always acceptable.

One virtue of the threshold/increment account is that it yields a plausible explanation of the examples (1)-(4) listed in section 6.1. (1) and (2) turn out to be acceptable because in both cases the conditional probability of the consequent given the antecedent is high enough, and also higher than the unconditional probability of the consequent. On the other hand, (3) and (4) turn out to be unacceptable, despite the high conditional probability of the consequent given the antecedent, because that conditional probability is just as high as the unconditional probability of the consequent. This explanation plausibly implies a direct relation between acceptability and probabilistic relevance: in an acceptable conditional the antecedent is relevant for the consequent in the sense that, assuming the former, the probability of the latter is higher than it would be otherwise.

From the logical point of view, however, the threshold/increment account is not ideal. This account is surely effective in avoiding logical principles that are dubious for inferentialists. At least three cases deserve attention: Monotonicity, Right Weakening, and Conjunctive Sufficiency. These three principles are invalid according to Douven's theory, which we take to be a desirable result. But the threshold/increment account is not equally effective in preserving logical principles that seem plausible for inferentialists. Here two examples are AND and OR. These two principle are not valid according to Douven's theory.

6.4 Rott's difference-making account

The third account, the *difference-making account*, has been developed by Rott from the idea that $p \to q$ holds when p makes a difference as concerns the credibility of q. The notion of difference-making can be spelled out in terms of possible worlds interpreted epistemically as belief states. More precisely, $p \to q$ is acceptable just in case (i) q holds in all closest worlds in which p holds, and (ii) it is not the case that q holds in all closest worlds in which $\neg p$ holds. While (i) expresses the well known Ramsey Test, (ii) is an additional condition devised to capture the intuition that q holds in virtue of p. The logic is developed accordingly in terms of acceptability preservation for all belief states.[20]

The difference-making account is inferentialist in the following sense: as long as one assumes that (i) and (ii) are satisfied just in case q can be inferred from p, one takes acceptable conditionals to correspond to valid arguments, in a sense of validity that leaves room for defeasible inferences.

Just as the threshold/increment account, the difference-making account cannot validate the embedded connexive principles. Nonetheless it qualifies as connexive,

[20] See [34] and [35]. The formulation provided here is not exactly Rott's formulation, which relies on the AGM formalism.

as it validates some version of the basic connexive principles. More precisely, it validates P1, P2, P11, and P4. Consider P1. If p is not necessary, (i) cannot be satisfied. If p is necessary, (ii) cannot be satisfied. In both cases, $\neg p \rightarrow p$ is unacceptable, so its negation is acceptable. A similar reasoning shows that P2 holds. Consider P11. If p is possible, (i) cannot be satisfied both for $p \rightarrow q$ and for $p \rightarrow \neg q$. It follows that $(p \rightarrow q) \wedge (p \rightarrow \neg q)$ is unacceptable, hence that its negation is acceptable. Note that we do not get this result on the supposition that p is impossible, which is why P3 does not hold. Now consider P4. If q is necessary, (ii) is violated both for $p \rightarrow q$ and for $\neg p \rightarrow q$. If q is not necessary, and (i) is satisfied both for $p \rightarrow q$ and for $\neg p \rightarrow q$, (ii) cannot be satisfied for both. So $p \rightarrow q$ and $\neg p \rightarrow q$ are never jointly acceptable, which makes the negation of their conjunction always acceptable.

The difference-making account, like the threshold/increment account, rules out conditionals with irrelevant antecedents. The intuitive difference between (2) and (3) in our initial list is explained by saying that (3), unlike (2), does not satisfy (ii). Moreover, (4) turns out to be unacceptable as well, given that again (ii) is not satisfied. However, this account does not explain the intuitive plausibility of (1), unlike the threshold/increment account, for (1) is predicted to be equally unacceptable as (3) and (4), and for the same reason: even in the closest worlds in which there is some tails in the first 5 coin tosses, there will still be at least 5 heads in the first 1.000 tosses.

As to the logic, the difference-making account is not entirely satisfactory. This account aptly avoids some logical principles that are dubious for inferentialists, such as Monotonicity, Conjunctive Sufficiency, and Right Weakening. However, it fails to capture other logical principles which are plausible instead, such as OR. Moreover, some of the principles validated by this account appear devoid of sound justification. A rather striking example is *Affirming the Consequent*, the inference from $p \rightarrow q$ and q to p. Suppose that q holds in all closest worlds (which means that q is acceptable). Then p must also hold is those worlds, for otherwise (ii) would be violated, against the assumption that $p \rightarrow q$ holds. This is quite odd. After all, $p \rightarrow q$ is meant to convey that q can be inferred from p, not the other way around.

6.5 The Incompatibility View

So far we have discussed three accounts of conditionals that qualify as inferentialist and connexive to some extent. As we have seen, none of these accounts combines inferentialism and connexivity in a fully satisfactory way, which should not come as a surprise, given that they were not designed specifically for the purpose of such combination. The aim of the rest of the paper is to show that there is at least one coherent alternative to the three accounts discussed above. Its core idea, which we call the *Incompatibility View*, is that a conditional holds when its antecedent is incompatible with the negation of its consequent.

The first clear formulation of the Incompatibility View goes back to the Stoics. In section 6.2 we saw how Sextus Empiricus describes Philo's view and Diodorus' view. Immediately after that, he lists a third view, which secondary sources attribute to Chrysippus:

> Those who introduce connectedness say that a conditional is sound when the opposite of its consequent conflicts with its antecedent. According to them, the conditionals just stated will be unsound, but 'If it is day, it is day' will be true.[21]

Here 'connectedness' translates συνάρτησις (*synárthesis*), from συναρτάω, which means "to join together", while 'conflicts' translates μάχηται, *máchetai*, from μάχομαι, which means "to fight". It is hard to tell how the conflict between p and $\neg q$ is to be understood, given that we know very little about Chrysippus' theory. But at least two remarks are in order. First, the conflict between p and $\neg q$ is not to be equated with the impossibility that p and $\neg q$ are true, otherwise the sentence (7) considered in section 6.2 would pass the test, whereas the last sentence of the passage above explicitly denies that (7) holds for Chrysippus. Second, it is not obvious that the conflict between p and $\neg q$ must be reducible to a purely formal relation.[22] What seems uncontroversial is that the following conditional is true for Chrysippus:

(8) If it is day, it is day

In this case his criterion is satisfied, for 'It is day' conflicts with 'It is not day'.

The Incompatibility View as described by Sextus Empiricus emerges again in Diogenes Laertius:

> A conditional is therefore true, if the contradictory of its consequent is incompatible with its antecedent. For instance, "if it is day, it is light": this is true. For the statement "it is not light", contradicting the consequent, is incompatible with the antecedent "it is day". On the other hand, a conditional is false if the contradictory of its consequent does not conflict with its antecedent, for instance "if it day, Dion is walking", for the statement "Dion is not walking" does not conflict with the antecedent "it is day".[23]

Here 'incompatible' and 'does not conflict' translate again μάχηται and οὐ μάχηται respectively, in line with the previous quotation. This phrasing of the view migrated into Latin philosophical terminology. In *De Fato*, Cicero explicitly addresses Chrysippus' doctrine, implying that for a conditional to be true it must be the case that its antecedent and the negation of its consequent *pugnant inter se*, which literally translates into "fight each other".[24]

The Incompatibility View survived through late antiquity and the middle ages, although often conflated with the strict account, for it is documented again in the XIV and XV century. This is a quote from Paul of Venice, in *Logica Magna*:

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[21] Sextus Empiricus, *Outlines of Scepticism*, II, 111, edited and translated by J. Annas and J. Barnes, in [36], p. 96.

[22] As observed in [2], pp. 107-108, some evidence suggests that Chrysippus' criterion included formal incompatibility, analytical incompatibility, and perhaps some sort of empirical incompatibility.

[23] Diogenes Laertius, *Lives of Eminent Philosophers*, VII, 71-73 (in [10], pp. 178-183, translation modified).

[24] Cicero, *De Fato*, 12 (in [5]).

> People say that for the truth of a conditional it is required that the opposite of the consequent be incompatible with the antecedent.[25]

A pupil of Paul of Venice, Paul of Pergola, phrases the view in his *Logica* by using the verb *repugnare*, which means ‘to contrast’:

> A conditional is true when the contradictory of the consequent is repugnant to the antecedent.[26]

Although fragmentary, the textual evidence preserved suggests that the Incompatibility View played a significant role in the tradition of logical and philosophical reflection on conditionals. The available historical record, however, does not go beyond some informal remarks in secondary sources, at least until the XX century.[27]

As far as we know, the first documented attempts to express in formal terms the Incompatibility View, as distinct from the strict account, are due to Orlov and Nelson. Orlov defines a compatibility relation between sentences in terms of conditionals understood along inferentialist lines: p is compatible with q when $p \rightarrow \neg q$ does not hold, that is, when $\neg q$ does not follow from p. Conversely, $p \rightarrow \neg q$ does hold when p is incompatible with q.[28] In a similar vein, Nelson treats incompatibility and entailment as interdefinable relations that can obtain between sentences in virtue of their meaning. Interestingly, Nelson presents his view as an alternative to Russell’s material reading and to C. I. Lewis’s strict reading of conditionals, just like Chrysippus’ view was originally an alternative to Philo’s view and to Diodorus’ view:

> The propositional function “p entails q” means that p is inconsistent with the propositional function that is the proper contradictory of q [...]. Entailment, not being defined in terms of truth-values, is a necessary connexion between meanings. I believe that this analysis of entailment comes much closer to expressing what “implies” means in ordinary discourse than does either material or strict implication.[29]

Whether or not Nelson’s formal theory fits exactly what Chrysippus had in mind, Nelson has much in common with Chrysippus. Chrysippus was both an inferentialist — in that he probably held the Stoic Thesis — and a connexivist, as suggested by the passage from Sextus Empiricus quoted above.[30] Nelson was also both an inferentialist — in that he defined conditionals in terms of entailment — and a connexivist, in that his semantics has straightforward connexive implications.

Arguably, this convergence is not accidental: the Incompatibility View produces a coherent integration between inferentialism and connexivity. On the one hand, the link with inferentialism is rather direct insofar as a valid argument is plausibly

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[25] Quoted from [3], p. 196.

[26] Quoted from [4], p. 124.

[27] [28] provides further details on the history of the Incompatibility View.

[28] [30]. He we rely on an unpublished English translation of Orlov’s paper, due to Anya Yermakova, which Graham Priest kindly shared with us.

[29] [29], pp. 444-445.

[30] Indeed, according to the characterization proposed in [26], connexive implication is defined precisely in terms of Chrysippus’ incompatibility criterion.

described as one in which the premises are jointly incompatible with the negation of the conclusion. This is surely the consensus view as concerns conclusive arguments, and can work as a fruitful guideline to think about defeasible arguments as well. On the other hand, the Incompatibility View substantiates the connexive principles in a straightforward way. Bracketing limiting cases (such as impossible antecedents) for the moment, it is clear that p can not be incompatible with itself, so it follows that $\neg p \to p$ and $p \to \neg p$ must be rejected, according to Aristotle's Thesis.[31] Under the same proviso, moreover, it can not happen that p is incompatible with both q and $\neg q$, so at most one of $p \to q$ and $p \to \neg q$ can hold, according to Abelard's First Principle and Boethius Thesis. And because incompatibility has to be a symmetric relation, it also follows that at most one of $p \to q$ and $\neg p \to q$ can hold, according to Aristotle's Second Thesis and Boethius Left Thesis.

6.6 The evidential account

Now we will sketch an account of conditionals, the *evidential account*, that is intended to develop the Incompatibility View in a coherent way. Its underlying idea is that $p \to q$ holds just in case p supports q, where the latter condition is spelled out in terms of incompatibility between p and $\neg q$. To articulate this idea we will adopt a modal semantics, although an alternative formulation can be given in a probabilistic semantics.[32]

In order to understand how the evidential account defines the incompatibility between p and $\neg q$, two distinct cases are to be considered. One is the case in which p and $\neg q$ are *absolutely incompatible*, where 'absolutely' rules out the possibility that p and $\neg q$ are jointly true. The other is that in which p and $\neg q$ are *relatively incompatible*, where 'relatively' implies that, although p and $\neg q$ can be jointly true, their combination is a remote possibility. In other words, to say that p and $\neg q$ are incompatible is to say that either they are absolutely incompatible or they are relatively incompatible. Both disjuncts can be spelled out in modal terms, provided that one assumes a suitably ordered set of possible worlds.

Let us start with absolute incompatibility. The simplest and perhaps most obvious way to define this relation is to equate it with the impossibility that p is true and q is false. That is,

Definition 6.1 p and $\neg q$ are absolutely incompatible iff there is no world in which p is true and q is false.

This is the characterization of absolute incompatibility that we provided in the original formulation of the evidential account.[33] On the assumption that absolute

[31] Here we are assuming that negation behaves classically, so that p is logically equivalent to $\neg\neg p$.

[32] The modal version of the evidential account is developed in [7], [33], and [9]. The probabilistic version is developed in [6] and in [8]. The symbol for the evidential conditional adopted in all these works is $\triangleright$, although here for ease of exposition we will retain the notation $\to$.

[33] [7].

incompatibility so defined suffices for the truth of $p \rightarrow q$, we get that any strict conditional entails the corresponding evidential conditional, that is, $\Box(p \supset q)$ entails $p \rightarrow q$. In particular, a crucial consequence of definition 6.1 concerns Necessary Consequent and Impossible Antecedent. If we call *non-contingent* any conditional in which the antecedent is impossible or the consequent is necessary, definition 6.1 is satisfied by all non-contingent conditionals. The underlying thought, in line with an established tradition which includes Adams, Stalnaker, and Lewis, is that necessary truth preservation *per se* is sufficient for the truth of a conditional.

However, as illustrated by the sentence (4) in our initial list, some cases of necessary truth preservation are potentially contentious in an inferentialist perspective, for they are cases in which there is no clear intuition to the effect that the antecedent supports the consequent. More generally, if the evidential account validates Necessary Consequent, it thereby implies that anything supports a necessary truth. Similarly, if the evidential account validates Impossible Antecedent, it thereby implies that an impossible truth supports anything.

Although there is nothing intrinsically wrong with Necessary Consequent and Impossible Antecedent, these two principles generate a tension within an account of conditionals based on the criterion of incompatibility, because they imply that p and $\neg q$ can be incompatible merely in virtue of some property — impossibility or necessity — that belongs to one of them independently of the other. This goes against a thought that may naturally be associated with the Incompatibility View, namely, that the incompatibility between p and $\neg q$ is *relational*: what is wrong with the combination of p and $\neg q$ must somehow depend on p and $\neg q$ taken together, that is, it must not arise from p or $\neg q$ taken separately.[34]

The point about relationality emerges clearly if we compare (1) with (4), for (4), unlike (1), fails to satisfy the constraint just stated. Another interesting illustration is provided by (7), the example used by Sextus Empiricus to show the difference between Chrysippus' view and Diodorus' view. In (7) arguably no incompatibility stems from the relation between the antecedent and the negation of the consequent, given that the negation of the consequent is the antecedent itself. Nonetheless, the antecedent of (7) is impossible, and its consequent is necessary.

In order to preserve relationality in the sense just explained, absolute incompatibility is *not* to be equated with the impossibility that p is true and q is false: the class of cases in which the former holds — due to the combination of the contents of p and $\neg q$ — should be a proper subset of the class of cases in which the latter holds. The second definition of absolute incompatibility that we will consider, therefore, is the following:

Definition 6.2 p and $\neg q$ are *absolutely incompatible* iff there are no worlds in which p is true and q is false, and

(a) p is possible;
(b) q is not necessary.

Here (a) and (b) rule out cases of vacuous necessary truth preservation, thus warranting that the incompatibility between p and $\neg q$ essentially depends on the relation

[34] See [29], p. 443.

between p and $\neg q$. Accordingly, (4) and (7) turn out to be cases in which the antecedent and the negation of the consequent are *not* absolutely incompatible.[35]

The rationale for this definition is somehow opposite to the rationale that one would employ for definition 6.1. Instead of assuming that anything supports a necessary truth, it is assumed that nothing supports a necessary truth. Similarly, instead of assuming that an impossible truth supports anything, it is assumed that it supports nothing.[36]

Now let us turn to relative incompatibility. This relation is defined as follows:

Definition 6.3 p and $\neg q$ are relatively incompatible iff there are worlds in which p is true and q is false, and

(c) p and q have the same value in some of the closest worlds;
(d) in the closest worlds in which p is true, q is also true;
(e) in the closest worlds in which $\neg q$ is true, $\neg p$ is also true.

Here (c) requires that p and $\neg q$ have different values at least in some of the closest worlds. (d) expresses the Ramsey Test, so it implies that $\neg q$ is false in the closest worlds in which p is true. Note that, given (d), the only interesting case ruled out by (c) is that in which p is false and q is true in all the closest worlds. So, (c) prevents the incompatibility condition from obtaining when (d) and (e) are satisfied only because p is very unlikely and q is very likely for independent reasons, as in the case of (3). Finally, (e) reverses the Ramsey Test, as it implies that p is false in the closest worlds in which $\neg q$ is true. To say that (c)-(e) are jointly satisfied is to say that the combination of p and $\neg q$ is a remote possibility.[37]

Definition 6.3 can be combined either with definition 6.1 or with definition 6.2. This means that the evidential account can be phrased in at least two ways. The first is classical:

Definition 6.4 p and $\neg q$ are incompatible iff there is no world in which p is true and q is false, or there is such world and the following conditions are satisfied:

(c) p and q have the same value in some of the closest worlds;
(d) in the closest worlds in which p is true, q is also true;
(e) in the closest worlds in which $\neg q$ is true, $\neg p$ is also true.

The second is non-classical:

Definition 6.5 p and $\neg q$ are incompatible iff either there are no worlds in which p is true and q and false, and

(a) p is true in some world;
(b) q is false in some world;

[35] [24], p. 145, considers an account of conditionals that equates their truth conditions with the right-hand side of definition 6.2. The same account is discussed in [16] and in [32].

[36] As explained in [23], the claim that nothing can follow from an impossible truth can be ascribed to the followers of Robert of Melun (ca 1100-1167). [24] also notes that this is idea is in line with the old conception of negation of cancellation.

[37] This explication of relative incompatibility is provided in [9].

or there are worlds in which p is true and q is false and

(c) p and q have the same value in some of the closest worlds;
(d) in the closest worlds in which p is true, q is also true;
(e) in the closest worlds in which $\neg q$ is true, $\neg p$ is also true.

The two variants of the evidential account differ as to the treatment of non-contingent conditionals: according to definition 6.4, all non-contingent conditionals are true, while according to definition 6.5 all non-contingent conditionals are false. Note that the second disjunct of definition 6.5 also requires that (a) and (b) are satisfied.

The choice between definition 6.4 and definition 6.5 ultimately depends on one's inclinations towards classical logic. As long as one is apt to regard vacuous necessary truth preservation as part of some intuitively correct notion of entailment, one will be willing to classify non-contingent conditionals as cases in which the antecedent supports the consequent. On the other hand, if one is inclined to think that necessary truth preservation alone does not suffice for entailment, one will regard non-contingent conditionals as cases in which the lack of a proper relevance link prevents the antecedent from supporting the consequent. As far as we can see, both lines of thought are consistent with the Incompatibility View, since definitions 6.1 and 6.2 overlap to a large extent. In particular, note that both definitions imply that (8) is true, in that its antecedent is absolutely incompatible with the negation of its consequent.[38]

6.7 Two key features of the evidential account

As a final point, we will show how the evidential account holds together inferentialism and connexivity. Since the evidential account is based on the Incompatibility View, this is one way to show how the Incompatibility view produces a coherent integration of inferentialism and connexivity.

The evidential account is overtly inferentialist. To say that p supports q in the sense of 'support' that matters to this account is to say that q can be inferred from p, or that p is a reason for q. As long as it is assumed that an argument is valid when its premises support its conclusion, the evidential account implies that a conditional is true just in case the corresponding argument is valid, which is nothing but the Stoic Thesis.

As in the case of the threshold/increment account and the difference-making account, the evidential account is inferentialist in a sense that is not purely deductive. The notion of incompatibility spelled out in definitions 6.4 and 6.5 is intended to cover both conclusive and defeasible reasons. More precisely, when the first disjunct holds — that is, p and $\neg q$ are absolutely incompatible — we say that p is a conclusive

[38] In [22] this second definition of absolute incompatibility is discussed as a possible interpretation of Chrysippus' criterion of incompatibility. However, as observed in section 6.5, it is not obvious that for Chrysippus incompatibility reduces to absolute incompatibility.

reason for q. When the second disjunct holds — that is, p and $\neg q$ are relatively incompatible — we say that p is a defeasible reason for q.

This feature emerges with clarity when one considers our examples (1)-(4). Both definitions 6.4 and 6.5 entail that (1) and (2) are true: (1) is true in virtue of the first disjunct, while (2) is true in virtue of the second disjunct. Moreover, both definitions entail that (3) is false, because although there are worlds in which its antecedent is true and its consequent is false, condition (c) in the second disjunct not satisfied. So, a principled distinction can be drawn between (2) and (3). Definitions 6.4 and 6.5 differ as to (4), of course, in that (4) is true according to the former but false according to the latter. But (4) is a contentious example, as explained above, and the difference between the two definitions with respect to (4) is orthogonal to the point just made about defeasible reasons.

Another fact to be noted is that the difference between the classical variant and the non-classical variant of the evidential account is neutral as to the logical principles discussed in sections 6.2-6.4. On both variants Monotonicity, Right Weakening, and Conjunctive Sufficiency fail, while AND and OR hold.[39]

Now let us turn to connexivity. First of all, note that the evidential account, at least in the modal version considered here, does not require syntactic restrictions on embeddings, so it is able to express the whole list of connexive principles discussed above. The classical variant validates P9-P16, as Iacona has shown elsewhere.[40] The non-classical variant, on the other hand, validates P1-P4. P1 and P2 hold because p and $\neg p$ cannot be true in the same world, so neither of the two disjuncts can be satisfied. P3 holds because, as long as p is not impossible, $p \rightarrow q$ and $p \rightarrow \neg q$ cannot be jointly true, due to (d) in the second disjunct. Finally, P4 holds because, as long as q is not necessary, $p \rightarrow q$ and $\neg p \rightarrow q$ cannot be jointly true, due to (e) in the second disjunct. The reason why the embedded connexive principles do not hold (neither unrestricted nor restricted) is that the non-classical characterization of absolute incompatibility implies that the instances of these principles may be false in virtue of the impossibility of their antecedent or the necessity of their consequent.[41]

The fundamental alternative that emerges from the two variants of the evidential account outlined above is the following: either one maintains classicality and restricts the connexive principles — as in P9-P16 — or one keeps the connexive principles unrestricted — as in P1-P4 — and allows only a restricted form of classicality. A case in point is *Identity*, the principle according to which $p \rightarrow p$ is a logical truth. Identity holds in the classical variant, while only a restricted version of it holds in the non-classical variant. We believe that this is exactly as it should be, given that the Incompatibility View can be specified in different ways, and Identity is not an essential part of inferentialism or connexivity per se.[42]

All things considered, the evidential account provides a coherent combination of inferentialism and connexivity, which can be developed in a proper formal frame-

[39] These facts are proved in [7] and in [9].

[40] [20] proves this fact by defining a formal semantics based on definition 6.4.

[41] We would like to thank Martina Calderisi for drawing attention to this point.

[42] [39] and [24], for example, provide different reasons for rejecting Identity. Also see [40] for an extensive treatment of this issue.

work. As long as inferentialism and connexivity are regarded as plausible claims about conditionals, as we take them to be, this suggests that perhaps there is something right in the old Incompatibility View.

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