The Synthetic Concept of Truth and Its Descendants

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Abstract. The concept of truth has many aims but only one source. The article describes the primary concept of truth, here called the synthetic concept of truth, according to which truth is the objective result of the synthesis of us and nature in the process of rational cognition. It is shown how various aspects of the concept of truth – logical, scientific, and mathematical aspect – arise from the synthetic concept of truth. Related to these aspects, i) the regression of truth is analysed and it is shown how the distinction between assertion and valuation resolves the regression, (ii) Tarski’s definition of truth is analysed and its role in the concept of truth is identified, and (iii) the truth predicate is analysed and it is shown why paradoxes of truth arise.

keywords: truth, the regression of truth, Tarski’s definition of truth, the truth predicate, paradoxes of truth

“The ideal subject of totalitarian rule is not the convinced Nazi or the convinced Communist, but people for whom the distinction between fact and fiction (i.e., the reality of experience) and the distinction between true and false (i.e., the standards of thought) no longer exist.”

Hannah Arendt

Many of the ambiguities surrounding the concept of truth stem from the fact that the concept has various aspects that are not sufficiently differentiated. Tarski’s T-schema [Tarski, 1933] is a classic example of this. T-schema is a set of T-sentences, the sentences of the form:

\[ T(⌜ϕ⌝) \leftrightarrow ϕ^* \]

where “T” is the symbol of the truth predicate, ϕ any sentence of a language L (usually the language we are considering), ⌜ϕ⌝ is the name of that sentence in a language ML (usually the metalanguage in which we consider L), while ϕ* is a translation of that sentence into ML. To get a concrete example of a T-sentence, I will take the English sentence “Svrco is afraid of thunder” (the language L will be part of the English language), and my native language as the language ML:

T(“Svrco is afraid of thunder”) ↔ Švró se boji grmljavine

where Svrco is afraid of thunder = Švró se boji grmljavine is a translation of the English sentence into my native language. Here the concept of truth appears in five places: as the truth value of
the left and right sides of the biconditional, as the truth value of the whole biconditional, as the meaning of the truth predicate symbol “T”, and as the truth value of the sentence “Svrco is afraid of thunder”. Only the last sentence belongs to the language $L$, while the other sentences and the symbol “T” belong to the language $ML$. However, all of them have a semantic source in the sentence “Svrco is afraid of thunder” of the language $L$. The left side of the biconditional through the symbol $T$ allows to speak in $ML$ about the truth value of the sentence “Svrco is afraid of thunder” of the language $L$, the right side of the biconditional is related to the truth value of the translation of that sentence into $ML$, while the truth value of the whole biconditional is related to the success of the translation. Thus, the key aspect of the concept of truth is related to the truth value of the sentence “Svrco is afraid of thunder” of the language $L$, while other aspects are connected to this primary aspect for various reasons. In what follows, I will focus on this primary concept of truth – the truth values of the sentences of the language $L$, leaving aside the truth values of the metalanguage in which I will carry the considerations. After analysing the primary concept of truth, I will consider other aspects of the concept of truth. Related to these aspects, in the last part of the article i) the regression of truth is analysed and it is shown how the distinction between assertion and valuation resolves regression, (ii) Tarski’s definition of truth is analysed and its role in the concept of truth is identified, and (iii) the predicate of truth is analysed and it is shown why paradoxes of truth arise.

There is a vast philosophical literature on the concept of truth. Although various aspects of the concept of truth have been addressed [Glanzberg, 2021], I have not come across a differentiation of the concept of truth as done in this article. An analysis of the concept of truth will be conducted on sentences. A convincing argument for such a choice was given, for example, by Quine [Quine, 1986].

1 The synthetic concept of truth

To determine the truth value of the sentence “Svrco is afraid of thunder” we must know the meaning of its parts. Knowledge of English grammar tells us which parts they are and what their linguistic meaning is: “Svrco” is the name of an object, and “is afraid of thunder” is a predicate expression. However, in order to determine the truth value of the above sentence, we must know exactly which object the word “Svrco” names and what the full meaning of the predicate expression “is afraid of thunder” is. Svrco is my only pet, and every connoisseur of English knows the full meaning of the word “is afraid of thunder”, despite the fact that we do not know clearly enough what the “full meaning of a predicate expression” means. Knowledge of these meanings is necessary but not sufficient to determine the truth value of the sentence “Svrco is afraid of thunder”. We still have to do an appropriate experiment, let nature give its contribution, to determine that it is a true sentence.

This example illustrates the basic cognitive situation of putting an object $a$ in an investigative framework (experimental apparatus) that results in one of two possible answers. I will term such a binary framework a predicate $P$. We apply the predicate $P$ to an object $a$ and describe the situation with the declarative atomic sentence “$P(a)$”. The result can take two values, yes and no. These are the so-called truth values of the language form “$P(a)$” termed True and False. True and False are designed by us as a part of the binary experiment design and selected by nature in the realization of the experiment. These binary experiments are the essence of our rational synthesis with nature. We make the question and offer two possible answers (binary experiment design), and nature selects an answer (realization of the experiment). The selected truth value is the value of this synthesis which discriminates what is and what is not. That is why I have termed this primary concept of truth the synthetic concept of truth.
The cognitive situation illustrated and described above, simple as it might seem, has a number of underlying characteristics and assumptions which need to be clarified. First of all, it reflects our innate approach to the world which we divide into objects (elements upon which something is done) and into predicates (which determine what is done). This division is not absolute – something that is a predicate in one context can become an object to which other predicates are applied in another context. This object-predicate dualism is a fundamental characteristic of the cognitive framework described here. It is reflected in language through the structure of the atomic sentence “P(a)”. Symbols “a” and “P” have different roles in the sentence. We use symbol “a” to name (mention) an object a. We use symbol “P” to say something about the object a. Because of these different roles, I say that symbol “P” symbolizes a predicate P. To my knowledge, Whorf is the first one to recognise that the object-predicate dualism is a prominent feature of Indo-European languages: “Our language thus gives us a bipolar division of nature. But nature herself is not thus polarized.” [Whorf, 1940]

A fundamental semantic assumption of the use of the atomic sentence “P(a)” in rational cognition is that “a” names an object. This rests on the assumption that it is possible to extract from the world something to be named. How we make the extraction and how we keep the connection between the name and the named in the flow of time is a very complex subject, and it will not be analysed here. One thing is for certain, the process of naming is also a kind of our synthesis with nature. I will term the named object the semantic value of the name.

The next fundamental semantic assumption of the use of the sentence P(a) in rational cognition is that the predicate symbol “P” symbolizes predicate P. This connection between the language form and reality is even more complex than naming, and it will not be analysed here. However, from the way we address nature through object-predicate construction it follows that no predicate is independent of us – it is the product of a cognitive interaction between us and nature. What is functionally crucial for predicates is that they are the full meanings of predicate symbols in the sense that they are procedures or processes, binary experimental frameworks, which applied to each object determine, through the intervention of nature, the result of the experiment – the truth value of the corresponding atomic sentence. In other words, we know the full meaning of a predicate symbol, if we know how to apply it to every object in order to, with the help of nature, get the result, True or False. Thus, each predicate determines, through the intervention of nature, a mathematical function (in the mathematical extensional sense) from objects to truth values. I will call this function the semantic value of the predicate (and of the corresponding predicate symbol). However, we must not equate a predicate and its semantic value. Otherwise, we would “destroy” the whole language mechanism of rational cognition. A predicate is part of the process of rational cognition while its semantic value is the final result of this process, in which nature is substantially involved. Reduced to an atomic sentence, it means that the resulting truth value gives unity to the atomic sentence: it makes the atomic sentence to be something more than just the concatenation of its parts, the predicate symbol and the name involved in the sentence.

In his book “Truth and Predication” [Davidson, 2005], Davidson points out the key problem of “unity of proposition” that the theory of truth and predication must solve. I consider this analysis provide the solution. Although I came to this solution in another way, it can be considered a solution that is obtained when we subtract metaphysics from Frege’s solution [Frege, 1891]. The essential difference between Frege’s and my approach to predicates is that Frege considers predicates (concepts, in his terms) to be metaphysical entities in the Platonic sense of the word [Frege, 1897, Frege, 1918], while I consider them to be binary investigative mechanisms that belong to our real activities.

As I have analysed one-place predicate symbols, I can also analyse multi-place predicate symbols.
The analysis of function symbols is similar to the analysis of predicate symbols. Every function symbol symbolises a function, a procedure that, when applied to objects, determines an object, with the help of nature. Thus, the semantic value of the function (and of the function symbol) is the corresponding mathematical (extensional) function between objects.

To conclude, the essence of the synthetic concept of truth is the following one. By dis-joining the world into objects and predicates, which we control through names and predicate symbols, we put binary questions to nature. By selecting one of two offered answers, nature brings its contribution to the framework, besides its contribution to the processes of naming and of predicating. In a binary experiment of applying predicate $P$ to object $a$, when nature selects an answer, True or False, it “says” something about itself. With this valuation of the language form “$P(a)$” (which describes and controls the binary experiment), we gain knowledge about nature. This is the starting point for the overall role of the concept of truth in our rational cognition.

Clearly, this concept of truth is not any kind of a deflationary conception of truth which says that a concept of truth is not important (various formulations can be found, for example, in [Stoljar and Damnjanovic, 2014]). The synthetic conception of truth is of crucial importance for rational cognition. Also, the synthetic concept of truth is not a kind of correspondence theory of truth where the truth value of the sentence is determined only by whether the sentence corresponds with reality or not (various formulations can be found, for example, in [David, 2020]). In the synthetic conception of truth the sentence itself, with its truth value, forms reality – reality is the result of this synthesis of us and nature. I consider that the synthetic concept of truth is the solution to the philosophical problem of truth – is there any connection between truth and reality and, if so, what is the connection. The synthetic concept of truth shows that there is a connection and precisely shows what the connection is.

2 The logical concept of truth

We can build various language structures over atomic sentences. The object-predicate dualism naturally leads to the first order language, which not only has a simpler and clearer semantics than other languages, but also proves to be the most important type of logical language. In what follows, I will assume this language. Each complex sentence of the language describes a particular experiment which is a combination of experiments described by atomic sentences. Eg. the sentence $P(a) \land Q(b)$ describes a binary experiment composed of the experiments described by the sentences $P(a)$ and $Q(b)$. This experiment applied to $a$ and $b$ yields True when both atomic experiments yield True, otherwise it yields False. This experiment can be considered as the result of applying a new predicate $R$ to the objects $a$ and $b$ which is defined as follows: $R(x, y) \leftrightarrow P(x) \land Q(y)$. Likewise, $\forall x P(x)$ can be considered a new experiment that gives the value True when for each valuation of the variable $x$ the experiment $P(x)$ gives the value True, while otherwise it gives the value False. A mathematical (extensional) function is connected with each linguistic construction of a sentence from simpler sentences. The function determines the truth value of the constructed sentence on the basis of the truth values of the sentences from which it is constructed. This connection of truth values gives recursive conditions which, together with the truth values of atomic sentences, determine a unique mathematical function that assigns, in a given evaluation of variables, a truth value to each sentence. This aspect of truth, the interconnectedness of the truth values of sentences of a language, I will term the logical aspect of the concept of truth. From this aspect follows a relationship of logical consequence between sentences, one of the crucial language mechanisms in the development of rational cognition. Likewise, from this aspect follows the property of the logical
truth of a sentence – it is the sentence whose truth is determined by the organization of the language regardless of its particular connection with reality. However, what is most important is that, in a first order language, the truth value of each sentence is entirely determined by the truth values of atomic sentences. According to the synthetic concept of truth, the truth values of atomic sentences are primitive semantic elements of language determined by the process of rational cognition. In this way, the truth value of each sentence is connected with reality in a completely determined way.

3 The scientific concept of truth

The first order language built upon interpreted atomic sentences has the external assumptions of its use. These are: (i) the fundamental assumption of the language use of names – every name names an object, (ii) the fundamental assumption of the language use of function symbols – every function symbol symbolizes a function which applied to objects gives an object, (iii) the fundamental assumption of the language use of predicate symbols – every predicate symbol symbolizes a predicate which applied to objects gives one of the two possible results, “True” or “False”, and (iv) the fundamental assumption of the language use of sentences – every sentence is true or false. In a real process of rational cognition, we use names for which we do not know completely what they name, predicate and function symbols for which we do not know completely what they symbolise, and quantified sentences for which we do not know if they are true or not. However, it is important to emphasize that regardless of whether the exterior assumptions are fulfilled or not, the logic of the language demands that when we use the language we assume that they are fulfilled. In thinking itself there is no difference whether we think of objects that really exist or we think of objects that do not really exist and whether the predicate symbols we use can be applied to such objects at all or not. That difference can be registered only in a “meeting” with reality. Furthermore, although semantic values of the complex language forms are determined by semantic values of the simpler forms from which they are built, in the process of rational cognition we invert this original priority. An assertion about a particular object is more confident and more determined rational cognition than an assertion about all objects. However, we cannot apply all primitive (undefined) predicates to all objects, because there are too many objects, potentially infinitely many. Furthermore, some objects disappear, some come into existence. So, we cannot know the truth values of all atomic sentences. We rely more and more on the regularities which we notice. These regularities are formed by universal and existential sentences (laws). These sentences gradually become the main basis for rational cognition, although we cannot perform completely the complex binary experiments they determine. Moreover, these sentences speak often about idealized situations and idealized objects using idealized predicates. For example, in classical mechanics, we analyse a motion of the so-called material particles which at each moment of time occupy exactly one point in space. Hence, we assert something about objects which even do not exist in the strict sense of this word. We make assertions about such objects without any corresponding atomic sentence we could verify experimentally. Despite this, such assertions are the result of a deeper analysis of real situations and, through a kind of synthesis, give us powerful knowledge of real situations. All this means that our real knowledge, regardless of the degree of its accuracy, is almost always only a fragment of some assumed semantically complete language. The whole dynamics of a scientific theory can be understood as the dynamics of completing and changing an appropriate language. However, this process is not chaotic, but it is, looking over longer periods, a constant advance in rational cognition of nature.\footnote{Even Kuhn’s scientific revolutions [Kuhn, 1962] can be interpreted as radical changes of established language frameworks.} That is because it has powerful regulatory mechanisms which control and drive it – the
exterior interaction with nature through experiments and the the logic of language. And at the core of these mechanisms is the synthetic concept of truth. It gives legitimacy and perspective to scientific research. I will term this aspect of the concept of truth the scientific aspect of the concept of truth.

4 The mathematical concept of truth

The concept of truth in mathematics essentially depends on the accepted philosophy of mathematics [Horsten, 2019]. Thus, the mathematical concept of truth presented here also depends on a certain philosophy of mathematics, which is elaborated in [Čulina, 2020b].

I consider mathematics primarily the internal organization of rational cognition. Building a logical language is one such organization. The first order language is a mathematical model constructed for the use in rational cognition just like natural numbers are constructed for counting. So, I consider logic is also part of mathematics. Major mathematical models arise from intuition about our internal activities and organization. It is from these concrete activities that the idea of an idealized mathematical world emerges. By choosing names, function symbols and predicate symbols, as well as setting certain specifications, we shape the initial intuition into one structured conception. However, here the role of functional and predicate symbols, as well as the truth values of sentences, is different than in rational cognition. Predicates are not investigative tool to address questions to nature, there is no intervention of nature, and thus no synthesizing role of truth values. Because we create a mathematical world we have a complete control in its design. We determine on which objects the predicate will give truth, in the same way as we we decide which character in a fairy tale will be good. For example, we can decide which natural numbers less than 100 will have some (unimportant) property $U$ (we will just enumerate such numbers). So, in mathematics, predicates are reduced to their semantic values, functions from objects to truth values. Likewise, when we describe a mathematical world by some set of axioms, inferring logical consequences from the axioms, we establish what is true in that world. This can be very creative and exciting work and it seems that we discover truths about some existing exotic world, but we only unfold the specification. The inferred sentences are not true because the world they describe is such, but that world is so conceived that those sentences are true in it. They are the conditions that the world must satisfy. I will term this aspect of the concept of truth, as a specification of the imagined mathematical world, the mathematical aspect of the concept of truth. Since I consider logic to be part of mathematics, the logical aspect of the concept of truth is also part of the mathematical aspect of the concept of truth. I would note that we have already encountered this mathematical aspect in logic on the example of a linguistic construction using the conjunction $\land$. This conjunction is directly associated with its semantic value, the corresponding Boolean function, without an intensional intermediate step.

5 The assertion-valuation distinction and the regression of truth

All previous considerations have been done in the appropriate metalanguage whose sentences also have their truth values. Using sentences of the language $ML$ I discussed the truth values of sentences of the first order language $L$. The reader will reflect on the correctness of my considerations, that is, on the truth values of my assertions. The insights she will thus gain are composed of sentences which also have truth values, which may be the subject of other sentences. And so on indefinitely. However, since the pattern is repeated in this infinite regression, it is sufficient to look at one step,
the transition from \( L \) to \( ML \), that is, to analyse the connection of the sentences \( T(\phi^-) \) and \( \phi \). Without loss of generality, we can concentrate on the connection between the sentences “Svrco is afraid of thunder” is a true sentence” and “Svrco is afraid of thunder”. The main difference in the use of these sentences is that when I say “Svrco is afraid of thunder”, the subject of my expression and thought is my dog Svrco, and when I say “Svrco is afraid of thunder” is a true sentence”, the subject of my expression and thought is the sentence “Svrco is afraid of thunder”. This is a typical use-mention distinction. In the first case I use the sentence “Svrco is afraid of thunder” to say something about Svrco and in the second I mention the sentence to say something about it. What is specific here is that we are talking about the truth of that sentence, where each of the above sentences has its own truth value. If, for example, we were talking about the number of letters in that sentence, nothing would be disputable. But, as far as the truth is concerned, there is also a difference between the above sentences. I will term it assertion-valuation distinction. Namely, the very way we use a (declarative) sentence is related to the transfer of information to consider the sentence true. When we use a sentence, we assert the sentence – not only do we say it or write it, but we also convey the information that we consider it true. So, when I assert “Svrco is afraid of thunder”, in addition to the information about Svrco, I convey the information that it is a true sentence. So, there is no need to assert it in a roundabout way with the sentence “Svrco is afraid of thunder” is a true sentence” (by which I again convey the information that this sentence is true). However, if someone considers the truth of that sentence he will not use it but will mention it and evaluate its truth. If he concludes that it is true, he will end his analysis with the assertion “Svrco is afraid of thunder” is a true sentence”. This assertion-valuation distinction is a mechanism for stopping or prolonging truth regression. The assertion aspect stops the regression, and the valuation aspect continues the regression. So if we agree on something, that’s where the regression ends. Usually the regression stops in the metalanguage because, if disputes do occur, they are disputes about the truth of the sentences of the language \( L \) and not about the truth of the sentences of the metalanguage, so they are resolved by the assertions of the metalanguage. If someone disputes what I have said about the truths of sentences of the language \( L \), he disputes the truth of the corresponding \( ML \) metalanguage sentence. But the subject of his analysis will again be the language \( L \) and the conclusion he draws will be the assertion of the metalanguage \( ML \) and not its metalanguage \( MML \).

As far as I know, the importance of the linguistic mechanism of assertion was first pointed out by Frege [Frege, 1897]. How subtle and important the concept of assertion is in Frege can be read in [Pedriali, 2017].

6 Tarski’s definition of truth

As analyzed in the introductory part of the article, Tarski’s T-scheme is a classic example in which various aspects of the concept of truth are mixed. This extends to Tarski’s definition of truth, too – some see the definition as an argument for the correspondence theory of truth, others for the deflationary theory of truth. A comprehensive analysis of Tarski’s work and various critiques of the work can be found in [Patterson, 2012]. In this section, Tarski’s T-scheme and Tarski’s definition of truth are analysed in relation to the aspects of truth differentiated in this article, especially in relation to the synthetic concept of truth.

Regarding the analysis of the concept of truth, the assertion-valuation distinction shows that truth value occurs in two ways, implicitly as part of an assertion or explicitly through the truth predicate symbol, i.e. through mentioning the truth value of a sentence. So to assert the sentence
T(⌜ϕ⌝) which explicitly says that the sentence ϕ is true is to assert the sentence ϕ, and vice versa. If we ignore the translation problems and consider that the metalanguage $ML$ is an extension of the language $L$, this means that all T-sentences are true, that is, to avoid regression, we can assert that for every sentence ϕ of the language $L$:

\[ T(⌜ϕ⌝) \leftrightarrow ϕ \]

The nature of the truth of T-sentences can be viewed in various ways, depending on how we view the truth predicate through which the truths of the left and right sides of the biconditional are equated. However, they all belong to the mathematical aspect of the concept of truth. It is common to consider each such T-sentence as a partial definition of the truth predicate. In this case, the T-sentences are analytical truths. So, this is a logical aspect of the concept of truth. This view is directly related to Tarski’s analysis of the concept of truth, which is more suited to a more general T-scheme $T(⌜ϕ⌝) \leftrightarrow ϕ^*$. Tarski’s definition of the truth of the language $L$ in the language $ML$ [Tarski, 1933] is a formally correct definition because it enables the elimination of the defined predicate symbol T in every sentence of the language $ML$. The definition is also a materially adequate definition in the sense that all T-sentences logically follow from it. However, Tarski’s definition of truth has the role of a content-wise definition only when we want to set the truth of the sentences of one as yet uninterpreted language $L$ by using the truth of the sentences of another language $ML$. This definition transfers the meaning, and thus the truth value of the sentence $ϕ^*$, to the truth of the sentence $ϕ$ via the appropriate T-sentence. That is why Tarski’s definition is so important in mathematical logic. However, for the interpreted language, it has no content-wise sense because it defines something that has already been determined. In such a context, this definition simply gives a translation from the language $L$ to the language $ML$ via the T-scheme: each sentence $ϕ$ of the language $L$ is translated into the $ϕ^*$ sentence of the language $ML$. If the translation is correct, it preserves the meanings and thus the truth values of the sentences. In this situation, Tarski’s definition is simply a mathematical construction of the translation function. Thus, it is a mathematical aspect of the concept of truth that makes it possible to connect the truths of sentences of two different languages. But whether Tarski’s definition is a substantive definition or just a mechanism of translation from one language to another, it only transfers the problem of the truth of a sentence of one language to the same problem of the truth of the corresponding sentence of another language. Instead of examining the truth of the statement “Svrcio is afraid of thunder”, we can now examine the truth of the statement “Svrcio se boji grmljave”. Since the translation is correct, it is one and the same problem. This is best seen when the metalanguage $ML$ is an extension of the language $L$, i.e. when we have a T-scheme $T(⌜ϕ⌝) \leftrightarrow ϕ$. Then Tarski’s definition translates the problem of the truth of the sentence “Svrcio is afraid of thunder” to the problem of the truth of the sentence “Svrcio is afraid of thunder”.

The problem with Tarski’s definition of the concept of truth and the interpretation of his contribution to the analysis of the concept of truth is as follows. Tarski says: “We should like our definition to do justice to the intuitions which adhere to the classical Aristotelian conception of truth – intuitions which find their expression in the well-known words of Aristotle’s metaphysics: ‘To say of what is that it is not, or of what is not that it is, is false, while to say of what is that it is, or of what is not that it is not, is true’.” [Tarski, 1944]. However, Frege shows [Frege, 1897] that it is not possible to give an absolute definition of truth, because the application of such a definition depends on the truth of definitions, so it is a circular definition. As a special case, he shows that a

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2If we were to use the more general T-scheme $T(⌜ϕ⌝) \leftrightarrow ϕ^*$ related to a metalanguage that is not an extension of the language $L$, due to the question of correctness of translation, the scientific aspect of the concept of truth would be present, too.
correspondence theory of truth is impossible because it reduces the problem “is a sentence true” to the problem “is it true that the sentence corresponds with reality”, which again leads to circularity. Tarski’s definition of the truth of a sentence is not an absolute definition of truth neither does it refine an intuition about truth as correspondence with reality. It is a relative definition of the truth of sentences in one language (object language) by the truth of sentences in another language (usually metalanguage). The definition enables a translation of the truth for sentences in one language into truth of sentences in another language, as Tarski explicitly states in his T-convention [Tarski, 1933]. Hence, in Tarski, the intuition about a correspondence theory of truth is realized as a correspondence of truth between two languages and not between language and reality. Tarski’s recursive definition of truth reduces the truth values of compound sentences to atomic sentences. Tarski’s and the synthetic conception of truth differ in the way they treat atomic sentences. Tarski finishes his definition by giving a translation of atomic sentences to metalanguage, and by this transferring the concept of truth from language to metalanguage. Contrary to this, in the synthetic conception of truth, the truth values of atomic sentences are undefined primitive elements determined by the process of rational cognition. In this way, the truth value of every sentence is connected with reality in a completely determined way. Thus, Tarski’s definition of the concept of truth correctly formulates recursive conditions that connect the truth of a constructed sentence with the truth of the sentences from which it is constructed, while by translating the truth of atomic sentences of language $L$ into the truth of sentences of metalanguage it ceases to be a content-wise theory of truth.

7 The truth predicate and the paradoxes of truth

The previous analysis of Tarski’s definition also gives the answer about the meaning of the truth predicate symbol $T$. The predicate is simply part of the description of the logic of the first order language $L$. Just as the syntactic structure of sentences is described starting from atomic sentences, so the semantic structure of sentences is described starting from the truth values of atomic sentences. Starting from the truth values of atomic sentences as given, it is described how the truth values of other sentences are related. Of course, this description was made in metalanguage using the truth predicate, whether we used the expressions “this sentence is true” or “the truth value of this sentence is True”. Thus, in the usual case when the metalanguage is an extension of the language, so there is no problem with the correctness of the translation, the truth predicate symbol is part of the logical vocabulary of the language $ML$, like connectives, quantifiers and the predicate symbol of equality. The only difference is in universality. The truth predicate symbol for the language $L$ can only belong to languages that extend the language $L$ in such a way that the language is part of their domain of interpretation. As we set, for example, the internal conditions (conditions that are part of the logic of the language and do not depend on the reality that the language speaks about) on the truth of the sentence $\varphi \land \psi$ in relation to the truth of the sentences $\varphi$ and $\psi$, so we also set the internal truth condition on the sentence $T(⌜\varphi⌝)$. According to the meaning of the truth predicate symbol $T$, this sentence is true when $\varphi$ is true and false when $\varphi$ is false, where $\varphi$ is a sentence of the language $L$. Thus, the truth predicate belongs to the logical concept of truth. This also means that all T-sentences $T(⌜\varphi⌝) \leftrightarrow \varphi$, where $\varphi$ is a sentence of the language $L$, are logical truths.

This analysis of the truth predicate also shows how the paradoxes of truth arise. It is a standard situation in science that atomic sentences of the language $L$ have a certain truth value that is the result of our rational cognition. On the other hand, in the metalanguage $ML$ we use the truth predicate symbol $T$ to describe the relations of truth values of sentences of $L$. So, the truth
predicate symbol \( T \) for the language \( L \) is not part of the language \( L \). Such a situation does not lead to paradoxes. Namely, according to the previously described truth condition on the logical symbol \( T \), in order to examine whether the atomic sentence \( T(⌜\varphi⌝) \) of the language \( ML \) is true, we need to examine whether the sentence \( \varphi \) of the language \( L \) is true, and its truth is completely determined by the truth of the atomic sentences of the language \( L \). Thus the truth value of the sentence \( T(⌜\varphi⌝) \) is unambiguously determined. This Tarski’s solution of the paradoxes of truth, given in [Tarski, 1933, Tarski, 1944], is achieved by an appropriate syntactic restriction: the metalanguage \( ML \) contains the symbol of the truth predicate of the language \( L \) and the language \( L \) itself does not contain it. Paradoxical sentences are simply forbidden by the very syntax of the language. This solution is quite satisfactory for scientific practice. However, as Kripke has clearly shown in [Kripke, 1975], natural language does not support such syntactic restrictions. In it the truth predicate is applicable to all its sentences \( (L = ML) \). Now, too, by the truth condition on the logical predicate of truth, the examination of the truth of the atomic sentence \( T(⌜\varphi⌝) \) is reduced to the examination of the truth of the sentence \( \varphi \), and the examination of its truth is reduced to the examination of atomic sentences. But now some of these atomic sentences can again be of the form \( T(⌜\psi⌝) \), so that the process does not stop but continues again. While for the standard language \( L \) which speaks of some natural phenomenon and does not contain its own truth predicate symbol, this procedure gives a unique answer, now we have no guarantee that the reduction procedure will stop at some step or that we will get unique truth values of sentences covered by such procedure. Let us consider the two simplest examples where the truth determination procedure is not successful:

the sentence \( L: \neg T(⌜L⌝) \) (The Liar)

the sentence \( I: T(⌜I⌝) \) (The Truth teller)

For the sentence \( L \) we have the following chain of reduction:

\[ L \leftrightarrow \neg T(⌜L⌝) \leftrightarrow T(⌜L⌝) \leftrightarrow L \leftrightarrow \ldots \]

It is easy to see that no evaluation along this chain satisfies the truth conditions: the assumption that \( L \) is true gives that \( L \) is false, and the assumption that \( L \) is false gives that \( L \) is true. Thus we cannot assign any truth value to the sentence \( L \). On the other hand, for the sentence \( I \) we get the following chain of reduction:

\[ I \leftrightarrow T(⌜I⌝) \leftrightarrow I \leftrightarrow \ldots \]

Now both evaluations, the evaluation according to which \( I \) is true and the evaluation according to which \( I \) is a false sentence, satisfy the truth conditions along the chain. So, this sentence can be both true and false in an equally (un)convincing way.

The paradoxes of truth stem precisely from the fact that the classical procedure of determining truth value does not always have to give a classically assumed (and expected) answer. Such an assumption is an unjustified generalization from common situations to all situations. We can preserve the classical procedure but we must reject universality of the assumption of its success. The awareness of that transforms paradoxes of truth to normal situations inherent to the classical procedure. In [Čulina, 2001] it is shown how this analysis leads to the solution of the paradoxes of truth, to the construction of a language in which the paradoxes can arise but also analysed and resolved. In [Čulina, 2020a] this solution has been applied to the various types of the paradoxes of truth that commonly occur in the literature.
8 Conclusion

In 1991, Milošević and Tuđman met in Karadordevo, in the former Yugoslavia. They talked behind closed doors, with no witnesses, and no record was left of the conversation. Did they then make an agreement on the partitioning of Bosnia and Herzegovina along so-called ethnic lines, and thus destroy so many human lives and cause so much human suffering? The synthetic concept of truth gives us the legitimacy to ask that question, and all of the above aspects of the concept of truth can help us get the answer one day.

References


