The Language Essence of Rational Cognition, with some Philosophical Consequences

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Dedicated to Ćuto

Abstract. This article analyses the essential role of language in rational cognition. The approach is functional – I only look at the effects of the connection between language, reality and thinking. I begin by analysing rational cognition in everyday situations. Then I show that the whole scientific language is an extension and improvement of everyday language. The result is a uniform view of language and rational cognition which solves many epistemological and ontological problems. I use some of them – the nature of ontology, truth, logic, thinking, scientific theories and mathematics, to demonstrate that the view of language and rational cognition developed in this article is fruitful and effective.

keywords: rational cognition, language, ontology, truth, logic, thinking, scientific theory, mathematics

1 Introduction

Perhaps many readers are surprised that I have such high expectations of an empty language-matter. But I have more right to be surprised that people have still drawn so few advantages from the fact that they could have regarded "language as a vehicle of human thoughts and the content of all wisdom and cognitions."

Johann Gottfried Herder [Herder, 1767]

A common view of the role of language in rational cognition and thinking is that it plays a passive role there: language is a medium for expressing and communicating thoughts, and for describing reality. The main protagonists of analytic philosophy, Frege, Russell, Wittgenstein, Carnap and Quine, considered language a suitable, even necessary, tool in their philosophical analysis. Suitable, because of its concreteness, necessary, because they did not see any other way to do analysis. However, language is only an aid to them in examining important epistemological and ontological problems. Moreover, language can be obscure unless we transform it into logically refined form. They did not consider language to be an essential part of rational cognition. The closest standpoint I could find in their writings is Quine's: "Thought, if of any considerable complexity, is inseparable from language - in practice surely and in principle quite probable." [Quine, 1957]. However, although he acknowledged the (probable) inseparability of language and reasoning, he did not draw the consequences from it.

The first philosophers to fully recognize the essential role of language in rational cognition and thinking were Hamman, Herder and Wilhelm von Humboldt in the second half of 18th century and the first half of 19th century, and Cassirer later, in the first half of 20th century. In the first half of the 20th century, linguists Sapir and Whorf came to the same conclusion. The works of Hamann [Haynes, 2007], Herder [Herder, 1772], von Humboldt [Humboldt, 1836], Cassirer [Casirrer, 1923], Sapir [Sapir, 1921] and Whorf [Carroll, 1956] provide a lot of substantial evidence that language affects our perception, thinking and action in an essential way, and that human intellect and language are inseparable. However, they did not systematically analyse the essential role of language in rational cognition and thinking.

This article analysis the essential role of language in rational cognition. The approach is functional – I only look at the effects of the connection between language, reality and thinking. I consider that this is the proper level of abstraction concerning these matters. I begin by analysing rational cognition in everyday situations. Then I show that the whole scientific language is an extension and improvement of everyday language.[1] The result is a uniform view of language and rational cognition (Section 2) which solves many epistemological and ontological problems. I use some of them – the nature of ontology, truth, logic, thinking, scientific theories and mathematics (Sections 3 and 4), to demonstrate that the view of language and rational cognition developed in this article is fruitful and effective.

2 The language essence of rational cognition

The whole structure of declarative language rests on names and predicate symbols together with the atomic sentences constructed from them. The analysis of such sentences is of the utmost importance for understanding how language is involved in rational cognition and thinking. Donald Davidson writes: "...if we do not understand predication, we do not understand how any sentence works, nor can we account for the structure of the simplest thought that is expressible in language. At one time there was much discussion of what was called the "unity of proposition"; it is just this unity that a theory of predication must explain. The philosophy of language lacks its most important chapter without such a theory, the philosophy of mind is missing its crucial first step if it cannot describe the nature of judgement; and it is woeful if metaphysics cannot say how a substance is related to its attributes." ([Davidson, 2005]).

Let's consider an everyday cognitive process, for example, a question "Is my family pet

[1] This is the language form of Einstein's claim that "The whole of science is nothing more than a refinement of everyday thinking" [Einstein, 1936]

Švrćo a dog". This cognitive process is the simplest type of an experiment, a binary experiment, in which there are only two possible answers, "yes" and "no". The experiment is done on an object named "Švrćo" through an investigative mechanism symbolized by the language form "is a dog". The experiment itself is described by the language form "Švrćo is a dog". The possible results of the experiment are "yes" and "no". It is enough to see Švrćo to conclude that the answer is "yes".

This example illustrates the basic cognitive situation of putting an *object* a in an investigative framework (experimental apparatus) that results in one of two possible answers. I will term such a binary framework a *predicate* $\underline{P}$. We apply the predicate $\underline{P}$ to an object a and describe the situation with the language form "$P(a)$" (notice that "P" here is not underlined which I will explain soon). Such a language form is termed (declarative) *atomic sentence*. The result can take two values, *yes* and *no*. These are the so-called *truth values* of the language form "$P(a)$" termed $True$ and $False$. $True$ and $False$ are designed by us as a part of the binary experiment design and selected by nature in the realization of the experiment. These binary experiments are the essence of our rational synthesis with nature. We make the question and offer two possible answers (binary experiment design), and nature selects an answer (realization of the experiment). The selected truth value is the value of this synthesis which discriminates what is and what is not. It provides that "unity of proposition", of which Donaldson writes. I will analyse this concept of truth, which I term *the synthetic concept of truth*, in more detail in Subsection 3.2.

The cognitive situation illustrated and described above, simple as it might seem, has a number of underlying characteristics and assumptions which are essential for the process of rational cognition. First of all, it reflects our innate approach to the world which we divide into objects (elements upon which something is done) and into predicates (which determine what is done). However, this division depends on context. Something that is a predicate in one context can become an object to which other predicates are applied in another context. This *object – predicate dualism* is a fundamental characteristic of the cognitive framework described here. It is reflected in language through the structure of the atomic sentence "$P(a)$". Symbols "a" and "P" have different roles in the sentence. We use symbol "a" to *name* (mention) an object a. We use symbol "P" to say something about the object a. Because of these different roles, I say that symbol "P" *symbolizes* a predicate, i.e. that "P" is the *symbolic part* of the corresponding predicate, rather than that it names the predicate. To emphasize this difference I will name "$\underline{P}$" the predicate symbolized by "P". This enables us to maintain a very important *use and mention* distinction, between using a predicate in a sentence to say something about an object (when we use symbol "P") and mentioning a predicate in a sentence to say something about the predicate itself (when we use symbol "$\underline{P}$").

To my knowledge, Whorf is the first one to recognise that the object – predicate dualism is a prominent feature of Indo-European languages: "Our language thus gives us a bipolar division of nature. But nature herself is not thus polarized." [Whorf, 1940]. He also recognizes that the dualism and the way we analyse nature is not inherent to nature but to our approach to nature: "We dissect nature along lines laid down by our native language. The categories and types that we isolate from the world of phenomena we do not find there because they stare every observer in the face; on the contrary, the world is presented in a kaleidoscopic flux of impressions which has to be organized by our minds – and this means largely by linguistic systems in our minds. We cut nature up, organize it into concepts and ascribe it significance as we do, ...” [Whorf, 1940].

Frege [Frege, 1892b] considers predicates (concepts) and objects to be fundamentally different entities. To him this division is absolute, not relative, dependent on context, as it is considered in this article. However, the essential difference between Frege's and my approach to predicates is that Frege considers predicates to be metaphysical entities in the Platonic sense of the word [Frege, 1918], while I consider them to be binary investigative mechanisms that belong to our real activities.

The language form ”$P(a)$” is not a passive description of the corresponding binary experiment – it is a part of the experiment. Although names for objects and symbols for predicates can be arbitrary, their presence in our cognitive framework is essential. Through names, we control our connection with objects and through predicate symbols, we control our connection with predicates. Moreover, objects and predicates do not exist by themselves – they exist as parts of our rational syntheses with nature. Since names and predicate symbols are a means of extracting objects and predicates in the cognitive framework, each name is part of the object it names and each predicate symbol is part of the predicate it symbolizes. Thereby, a particular syntactic form is not important. What is important is the very presence of the form together with the condition that different objects and predicates have different corresponding language forms.

To my knowledge, von Humboldt is the first to recognize the importance of the previously described connection between language forms and the formation of concepts, and who finds in this relation the key to understanding why language is essential for thinking: ”Language is the formative organ of thought. Intellectual activity, entirely mental, entirely internal, and to some extent passing without trace, becomes, through sound, externalized in speech and perceptible to the senses. Thought and language are therefore one and inseparable from each other. But the former is also intrinsically bound to the necessity of entering into a union with the verbal sound; thought cannot otherwise achieve clarity, nor the idea become a concept. The inseparable bonding of thought, vocal apparatus and hearing to language is unalterably rooted in the original constitution of human nature, which cannot be further explained ... without this transformation, occurring constantly with the help of language even in silence, into an objectivity that returns to the subject, the act of concept formation, and with it all true thinking, is impossible.” [Humboldt, 1836]. Umberto Eco says this poetically in the last sentence of the 1980 novel *The Name of the Rose*: ”Stat rosa pristina nomine, nomina nuda tenemus.”[2]

A fundamental semantic assumption of the use of the atomic sentence ”$P(a)$” in rational cognition is that ”a” *names* an object. This rests on the assumption that it is possible to extract from the world something to be named. How we make the extraction and how we keep the connection between the name and the named in the flow of time is a very complex subject, and it will not be analysed here. One thing is for certain, the process of naming is also a kind of our synthesis with nature. When I use the name ”Švrćo”, I exactly know what is named: my dog Švrćo. However, even in everyday situations, we use names for which

[2]”Yesterday's rose stands only in name, we hold only empty names.”

we don't know the exact object they name, for example, the name of a person we don't know. Even worse, it is possible that such a person does not exist, as it the case today with fake profiles on the internet. In the same unwarranted way, we extend the language used in everyday situations to other situations, when we are involved in science and mathematics, or when we talk fairy tales to children. However, we think "with names" in the same way, whether we know what they name or not and whether they name anything at all. For example, when we are involved in the fairy tale *Snow White and the Seven Dwarfs* we think, discuss and make conclusions as if all the characters in the story exist, because we are "tuned" to think in this way in semantically clear everyday situations. Only, when we step out of the language of the story (and use another language) we acknowledge that there are no such objects. Concerning names, the moral is that when we use language we assume that every name names an object, no matter how this connection is achieved and whether it is achieved at all.

I consider that naming, together with the fundamental assumption of its use, is a key primitive element of language. I think it is wrong to minimize the importance of naming as in Russell's theory of descriptions [Russell, 1905], in Quine's reduction to values of variables [Quine, 1948] or more radically in Quine's reduction to "ideal nodes at the foci of interesting observation sentences" in his naturalized epistemology [Quine, 1990].

Russell attempts to avoid names by using descriptions. However, the primary role of language is connection with the world, and naming is one of the key elements of the connection. If a description identifies an object then it can be a definition of its name. However, usually, we cannot define all names – we are forced to choose some names as primitive names. If a description does not identify an object then it gives a roundabout and complicated way to talk about the described objects, if there is any such. However, I consider that the descriptive translation is not the way we use language, even in science. We use names simply and directly, although in this way we are exposed to the danger that they do not name anything. In some situations, there is no danger at all, for example in fairy tales and in mathematics (as will be discussed later in Subsection 4.2). In some situations, most notably in science, when we discover that a name does not name anything then we revise the language. Therefore, such an unwanted situation is a regular stage in the development of scientific theory.

Concerning Quine's "to be is to be a value of variable" [Quine, 1948], in my view it is the same as to say "to be is to be named". The corresponding Quine's ontological commitment is just the fundamental assumption of the use of names. In his naturalized epistemology [Quine, 1951, Quine, 1990], Quine considers observation sentences as the primary elements of rational cognition, and their dissection to predicate symbols and names as something artificial, even unnecessary, just a convenient organization of knowledge. Contrary to this, I consider that names and predicate symbols are primary language elements through which we synthesise our rational cognition into the objective truth values of atomic sentences constructed from them.

The next fundamental semantic assumption of the use of the sentence $P(a)$ in rational cognition is that the predicate symbol "P" symbolizes predicate $\underline{P}$, that is to say, symbol "P" is the symbolic part of predicate $\underline{P}$. This connection between the language form and reality is even more complex than naming, and it will not be analysed here. However, from the way we address nature through object-predicate construction it follows that no predicate is independent of us – it is the product of a cognitive interaction between nature and us. With predicate symbols, as with names, we encounter uncertainty, too. When I use the predicate symbol "to be a dog", in standard situations I know immediately how to perform the corresponding experiment. However, it could happen that in some exceptional situations I don't know how to determine if an object is a dog. We can give a description, even a definition, of this predicate using other predicates. However, we cannot define all predicate symbols – we are forced to choose some predicate symbols as primitive ones. Usually, our everyday predicates are primitive predicates. For example, we certainly do not use a definition of predicate symbol "to be a dog" as the primary way of learning that predicate. From the moment of birth we form the predicate, I would say almost by perception, as part of our ability to differentiate beings. The mechanism of the predicate is deeply rooted in our sensory world, and only later do we complete it with determinations which range from everyday experience to advanced theoretical knowledge (for example about dog's genetic code). The same happens in science. We use various predicates for which sometimes we are uncertain about how to apply them to various objects. In other words, we have predicate symbols which refer to incomplete predicates. This situation will be analysed in more detail in Subsection 4.1. Here I just want to stress that in the same unwarranted way as with names, we extend the use of predicate symbols from everyday situations to other situations. However, it does not prevent us to think with predicate symbols as they always refer to completely determined predicates. So, concerning predicate symbols, the moral is that when we use language we assume that every predicate symbol symbolizes a predicate, without considering how this connection of language and reality is achieved and whether it is achieved at all.

Two more considerations about predicates are needed. The first one is about the difference between a predicate and its semantic value. Every (fully determined) predicate is a kind of procedure which determines, through the intervention of nature, a mathematical function (in the mathematical extensional sense) from objects to truth values. I will call this function the semantic value of the predicate (and of the corresponding predicate symbol). However, we must not equate a predicate and its semantic value. Otherwise, we would destroy the whole language mechanism of rational cognition. A predicate is part of the process of rational cognition while its semantic value is the final result of this process, in which nature is substantially involved. Reduced to an atomic sentence, it means that the resulting truth value gives unity to the atomic sentence: it makes the atomic sentence to be something more than just the concatenation of its parts, the predicate symbol and the name involved in the sentence.

The second consideration is about situations where we do not use predicates as an investigative tool to address questions to nature. Commonly, these are situations which we create and over which we have control, for example, in designing a game, a story or a mathematical world (as I will explain later in Subsection 4.2). Then, for some predicates, we directly decide on which objects they give $True$ and on which objects they give $False$. For example, we can decide which character in a fairy tale will be good or which natural numbers less than 100 will have some (unimportant) property U (we will just enumerate such numbers). This is another use of predicates in which we directly reduce them to their semantic values. The role of these predicates in our rational activities is quite different than the role of predicates which are investigative mechanisms. However, it is convenient to treat all predicates uniformly. Formally, we can achieve this by considering that the list of truth values decisions is the corresponding investigative mechanism for this type of predicate (when we apply the predicate to an object, we determine the truth value by looking at the list).

In the previous considerations, one more assumption is implicitly present – the semantics is two-valued. We assume that predicates applied to any object give truth or falsehood. However, it is possible that in some situations a binary experiment does not succeed in giving an answer (for example, truth paradoxes witness such situations). Because of this, the classical language has the additional assumption that it is a two-valued language: every sentence is true or false but not both. For atomic sentences, this assumption follows from the fundamental assumption of the language use of predicates. For compound sentences, I will analyse the assumption soon.

In the previous analysis the semantic determinations of the language were maximally abstract. The semantics is reduced to so-called *semantic values* of language elements: (i) to every name we attach only the object it names, (ii) to every predicate symbol we attach, through an investigative framework, only the mathematical function from objects to truth values, and (iii) to every sentence we attach only the truth value. Additional semantic determinations are possible. For example, when we are thinking about the meaning of a sentence, whatever it is, it is certainly not its truth value, but a kind of combination of the meaning of the parts of the sentence. The semantic reduction of predicates and sentences into truth valuations and of names into what is named is a maximal abstraction. I shall name this type of abstraction *Frege's abstraction* [Frege, 1891, Frege, 1892c, Frege, 1893] (see page 10). I consider that it is just a proper level of abstraction which, on the one side, explicates all precise effects, and on the other side, hides all complexities and obscurities of language use in the process of rational cognition.

As we have analysed one-place predicate symbols, we can also analyse multi-place predicate symbols. The analysis of function symbols is similar to the analysis of predicate symbols. Every function symbol symbolises a *function*, a procedure that, when applied to objects, determines an object, possibly with the help of nature. Thus, the semantic value of the function (and of the function symbol) is the corresponding mathematical (extensional) function between objects.

Further analysis requires a specification of language. I chose the interpreted first-order language. Through the search for a language better adapted to precise and effective thinking, this language evolved from natural language as part of the development of mathematics. It has precise syntax and (abstract) semantics, and has two important mechanisms which make it effective – symbolization and the use of variables. There are various artificial languages of declarative type, but the first-order language is the dominant one, in my opinion rightly. An argument for this claim is that when we describe other languages and their logics, we describe them in the first-order language. This means that it is the most natural and the most powerful: we can translate all these languages in the first-order language. Furthermore, the first-order language has a simpler and clearer semantics than other languages.

Basic building blocks of the first-order language are atomic sentences which I have already analysed. Consequently, all the assumptions of the use of atomic sentences are now the assumptions of the use of the first-order language.

We can combine atomic sentences, the descriptions of basic binary experiments, into composed sentences which determine new binary experiments. For example, we can combine atomic sentences "Švrćo is a dog" and "Micika is a cat" into a new sentence "Švrćo is a dog and Micika is a cat". The new binary experiment is realized in such a way that the first binary experiment is realized on Švrćo, the second on Micika, and the final answer is "yes" if the answer to each particular experiment is "yes". Otherwise, the final answer is "no". Such a combination of sentences (binary experiments) A and B is described by the language form $A \wedge B$. Because of the level of Frege's abstraction, we abstract from various uses of the symbol "and" in the natural language just the combination of truth values. That is the main reason why symbol "$\wedge$" stands instead of "and". The connective "$\wedge$", like any other logical connective, regardless of whether it is abstracted from a natural connective or not, is determined by the description of how the truth value of the composed sentence depends on the truth values of sentences from which it is composed and on nothing else. Its semantic value is just the mathematical function which connects these truth values. This is another important feature of the language described here: its *extensionality*. Like a relationship between logical connective $\wedge$ and natural connective "and" there is a relationship between $\vee$ and "or", $\neg$ and "not", $\rightarrow$ and "if... then ...", $\leftrightarrow$ and "...... if and only if ...". The standard result of logic is that any other finite extensional combination of sentences to a new sentence can be realized by these connectives.

Why do we need these combinations at all, given that there is nothing new in them concerning rational cognition which is not present in atomic binary experiments? One reason is because of simpler utterance. For example, the connective $\neg$ ensures that the result of an experiment can always be "yes". Namely, when we assert something, let's say $P(a)$, we implicitly assume that this sentence is a true sentence. This comes from the way we use sentences in everyday situations. The connective $\neg$ ensures that in every experiment we can assert something, A or $\neg A$. Another reason for introducing connectives is that we can define new predicates by means of composed sentences. For example, we can describe new predicate $\underline{N}$ by sentence $N(a,b) \equiv P(a) \wedge Q(b)$. In this way, the language gives us a mechanism of abstraction which is indispensable to control the complexity. Furthermore, these combinations enable us to assert something even in situations when we do not have complete information. For example, I don't know where Švrćo is now, but owing to the connective "or" I can assert that he is in the house or in the garden. However, the main importance of combining binary experiments is to recognize and determine a regularity that is repeated in certain types of combinations. We recognize the simplest such regularity when we notice that every time one binary experiment of applying a predicate to an object gives the answer "yes", another binary experiment of applying another predicate to the same object also gives the answer "yes'. For example, every time when we assert that an object is a dog, we or somebody else, sooner or later, will also assert that the object is mortal. We combine experiments "Švrćo is a dog" and "Švrćo is mortal" into experiment "If Švrćo is a dog then Švrćo is mortal", or in a symbolic form $D(\check{S}) \rightarrow M(\check{S})$, where "$D$" stands for "is a dog", "M" stands for "is mortal" and "$\check{S}$" stands for "Švrćo". However, if we apply this combined experiment to my neighbour's dog Marley, the answer will be again "yes", or to my former dog Odi, too, etc. We can capture in a simple way this regularity by using a technique of variables. Variables are names of intentionally unspecified objects with the purpose of gaining generality. In a neutral context, it is common to use for variables the last letters from the alphabet, x, y, $\ldots$. Hence, we think that an experiment $D(x) \to M(x)$ will give the answer "yes" regardless of what object x we apply it to. The mechanism of variables enables us to describe a potentially infinite number of similar experiments by a simple language form. We combine potentially infinite applications of the same combined predicate to various objects into one new experiment which we validate as true if and only if all of the potentially infinite applications give the value true. For this combination we use the word "for all", that is, its logical analogue, symbol $\forall$, and we describe the combined experiment by the language form $\forall x(D(x) \to M(x))$, the so-called universal sentence. Because this experiment usually involves a large (potentially infinite) number of atomic experiments, it cannot be completely realized by doing all of these particular experiments. Thus, we cannot ever be sure that $\forall x(D(x) \to M(x))$ is true, but finding a false particular experiment we can be sure that it is false. The same type of combined experiment is behind existential sentences, when we use the word "there is", that is, its logical analogue, symbol $\exists$. The symbols $\forall$ and $\exists$, the so-called *quantors*, serve for this type of combination of sentences which is different from the combinations by connectives. It can be shown that any other combination of such a type can be realized by these two quantors and connectives. By means of quantors, we gain composed experiments which involve, in principle, an infinite number of atomic experiments. Quantors enable us to express or to form, to determine by language, regularities observed or foreseen in a cognitive interaction with nature. We have not devised or discovered those regularities of nature. They are also part of our synthesis with nature in the process of rational cognition.

However, quantification poses the so-called problem of induction [Hume, 1740]. Let's consider the simple form from the previous paragraph, $\forall x(P(x) \to Q(x))$, where "P" and "Q" are certain predicate symbols over potentially infinite domain. We can investigate the truth value of $P(x) \to Q(x)$ for every value of x (in principle) but we cannot do it for all (potentially infinite) values. So, $\forall x(P(x) \to Q(x))$ describes an experiment we cannot perform. Because this is a situation in which we can possibly get the answer "no" but never the answer "yes", we can conclude that this is not a binary experiment at all, and we could exclude this type of sentences from language. However, this is like a too strong cure which would kill the patient (science) together with the illness. We could not express regularities which we observe and which are the main sources of knowledge, as the history of science confirms. As C. D. Broad said "induction is the glory of science and the scandal of philosophy" [Broad, 1952]. Better option is historically chosen. It is again just an extension of the use of language in everyday situations: we accept such universal and existential sentences (and corresponding experiments) despite all uncertainty they bring. Besides the enormous evidence of the fruitfulness of this approach, we can also give an argument to support it. We cannot perform all the experiments on which the truth value of a universal sentence depends. However, all these experiments have a definite truth value. Therefore, the universal sentence also has a definite truth value, although we might not know what this value is. Here, we are in the same position as with naming and predication, when we name something although we do not know what, or when we use a predicate symbol although we do not know how to apply the corresponding predicate. As with naming and predicating, we extend the use of language in ordinary situations and assume that every sentence of the interpreted first-order language is true or false, regardless of the way we find its truth value, and even regardless of whether we can find it at all.

Previously, I did not say anything about meanings of language forms. It is a very important but difficult topic. However, concerning rational cognition, my opinion is Frege's opinion: semantic values (reference, in Frege's terms) yield to knowledge while the meaning (sense, in Frege's terms) is the means of achieving the semantic values. Frege is the first who analyses the meaning and semantic values systematically and gives priority to semantic values: "...the reference and not the sense of the words as the essential thing for logic." and "The reference is thus shown at every point to be the essential thing for science." [Frege, 1892a]. Although the whole art of constructing and using language lies in the faculty of meaning, I consider the reduction of semantics to semantic values a key to the success of modern logic. Since Frege is the one who insisted that reference is essential for logic and science I termed this reduction as *Frege's abstraction*.

To conclude, the language essence of rational cognition is the following one. By disjoining the world into objects and predicates, which we control through names and predicate symbols, we put binary questions to nature. By selecting one of two offered answers, nature brings its contribution to the framework, besides its contribution to the processes of naming and of predicating. In a binary experiment of applying predicate $\underline{P}$ to object a, when nature selects an answer, True or False, it "says" something about itself. With this valuation of the language form "$P(a)$" (which describes and controls the binary experiment), we gain knowledge about nature. This is the starting point for the overall rational cognition. The first-order language built upon these atomic sentences has the external assumptions of its use. These are: (i) the fundamental assumption of the language use of names – every name names an object, (ii) the fundamental assumption of the language use of function symbols – every function symbol symbolizes a function which applied to objects gives an object, (iii) the fundamental assumption of the language use of predicate symbols – every predicate symbol symbolizes a predicate which applied to objects gives one of the two possible results, "true" or "false", and (iv) the fundamental assumption of the language use of sentences – every sentence is true or false. Furthermore, the semantics is reduced to semantic values.

3 Consequences for ontology, truth, logic and thinking

3.1 Ontology

Because rational cognition is the synthesis of us and nature through the use of language, ontological assumptions, hence the most general assumptions about the world, are nothing else but the external assumptions about the use of language. When we say that there are objects in the world and that they have some properties, it seems that we say something general about the world. Truly, we say something about the language we use in rational cognition of the world: we assume that every name denotes something which we can extract from the world by naming, that every function symbol symbolizes a function, and that every predicate symbol symbolizes a binary investigative framework. Of course, because we synthesize by language rational cognition of the world, these assumptions about the use of language are at the same time assumptions about the world. However, these assumptions are not assumptions about the world itself, but about its connection with language in a process of rational cognition. Likewise, the truths that we achieve are not truths about the world itself – they are truths of our rational synthesis with the world. In this way, metaphysics disappears. Instead, we have our own language activity in rational cognition of the world.

3.2 Truth

The truth values *True* and *False* of atomic sentences are crucial elements of the language synthesis of rational cognition. They are parts of the binary experimental design that lies behind any such atomic sentence. Nature's selection of one of the values in the realisation of the experiment is nature's imprint into our framework. So, offered by us and selected by nature, the truth value of the sentence has an objective cognitive value which discriminates what is and what is not.

Clearly, this concept of truth, which I termed *the synthetic concept of truth*, is not a deflationary, disquotational, minimalist or any other conception which says that a concept of truth is not important. The synthetic conception of truth it is of crucial importance for rational cognition.

Also, the synthetic conception of truth is not a kind of correspondence theory of truth where the truth value of the sentence is determined only by whether the sentence corresponds with reality or not. In the synthetic conception of truth the sentence itself, with its truth value, forms reality, which is the result of the synthesis of us and nature.

I consider the synthetic concept of truth to be a semantic concept of truth but with an essential difference from Tarski's semantic concept of truth. Tarski says: ”We should like our definition to do justice to the intuitions which adhere to the classical Aristotelian conception of truth - intuitions which find their expression in the well-known words of Aristotle's metaphysics: 'To say of what is that it is not, or of what is not that it is, is false, while to say of what is that it is, or of what is not that it is not, is true'.” [Tarski, 1944]. However, Frege shows [Frege, 1897] that it is not possible to give an absolute definition of truth, because the application of such a definition depends on the truth of definiens, so it is a circular definition. As a special case, he shows that a correspondence theory of truth is impossible because it reduces the problem ”is a sentence true” to the problem ”is it true that the sentence corresponds with reality”, which again leads to circularity. Tarski's definition of the truth of a sentence is not an absolute definition of truth neither does it refine an intuition about truth as correspondence with reality. It is a relative definition of the truth of sentences in one language (object language) by the truth of sentences in another language (usually metalanguage). The definition is a translation of the truth for sentences in one language into sentences in another language, as Tarski explicitly states in his T-convention [Tarski, 1933]. Hence, in Tarski, the intuition about a correspondence theory of truth is realized as a correspondence of truth between two languages and not between language and reality. Tarski's recursive definition of truth reduces the truth values of compound sentences to atomic sentences, as indicated in Section 2. Tarski's and the synthetic conception of truth differ in the way they treat atomic sentences. Tarski finishes his definition by giving a translation of atomic sentences to metalanguage, and by this transferring the concept of truth from language to metalanguage. Contrary to this, in the synthetic conception of truth, the truth values of atomic sentences are undefined primitive elements determined by the process of rational cognition. In this way, the truth value of every sentence is connected with reality in a completely determined way. Therefore, I consider that the synthetic conception of truth is the solution to the philosophical problem of truth – is there any connection between truth and reality and, if so, what is the connection. The synthetic conception of truth shows that there is a connection and precisely shows what the connection is.

The synthetic concept of truth is elaborated in more detail in [Čulina, 2020b].

3.3 Logic

In a broad sense of the word *logic*, when we are asking what the logic of some mechanism is, we are asking how the mechanism is applied and how its parts are connected. Here, our "mechanism" is rational cognition, so the narrow sense of the word *logic* is that it is the logic of rational cognition. Because of the essential connection of rational cognition with language, it means that logic is primarily the logic of a language, how we apply the language, and how its parts are connected. It means that logic is not an objective science about propositions, thoughts, absolute truths or some other universal metaphysical ghosts, but it is a normative science of the organization and use of the particular language. The existence of various languages means the existence of various logics. Aristotle's traditional logic is the logic of the natural language, and the lack of the language precision puts limits on traditional logic. The first-order language is a precise language and it has a precise logic. Its logic is just its inner organization together with external assumptions of its use. It is important to emphasize that regardless of whether the exterior assumptions are fulfilled or not, the logic of the language demands that when we use the language we assume that they are fulfilled.

The inner organization of the first-order language consists of the syntactic structure of language forms, and their semantic values. These values rest on the exterior assumptions about the use of the language – every non-logical symbol of the language has a corresponding semantic value. The syntactical structure is determined by the rules of the construction of more complex language forms from simpler ones, starting with names for descriptions and with atomic sentences for sentences. In these constructions we use special symbols which identify the type of the construction. They are logical symbols (connectives, quantors, symbol for identity), because their semantic values are not determined by the reality (the interpretation) which is the subject of the language but by their use in the language constructions. From this inner organization of the language follows a relationship of logical consequence between sentences: the inner logic of the language (based on the exterior assumptions about the use of the language) obligates us that when we accept the truth of one set of sentences we must accept the truth of some other sentences. The concept of logical consequence is one of the crucial language mechanisms in the development of rational cognition, as it will be elaborated further in Section 4.1. Likewise, logical truths are sentences whose truth is determined by the inner organization of the language and the external assumptions of the language use, regardless of their particular connection with reality. Therefore, in traditional philosophical terms, logical truths are not necessarily a priori truths in an absolute sense of the word because they depend on the external assumptions of the language use, i.e. they depend on reality. Only under the condition that these assumptions are fulfilled, we can consider logical truths to be a priori truths. Furthermore, logical truths are not even analytical truths in an absolute sense of the word. They are primary synthetic truths of the constructed language. However, inside the language, we can consider logical truths to be analytic truths because we establish them by analysing the given language and not reality.

3.4 Thinking

Thinking is essentially related to language because we synthesise rational cognition through language and rational cognition is by definition cognition pursued by thinking. This connection is general, not only in relation to rational cognition. I will show it on an example. Let's imagine a good football player while playing football. His activities involve thinking about the situation. Moreover, he uses a theory which involves rules of the game, different strategies and tactics, as well as the directions of a coach. However, because of his experience, practice and natural talent, the football player in a particular situation thinks almost instinctively. I will term such thinking as immediate thinking. During such thinking, it seems that he does not use language. After the game, the football player can describe his play, analyse it, and think about other possibilities he could have realized. While doing this he uses language. The language is not just a means of transmitting information (for example, to a coach): it makes the analysis of the match possible. Even in the descriptive part, the language is crucial, because it enables the football player to extract from reality what was essential for the game. He does not mention that birds fled over the stadium, nor the hairstyle of an assistant referee, but he uses words, like "rival player", "empty space", "pass", "offside" etc. With these words, he abstracts from reality what is important for a description and an analysis of the game. Moreover, the knowledge that the football player has about the game and his behaviour in the game is also articulated and preserved in language. Could he acquire, preserve and use this knowledge without language? In any complex form, I consider the answer is no. Does this knowledge affect his perception and action (immediate thinking) during the game? I consider, yes. Could the football player analyse the game without language? It is hard for me to imagine how it could be possible. I consider that it is impossible to think about reality without abstractions and, concerning thinking, it is impossible to make abstractions without language. Concerning thinking, the abstractions are the language abstractions. Abstract thinking is, in its effect, just the construction and use of language.

4 Consequences for scientific theories and mathematics

4.1 Scientific theories

Although when we use the first-order language we assume that all the exterior assumptions of its use are fulfilled, usually they are not fulfilled, even in everyday situations. In a real process of rational cognition, we use names for which we do not know completely what they name, predicate and function symbols for which we do not know completely what they symbolise, and quantified sentences for which we do not know if they are true or not. Furthermore, although semantic values of the complex language forms are determined by semantic values of the simpler forms from which they are built, in the process of rational cognition we invert this original priority. An assertion about a particular object is more confident and more determined rational cognition then an assertion about all objects. However, we cannot apply all primitive (undefined) predicates to all objects, because there are too many objects, potentially infinitely many. Furthermore, some objects disappear, some come into existence. So, we cannot know the truth values of all atomic sentences. We rely more and more on the regularities which we notice. These regularities are formed by universal and existential sentences (laws). These sentences gradually become the main basis for rational cognition, although we cannot perform completely the complex binary experiments they determine. Moreover, these sentences speak often about idealized situations and idealized objects using idealized predicates. For example, in classical mechanics, we analyse a motion of the so-called material particles which at each moment of time occupy exactly one point in space. Hence, we assert something about objects which even do not exist in the strict sense of this word. We make assertions about such objects without any corresponding atomic sentence we could verify experimentally. Despite this, such assertions are the result of a deeper analysis of real situations and, through a kind of synthesis, give us powerful knowledge of real situations. Their effectiveness, if not our sense of their importance, testifies that they are essential for a deeper understanding of nature. All this means that our real knowledge, regardless of the degree of its accuracy, is almost always only a fragment of some assumed semantically complete language. In the process of rational cognition, we decrease unspecified parts of the language, even change the semantic values that had been already formed. This construction of the language web is a very creative process, full of imagination and beauty, which gives us the freedom to investigate even the most fantastic conceptions. However, this process is not chaotic, but it is, looking over longer periods, a constant advance in rational cognition of nature. That is because it has powerful regulatory mechanisms which control and drive it – the exterior interaction with nature through experiments and the inner logic of the language. Namely, for a theory to be a scientific one, at least some names and some function and predicate symbols must have an exterior interpretation, an interpretation in the exterior world, not necessarily a complete one. This partial external interpretation enables us to perform at least part of the binary experiments described by atomic sentences. This allows nature to put its answers into our framework, so that we can test our conceptions experimentally. Without this part the theory is unusable. On the other hand, the language disciplines us in a way that we shape our cognition and understanding into a set of sentences which we consider to be true. In an ideal case, we choose a not too big set of sentences we are pretty sure to be true, the axioms of the theory. Then, we are obligated, by the logic of the language, to consider true all sentences which logically follow from the axioms. So, another rationalized part of our conceptions consists of a set of sentences we consider to be true and to which we try to give an axiomatic organization. Therefore, a scientific theory about nature is a junction of a set of sentences (the sentence part of the theory) and partial external interpretation of the language (the interpreted part of the theory). From the axioms of the theory, we logically deduce the truth values of sentences. Particularly, we deduce the truth values of atomic sentence which belong to the external interpretation and which are, therefore, experimentally verifiable. If the truth values do not coincide with the truth values which nature gives, then the theory is wrong. If they are identical, it makes the theory trustworthy but, as we know, it is not proof that it is right. As Popper emphasizes, theories must be experimentally verifiable so that they can be falsifiable. In this interaction of the sentence part and the externally interpreted part of a theory, the real dynamics of the theory takes place: the axioms, as well as the interpreted parts, evolve, even change, and the same happens with the whole language framework.

It must be emphasised that we develop theories in the same way, regardless of whether we find them, in the end, wrong or not. Language enables the construction of a virtual world as well as a "real" world. Inside language, all these worlds are equally real. Only if we have an externally interpreted part of a theory then, through an interaction with nature, we can classify the theory as right (true) or wrong (false). For a theory which has no verifiable part, it is meaningless to classify it as a true or false theory.

An understanding of rational cognition, as the construction and the use of language, enables us to perceive its structure and limits. It is true that, to cite Wittgenstein: "The limits of my language mean the limits of my world." [Wittgenstein, 1921]. However, if we do not look at language as something static, then there are no static limits either. Language is a very flexible structure, which can support even the most unusual conceptions. In the dynamics of language development the limits of language change, too.

Carnap [Carnap, 1942] also constructs an ideal language for science. However, he poses very rigid conditions. Predicates must be empirical (completely feasible) or theoretical (non-empirical) but with correspondence rules which reduce such predicates to empirical predicates ([Carnap, 1966], Part V). In the approach developed here, science is the construction of the language which is not semantically complete in any phase of the construction. Popper [Popper, 1959] reacts on Carnap's rigid framework insisting on a minimal condition for a scientific theory, that it must be falsifiable. His view is attractive to scientists because it enables much-needed freedom to their work. However, what he says about the structure and the growth of scientific knowledge is not constructive enough. I think that the view of science as the construction of a language framework does not limit the scientific freedom while on the other side it is constructive – it provides the means for the hygiene, analysis and guidance of scientific ideas and conceptions. Popper would not agree that science is a linguistic activity, but by the very nature of rational cognition, it is a linguistic activity. As Whorf says [Whorf, 1941]: "We don't think of the designing of a radio station or a power plant as a linguistic process, but it is one nonetheless.". Even Kuhn's scientific revolutions [Kuhn, 1962] can be interpreted as radical changes of established language frameworks.

4.2 Mathematics

I consider mathematics an internal organization of our rational activities, above all rational cognition, a thoughtful modelling of that part of the process of rational cognition that belongs to us. For example, the first-order language is a mathematical model constructed for the use in rational cognition just like natural numbers are constructed for counting. This model is the result of thoughtful modelling of intuition about our natural language. Thoughtful modelling of other intuitions about our internal human world, for example, intuitions about quantity, symmetry, flatness, nearness, etc., leads to other mathematical models. Thereby, by our internal human world, I consider our behaviour as a biological species in an environment in which we exist, a space and time of our immediate senses and activities which we organize and design by our human measure. As an illustration for further considerations, I shall briefly display two classical models, natural numbers and real numbers, and one recent, set theory. A much more elaborate analysis of the nature of mathematics can be found in [Čulina, 2020a].

Natural numbers are the result of modelling our intuition about the size of a collection of objects. We measure the collection by process of counting, and natural numbers are objects for counting. To start counting we must have the first number, to associate it to the first object in the collection. To continue counting, after each number we must have the next new number, to associate it with the next object in the collection. It means that for counting it is not important how natural numbers are made, but only the structure of the set of natural numbers which enables us to count is important. Although it is not important how natural numbers are made, it seems that they exist in the same way as chess figures, in the sense that we can always realize them in some way. However, the structure of natural numbers, as opposed to the structure of chess, brings in itself an idealization. To be possible to continue counting forever, each natural number must have the next natural number. Therefore, there are infinitely many natural numbers. So, although we can say for small natural numbers that they exist in some standard sense of that word, the existence of big natural numbers is in the best case some kind of idealized potential existence.

Through real numbers we organize and make precise our intuition about the process of measuring. Real numbers are imagined as the results of such measuring. However, we can imagine situations in which an idealized process of measuring never stops – we generate a potentially infinite list of digits, with no consecutive repetition of the same group of digits after some step. If we want to have the results of such measurement processes, we must introduce, in addition to rational numbers, new results of measuring – irrational numbers. As opposed to natural numbers whose existence we can understand at least as some kind of an idealized potential existence, we cannot explain the existence of irrational numbers in this way. Although we can approximate irrational numbers by rational numbers with arbitrary precision, their existence is outside our means of construction. We have just imagined irrational numbers and they exist only in our imagination.

In the consideration of any objects, the consideration of the sets (collections) of these objects naturally occurs. Moreover, sets are often necessary for specification. For example, the specification of natural numbers requires the axiom of induction, which, in its full formulation, needs the notion of a set. Likewise, the specification of real numbers requires then axiom of continuity (completeness), which also needs the notion of a set. In what way are there the set of natural numbers and the set of real numbers, when both are infinite? Moreover, when we think of sets, we also consider sets of sets. If we want to have an elegant, rounded and universal theory of sets, infinite sets are naturally imposed on us, truly the whole infinite hierarchy of such sets, together with the infeasible operations on them. How can we understand the existence of such sets and operations? Should we reject this theory, which has proven to be very successful, because its objects can be realized only when they are finite? Hilbert, who certainly knew what is good mathematics, said of Cantor's theory of infinite sets: "This appears to me to be the most admirable flower of the mathematical intellect and in general one of the highest achievements of purely rational human activity."[Hilbert, 1925].

Major mathematical models, like the three previously described, arise from intuition about our internal activities and organization. It is from these concrete activities that the idea of an idealized world emerges. Due to the essential role of language in thought processes, we can only realize this idea by building an appropriate language. By choosing names, function symbols and predicate symbols, we shape the initial intuition into one structured conception. Since the conception goes beyond our real capabilities, the constructed language has only partial interpretation in our internal world. Since interpretation is partial, and because the domain of interpretation is usually infinite, we cannot determine the truth of all sentences of the language. Therefore, we must further specify the conception by appropriate choice of axioms. Thus, the final mathematical model is a junction of axioms and partial internal interpretation of language. The key difference with scientific theories is that the interpretation here is in our internal and not in the external world. Inferring logical consequences from the axioms, we establish what is true in the mathematical model. This can be very creative and exciting work and it seems that we discover truths about some existing exotic world, but we only unfold the specification. For example, In the model of real numbers, we deduce the existence of Euler number e, the irrational number to whom the sequence $\left(1+\frac{1}{n}\right)^n$ is closer when we take bigger and bigger natural number n. The number e certainly does not exist in the same way as my dog Švrćo. It exists in the same way as an idealized material point in classical mechanics, as non-existing phlogiston in a wrong theory about chemical reactions, and as Snow White in the classical fairy tale *Snow White and the Seven Dwarfs*. The "magic" of language enables us to imagine a world and existence of various objects with various properties and relationships in that world. We specify this world by axioms in an adequate language for which we assume that it is semantically complete language. Because of this assumption, in thinking itself there is no difference whether we think of objects that really exist or we think of objects that do not really exist. That difference can be registered only in a "meeting" with reality, ie through experimentally verifiable part of the language. If the language does not have such a part, and that is the case with mathematics, then the objects we are talking about exist only within the conception (story), although they do not exist in the external world. Equally, if the language has not an experimentally verifiable part then the sentences we consider true within the conception are not true in the external world. We cannot experimentally verify that $|| + || = ||||$ $(2 + 2 = 4)$, not because it is an eternal truth of numbers, but because it is the way we add tallies. Likewise, we cannot experimentally verify that $(x^2)' = 2x$ because it is the consequence of how we imagined real numbers and functions. Mathematical objects are, possibly, objects extracted from our inner activities, and mathematical truths are, possibly, truths about our inner activities. We are free to imagine any mathematical world. The real existence of such a world is not of primary importance: all that matters is to be a successful thought tool in the process of rational cognition. In Cantor's words, "the essence of mathematics lies precisely in its freedom" [Cantor, 1883]. The only constraint is, inside classical logic, that conceptions must not be contradictory. For Hilbert, in mathematics to exist means to be free of contradictions. In Hilbert's words: "the proof of the consistency of the axioms is at the same time the proof of the mathematical existence", [Hilbert, 1900]. In Dedekind's words, "numbers are free creations of the human mind; they serve as a means of apprehending more easily and more sharply the difference of things" [Dedekind, 1888]. These views are in sharp contrast with historical views that mathematical truths exist really in some way and that we discover them and not create them. Historically, this change of view occurred in the 19th century with the appearance of non-Euclidean geometries. The new philosophical view of mathematics has freed the human mathematical powers and it has caused the blossom of modern mathematics. It is a nice example of how philosophical views can influence science in a positive way.

The view on the nature of mathematics as a free human creation belongs to the tradition which has begun with Dedekind, Cantor and Hilbert. It has spread into the mathematics community through works of Emmy Noether, van der Waerden and Bourbaki.

However, how is it possible that something imagined can contribute to the rational cognition of the world? I will illustrate it with numbers. Real numbers are imagined objects which can be only approximately realized in our inner world. However, in the process of measuring, we connect them with the external world, enabling nature to select one of the offered numbers as its answer. The number itself is not real (it does not belong to the external world) but nature's selection is real. Numbers belong to our experimental framework of rational cognition but nature's selection of the number is a truth about the world. For example, in the process of measuring the speed of light, between all numbers nature selects the number c. The selected number c possibly exists as our inner construction. Whether it is a rational or irrational number depends on the choice of units of measurement. However, that c is the speed of light is an idealized truth about the world which is synthesized in the process of measuring. Idealized, because we assume that c is the result of an idealized process of measurement to which the actual measurement is only an approximation. In the same way, the simple assertion about natural numbers, that $2 + 2 = 4$, is a true sentence about the imagined world of natural numbers, and not a truth about the external world. However, through the real process of counting, we can use assertions about numbers to obtain a synthesizing assertion about the external world. For example, when we put two apples in a basket which already contains two apples, we predict that there will be $2+2 = 4$ apples in the basket. This is the prediction about reality deduced from the mathematical assertion that $2+2 = 4$ and the assumption that the mathematical model of counting and adding one set of things to another set is applicable in this situation. However, we must distinguish the mathematical assertion that $2+2 = 4$ from the assertion about the reality that when we add 2 apples to 2 apples there will be 4 apples. The best way to see the difference is to imagine a situation where we add two apples to two apples and get 5 apples. It would mean that it is not always true, as we have thought, that adding 2 apples to 2 apples gives 4 apples, but in some situations, according to as yet unknown physical laws, an additional apple emerges. However, this situation would not have any influence on the world of numbers. In that world, it is still true that $2 + 2 = 4$. It only means that in some real situations we cannot apply the mathematical model of counting and addition. The natural numbers model, as well as the language model of rational cognition, or any other mathematical model, have their assumptions of applicability. For the number model, we assume that we can associate the number of elements to a collection of objects by the process of counting. For the language model of some phenomena, we assume that every name names an object, that every function symbol symbolizes a function, that every predicate symbol symbolizes a predicate involved in the phenomena and that any sentence has a truth value. Only when the assumptions of the model of counting are fulfilled can we employ the mathematics of numbers to the real world. Likewise, only when the assumptions of the language model are fulfilled can we employ the mathematics of the language, its internal meanings and logical relations, to the phenomena. However, the question of the applicability of a mathematical model in some situation, is not a mathematical question at all. If a mathematical model is applicable, and it is an assumption about reality and not about the model, then we have at our disposal the whole mathematical world that can help us in asserting truths about the real world. We have at our disposal an elaborate non-verifiable language which we can connect with a verifiable language through assumptions about the applicability of the model. If a contradiction occurs in interaction with the verifiable part of the language, it does not mean that the mathematical model is false (the concept of the real truth and falsehood does not make any sense for the model) but that the assumptions about its applicability in that situation are false.

Quine in his naturalized epistemology ([Quine, 1951],[Quine, 1990]) considers that every part of the web of knowledge is liable to experiment, including logic and mathematics. That is true, but there are qualitative differences between science on one side and logic and mathematics on the other side. Experimental evidence can affect the truth values of scientific sentences but not the truth values of mathematical and logical sentences. It can only question the applicability or adequacy of mathematical models and language frameworks in some parts of science. Scientific theories are true or false of something while mathematical models, including the language models, are good or bad of something.

References

[Broad, 1952] Broad, C. D. (1952). *Ethics and The History of Philosophy.* Routledge.

[Cantor, 1883] Cantor, G. (1883). *Grundlagen einer allgemeinen Mannigfaltigkeitslehre.* Teubner, Leipzig.

[Carnap, 1942] Carnap, R. (1942). *Introduction to Semantics.* University of Chicago Press.

[Carnap, 1966] Carnap, R. (1966). *Philosophical Foundations of Physics.* Basic Books.

[Carroll, 1956] Carroll, J. B., editor (1956). *Language, Thought, and Reality: Selected Writings of Benjamin Lee Whorf.* The M.I.T. Press.

[Casirrer, 1923] Casirrer, E. (1923). *Philosophie der symbolischen Formen, Band 1: Die Sprache.* Cassirer, Bruno.

[Davidson, 2005] Davidson, D. (2005). *Truth and Predication.* Harvard University Press.

[Dedekind, 1888] Dedekind, R. (1888). *Was sind und was sollen die Zahlen.* Vieweg, Braunschweig.

[Einstein, 1936] Einstein, A. (1936). Physics and reality. *Journal of the Franklin Institute*, 221(36):349 – 382.

[Frege, 1891] Frege, G. (1891). Funktion und begriff. *Vortrag, gehalten in der Sitzung vom 9. Januar 1891 der Jenaischen Gesellschaft für Medizin und Naturwissenschaft.*

[Frege, 1892a] Frege, G. (1891-1892a). Ausfuhrungen über sinn und bedeutung. Published posthumously in [Hermes et al., 1969].

[Frege, 1892b] Frege, G. (1892b). Über begriff und gegenstand. *Vierteljahresschrift für wissenschaftliche Philosophie*, 16:192 – 205.

[Frege, 1892c] Frege, G. (1892c). Über sinn und bedeutung. *Zeitschrift für Philosophie und philosophische Kritik*, 100:25–50.

[Frege, 1893] Frege, G. (1893). *Grundgesetze der Arithmetik, Band I.* Verlag Hermann Pohle, Jena.

[Frege, 1897] Frege, G. (1897). Logik. Published posthumously in [Hermes et al., 1969].

[Frege, 1918] Frege, G. (1918). Der gedanke. *Beiträge zur Philosophie des deutschen Idealismus*, 2:58 – 77.

[Haynes, 2007] Haynes, K., editor (2007). *Hamann: Writings on Philosophy and Language.* Cambridge University Press.

[Herder, 1767] Herder, J. G. v. (1767). *Über die neuere deutsche Literatur: Fragmente.* J. F. Hartknoch.

[Herder, 1772] Herder, J. G. v. (1772). *Abhandlung über den Ursprung der Sprache.* Voß.

[Hermes et al., 1969] Hermes, H., Kambartel, F., and Kaulbach, F., editors (1969). *Gottlob Frege: Nachgelassene Schriften.* Meiner Verlag.

[Hilbert, 1900] Hilbert, D. (1900). Über den zahlbegriff. *Abhandlungen der Königlichen Gesellschaft der Wissenschaften zu Göttingen*, 8:180–183.

[Hilbert, 1925] Hilbert, D. (1925). On the infinite. *Mathematische Annalen*, 95:161–190.

[Humboldt, 1836] Humboldt, W. v. (1836). *Über die Verschiedenheit des menschlichen Sprachbaus und ihren Einfluss auf die geistige Entwicklung des Menschengeschlechts.* Bonn, F. Dümmler.

[Hume, 1740] Hume, D. (1738 – 1740). *A Treatise of Human Nature.* John Noon.

[Kuhn, 1962] Kuhn, T. S. (1962). *The Structure of Scientific Revolutions.* University of Chicago Press.

[Popper, 1959] Popper, K. (1959). *The Logic of Scientific Discovery.* Hutchinson & Co.

[Quine, 1948] Quine, W. V. O. (1948). On what there is. *Review of Metaphysics*, 2(5):21 – 38.

[Quine, 1951] Quine, W. V. O. (1951). Epistemology naturalized. *The Philosophical Review*, 60:20 – 43.

[Quine, 1957] Quine, W. V. O. (1957). The scope and language of science. *British Journal for the Philosophy of Science*, 8:1 – 17.

[Quine, 1990] Quine, W. V. O. (1990). *Pursuit of Truth*. Harvard University Press.

[Russell, 1905] Russell, B. (1905). On denoting. *Mind*, 14(56):479 – 493.

[Sapir, 1921] Sapir, E. (1921). *Language: An introduction to the study of speech*. Harcourt, Brace and Company.

[Tarski, 1933] Tarski, A. (1933). *Pojęcie prawdy w językach nauk dedukcyjnych*. Towarzystwo Naukowe Warszawskie.

[Tarski, 1944] Tarski, A. (1944). The semantic conception of truth. *Philosophy and Phenomenological Research*, 4:341–376.

[Čulina, 2020a] Čulina, B. (2020a). Mathematics - an imagined tool for rational cognition. unpublished.

[Čulina, 2020b] Čulina, B. (2020b). The synthetic concept of truth. unpublished.

[Whorf, 1940] Whorf, B. L. (1940). Science and linguistics. *Technology Review*, 42:229 – 231. 247 – 248.

[Whorf, 1941] Whorf, B. L. (1941). Language, mind and reality. *The Theosophist*, 63:1.1281 – 1.1291. 2.25 – 2.37.

[Wittgenstein, 1921] Wittgenstein, L. (1921). *Logisch-Philosophische Abhandlung*, volume 14. Annalen der Naturphilosophie.