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An Overview of Black Holes

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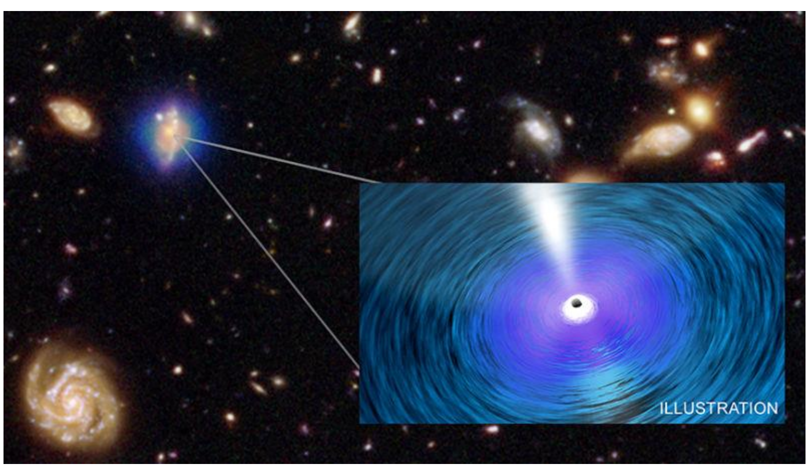
ABSTRACT

Black holes are one of the fascinating objects in the universe with gravitational pull strong enough to capture light within them. Through this article we have attempted to provide an insight to the black holes, on their formation and theoretical developments that made them one of the unsolved mysteries of universe.

Keywords: Black Holes, General Theory of Relativity, Schwarzschild Metric, Schwarzschild Radius, Kerr Metric, Kerr-Newman Metric, Chandrasekhar Limit, Tolman-Oppenheimer-Volkoff Limit, Reissner-Nordstorm Metric, Gravitational Collapse, Singularity

Introduction

Black hole is a region in space-time where gravitational field is so immense that it is impossible even for light or any other electromagnetic radiation to escape. Hence, it is not visible to us. However, it is predicted through its interaction with other matter in its neighborhood. They are the prediction of Einstein's General Theory of Relativity. Early on 18th century, Mitchell and Laplace had suggested about the objects with intense gravitational pull strong enough to trap light within it, but black holes appeared in the equations of physics, after Schwarzschild gave the solution of Einstein's field equation in early 1916. The discovery of pulsars in 1967 ignited back the interest on gravitationally collapsed bodies to the scientists, and finally in 1971, Cygnus X-1, first black hole was detected.



(An Illustration of black hole on top of data of distant galaxies. Image Credit: X-ray: NASA)

The size of black hole depends upon the remnant core of the star after the formation of supernovae explosion. So their sizes vary from micro, stellar size to super massive. It is generally believed that most of the galaxies have supermassive black holes at their centre. Our very own galaxy, Milky Way has supermassive black hole at its centre, commonly known as Sagittarius A*. The term black hole became popular in physics after it was publicly advertised by John Wheeler in 1967.

Formation of Black Holes

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Stars relieve on the Hydrogen as fuel during their entire lifetime. Once the hydrogen gets consumed, then the star begins to contract in itself as the gas particle move away from each other to maintain stars gravitational pull towards them. During this time the star heats up, and some fraction of its outer layers are ejected in the space which is commonly known as Supernova explosion. After the explosion the remaining inner core of the star begins collapsing in itself and this final remaining core determines the coming fate of the stars.

Chandrasekhar discovered that stars having the core above the mass of Chandrasekhar limit (1.4 Solar Masses) would be unable to overcome the above gravitational collapse and ultimately result in singularity, where the gravitational field would be strong enough to trap light.

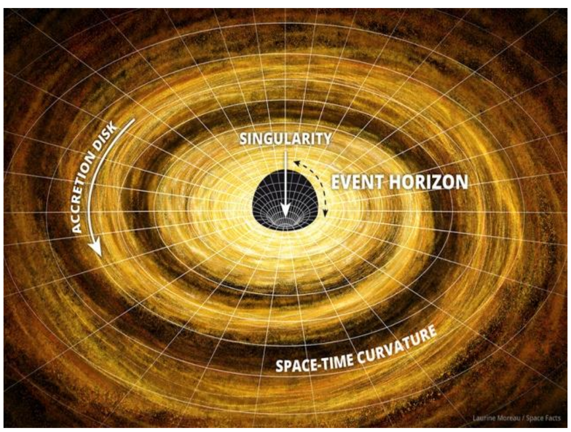
The stars with mass less than Chandrasekhar limit then, would reach their final stage being white dwarf. However, the stars with mass slightly more than that of Chandrasekhar limit would also have the chance of forming Neutron stars- highly dense stars composed of neutrons, and they are stable in nature. Finally R. Oppenheimer developed sensible explanation, for the instability of stars under gravitational collapse which would result in the singularity as hypothesized by Chandrasekhar.

Most of the black holes are formed due to the gravitational collapse of dying stars, which occurs when the star's internal pressure is not sufficient enough to resist their very own gravity.

There is no any significant difference between the gravitational field of black hole and any spherical object of the same mass, as it is ruled by Birkhoff's theorem that only the vacuum solution is spherically symmetric.

For their description we use different metrics according to the property they exhibit. Black holes lacking electric charge and angular momentum are described by the Schwarzschild metric. Uncharged but rotating black holes are described by using Kerr Metric. Charged but non-rotating and spherically symmetric black holes are described by Reissner-Nordstorm Metric. Whereas both charged and rotating black holes are described using the Kerr-Newman Metric. Black holes formed from the gravitational collapse of stars are thought to have neutral charge due to the large scale of electromagnetic force. The singularity for non-rotating black hole is known as point singularity whereas the singularity for rotating black holes is known as Ring singularity.

(An Illustration of Black Hole. Image Credit: Space Facts)



Theoretical Developments

Einstein published his General Theory of Relativity in 1915, with his field equations,

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu}R + g_{\mu\nu} \wedge = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Where,

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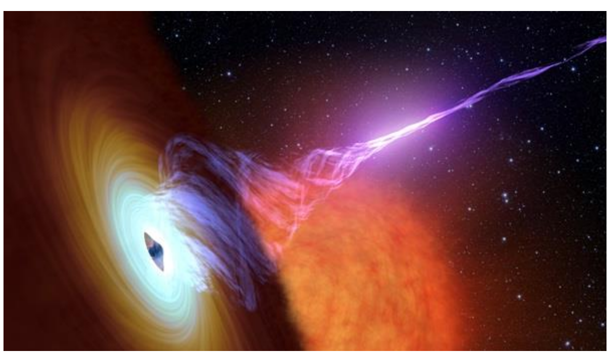
R = scalar curvature,
$R_{\mu v}$ = Ricci Tensor,
$g_{\mu v}$ = metric tensor,
Λ = cosmological constant,
G = Newton's Gravitational constant,
c = speed of light in vacuum,
$T_{\mu v}$ = Energy Stress Tensor
pointing out that the presence of mass would deform the space-time in a phenomenal way that the path taken by the particle world would bent towards the mass.

Few months later, Karl Schwarzschild found the solution for Einstein's field equations of a point and spherical mass with his solution,

$$c^2 d\tau^2 = \left(1 - \frac{r_s}{r}\right) c^2 dt^2 - (1 - \frac{r_s}{r})^{-1} dr^2 - r^2(d\theta^2 + sin^2\theta d\varphi^2)$$

Where,
$r_s = \frac{2GM}{c^2}$ Is the Schwarzschild radius for the object of mass M,
τ = proper time if $d\tau^2$>0,
t = time co-ordinate,
r = radial co-ordinate,
θ = colatitude,
φ = longitude
Concluding that any matter could be theoretically drawn into a point of singularity, i.e. of zero volume and infinite density. Schwarzschild further determined the boundary of such singularity by his above equation of radius, known as Schwarzschild's radius or the radius of event horizon-a place where time would eventually stop for the external observer, but not to the in falling one.



(An Illustration of black holes releasing jets of radiation. Image Credit: Astronomy Magazine)

In 1931 young Indian physicist Subhrahmanyam Chandrasekhar by using Special Relativity calculated that for non-rotating mass above Chandrasekhar limit

$$M_{limit} = \omega_3^0 \frac{\sqrt{3\pi}}{2} \left(\frac{\hbar c}{G}\right)^{3/2} \frac{1}{\mu_e m_H}$$

$\omega_3^0 \approx 2.018236$ is constant,

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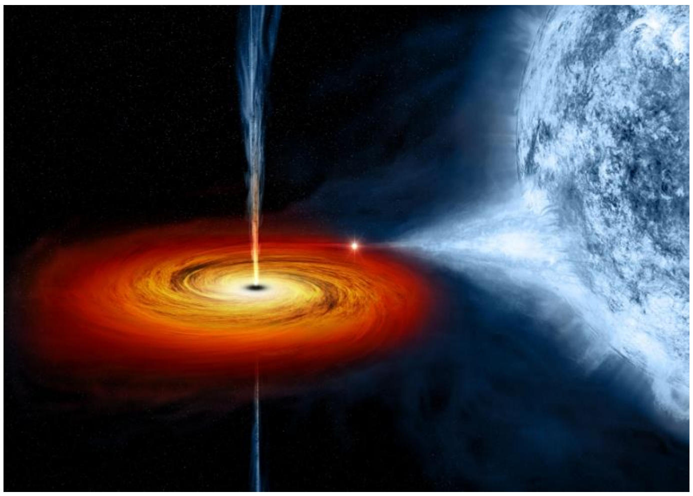
$\left(\frac{\hbar c}{G}\right)$ is the planck's mass,
$\hbar$ = reduced Planck's constant,
μ_e = Avg. mol. Weight per electron, and
m_H = Mass of Hydrogen atom
Has no any further stable solution and eventually result in the gravitational collapse.
But since white dwarfs-objects in the space slightly massive than the Chandrasekhar's assumption would collapse into a stable neutron star obeying Pauli's Exclusion Principle, concrete explanations for black holes weren't seen in the physics till then.



(Illustration showing a black hole pulling the matter from its nearby star. Image Credit: NASA)

It was in 1939, when Robert Oppenheimer and other hypothesized that neutron stars exceeding the Tolman-Oppenheimer-Volkoff limit would, follow the Chandrasekhar's assumption, ultimately collapsing into the black holes, and no laws of physics would prevent it from being so, described by TOV equation $\frac{dP_{(r)}}{dr} = -\frac{G}{r^2}\left(\rho_{(r)} + \frac{P_{(r)}}{c^2}\right)\left(M_{(r)} + 4\pi r^3 \frac{P_{(r)}}{c^2}\right)\left(1 - \frac{2GM_{(r)}}{c^2\, r}\right)^{-1}$
Where,
P = Pressure, and ρ = density at given values of r.

Debates were ongoing in the physics community, and in 1965 Roy Kerr gave the exact solution for rotating black holes with his metric

$$c^2 d\tau^2 = ds^2 = -\left(1 - \frac{r\, r_s}{\Sigma}\right) c^2 dt^2 + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 + \left(r^2 + a^2 + \frac{r a^2 r_s}{\Sigma} sin^2\theta\right) sin^2\theta\, d\phi^2 - \frac{2 r_s\, r a^2\, sin^2\theta}{\Sigma}\; c\, dt\, d\phi$$

Where,
r, θ, and ϕ are elements of spherical co-ordinates system,
a, ∑, and ∆ are introduced for convenience defined as

$$a = \frac{J}{Mc}$$

$\sum = r^2 + a^2 + cos^2\theta$
$\Delta = r^2 - r_s\, r + a^2$
$r_s = \frac{2GM}{c^2}$ Is the Schwarzschild radius for the object of mass M

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Further in 1965 Erza Newman gave axis symmetric treatment considering both the rotation and electrical charge of the black hole.
Later on, No Hair Theorem was evolved by the work of Werner Israel, Brandon Charter, and David Robinson, which completely described the stationery black hole in terms of three independent parameters, Mass, Angular Momentum, and Electric Charge, as given by the Kerr- Newman metric,

$$c^2 d\tau^2 = -\left(\frac{dr^2}{\Delta} + d\theta^2\right)\rho^2 + (\,cdt - a\,sin^2\theta\,d\phi)^2\,\frac{\Delta}{\rho^2} - \left((r^2 + a^2)d\phi - acdt\,\right)^2\,\frac{sin^2\theta}{\rho^2}$$

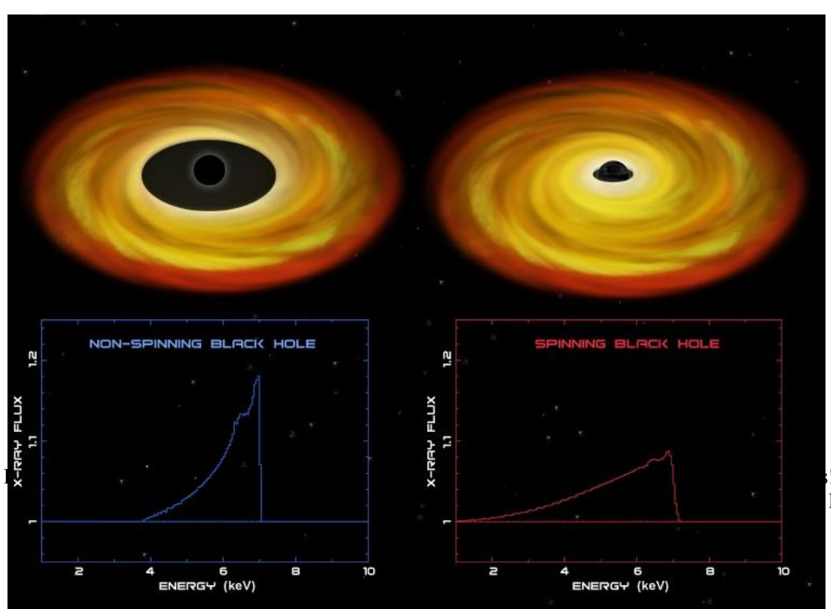
Where,
r, θ, and ϕ are elements of spherical co-ordiante system,
$\rho^2 = r^2 + a^2cos^2\theta$,
$\Delta = r^2 - r_s\,r + a^2 + r_Q^2$

These parameters were essential as they would help to locate the black holes. A charged black hole, like any other charged bodies would repel charge in its vicinity. The ADM mass, often known as the gravitational analogue of Gauss Law, gives the total mass inside the sphere containing the black hole. Angular Momentum can be calculated by the process of frame dragging, as space time is slightly dragged within the surrounding of any rotating mass, hence any other objects in its immediate neighborhood would tend to be aligned in the direction of its rotation.
Similarly, black holes spherically symmetric black holes having charge but no rotation is described by Reissner-Nordstorm metric

$$ds^2 = \left(1 - \frac{r_s}{r} + \frac{r_Q^2}{r^2}\right)c^2dt^2 - \left(1 - \frac{r_s}{r} + \frac{r_Q^2}{r^2}\right)^{-1}dr^2 - r^2d\Omega^2$$

with $d\Omega^2$ defined as
$d\Omega^2 = d\theta^2 + sin^2\theta\,d\phi^2$

Further in 60's, the work of Stephan Hawking, Roger Penrose and other concluded the existence of singularity within a black hole as a consequence of General Theory of Relativity. They even led to formulation of black hole thermodynamics, by a conventionally easy way of relating, mass to its energy, surface gravity to its temperature, and area to its entropy. Further in 1974, Hawking showed that like a black body- a black hole must radiate radiation with a certain temperature proportional to its surface gravity.



Many mathematical treatments have been seen till today in for the black holes, but their core lies within the above metrics.
(An Illustration of Spinning and Non-spinning black holes. Image Credit: Physics of the Universe)

Consequences

General Theory of Relativity predicts several consequences for the black holes. The first one is gravitational time dilation, i.e. for an observer sitting far away from the event horizon of black hole, any clock near the event horizon would appear to tick slower, and hence any object approaching the event horizon would take infinite time to reach it. Another considerable effect is the gravitational redshift. Any object getting close to event horizon would emit the light red and dim to an observer far away, and as it reaches event horizon, it becomes so dim that after that it can never be seen again. All these effects are observable for external observer only, whereas any observer crossing the event horizon would never feel any of such effects. Once it crosses the event horizon, then all information possessed by it, is lost forever, which is commonly known as Black Hole Information Paradox.

Conclusion

Black holes are one of the most fascinating objects in the universe and have long puzzled the scientists with their absurd nature. Even after their formation, they can grow continuously by absorbing the materials from its neighborhood. Once any matter crosses the event horizon, then all information possessed by it is loosed forever to the singularity of black hole. Due to the immense gravitational pull, matter hovers around the event horizon and is seen in the form of disk, commonly known as accretion disk. These matter particles revolving in the accretion disk are heavily compressed and hence the friction developed among them, converts the kinetic energy of the particle into heat and radiation. Thus, by observing those radiations, scientists predict the presence of black hole. Depending upon the nature of black holes, we treat them by using different mathematical metrics for their complete description. Einstein famously remarked that space and time are woven together in flexible fabric. Black holes with their enormous gravitational pull distort the space and time, and have always remained as one of the primary focus of study for the scientists.

To study them more precisely a project entitled as Event Horizon Telescope is working, which is thought to provide us with the first picture of event horizon of black hole. It is a worldwide collaboration of scientists using powerful radio telescopes through a large network, which ultimately acts as a virtual telescope almost as big as the size of our earth. Scientists believe that solving the puzzles of black holes would reveal many realms of the cosmos.

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