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Axioms, Definitions, and the Pragmatic *a priori*

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AUTHOR'S NOTE

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1. Introduction

1 "This is the day for doubting axioms" (EP1: 242 [1886]), exclaimed Peirce, who also predicted that the tendency to question the exact truth of axioms is "likely to teach us more than any other general conception" (*ibid.*: 215 [1884]) in the coming years. His proclamation has been borne out, but perhaps not in the way intended: the foundations of mathematics in the early twentieth century was largely concerned with axiomatization and the philosophical interpretation of this process. However, these formal endeavours were estranged from the pragmatist (or pragmaticist) sentiments of Peirce and (especially) Dewey,[1] who were not so interested in the formal "foundations" of mathematics. Instead, they viewed the axioms of the mathematical sub-disciplines as amenable to change and reflective alteration, ultimately responsible to the demands of natural science.

2 The question of axioms' epistemic status and metaphysical interpretation is of no small importance to philosophy. Indeed, since its very initial stirrings, and throughout the

canon, Western philosophy has been enamoured with the seeming undeniable and rigorous method of the mathematicians. Peirce notes that metaphysics, in particular, has imitated geometry, and with the rejection of the (Euclidean) geometrical axioms, so must the metaphysical axioms be thrown out (*ibid.*: 242 [1886]; 296 [1891]). Instead he claimed that we should have no expectation that the fundamental assumptions of geometry be perfectly exact or that every phenomenon in all details follows any law precisely (*ibid.*: 216 [1884]; 296 [1891]).

3 Peirce's denial of the exactitude of axioms is derived from his general views about inquiry and spontaneity, but his focus on geometry is derived from his historical context. From Euclid to Kant, the rules of geometry were supposed to be self-evident truths about space; not only were they necessary and indubitable, but they were also directly applicable to the world. This standpoint began to break down in the 19th century with the investigation of more general geometrical systems and their subsequent applications to relativistic physics. It has become clear that instead of being self-evident truths which are immediately known to reason, the axioms of any geometrical (mathematical) system are postulates, adopted because of the fruitfulness of their consequences. With this change in how mathematical axioms are conceived, philosophers lost their canonical example of immediate knowledge of universal principles (Dewey 1939: 141). At the same time, this has liberated mathematicians from the restriction of their postulates to existential matters, resulting in a proliferation of abstract mathematical work not directly concerned with scientific applications.

4 The alternative view espoused by Peirce and Dewey holds that the axioms of logic, metaphysics, and mathematics – like those of the natural sciences – are not logically necessary premises. Dewey summarizes Peirce's notion of axioms as leading principles, which are

> formulations of *operations,* which (a) are hypotheses about operations to be performed in all inquiries which lead to warranted conclusions; and (b) are hypotheses that have been confirmed without exception in all cases which have led to stable assertions; while (c) failure to observe the conditions set forth have been found, as a matter of experience of inquiries and their results, to lead to unstable conclusions. (Dewey 1939: 156)

5 In fact, this can be taken as a succinct statement of a central thesis of (Peircean) pragmatism, in which axioms (postulates) are hypothetical, operational, tentative, and judged based on their consequences in long-run inquiry.

6 Part of the goal in this paper is to outline how axioms and definitions play a similar role in mathematics as other pragmatic *a priori* (or constitutive) elements do in empirical science. This is achieved, firstly, by excavating the roots of the pragmatic *a priori* in Peirce and Dewey (and through Lewis), and mapping the relevant notions onto mathematics. In particular, Dewey's emphasis on the continuity of knowledge practices is argued to be equally applicable to mathematics as it is to natural science. This historicity of knowledge is not only compatible with the pragmatic concept of apriority, but it also has the advantage of recognizing and accounting for the presence of conceptual change in these disciplines. It is hoped that this work is a contribution to the pragmaticist philosophy of mathematics, as it navigates between foundationalism and relativism, while appreciating apriority and contextualism.

2. The Self-Evident, the Necessary, and the Rational *a priori*

7 Many philosophers have held the idea that demonstration (or knowledge) must rest on ultimate and indubitable propositions. Descartes is Peirce's kicking-boy for this position, but he is not the only representative. To accept what we do not doubt is no real commitment; indeed, it is no choice at all, because "to accept propositions which seem perfectly evident to us is a thing which, whether it be logical or illogical, we cannot help doing" (EP1: 126 [1878]). For the pragmatist, doubts must be expressed in some meaning system for them to be intelligible as doubts – we cannot doubt everything all at once, like Descartes. In fact, pragmatists generally hold that what is undoubted in one context does not have to remain undoubted in all other contexts, since the undoubted in an inquiry may itself come to be the focus of an investigation. As such, the "doubted" versus "undoubted" distinction is always contextually contingent (Popp 1998: 60).

8 Rationalist philosophers have typically held that the foundational universal principles are known immediately to the faculty of reason (or rational intuition); such knowledge is logically necessary and *a priori*. The foundation for this (rationalist) conception of the *a priori* – the intuitive apprehension of first principles through the power of reason – is no longer tenable, says Dewey. The ground of the necessary *a priori*, and of deductive reason, was thought to be this faculty which had the power of direct apprehension of self-evident, axiomatic truths; this notion was derived from (one way of understanding) that pinnacle of the axiomatic method: Euclidean geometry. However, if the methodology of metaphysics is based on that of geometry, once we reject the classical interpretation of the latter, it raises doubts about its use in the former.

9 On the contrary, Peirce holds that the essence of the *a priori* method (in metaphysics) is simply to think as one is inclined to think (EP1: 121 [1877]). Relying on an indubitable faculty (intuition) reduces the self-evident to simply the obvious (to the thinker involved); as such, this method is very susceptible to the reproduction of bias. At one point, Peirce says that the only logical criteria by which this method can be corrected is consistency (*ibid.*: 138 [1878]) – and this is a very weak condition for the production of axioms which are meant to form the ground of all additional knowledge. In fact, this results in a sort of sterile conventionalism: this rational-logical method is reliable in producing analytic statements, either through stipulative definitions of term-use or tautologies. Similarly, tautologies are necessarily true – or true in all possible worlds – but this contributes only very general restrictions on the structure of the actual world (Popp 1998: 16). Neither sort of these kinds of statements, although *a priori*, satisfy our desire for metaphysical or scientific axioms which provide a basis for synthetic knowledge; one could say that they are devoid of content (EP1: 18 [1868]).

10 Although their criticisms of the rationalist *a priori* are well-founded, Dewey and Lewis (and Popp) tend to equate the *a priori/a posteriori*, analytic/synthetic, necessary/contingent, and conceptual/empirical distinctions (Rosenthal 1987: 121). Peirce faces similar problems when trying to decide if necessity and fallibilism applies to mathematics.[2] Stump also argues that because there is no "sharp" distinction between the analytic and the synthetic, "it is also impossible to make a sharp divide between a priori and empirical statements" (Stump 2015: 111). More recent work by, e.g., Sullivan 2018, does a good job at disentangling these equivalences. The separation between

these dichotomies opens up room for a much more subtle interaction between, for example, the empirical and the *a priori*. For now, we will note that the choice of axioms or definitions is based on much more than simple consistency. In particular, it involves similar pragmatic considerations as in the choice of scientific postulates. Both can be corrected in light of new theoretical and empirical results and changes in desired ends and practices.

11 The main argument for supposing that there is a (unique) logical basis for knowledge is in order to avoid an infinite regress or a vicious circle. However, if our starting point consists of hypotheses, which are not supposed to be indubitable, then we can avoid both problems. By adopting a framework of inquiry that is fallible, it can grow with inquiry; we do this by interpreting the *a priori* as something which can be reconstructed in light of experience (Popp 1998: 60). Since we cannot start from nothing, we have no choice but to use our prior conclusions in the course of subsequent inquiry. These factual and conceptual tools are of indispensable practical value moving forward. However, they are not indubitable or necessary, and they are not exempt from reexamination and reconstitution. That our hypotheses have provided an adequate basis for previous investigations is not a logical demonstration that they will continue to work in new situations (Dewey 1939: 140-1). This approach – which we shall call the pragmatic *a priori* – relies on (hypothetical) postulates, rather than (indubitable) axioms.[3]

12 There is another prevalent notion of the *a priori* in the post-Kantian era: the constitutive (or relativized, or dynamic) *a priori*, which emerged from the work of the logical positivists Schlick, Reichenbach, and Carnap. Recent commentators include Friedman 2001, Stump 2015,[4] and Sullivan 2018. These two visions of the *a priori* share some commonalities in that they reject the classical 'rationalist' position and entertain the *a priori* as contextual and hypothetical. This commonality provides one avenue for rapprochement between logical positivism and pragmaticism.[5] They both also offer a way forward for naturalists and moderate rationalists to defend the *a priori* without invoking Kant or Descartes. It also begs addressing whether difficulties apply to both equally, e.g., Quine's holism, incommensurability, etc. However, before we delve into an explication of the *a priori* from the pragmatists, we need to explain the role of continuity in the process of inquiry, because our hypotheses build on what comes before and so there cannot be abrupt or radical changes in the development of (scientific) knowledge.

3. Continuity

13 Continuity is a central idea to the thought of both Peirce and Dewey. Although both reject the rationalist account of the *a priori*, neither do they adhere to a classical empiricist train of thought. It would be better to say that pragmatists are naturalists, and therefore the locus of truth is to be found in nature, not the mind (although it *is* contextualized in and through inquiry and/or reflection). Dewey explicitly claims his theory of logic to be naturalistic, so that the first principles of logic "represent conditions which have been ascertained during the conduct of continued inquiry to be involved in its own successful pursuit" (Dewey 1939: 11). There is a continuity in the temporal progression of logic as a result of our cumulative advancement through inquiry and reflection. The conclusions of any special inquiry give rise to the

developments of further inquiry *(ibid.)*. The same can be said of mathematical development, which is both cumulative and progressive, resulting from both scientific investigation and conceptual work.

14 Since there is temporal continuity and progression, it is essential to Dewey's naturalistic theory of logic that there is no sharp distinction between the lower (earlier, less complex) and the higher (later, more complex). The same must hold for mathematics: there is no hard separation between the biological and cultural origins of mathematical activity and the later abstract and rigorous mathematical theory. Logic, mathematics, science, and metaphysics are fundamentally embedded in the biological and cultural matrix in which they were born, grew, and live (*ibid.*: 20). Indeed, Dewey claims that "the habit that operates in an inference is purely biological" (*ibid.*: 12), since the intellectual operations are foreshadowed in behaviour of the biological kind, and the latter prepares the way for the former (*ibid.*: 43).

15 However, this does not imply that there is no difference between *a priori* and *a posteriori* justification; the continuity between the higher and the lower is diachronic, while the epistemic distinction is synchronic and dynamic – perhaps relative to a theory. Similarly, the epistemic status of particular axioms or definitions within mathematics can change over time, but the entire enterprise emerges from within the biopsychosocial history of humanity. It should also be noted that the presence of revolutionary theoretical changes is not contrary to Dewey's project. He acknowledges that "the objective subject-matter of inquiry undergoes temporal modification" (*ibid.*: 119), which is explained by Brown as the addition of "new meanings and connections to the qualities, events, and things of our experience" (Brown 2012: 298), not unlike a Kuhnian paradigm change. However, the continuity that Dewey speaks of is more coarse-grained and anthropological than the Kuhnian analysis which detects revolutionary changes in history.[6]

16 Of course, there does seem to be a practical distinction between biological and psychological activity, but the point is that this is not an absolute difference. There is a transition, in our evolutionary past, from organic behaviour into the reflective and the self-conscious, but this is not a discontinuity in the same way as if Reason or Intuition are used to explain this difference (Dewey 1939: 44). It is not as if there is a moment in time, before which was purely organic, and afterwards we had access to the rational *a priori*. New forms and structures can emerge – there can be progress – without the assumption of disjointedness. This continual progression of inquiry precludes the sudden appearance of a radically new perceptual faculty, but this does not mean that the indispensability of logical principles is denied; the issue becomes one about their origin, development, and use. The answer to the first two is the combination of biology and culture, while the third has its ultimate explanation in the manipulation of existential matters and their function in these operations.

17 This continuity is also expressed by Dewey's linking the origin of science to common sense and practical matters. There is no justification is asserting a distinction between the "productive" knowledge of artisans and the "rational" work of scientists because this presupposes an absolute difference between experience and the rational by putting limits on what experience must be (*ibid.*: 37-8). Dewey explains that logic is experiential, in that it is distinct from *a priori* speculation and intuition (*ibid.*: 39). The same can be said of mathematical experience, and this is within the spirit of Dewey's approach. To deny that there is such a thing as mathematical experience is to put an *a*

priori limit on what experiences can be and also negates the continuity between colloquial mathematical activities – like counting, measuring, and fitting-together – and the theorem-proving, computational, or theorizing activity of professional mathematicians.

18 This approach fits well with Dewey's naturalistic, evolutionary, and operational approach to language, which he takes in a very wide sense, including oral and written speech as well as gestures, rites, and ceremonies, monuments, and products of the industrial and the fine arts. A tool or machine is not only a physical object, but also a mode of language, because it says something about its function (*ibid.*: 46). In the same way, we could expand mathematics to include not just the modern axiomatic systems, but also things like computing, architecture, carpentry, masonry, music, etc., because they reflect mathematical (geometrical and arithmetic) principles.

19 In addition, the meaning or representative capacity of language is a conventional matter but it is not merely stipulative; it requires agreement in action or social use. For example, the meaning of legal words is defined by a court and by judges, but (under common law) these motions are to be interpreted in future actions by society; this subsequent behaviour is what actually determines the meaning of the words in question *(ibid.)*. Meaning is essentially tied to social action, and a physical sound or mark is a part of language only through its operational force. It "functions as a means of evoking different activities performed by different persons so as to produce consequences that are shared by all the participants in the conjoint undertaking" (*ibid.*: 48). Over time, conceptions of the law are formalized through these ordinary interactions; they are not imposed absolutely prior, or external, to the proceedings, and they are not fixed or eternal once imposed. When these legal conceptions are formalized, they are formative and constitutive because they regulate and structure the proper conduct of the relevant activities (*ibid.*: 102). The same will hold of logical principles, as well as mathematical and scientific principles. The "independence" of the rules of law, or science, logic, or mathematics, is intermediate rather than complete and final. They have their origin in experience, while also modifying future interactions with existential matters (*ibid.*: 103). These principles are derived from experience and from the examination of methods of our previous inquiries, and at the same time, they are operationally *a priori* with respect to future investigations (*ibid.*: 14).

4. Inquiry, Postulates, and the Pragmatic *a priori*

20 The formalized principles of previous inquiry, at a certain stage of investigations, ought to be treated as established laws (EP1: 216 [1884]) – this is something like the Kuhnian stage of normal science. During this time, the consequences of the assumed rules are teased out and tested, since to continually doubt one's presuppositions make it nearly impossible to proceed. At a later stage, once we have collected adequate information and reasoning regarding these assumptions, we can either formulate a guiding principle, or hypothesis, which solidifies the previous principles, or we are forced to alter them and shift our paradigm. This procedure reflects the continuity of inquiry and involves Peirce's circle (or triad) of inference: hypothesis (abduction), deduction, and induction. In one sense, we begin with abduction (based on prior knowledge), proceed with deduction of their consequences, and then inductively perform

experimental observations. However, this is not a closed loop, nor a vicious circle, because we have acknowledged that there is no discontinuity in this development. This is contrary to a strong reading of Kuhn leading to incommensurability between progressive theories.

21 The deductive consequences of a set of hypotheses are not merely formal, but essentially have to do with the habits that they induce and their effects on existential matters. Guiding principles are not simply premises for a series of arguments but are the conditions that provide direction for further testing (Dewey 1939: 13). The essence of belief, to a pragmatist, are the habits which emerge from one's conclusions. Indeed, to Peirce, if two beliefs do not differ in the rule of action that they prescribe, then they are not different at all (EP1: 129-30 [1878]). In the same vein, it is impossible that "we should have an idea in our minds which relates to anything but conceived sensible effects of things. Our idea of anything is our idea of its sensible effects" (*ibid.*: 132). These ideas are at the core of what pragmatism proposes. What matters is the operational character of our hypotheses, not whether we take them to be true. If we take our axioms to be true or unquestionable, then the history of science, says Dewey, shows that inquiry is thereby obstructed (Dewey 1939: 142). It is because our assumptions are provisional, and revisable upon further inquiry, that we make continual progress in the sciences. We can call these tentative presuppositions postulates, rather than axioms, to reflect this difference. In one sense, to postulate a proposition is simply to hope that it is true (EP1: 300 [1892]). In a stronger sense, Peirce says that it is "the formulation of a material fact which we are not entitled to assume as a premise, but the truth of which is requisite to the validity of an inference" (*ibid.*: 301). We can hope that anything can be true, and postulate it accordingly, but its justification is to be found in its functioning as a starting point for our inferences, ultimately leading to a habit, or operation, in relation to experience (*ibid.*: 302).

22 A postulate is a provisional assumption, to be judged according to its consequences, i.e., operationally. According to Dewey, they are not true or false in themselves, but their meaning is to be evaluated through their implications. This is a means-to-end attitude and is contrary to the rationalist (or classical) attitude towards axioms, under which they are assumed to be self-evidently true and the ultimate ground of all our deductions (Dewey 1939: 16). Instead, we have an immense freedom in laying down postulates, which is "subject only to the condition that they be rigorously fruitful of implied consequences" (*ibid.*: 10). Initially, a postulate is implicit in an inquiry, but later it is recognized formally; now, it has been stipulated, and it commits those who stipulate it to its consequences. As such, a postulate involves the assumption of responsibilities, namely, obligations to act in specific ways (*ibid.*: 16-7), through which it is evaluated and reevaluated. Therefore, even after it has become formalized as a convention of behaviour, it is not arbitrary, nor is it simply a mere linguistic convention – it is the means by which an inquiry is practiced and stable beliefs are attained. Since we are always building on and reconstructing our previous work, we may have to proceed for a considerable time in order to make explicit the postulates that are involved.

23 We can see now that a postulate, although it is not assumed to be self-evident or logically necessary, is practically necessary and it is (temporally and logically) prior. The pragmatists Peirce and Dewey are not committed to the rationalistic or axiomatic *a priori* via intuition but have reconceived the *a priori* in terms of postulation – to be

cashed out in terms of the operations and rules of action that they engender. The fundamental laws of any science are of such a nature; they are *a priori* not because they are logically necessary, but because they circumscribe the potential field of inquiry and define its limits; without them, investigation becomes impossible (Lewis 1923: 169). The pragmatic *a priori* is hypothetical and operational, rather than categorical and intuitive.

24 According to Lewis, previous approaches to the *a priori* make two mistakes. First, they misconstrue the *a priori* as necessary truths (of the mind) and second, they claim that the *a priori* is entirely independent of experience. The pragmatic *a priori* is necessary, as opposed to contingent, but not as opposed to voluntary; it is necessary only to a specific context of inquiry. It is independent of experience, not because it puts constraints on our sensory data, but because it does not say anything directly about experience (*ibid.*: 166). It is dependent on experience in the sense of being amenable to alteration based on further reflection on our experiential input; the ultimate criteria of laws (of logic, metaphysics, science, or mathematics) are pragmatic and operational. For example, the laws of logic are canonical examples of *a priori* principles, as necessary truths independent of experience. They are *a priori*, because they are required for us to make sense of deductive reasoning and because they say nothing specifically about the natural world or our perceptions of it. Lewis conceives of them as "the parliamentary rules of intelligent thought and speech" (*ibid.*: 167). They dictate the rules of cognitive processes – of definition, classification, and inference – not operations of the objective world. In the same way, the fundamental laws (or postulates) of any discipline are *a priori*.

25 We can maintain an attitude of fallibility about these laws, since if they continue to fail in performing their function, we begin to doubt their pragmatic validity, and can reevaluate them. However, no particular event will force us to reject our *a priori* postulates – we can say that the pragmatic *a priori* is revisable but not refutable. Lewis takes this as the dividing line between the *a priori* and the *a posteriori*, namely between those "principles and definitive concepts which *can* be maintained in the face of all experience and those genuinely empirical generalizations which *might* be proven flatly false" (*ibid.*: 172). Here we see how the pragmatists weave in between rationalism and empiricism: there are principles that have no direct bearing on experience but are still not indubitably true, being subject to re-analysis in light of further experiences.

26 Similar ideas have been defended somewhat more recently. For example, Hilary Putnam argues that "there are statements in science which can only be overthrown by a new theory – sometimes by a revolutionary new theory – and not by observation alone" (Putnam 1976/1996: 95). He calls these *contextually a priori* and argues that as a correlate, it implies that *a posteriori* statements do not always having specifiable confirming or disconfirming conditions *(ibid.)*. Ullmann-Margalit and Margalit (1982: 436) use the notion of conclusive presumption to defend the distinction between revision and rebuttal, arguing that even if no statement is immune to revision, there may be some which are immune to rebuttal. Similarly, Chang (2004: 224) argues that there "is a real sense in which elements of the inner layers support the elements of the outer layers, and not vice versa," and so there can still be hierarchical justification without resulting in foundationalism or infinite regress.

27 One objection might appeal to the fact that because our theories are underdetermined by observation, and therefore we can always appeal to auxiliary hypotheses to maintain our position, any hypothesis can be seen as *a priori* on this account. However, this

strengthens our position, because it implies that such pragmatic *a priori* (or constitutive) assumptions are necessary in order to proceed. Of course, another theory can be put forward with different postulates, within which our previously non-empirical basis becomes open to test against experience. This is simply an acknowledgement of the dynamic, contextual, and relative nature of this conception of the *a priori*. Additionally, if our observations are theory-laden, then the *a priori* restricts the potential meanings of our terms and puts constraints on our (perceptual) experience. However, this is not the same as saying that it can be proven false – just that it may turn out to be inadequate as a framework to explain our experience and that we ought to expand our paradigm to resolve certain puzzles that have arisen. For example, "Euclidean geometry was always revisable in the sense that no justifiable canon of scientific inquiry forbade the construction of an alternative geometry; but it was not always 'empirical' in the sense of having an alternative that good scientists could actually conceive" (Putnam 1976/1996: 95). In fact, the possibility of encountering anomalies which cause us to question and dictate our prior assumptions, and the subsequent adoption of a new theory, means that these limits on our experience are not strict.

5. Definition

28 The main thesis in this section is that definitions are the kind of thing that are pragmatically *a priori*, even though this has some minor conflicts with Peirce and Dewey's comments on the subject. Peirce is largely critical of Descartes for focusing on definition as a basis for knowledge. When encountering vague ideas, Descartes (and the modern rationalist philosophers who followed him) resolve the ambiguity by requiring an abstract definition of every important term. Through this method, we could obtain "clear and distinct notions" through apprehending the contents of an idea, i.e., whatever is contained in its definition (EP1: 125-6 [1878]). Peirce takes a rather dim view of definitions as a source of knowledge, saying that "[n]othing new can ever be learned by analyzing definitions" (*ibid.*: 126) (implying that they are manifestly tautological), while at the same time admitting that this is a useful activity for ordering our thought in an economical way. For Peirce, the first step toward clearness of apprehension is familiarity, and the second involves the formation of a definition *(ibid.)*.

29 These ideas are pervasive in modern philosophy of mathematics, where thinkers often want to avoid any sort of essentialism and so they take a merely nominal and practical standpoint when it comes to the act of definition: definitions make our proofs shorter – they are simply abbreviations. Lewis claims that the act of defining is legislative because it is arbitrary – meaning is assigned to words as a matter of choice. Because of this attitude, he argues that definitions (as analytic propositions) are necessarily true and legislative of the domain in question (Lewis 1923: 167). On the other hand, he says the definitions can be rejected, or unsuccessful, because "classification thus set up corresponds with no natural cleavage and does not correlate with any important uniformity of behavior" (*ibid.*: 169). If definitions can be modified or rejected on the basis of pragmatic considerations of their consequences, then they are false, according to a pragmatic conception of truth. If they can be false, or actively doubted, then they are not necessarily true, and thus, not analytic. As such, we should admit that

definitions, like other hypotheses, are amenable to truth or falsity. Indeed, if they are not assumed to be true, they are of no value, and we cannot reason with them – thus defeating their purpose. If they are the kinds of things that have a truth value – as they do in mathematics – then may be the kinds of things that capable of being justified.[7]

30 Definitions are *a priori* because (formal) science is based upon concepts that have been defined. At the very least, classification of objects into kinds is a necessary precondition for thinking, and if definition is classification (as Lewis claims), then definitions are necessary for the undertaking of investigation, i.e., they are pragmatically *a priori*. Indeed, there is no principled difference between definitions and laws in these regards, and laws sometimes function as definitions, e.g., F=ma or ZFC set theory.[8] Such "definitive laws" (postulates) are *a priori*, because without them we cannot enter into the subsequent inquiry. However, these definition-laws, as we know, are amenable to change and abandonment if "the structure which is built upon them does not succeed in simplifying our interpretation of phenomena" (Lewis 1923: 170). We can say exactly the same thing regarding, for example, the axioms of a set theory, which also act as an implicit definition of sets. Additionally, definitions are – technically speaking – a specific form of axiom, and so our comments equally apply to axioms as they do to definitions. If they fail to perform their duty in mathematical activity, they are rejected or modified, thereby rendering the previous version false.

31 There is a mistake here, I think, because Lewis is talking of the attribution of a meaning to a name, rather than a name to a meaning, because his prototype of definition is stipulative. It is fairly arbitrary what name is given to an idea, but it is completely up to us which ideas get names, and this is decided on pragmatic grounds in the light of experiential data and conceptual needs. Lewis makes his argument based on the improper conflation of aprioricity, necessity, and analyticity: "If experience were other than it is, the definition and its corresponding classification might be inconvenient, fantastic, or useless, but it could not be false" (*ibid.*: 167). Indeed, a pragmatist should regard a definition that is inconvenient, fantastic, or useless as not (much) better than one that is false. The meaning assigned to words is basically a matter of choice and a relatively trivial matter, of course. However, it is the result of much investigation and reflection to decide which ideas are important or fruitful and require (or deserve) definitions. Often, it is only after some result has been proved that a name is assigned to a (mathematical) concept. One could adopt a merely nominal approach to such a practice, but there must have already been substantive work done to justify the adoption of this definition.

32 Both Peirce and Dewey usually discuss mathematics (and mathematical definitions) in how it relates to natural science, rather than as an activity internal to mathematical discourse. Dewey describes definitions as ideal and ideational, because "they are not intended to be themselves realized but are meant to direct our course to realization of potentialities in existent conditions-potentialities" (Dewey 1939: 303). In other words, (mathematical) definitions are pragmatically justified on the basis of their applications to existential matters. For example, a circle is never matched exactly by matter, but it still has a useful function in directing our inquiry. However, Dewey does recognize the logical (methodological) importance of the act of defining. Conceptual meanings, he says, represent possibilities of solutions, like in the case of the circle. They perform this function because they are constituted by characters that are necessarily interrelated because they form a single concept. The value of this structure is its capacity to be

substituted in a series of inferences (*ibid.*: 343). This last point is consistent with a nominalist account of definition, since the function under this approach is simply as a shorthand, substitutable *salva veritate* in further discussions. However, the necessary interrelationship between the components of a definition allows Dewey to conclude that definitions play an indispensable role in inquiry. He explains: "how and why a given selection and conjunction of the terms of a definition is logically grounded instead of being arbitrary" (*ibid.*: 55). Therefore, in opposition to Lewis, Dewey admits that definitions are not necessarily true and are justified on the basis of its applications in further investigations. In addition, they are *a priori*, since there is "something of the nature of contrary-to-factness in all definitions" (*ibid.*: 303) and they "exercise jurisdiction in further inquiries" (*ibid.*: 350). Like all such instruments, they are adopted on the basis of prior activities and modifiable by further use. In such a manner, they function as boundaries between past and future inquiry *(ibid.)*.

33 Dewey consistently argues against an Aristotelian conception of logic. In this context, he rejects the ontological (essentialist) interpretation of definitions which uses differentia to pick species from within a genus. Similarly, if we take it to be a purely linguistic matter, then this "leaves the *combination* of the defining words wholly unexplained and ungrounded" (*ibid.*: 355), as argued above. Instead, Dewey proposed that the conjunction of characteristics in a definition does not form the meaning of a category but is actually normative of what a thing should have if it is to be part of that category. For example, certain traits define shipness, not existential ships (*ibid.*: 356). This is part of the reason (other than their abstraction) why Dewey does not hold to an existential (or extensional) notion of mathematical definitions. For example, "circle, ellipse are not kinds of conic sections, but are ways of being the abstract universal in question" (*ibid.*: 361). This is consistent with his operational-functional approach.

6. Mathematics

34 The puzzle of how an abstract, deductive science could be so effective has asked for a long time – Peirce raised the issue almost 140 years ago (EP1: 227 [1885]). His answer was that all deductive reasoning involves some degree of observation, in the form of structural analogies with the existential matters under investigation. Through experimentation upon these imaginary representations, we can discover hidden relation among the parts of what is represented *(ibid.)*. However, such an idea is still consistent with mathematics being *a priori*, since its laws do not prevent any specific perceptions and are compatible with whatever may happen in the natural world. The pragmatists, as we have seen, reject the notion of self-evident "first truths" which are externally imposed, but agree that principles of logic and mathematics are operationally necessary for any scientific inquiry.[9] According to Dewey, the postulates of geometry are "not self-evident first truths that are externally imposed premises but are formulations of the conditions that have to be satisfied in procedures that deal with a certain subject-matter, so with logical forms which hold for *every* inquiry" (Dewey 1939: 17). Lewis also draws an analogy between logical and mathematical rules, both being limited by just consistency, whereas applications must be useful. He tends to side with logicism, reducing mathematics to analytic truths, a prime example of a priori knowledge (Stump 2015: 100). This may seem to impose a separation between the formal rules and the rules for matter, but this is contrary to the commitment to

continuity that characterizes both Peirce and Dewey. To resolve this tension, we must tell a story about the continuity of mathematical development.

35 Histories of mathematics often begin with the ancient Greeks, with maybe passing mention of the Mesopotamian, Egyptian, or Chinese contributions. This focus reflects a bias towards the written and deductive forms of reasoning that the Greek thinkers are lauded for. However, it could be argued that mathematics did not start a mere 2 500 years ago but descends backward into prehistory and beyond. There are basic forms of mathematical reasoning – like counting and spatial recognition – in a variety of animals, and these are part of the biological matrix out of which mathematics evolved.[10] Activities like chanting, dancing, weaving, divination, art, and even language and ritual, reflect stepwise forms of behaviour, based on the recognition and repetition of patterns, both natural and imagined. Some of these even date to before the emergence of our species. Before written languages existed, there were complicated forms of enumeration, music, astronomy, and architecture which co-opted these forms of thinking and helped formulate mathematical principles and practices. They were also bound up with other forms of culture, like governance, commerce, warfare, and religion, and they engendered specific forms of socially constructed habits. This was the sort of biological-cultural matrix out of which the earliest forms of written mathematics in Asia and Africa emerged.

36 Of course, the idea of a proof may have been primarily a Greek invention, but in line with the pragmatic synechism, there is some measure of continuity between the earlier (lower) forms of mathematics and the later (higher) ones. Indeed, the evolution of proof methods using both natural and symbolic languages is a demonstration of this continuity within mathematics proper. Each novel method was incorporated into a previous collection of practices and meanings and directed towards both new and old applications. Indeed, if we take seriously Dewey's notion of language as including all sorts of rites and rituals, as well as the associated tools, then there is no real distinction between written and non-written forms of mathematics. We can see now that there was no point in time that mathematics became operationally *a priori* – it was always necessary for a variety of inquiries and there was no moment of revelation which opened thinkers to the realm of mathematics. This narrative should also make it clear that there are forms of experience that are mathematical and that mathematical practice has undergone a long history of development and innovation, all the while maintaining a sense of continuity with previous forms. Mathematics, therefore, is in continual development; there may be so-called technical and conceptual revolutions, but there is no shift so radical that it completely breaks away from previous thought.

37 If we restrict ourselves to formal mathematics, say, from Euclid onward, then we can make more specific claims about the role of axioms and definitions. In such mathematical works, there are usually definitions of concepts and operational axioms; the former are typically explicit, while the latter are sometimes assumed to be known. However, there is no perfect distinction between axioms and definitions since the rules under which we operate determine the possibilities and extent of the concepts. Indeed, as mentioned, definitions are a specific type of axiom. For example, if we assume that we are working in a flat two-dimensional Euclidean space, then we can define a square to be a closed figure with four equal geodesic ("straight") sides and four equal internal angles. We can then prove that these angles are equal to right-angles, i.e., the sides are perpendicular to each other. However, if we inhabit the surface of a sphere, which is

also 2-dimensional, then this result will not hold. For example, we can partition a great circle into four equal lengths to get a "square," since each side is one-quarter of the circumference – a geodesic – and each angle is now equal to two right-angles. Indeed, this "square" is also a circle, since all points on it are equidistant from a point (in fact, two points) in the space. Of course, we may not want to count such a figure as a square, since it does not perform the function for which we use such a concept. In this case, we would alter our definition of a square in light of this new finding.

38 We might even want to call this an observation, which, as such, forms part of our mathematical experience. Another example, this time from arithmetic, will further demonstrate the point. Consider the well-worn example that 5+7=12. It might appear that there is no choice in the matter, and therefore that this is a necessary truth, even self-evident. However, what if we assumed the normal rules of arithmetic, but supposed that 5+7=10? If we can still subtract and have associativity, then we can take 5 away from each side of the equation to obtain 7=5 and repeat to get 2=0. Now, this might seem patently absurd, but it is a true statement in any ring of characteristic 2, e.g., in binary arithmetic. The utility of such an assumption seems dubious until you realize that this is the arithmetic of binary logic and computing.

39 There is nothing qualitatively different about these different systems; the 2-dimensional Euclidean plane and the surface of a sphere (or natural number and binary arithmetic) are abstract models with their own rules and concepts. They may or may not have domains of applicability in the world of perception (they do). The axioms and definitions are necessary to deduce further results, and these consequences may induce us to alter our assumptions or move to another system. None is self-evident or necessarily true, and all of them are *a priori*, having no direct bearing on the world of perception. If we observe that, for some objects, 2+2=5, then this is no rebuttal of the truth in the natural numbers that 2+2=4. In such an instance, "we should be obliged to become a little clearer than is usual about the distinction between arithmetic and physics, that is all" (Lewis 1923: 168).

40 We can characterize the mathematical method using the same language that Peirce does for science. We encounter various examples of phenomena, which we classify according to our previous rubrics. We hypothesize (through definition) about a class of objects, from which we prove results. It may be that some of the traits involved in the definition are extraneous or they may be too "loose," and so we alter our assumptions (of either the rules of operation or the definition of the classes) and proceed as before. Conjectures are the result of inductively obtained experimentation and the consideration of possible counter-examples.[11] Theorems are proved deductively using hypotheses, in the form of definitions or axioms. These theorems are not merely formal, because they act as guiding principles which provide the conditions for further testing (conjecture). They also lead to certain habits of thought and action – even if these are entirely within the domain of pure mathematics – through which we can pragmatically determine their value.

41 There is freedom of choice when deciding to pursue any line of inquiry and our starting point is nothing other than what we are inclined to think at the time – yet these assumptions are fallible and open to evaluation in the light of further inquiry. They are *a priori* not because they are logically necessary or self-evident, but because they define the limits of inquiry. It is true that we must sometimes doubt our axioms, but mathematics – while broadly directed towards natural science – is not beholden to the

sensory world. Instead, the rigour of mathematics must be complemented with self-reflection, not fettered by claims of self-evidence and necessary truths. Just like in natural science, starting from so-called undeniable claims is no more than a personal and historical bias, and in mathematics and science such assumptions ought to be liberated through the freedom of inquiry. This is a sentiment that both Peirce and Dewey would be sympathetic to.

7. Conclusion

42 The notion of the pragmatic *a priori* originated with the work of Peirce and Dewey and was expanded and commented on by C.I. Lewis and others. It exhibits many similarities with the constitutive approach, which acknowledges the contextualism of the *a priori* by relativizing postulates to theories. However, the pragmatic approach offers two advantages: (1) the emphasis on continuity, and (2) the evaluation of presuppositions in concrete or material situations. Both are improvements over the rational approach, but the pragmatic *a priori* – with these two advantages – makes more room between extreme forms of absolutism and relativism for our scientific and mathematical "foundations." An important caveat is that we must be more cautious than earlier proponents in equating the *a priori/a posteriori*, analytic/synthetic, and conceptual/empirical distinctions. This opens up a route to explaining how definitions and other conventions are not merely arbitrary stipulations or abbreviations, along with possibility of their being an example of genuine (pragmatic) *a priori* knowledge – especially if they have a truth value and are capable of justification.

43 The adoption of continuity between lower and higher forms of inquiry allows us to more easily see the relevance of historical, social, and cognitive factors in the philosophy of mathematics and science without committing us to psychologism or social constructivism. In this way, this work joins up nicely with later work done on scientific and conceptual change and the social component of knowledge production. Dewey's situationism provides a middle path between atomism and holism, while replacing universalism with contextualism (Brown 2012: 268). Furthermore, a contextual and pragmatic approach to the *a priori*, along with the continuity in material practices, gives a better account of mathematical (and scientific) progress than either the rationalist *a priori* or the strong social constructivist projects. In mathematics, this line of thinking dovetails with the more historically and socially cognizant philosophies of mathematical practice.[12] The priority of methodology over logic or metaphysics also coincides with the general tendency in philosophy of science and mathematics. In these respects, the works of Peirce and Dewey – especially their conception of the pragmatic *a priori* – ought to be of ongoing interest to epistemologists. It is my hope that this paper provides an entryway into a pragmaticist philosophy of mathematics that avoids the pitfalls of both classical rationalism and empiricism, while maintaining a robust relationship with the history and practice of (mathematical) inquiry.

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NOTES

1. There has been much more work done on Peirce's philosophy of mathematics, e.g., Moore 2010, than on Dewey, hence relative emphasis on the latter in this essay.
2. See Haack 1979 for an investigation into the relationship between fallibilism and necessity in Peirce's thinking about mathematics.
3. Arthur Pap's functional *a priori* was also strongly influenced by pragmatism, tracing through Lewis, Dewey, and Peirce. Poincare's conventionalism is also a contributor to these issues. See Stump 2015 for a fuller history.
4. Stump 2015 eschews the use of "a priori" (xiv, 3, 6, 146, 167), instead focusing on the "constitutive" element in science – but gives a good history of its use in the post-Kantian era. He claims to be defending a pragmatic theory of the *a priori*, although different from that of Stump (2015: 16).
5. See Franco 2020 for a comparison between the (neo-Kantian) relativized *a priori* of Reichenbach and Lewis.
6. For a comparison between Kuhn and Lewis, see Mayoral 2017.
7. If and how definitions and axioms are capable of being justified requires more work, but it must be at least partially based on their epistemic virtues, especially fruitfulness.
8. There is a difference, however, between physical and mathematical postulates, in that the former have as their end the manipulation of existential matter, whereas the latter do not. There is also the issue of distinction between explicit and implicit definitions, which we do not touch on here.
9. This is the basis for the indispensability argument in the philosophy of mathematics.
10. For a biological-cognitive account of how mathematics emerged, see Lakoff & Núñez 2001.
11. This process is expertly and insightfully exhibited in Lakatos 1976/1999, even if there are historical inaccuracies in the retelling.
12. See Ratner 1992, who draws favourable comparisons between Dewey and later philosophers of mathematics (e.g., Polya, Lakatos, Putnam, and Kitcher).

ABSTRACTS

Peirce and Dewey were generally more concerned with the process of scientific activity than purely mathematical work. However, their accounts of knowledge production afford some insights into the epistemology of mathematical postulates, especially definition and axioms. Their rejection of rationalist metaphysics and their emphasis on continuity in inquiry provides the pretext for the pragmatic a priori – hypothetical and operational assumptions whose justification relies on their fruitfulness in the long run. This paper focuses on the application of this idea to the epistemology of definitions and an account of progress in mathematics, although it has broader implications for the study of conceptual change and the function and basis of presuppositions in the sciences.

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