Complex, Dynamic and Contingent Social Processes as Patterns of Decision-Making Events – Philosophical and Mathematical Foundations[1]

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Abstract. This work presents a post-positivist research framework for explaining any surprising or anomalous fact in the evolutionary path of a complex, dynamic, and contingent social process. Firstly, it elaborates on the reconciliation between the ontological and epistemological assumptions of Critical Realism with the principles of American Pragmatism. Next, the research method is presented: theoretical propositions about a social structure are translated into a set of grammar rules that acknowledge patterns of sequences of events, either involving individual action or interaction between actors within a real social system. The result is a discrete mathematical model for a concrete category of social process based on these rules. Finally, data-grounded refinement of the theory is possible through comparison between cases belonging to the same category, but differing in some contingent pattern of sequences of event outcomes. Consequently, their grammars differ in some pairs of context-sensitive rules that explain this surprising fact, and the derivation of this alternative historical trajectory of event outcomes becomes an extension to the early category of social process. In this sense, the proposed framework suggests a hierarchy of classes of grammars for middle-range explanations based on the ontological assumption of the generative nature of social reality.

Keywords: Category Theory; Critical Realism; Generative Grammar Theory; Pragmatism; Social Ontology.

1 Introduction

The philosophies of science deal with the foundations, methods, and meanings of research practices, but while they are not inherently metaphysical, they eventually raise questions concerning the existence and nature of reality that lie between the opposite metaphysical positions of Idealism and Realism. Idealist metaphysics claims that the human mind interprets events and creates reality such that ideas are the only reliable form of knowledge. Knowledge derives only from abstract reasoning, and both truth and values are absolute and universal. Knowing is the same as the logical deduction of

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implicit theoretical results. In contrast, realist metaphysics claims that reality exists objectively and independently of the human mind, and that science can often reveal the truth, which is also absolute and universal. Knowledge derives from human perception of real facts based on natural laws. In this sense, while epistemology is concerned with knowledge and truth related to reality, metaphysics is concerned with the existence and nature of reality itself.

Between these two opposite metaphysical stances, the North-American philosophers Charles Peirce, John Dewey, William James and others proposed such a mid-term view in their *Pragmatism* (White, 1973). Born from Peirce's criticisms to Immanuel Kant's Idealism, which claims for the abstraction of ways to comprehend reality even though reality never fits such abstractions, Pragmatism proposes that the metaphysics of reality exists, but it is in continuous change, such that ideas and words are tools for learning and prediction through action on real problems. Throughout the 19th and 20th centuries, varieties of this philosophical view flourished in works of Alfred Whitehead, George Mead, Willard Quine, Hilary Putnam, Richard Rorty and others.

The pragmatic theory of truth acknowledges that phenomena exist independently of the mind and perception of people, but their conceptualization depends on the linguistic practice embedded in culture, history, and human needs – although it does not depend on the subjectivity of the individuals. Both truth and values are situational; that is, there is no absolute truth, but only contingently valid truths. The only way for an individual to create new knowledge about reality is by interacting with its environment to solve practical problems through experience. The truth about this new knowledge may be acknowledged if it works to accomplish tangible goals based upon some assumptions coming from individual experience in a specific empirical setting. Peirce (1878) firstly outlined what is known as the *pragmatic maxim*: consider the practical effects of the objects of your conception; then, your conception of those effects is the whole of your conception of the object.

In the social sciences, along with the conflict between Idealism and Realism, there is the opposition to the mainstream epistemology in *Social Positivism*. Based on a naïve or direct realism, Social Positivism acknowledges that the attributes of real objects are directly observable and measurable. It also claims that society and social phenomena operate according to universal laws. Through deductive logic, testable hypotheses follow these universal laws in the form of conjunctions of variables, ready to be verified against empirical data, generating predictable consequences, much like physical and natural sciences. Social positivists assume causality as the refutable, recurrent occurrence of a pair of subsequent events, which is considered valid for an entire population so that it is possible to predict the effect of the set of hypothesized causal factors in any empirical setting. Positivist social science can either explain or predict any phenomenon in such a trivial way.

Criticizing the use of reductionist and empiricist assumptions in the social sciences, there is the anti-positivist stance, known as *Interpretivism*. Firstly, it claims that social reality is inexplicable in terms of objective rationality, that is, using the method of the natural sciences, since researchers cannot detach their values and beliefs from their research. Secondly, interactions among human actors promote the learning of concrete situations, resulting in many variations in adaptive behaviours and in the resulting social forms. There is no universal and definite truth to reveal or explain, but many meanings shared between social actors to be interpreted in relation to the concrete

situation. Those who disagree with social positivists argue that the inference of causality from empirical relations is not reasonable at all, such that neither prediction nor explanation is worthwhile, but only subjective interpretation.

Between these opposite philosophies of the social sciences, there is a post-positivist stance called *Critical Realism* (Bhaskar, 1975). Firstly, it proposes a stratified ontology based on the duality of structure and agency. Secondly, it advocates for an objectivist epistemology to explain complex, dynamic, and contingent phenomena based on the logic of retroduction rather then the logic of falsification.

In this stance, the positivist notion of a linear combination of factors is not a causal explanation for the observed empirical effects but rather a descriptive relation that does not rely on a theoretical statement about how it comes into being. In fact, positivism requires the control of conditions for both the system closure and accurate prediction of true consequences. However, there is no way of explaining the emergence and evolution of social forms and processes over time in empirical settings using this approach. Conversely, this is a problem that Critical Realism aims to tackle. Although social forms are regarded as inherently meaningful and subject to change over time, they possesses an enduring structure that makes it possible to interpret and quantify their subjective and objective features.

This work assumes that Critical Realism shares many assumptions with Pragmatism in the search for a post-positivist philosophy of the social sciences. However, it further recommends reconciling these assumptions in the form of a Pragmatist Critical Realism. Additionally, it describes a research methodology (Braga, 2018, 2016) that goes beyond mere description of social phenomena to a problem-solving approach based on systematic analysis of empirical evidence, while acknowledging the emergent nature of patterns of actions and interactions in evolutionary social processes. The work also describes a formal model and analytical procedure based on the assumptions of Pragmatist Critical Realism, in line with existing operationalization of the assumptions of Social Positivism. A discrete mathematical model of social processes is derived from Formal Language Theory, which extends Set Theory but relies on Generative Grammar Theory (Chomsky, 1956, 1959) to the study of linguistic systems rather instead of pure Automata Theory (Church, 1937; Turing, 1936). The model also applies Category Theory (Eilenberg & MacLane, 1945; MacLane, 1948) to translate those generative principles into a model for empirical research.

Generative Grammar Theory is a framework for studying the hypothetical innate linguistic structure, which adopts the concept of grammar as the formal system of rules to generate sentences of a given language through patterns of combinations of words. This approach is consistent with Analytic Philosophy's approach to science, as it applies the mathematical model from Formal Language Theory to the study of natural languages. Although Chomsky characterizes his theory as “Cartesian linguistics,” he has been influenced by pragmatist philosophy. In an interview published as a book, Chomsky (1977, p. 71) admitted “*the philosopher to whom I feel closest and whom I’m almost paraphrasing is Charles Sanders Peirce.*” This influence is evident in his model of natural language acquisition relying on the abductive mode of inference, although in the same book he also criticized the lack of formalism in the works of the father of Pragmatism. The young Chomsky was likely exposed to Pragmatism in a school where his father was principal. This school was based on the progressive ideas for education of John Dewey (Chomsky, 2003): both “learning” as a social and interactive process

and "school" as an institution by which social reform occurs. Although Chomsky does not considers himself a pragmatist, his linguistic and political works still rely on Peirce's abductive inference to explain how human mind generates linguistic sentences, and on Dewey's definition of democracy as a participatory and decentralized regime to argue for how anarchism can shape society.

Category Theory (CT) is a mathematical framework that allows for the modelling and comparison of abstract structures called *categories*, in terms of a labelled directed graph. The nodes are named *objects* and arrows are named *morphisms*. CT has already been applied to theory formulation (Spivak, 2013), separating the abstract structure from both the theoretical propositions (concepts and relations between them) and the empirical evidence (data and their types). The objects and the morphisms among them map into concepts and relationships of the theoretical framework in use and into the sets of values for their instances. The proposed categorical approach to make science supports the description of domain-invariant mechanisms. Pragmatism is also the philosophical foundation for the work of Eilenberg and Maclane (Krömer, 2007).

While a categorical framework has been used for modelling cognitive development (Halford & Wilson, 1980), this paper proposes that Generative Grammar Theory can contribute further to this approach. It assumes that instances of a category of social process result from a real-world automaton. In the following sections, the comparison between American Pragmatism and Critical Realism leads to workable definitions of *structure* and *agency*, using the ontological notion of *process* as a chain of decision-making events, and underlining its evolutionary and emergent nature. Category Theory and Generative Grammar Theory are both used to define the mathematical formalism for representing the computational nature of social structures in the real world.

2 The Comparison between Pragmatism and Critical Realism

There is no unified pragmatist philosophy, but rather numerous lines of thought. Peirce, Dewey, and James have held different assumptions about the nature of reality and the ways of knowing it, resulting in distinct theories of truth that coexist within each branch of the family tree. In this section, a brief introduction to the assumptions of the various branches of Pragmatism is presented. The goal is to argue for conciliation between American Pragmatism and Critical Realism in the search for a research framework capable of assessing emergent social forms through their constitutive social processes.

Firstly, John Dewey (1929), in line with naturalism, claims that emergence is nature actualizing its potentialities. The ontological structure of reality is defined by humans and is itself an emergent result of human experience. This notion of emergence is a tool to assess reality and solve practical problems, which is in line with the core assumption of *instrumentalism*. The human mind is an instrument for coping with an environment that exists independently of it but is mediated by it. Theorization is the way to do so; theory is a tool to create empirical knowledge from the sense experience instead of being representative knowledge by itself. The statements must be operationalized in the form of human action in conjunction with their expected consequences to become a thesis under scientific assessment.

Based on John Dewey, the school of Metaphysical Realism proposes a metaphysical framework based on an ontology of events and relationships rather than substances and

discrete properties. Dewey (1929) firstly proposed the evolutionary nature of reality in such a way that there is one reality, but many ways to assess it. Everyone searches for truth based on their own experience. Learning from experience is the primary approach to solve human problems, and human knowledge is a tool for adapting to reality and controlling it. The assumption of *fallibilism*, or the provisional nature of the truth of a belief or statement, exists because reality is in continuous process of change. Truth relies on both individual interests and the context of the evolving phenomenon. Dewey's naturalistic model of emergence acknowledges both the levels of ontological analysis and the invariant truths that are comparable and assessable. Nonetheless, any metaphysical ground-map is held as a tool only, not a true representation of reality. This point of view, known as *objective relativism*, opposes *epistemic relativism*, in which truth is relative to the context and the conceptual framework of the researcher for the same set of facts.

Secondly, Alfred Whitehead (1929) offers an event-based, process ontology, which relies on temporal relations of dependence between events in sequence. For him, reality consists of a web of interrelated processes rather than a structure of material objects. Any process is defined by its relations with other processes in such a way that all of our decisions and actions have consequences for the reality around us. Any entity has either a "defining essence" (if it is a thing) or a "core identity" (if it is a person) that is basically the same and seems unchanging over time. However, they change all the time in minor attributes. All entities result from experience; there is no irreducible raw matter like the enduring, unchanging objects of scientific materialism. The material substances (or the essences) of things and identities of people do not define them. At any moment in time, quite the opposite occurs, which is denoted by a type of event, even though any formal language cannot name all events. An entity is both the synthesis and the reaction to the reality around it, that is, in terms of the existing external relations with other entities at any moment, or in terms of the conditioning of the surrounding structure.

Based on Whitehead's ideas, the school of Structural Realism puts forward a process philosophy that defines "truth" as emerging from dynamic patterns of events, rather than static structures of substance. Most of the philosophers in this school merge two modes of perception of reality, which they refer to as "causal efficacy" (physical) and "presentational immediacy" (conceptual), into a "symbolic reference." According to this philosophy, entities are viewed as spatiotemporal sequences of events (or processes) that develop in relation to other events (or processes) and may overlap each other in an emergent manner. They become "complexes of occasions of experience," rather than being "dialectical determinations." Order exists among potentials that recognize novelty and provide purposes to entities. Causes precede effects in time. Therefore, a hierarchical ontology is also necessary in this sense.

In his turn, George Mead (1934) proposed a philosophy of action that consists of a pragmatic method to interpret social interactions based on qualitative techniques. The philosophy aims to test the theoretical proposition's truth by evaluating its utility in solving practical human problems. This pragmatic definition of truth is more subjective, based on the efficacy of a course of action in achieving individual goals within a given context. Pursuing the truth involves discovering processes that work to reach human ends, but these processes must be empirically tested since they are a function of their practical outcomes. Pragmatism is about making decisions in a way that is consistent with the aimed goals.

Inspired by Mead, the school of Symbolic Interactionism proposes a metaphysical framework for describing how individuals interact with one another to create symbolic systems. These systems make the conditioning of the individual behaviours and explain the way society is produced and reproduced through repeated social interactions. People act towards things based on the meaning they assign to them, which is derived from social interaction through a process of communication with peers. People create and transform the meaning of things through an interpretation process, and grasp their situation before they can perceive their external environment through interaction and reflection. The reasons for human action result from the contextual conditions shaping the present situation, including past event outcomes and environmental exogenous conditions. Once action occurs, interpretation is then made upon action.

Finally, Richard Rorty (1996) proposes a linguistic or analytic pragmatism in which the meaning of words is a function of how they are used rather than what people intend to describe with them. Rorty's work focuses on language rather than experience, but he is against representationism, denying that the analysis of language discloses meaning and truth about reality. He acknowledges the notion of *ontological relativism*: all ideas and perceptions about reality come to our awareness in terms of our mental language, which specifies how objects within reality are built from our sense-data (Quine, 1969). Reality, which exists independent of our perceptions of it, can be thought of as a true language, but there is no objective way to fit the language of reality into our mental language. Likewise, if there are many ways to translate between two mental languages, then there are many ontologies that can be consistently applied to represent reality. The role of language is not to describe reality, but rather to guide actions and achieve goals within the community sharing it.

The school of Neopragmatism acknowledges contextual conditions of the historical emergence of phenomena (*historicism*), assumes that natural sciences explain natural phenomena accurately (*naturalism*), and recognizes the role of change through human agency in complex, dynamic and contingent phenomena (*relativism*). Like other branches of Pragmatism, it aims to creat beliefs and habits that enable people to adapt to their environment, rather than creating representations of reality. This branch of Pragmatism relies on Rorty, but it approaches a postmodernist line of thought instead.

These schools of thought tackled reality toward a pragmatist view, which is still compatible with other lines of thought. This work assumes that Critical Realism also seems to contribute even more to this view because of its intersection with pragmatist principles and its stratified social ontology. The following sections explain how these intersections can support this work's goal.

2.1 The Case for a Realist Social Ontology

Critical Realism acknowledges the assumption that the social sciences can accurately describe social phenomena. However, there is a belief in the coexistence of observable and unobservable features such that there is the need for a stratified ontology in three distinct levels: the real domain, the true (or actual) domain, and the empirical domain.

Critical Realism takes a realist perspective on the nature of reality and approaches complex phenomena and evolutionary processes. While classical pragmatists are anti-representationalists, many of them acknowledge that the choice of theoretical

framework and ontology is necessary for solving real problems. They hold an ontological belief that, although the human mind can never fully assess objective reality, it can refine a theoretical proposition in the face of surprising or anomalous facts. Critical Realism shares this belief and proposes a stratified ontology that fills in the blanks about the nature of reality before any empirical testing takes place.

In line with the core assumptions of utilitarianism and instrumentalism, Pragmatism needs a kind of Realism; it considers that scientific statements are objectively true, but their objectivity derives from the theoretical framework and language in use to observe the reality. Ontology as a formal language frames the complexity of reality into the given theoretical concepts and the system of relationships between them, but it is often relative to the researcher's goals. Critical Realism provides a kind of non-naïve realist ontology to play this role, rejecting radical empiricism in favour of useful metaphysical principles.

Charles Peirce's model of categories, as general attributes present in all phenomena, aligns with epistemological realism rather than nominalism. This is because abstract concepts are considered just as real as concrete objects of knowledge. Peirce's notion of science is not solely focused on the refinement of categories and rule-like laws. Instead, he recognizes that observations and measurements are not always precise, and categories and laws are part of a dynamic process of change over time that requires the interpretation of empirical evidence against a theoretical structure.

Peirce's Pragmatism rejects abstractions deduced from theoretical principles relying on a closed system. Instead, he emphasizes the importance of facts observed after the course of practical actions performed in an open system. According to Peirce, human experience is about interacting with reality rather than representing it, as scientific language cannot be absolutely precise at all times.

On the contrary, Alfred Whitehead's Structural Realism relies more on empiricism and rationalism than Critical Realism does. His realism is based on a formal categorical scheme, in the same way that logic relies on metaphysics. Using an analogy, metaphysics defines categories with objects and links between them, much like mathematics defines domains of numbers and functions between them. This assumption is not a point of conflict between pragmatists and critical realists as long as there are no truth claims about reality that rely upon a metaphysical model. For pragmatists, ontology is only a tool and not an exact representation of reality.

Nonetheless, Pragmatism is not a radical stance against representationalism: if there is no consistent way of fitting reality into a formal language nor of translating between languages, then there is no one-to-one mapping of terms in the first language into terms in the other. In fact, there are many ontologies consistently describing the same reality and researchers are always improving their representability. After all, representations adopted by an individual influence their conclusions about reality. Social research begins with an indeterminate situation to transform it into a sufficiently determinate one. This new situation must enable coherent actions through the creation of a "language game" (Wittgenstein, 1953) consisting of a set of rules and conceptual objects to which it refers in a specific context of use. In this sense, scientific statements only make sense in the context in which they were formulated despite the fact that reality has its own (partially accessible) language. Again, it is the case for a realist ontology.

2.2 The Objective Nature of Ontology and Epistemology

The objectivist epistemology of Critical Realism assumes that reality exists objectively and is accessible by measurement instruments in closed systems or controllable environments. Nevertheless, reality is also socially constructed and evolves as an open system. The goal of a critical realist research project is to comprehend the causal powers and liabilities of objects rather than to identify relations among their observable attributes. It accepts that each object influences other objects through its causal powers or capabilities and receives the influence of other objects through its liabilities or susceptibilities. The structure among objects generates a type of event after the activation of a causal mechanism, and a specific event outcome occurs when an object exerts influence on another object. Any outcome is the observable consequence of an action of an individual, an interaction within a group of individuals, or even with another existing thing. Any type of individual action or social interaction maps to a set-theoretic relation with other past outcomes, such that there is always a co-domain of all possible alternative event outcomes in each system state. Realist ontology is needed for this process, but an objectivist epistemology is also necessary to grasp this process over objects of knowledge based on sequences of events of mutual interaction.

Consider a phenomenon showing a pattern of behaviour by influence of the causal powers of its constitutive objects. It may also be subject to scientific inquiry to improve the understanding of the generative mechanisms taking place in this specific empirical setting. Critical Realism assumes any social phenomenon as continually developing and differentiating through space and time; however, locked-in by some configurations of contextual conditions in the empirical setting of the instances under investigation. The generative mechanism sets up a propensity or tendency that may or may not occur under specific contextual conditions. Once activated, it may or may not result in an observable effect if another opposing generative mechanism has neutralized it, or even if it remains unnoticeable. Both agents and structures own powers and liabilities to generate specific events, but their mechanisms are slightly dissimilar. On the one hand, agents both exert actions on and suffer the actions from other agents. On the other hand, the system's structure both exerts influence on and suffers influence from the actions of agents. Inside a social system, the agents become social actors and theoretical objects such as social forms and social processes result from the collective action of actors under the influence of the surrounding social structures. This has the benefit of tackling the problem of the dualism between structure and agency. Actions can be more relevant to understanding if they are considered as an empirical manifestation of the activated mechanism. Either individual or collective action must be regarded as the unity of analysis by default.

Pragmatism agrees that things are real if they have causal powers and liabilities such that ontological existence and causal efficacy are equivalent notions. Nevertheless, it is always possible to acknowledge observable effects as the consequence of a non-existent entity. If the requirement to be real is the possibility of demonstration of the existing causal powers and liabilities, then that is the case for a philosophical problem known as *downward causation.* Pragmatism adopts a kind of "mild realism" when it recognizes that the observed effects rely on the theoretical framework in use. Pragmatists are not worried about the independent existence of the emergent forms and properties, but with the causal efficacy of them: they are not exact representations of phenomena, but tools

to explain past outcomes and to predict future events. This is not a point of conflict with Critical Realism, except if the emergent forms become causal explanations to observed effects without being translated into processes, which are sequences of observable event outcomes and the respective generative structure or mechanisms that effectively exist between them.

2.3 The Subjective Nature of Knowledge

Critical Realism agrees with Social Positivism on the assumption of the existence of a social reality independently of our knowledge of it. Even though our knowledge about social phenomena may always be subject to empirical verification, it is fallible and theory-laden. The definition of theoretical objects, such as social entities and processes that are the building blocks for a critical realist explanation, differs from the definition of constructs: objects are not a measure of real things but real things themselves. Realists accept that theoretical structures mediate the data collection in fieldwork. The structure shapes the perceptions of the subjects in any social research about real objects: some data can properly typify objects through their attributes and events, but other data do not fit well. In line with Pragmatism (Dewey, 1929), the researcher's bias manifests in his hypothesis about the operating structures in a concrete situation, resulting from his experience. Social phenomena are concept-dependent: Critical Realism acknowledges their objective existence, but the social researcher's interpretation of empirical evidence is still necessary. Furthermore, social science is a kind of social practice that is under the influence of the content of the existing relationships between the actors enrolled in the research. The practice of science does not search for the absolute and conclusive structure of reality.

Contrary to Idealism, both Realism and Pragmatism claim that the true knowledge of reality is possible. On the one hand, Idealism argues for a *coherence* theory of truth based on the logical consistency within a set of theoretical statements, that is, a system of beliefs that support one each other. However, this assumption may imply circularity and relativism. On the other hand, Realism argues for a *correspondence* theory of truth, which holds that any theory can only represent the object of study by establishing some level of correspondence between theoretical propositions and attributes of the object. Reality exists independently of human action and thought, and there is the need for empirical testing through systematic research methods. As a third way, Pragmatism argues for a theory of truth in which reality is irreducible to logical propositions; nevertheless, any theory can fit human actions leading to the accomplishment of some human goal. This kind of philosophy denies the possibility of a complete correspondence between theory and reality. Theoretical propositions are true in a given context, but if and only if the beliefs upon which they depend are necessary to accomplish a human goal through some specific course of action or system of practices. Truth is about scope and degree rather than about logical consistency or empirical verifiability. Some correspondence between human beliefs and the state of reality is viable since it has a rational structure accessible to human perception and cognition; however, the aim of any researcher is to describe the process of interaction with the reality that is reproducible in some empirical setting rather than to describe reality itself.

Science can provide representations of the structure of reality that are progressively accurate through either the refutation or the refinement of existing theories: many things are real even though their corresponding concepts result from a system of practices. For pragmatists, verifying the necessity of any theoretical statement is conceivable by the application of John Stuart Mill's method of difference. Any kind of emergent form or property is real if it provides a better representation of the phenomenon rather than the mere description of its underlying configurational pattern of events in isolation, that is, it has ontological existence due to the existing theoretical commitments in a community of researchers and practitioners. No ontology is detachable from epistemology. In this way, emergence is an abstract notion and the set of structures that represent reality is the product of human thought and research practice.

2.4 Crafting the Best Explanation using the Logic of Retroduction

In terms of methodology, Critical Realism places itself between Positivism and Anti-positivism. On the one hand, the positivist, extensive research approach makes use of large-scale empirical data and statistical analysis in the search for population-wide similarities and regularities. On the other hand, the anti-positivist, intensive research approach makes use of both case study evidence and qualitative analysis in the search for situationally constrained interpretations. As a third way, a critical realist explanation is any system of relations among theoretical concepts understood as structured objects owning causal powers and liabilities; structures generate patterns of events in specific empirical settings. In other words, this research approach comprises the specification of the social phenomenon as a category of social process, that is, a particular pattern of sequences of observable types of events resulting from the contingent activation of non-observable generative mechanisms known as “causal processes” or “deep structures”. Critical Realism argues that causal explanation needs the disclosure of the complexity of causal relationships through the analysis of a chain of events; however, it assumes a derivation path of generative mechanisms and their contextual conditions of activation, grounded in empirical facts. This kind of methodology provides causal explanations in terms of a contingent tendency instead of a universal law; it also provides the analytical generalization of the findings for the theory instead of the statistical generalization for the population.

Contrary to Social Positivism, Critical Realism considers *contradictory facts* in the scope of the social sciences. First, it assumes an anomalous or surprising fact as a new event outcome contradicting the prediction of the currently most accepted theory. Next, it recognizes some contextual conditions for the explanation of this new event outcome. This approach needs the empirically grounded refinement of an existing social process model to explain all the observed contradictions based on contextual conditions. If there are different interpretations of empirical evidence in terms of distinct derivations of the rules of behaviour for the same instance of a category of social process, then competing explanations exist. The analytical decomposition of a sequence of events in terms of processes and their generative mechanisms instead of a mere descriptive interpretation in terms of collective constructions of actors in social interaction situations is not only possible; it can also provide the so-called “best explanation” for the surprising fact.

Critical Realism rejects the notion of scientificity based on the principle of objective rationality in order to consider a theoretical statement to be scientific. It instead advises the notion of *epistemic relativism*: causal explanations of social phenomena denoted by historically and spatially constrained generative mechanisms instead of universal and perpetual laws of cause-and-effect. The scientificity of a theoretical statement about a contingent behaviour does not rely upon the *logic of falsification* but upon the *logic of retroduction* borrowed from the pragmatists: the reasoning procedure, which makes use of the three modes of inference (abduction, deduction and induction). Initially defined as "hypothesis" in a paper of Charles Sanders Peirce (1878), abduction, together with induction (and contrary to deduction), is one of the two modes of inference that does not offer foregone conclusions.

First of all, the retroductive procedure apprehends a kind of surprising or anomalous fact as an unexpected new event outcome and draws out an explanatory hunch about the contextual conditions enabling this occurrence. Once proposed, the explanatory hunch is taken backward in the sequence of event outcomes to explain the surprising fact by hypothesizing what new mechanism could have generated it. The explanation relies on a chronological order or configuration of past event outcomes that turns out to be the contextual conditions of activation of the hypothesized mechanism. Both the mechanism and the conditions come from any known theoretical structure in a related field of human knowledge using *abduction*. In the second stage of inquiry, the researcher derives the observable effects, that is, some predictable consequences – other new event outcomes preceding the surprising or anomalous fact – from the hypothesis by means of *deduction*. Finally, in the third stage, the researcher tests the hypothesis by contrasting its predictions against empirical data by means of *induction*.

A retroduction-based methodology must acknowledge unexpected conflicts between the currently most accepted theory and the empirical facts that recurrently take place in a specific empirical setting. Retroduction (by means of abduction) is a knowledge-extending tool for drawing explanatory inferences because it is often (not always) capable of making claims that do not follow logically from the premises. Neither deduction nor induction can provide any new ideas in the same way abduction can do.

Pragmatism and Critical Realism are already guiding social researchers, but the latter seems to adopt many ideas of the former together with the stratified ontology to grasp emergent phenomena. Nevertheless, a process model enabling the operationalisation of this stratified ontology using a systematic, retroductive research methodology is still an open problem. In fact, the next sections propose a solution based on two theories from mathematicians inspired by the philosophy of Pragmatism.

3 Defining Social Structure using Category Theory

Category is a concept originated from the Greek philosophy, denoting a class of things regarded as having one or more common attributes. Each category is characterized by a single quality that can be predicated to a kind of object. This quality is observable in each instance of the category in the form of a generic and indeterminate term, which helps determine the substance of the object.

Aristotle introduced the concept of category as a term used in logical reasoning to eliminate ambiguities in statements and allow the application of logical rules without

variation of meaning. According to him, every existing object is under one of ten categories, which enumerate everything that is expressible without composition or structure, whatever thing that can be either the subject or the predicate of a statement. Other philosophers such as Kant and Hegel attemped to propose a better list of categories, but it was Charles Sanders Peirce (1868, 1885) who redefined the concept of category as "a predicate of predicates" that must be few in number, "just as the chemical elements are" – precisely in the number of three.

Eilenberg and MacLane introduced the Peircian notion of category to mathematics. Working within a categorical framework has advantages over a set-theoretic framework for mathematical analysis. A category is a collection of objects with some relationships or statements relating them to each other. It is a structure of statements that group objects together by comparing them using specific criteria or frameworks. These statements are also objects that can have statements between them in higher levels of statements or dimensions. In mathematics, categories do not focus on the elements of a set. Instead, they define properties of elements by describing how various sets relate to each other.

Any category C is a mathematical structure consisting of *objects*, $\text{obj}(C) = \{N_1, \ldots, N_n\}$, with *morphisms*, $\text{hom}(C) \subset \{f_{i,j} : N_i \to N_j\}$, which is such a relationship between objects. Both constitutive elements of a category are representable as nodes and arrows in a directed graph for visual representation. Nevertheless, a category is not merely a directed graph; it defines the operation of *composition* of morphisms with respect to its properties of *identity* and *associativity*. As a result, there is the principle of *equivalence* between one composition of morphisms and any other morphism linking the first object with the latter. For instance, given a pair of morphisms $f : N_1 \to N_2$ and $g : N_2 \to N_3$, the composition $g \circ f$ (also described as $f \,.\, g$ or $f \,;\, g$) is another morphism $g \circ f : N_1 \to N_3$ such that $\text{hom}(N_2 , N_3) \times \text{hom}(N_1 , N_2) \to \text{hom}(N_1 , N_3)$. By the axiom of *identity*, $id_i : N_i \to N_i$ is the identity morphism for N_i, such that for $f : N_1 \to N_2$, the equivalence relation $id_1 \circ f = f = f \circ id_2$ holds. By the axiom of *associativity*, given $h : N_3 \to N_4$, the equivalence relation $h \circ (g \circ f) = (h \circ g) \circ f$ also holds.

Likewise, *functor* is any mapping between a pair of categories that preserves their structure, namely the arrows between the nodes and all equivalence relations. In other words, any functor is a set of morphisms linking morphisms from the source category with morphisms from the target category. Consider a category C in which objects map to sets, and morphisms map to set-theoretic relations (i.e., *functions*) from a domain set into a codomain set. A *faithful functor F* links C to the category for sets $F : C \to \text{SET}$, providing sets with further external structure because they assign to each object of C its underlying set, and to each morphism of C, its underlying function. This is a *concrete category* (C, *F*), where $C = (\text{obj}_C, \text{hom}_C, \text{id}, \circ)$, $\text{obj}_C = \text{obj}(C)$ is the set of objects of C, $\text{hom}_C = \text{hom}(C)$ is the set of morphisms of C, $\text{id} : \text{obj}(C) \to \text{hom}(C)$ is the identity function for objects of C, and $\circ : \text{hom}(C)^2 \to \text{hom}(C)$ is the composition function for arrows of C, in addition to the faithful functor *F*. Each morphism is a mapping between a pair of objects, that is, $f_{i,j} : N_i \to N_j$, which encapsulates the internal structure of the underlying set-theoretic relation that exists between them.

In categorical social research, the structure of an abstract category must be mapped into a structure of theoretical concepts and their relationships as well as into a structure of types for the evidence data set – both defining the empirical model. A pair of functors

between each abstract category's morphism (and for their objects) is necessary to link equivalent structural relations (and their concepts) to its data type (and their domain value sets). Each *implementation* functor from an abstract category into the theoretical structure category operationalizes theory for empirical test. Each *instantiation* functor from that abstract category into the data set category classifies empirical evidence about the phenomenon under studying.

Thus, a concrete category C implements that abstract category T for the theoretical structure plus the "specificities" of the empirical setting – configurations of past event outcomes that turn out to be the historical context of a derivation path (an evolutionary trajectory). The *implementation* functor defined as L: C $\rightarrow$ T makes both the objects and morphisms of C get their meaning by mapping into the objects and morphisms of T.

Next, the *instantiation* functor is a faithful functor F: C $\rightarrow$ SET, which maps the categorical form of the above empirical model into the set-theoretic form of the empirical data for all instances of the phenomenon under studying. Precisely, in the concrete category C, for any object c $\in$ C, F(c) returns the set of all possible values of type c in the data set. Concisely, setting up the categorical structure C to describe a phenomenon offers an *interpretation* of the empirical data with respect to a theoretical view.

Consider an example of the application of the concept of functor for the definition of a social category. A category C with a functor L from C into another category named after a specific social phenomenon (e.g., an institution, a social process, a language) is the description of a structure (e.g., family, competence development process, English language). The second category is a vocabulary (or *lexeme*) which provides meaning to each object and morphism of the first category C within a community. In the case of the institution of family, the pair (C, L) is a social category if and only if L:C $\rightarrow$ Family, in which the abstract structure maps into theoretical concepts and their relationships. In the same sense, a phrase in English belongs to a linguistic category C if and only if L : C $\rightarrow$ English, in which the instance of this phrase structure maps into the lexeme of English words and syntactic expressions. It is still necessary to separate the categorical structure from the theoretical concepts and their relationships. The equivalence between distinct categories means either different categories mapping into the same theory (by concurrent hypotheses), or one abstract category mapping into different theories (by analogy). If two categories are partially equivalent, then there are some morphisms mapping into other morphisms or compositions of morphisms, but not all of them.

A *commutative diagram* can be used to define the equivalence between an individual morphism and a composition of morphisms from the first object into the last one. Take the example above, in which a concrete category C can be the implementation of either English or family categories. The morphism g $\circ$ f : $N_1 \rightarrow N_2$ may define either a phrase or a parenting relationship as <"a person," "has as father," "a man">, in which f : $N_1 \rightarrow N_1 \times N_1$ means <"a person", "has as parents", "a couple of persons"> and g : $N_1 \times N_1 \rightarrow N_2$ means <"a couple", "has", "a man">. As well, "a couple of persons" meaning "a pair of persons of distinct genders" can assume a definition based on the Cartesian product $N_1 \times N_1$ (Fig. 1). Finally, if N_2 is a subset of N_1 and it is the set of all persons that are "male" in gender, then there could be another morphism h : $N_1 \rightarrow N_2$ not related to g $\circ$ f meaning that the set N_2 has all the elements of the set of N_1 that are male in gender.

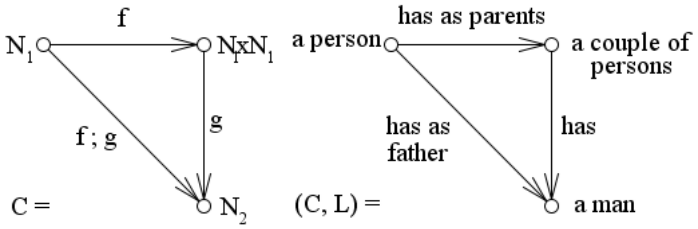

Fig. 1 An implementation of the structure of family into a category as a commutative diagram.

The concept of *consistency* means that the relation described by the symbol system must reflect the corresponding structure or process hypothesized for the phenomenon under studying (Halford & Wilson, 1980). A commutative diagram denotes consistency between the symbolic system and the real system. The existence of consistency requires *structural isomorphism* between the categories for both systems, that is, the map between them *commutes*.

4 Defining a Social Process using Generative Grammar Theory

The category of *process* can describe any pattern of sequences of events that takes place over time in a specific system. On the one hand, a type of *event* describes any fixed, finite domain set of observable and mutually exclusive event outcomes. On the other hand, the concept of *system* represents any working structure of relations among the types of events occurring in sequences, in which many kinds of processes of change in the state (or the configuration of conditions) of the system may occur.

At any time, there may exist many concurrent sequences of events within the same system, which are instances of the same category or distinct categories of process. Any single event outcome may take part in more than one instance of any of the categories of processes of that system. An emergent form or entity cannot be described as a whole, except in terms of its constitutive processes, that is, the patterns of sequences of events that may occur within it.

The structure of a system comprises relations between subsequent types of events, or rules that represent one evolutionary step taking place in instances of some category of process. If there are more than one alternative state transitions from the same system state producing event outcomes (i.e., their sets of instances) that are mutually exclusive, then it is a non-deterministic state transition. In other terms, there is uncertainty on the development of the process in this state of the system. This situation is not the same as an event that takes part in more than one process (or instance): it generates more than one separate state transitions in concurrent sequences of events developing at the same time within the same system.

Types of rules representing subsequent types of events can compose with other types of rules becoming a pattern of sequences of events. A category of process may contain a pair of nested categories of processes recursively until there are only rules between pairs of types of events. This hierarchy of nested processes is a taxonomical dimension that is orthogonal to the time dimension in the chains of events that comprise instances

of a category of process. One evolves along with the other. There is an analytic way of describing it using a formal language, that is, using a mathematical model deduced from a class of grammar, much like what has been done in the Generative Grammar Theory (Chomsky, 1956, 1959). The assumption here is that the structure of the phenomenon under inquiry works as an automaton acknowledging symbolic patterns valid in a formal language.

The description of a process-like phenomenon using a generative grammar locates it in the hierarchy of classes of automata with increasing algorithmic complexity levels; it is necessary to compute the patterns of relations between their symbols. In the lowest complexity level, *regular grammars* restrict all production rules to the forms A → a, B and A → a; event outcomes are non-terminal symbols representing the state transitions relying on information available in the current system state only, which is memoryless. Next, the *context-free grammars* have at least one rule with more than one nonterminal symbol at the right side like A → B, C; the state transitions denote a pair of patterns of sequences of events unfolding over time, or a process encompassing a pair of nested processes. This structure assures the occurrence of a second pattern of events (C) that is yet to come into being, which requires the stack of a pushdown automaton to keep the memory of the next possible derivations. Finally, *context-sensitive grammars* have at least one rule with a nonterminal symbol (A) at the left side as condition for a rule like A, B → A, C to apply; all past event outcomes may have the power to influence events yet to come into being, which needs to acknowledge the context in the derivation path (i.e., the antecedent non-terminal symbol A).

Table. 1 The computational complexity of the main classes of generative grammars.

Grammar Class	Computational Complexity	Automaton Class	The Form of the Production Rules
Regular	Linear, O(n)	Finite-state	Left Linear Form (LLF) or Right Linear Form (RLF) – **A → a, B** (or **A → B, a**) and **A → a**
Context-free	Linear, O(n) & Polynomial, $O(n^3)$	Pushdown	Chomsky Normal Form (CNF) – **A → a** and **A → B, C**
Context-sensitive	Exponentials, $O(2^n)$	Linear Bounded	Penttonen Normal Form (PNF) – CNF's rules plus **A, B → A, D**

The term *contextual* means a result depending on the information setting, concerning the situation or location in which the collection of evidence took place. The rule linking a pair of nested processes or events that take place one after the other under specific contextual conditions is an information. The term *contingent* means a result that is not certain to occur, but it is only possible. Indeed, even if a contingency is going to occur, it is only incidental, that is, it depends on something that still does not exist, but may be yet come into being. The occurrence of a new event outcome may result from some sequence of past event outcomes such as contextual conditions. However, it may still be contingent upon structural relations that are going to change in the future because the system is open; this new structural relation becomes part of the system enabling a new mechanism that activates under specific configurations of contextual conditions only. In any way, it is not relevant to the sequential analysis: a structural change could

simply be regarded as *latent* because it becomes operational after a specific context comes into being.

Any category of processes has an unobservable structure, only partially intelligible from the observed pattern of sequences of events. The focus of categorical-generative analysis is the transition from an event outcome to another as the result of one or more sequential configurations of past event outcomes much like explanatory hypotheses of these occurrences in the empirical setting under investigation. These state transitions are tendencies that may happen (or not) depending on the occurrence of these specific configurations. The past event outcomes occurring over time potentially remain as the contextual conditions influencing a new event outcome in the development of a process instance. It may be impossible to determine the moment of a transition because of contextual conditions and generative mechanisms that are unknown or not accurately specified. In this case, the formal grammar is non-deterministic despite the assumption of determinism regarding the real-world phenomenon.

Under the assumption of a realist metaphysics, it is not the case for context-sensitive grammars. Under the assumption of an idealist metaphysics, by which event outcomes yet coming into being eventually influence the current system state transition, it may be the case. For realists, all processes must be further decomposable in pairs of nested processes until only observable event outcomes take part in the analysis because this derivation has an inverse based on an inductive algorithm. Any class of formal grammar conforming to this constraint allows the recursive formal definition of a category of process, that is, in terms of the composition of categories of nested processes and types of events recursively.

Using mildly context-sensitive grammars, precisely in the algorithmic complexity level of *indexed grammars* (Aho, 1968), each event outcome, denoted by a terminal symbol, takes part in a finite set of mutually exclusive event outcomes comprising a type of event. On every occasion, if the instantiation of a type of event occurs, then one and only one of the alternative outcomes in its domain set may occur. Each system state is representable as a non-terminal symbol, that is, each system state refers to a pattern of chains of event outcomes that is either a category of a nested process or a single type of event.

Any scientific description using these ontological concepts of system, process, and event is somewhat complicated; however, it relies upon qualitative data, a formal model and a systematic procedure of pattern recognition using the theory generation approach. The goal is the explanation of surprising or anomalous facts as new event outcomes. The rigour of the procedure and the preciseness of the model imply benefits for the reproducibility of the results in other empirical settings.

This section provided a categorical structure for social processes, implementing the mathematical model of formal grammars for social research. Braga (2020) describes how this categorical-generative approach applies to the problem of modelling process-like phenomena based on an important research question in Organizational Economics and Strategic Management: the Theory of the Firm.

5 Conclusions

Pragmatism is a general-purpose philosophy claiming that the truth of a belief relies on its usefulness to guide a course of action to reach the believer's goal. It has influenced a number of schools of thought, most of them approaching the realist metaphysics such as Metaphysical Realism, Structural Realism, and perhaps Critical Realism. The latter diverges with many branches of Pragmatism on the role of ontology in making truth assertions; however, not necessarily on its usefulness to reach the goals of the research.

This work offers a pragmatist-inspired mathematical model based on Category Theory and Generative Grammar Theory to build up some analytical tools. These tools assume an underlying generative structure of reality to solve a practical problem based on the occurrence of surprising or anomalous facts that require further explanation.

Before using grammars as a model for scientific inquiry, it is necessary to ensure that the mathematical properties of this formalism align with the ontological and epistemological assumptions of the adopted philosophical paradigm. The goal of refining the grammar with empirical data is to replace alternative state transitions with equivalent context-sensitive rules, but this is often difficult due to the complexity of emergent, ever-evolving phenomena and incomplete theoretical statements. By identifying contextual conditions for each set of alternative rules, the predictability of their event outcomes can be increased, thereby improving the explanatory power of the grammar at the expense of increased computational complexity.

Category Theory can support the description and analysis of theoretical knowledge against empirical evidence with the benefit of graphical representation of categories. Because context-free grammars have a tree-like hierarchy of composition rules when applied to any derivation path, there is a graph-like representation that synthesizes all possible derivations. It is comparable with other graphical structures using colouring graph algorithms. This procedure of graph analysis can describe the matching parts of a pair of categorical structures, calculating a similarity index between them. Therefore, structural comparison between multiple cases belonging to the same category of social process can support the data-grounded refinement of a theory; conciliating differences in some parts of their graphical structures due to contingent developments that often need further explanation. In other words, the assessment of the structural equivalence of grammars by comparing their respective categorical structures, which are all possible derivations, is the final task of this analytical approach.

A *categorical-generative revolution* unifying all social sciences in the same way the marginal revolution initiated the mainstream economics in the mid-1950s is likely. Both Theory of Computation and Discrete Mathematics offer the foundations for the social sciences much like Theory of Probability and Statistics have provided the grounds for mainstream economics. In this case, abstract discrete structures like sets, rings, groups and grammars are the mathematical models that replace the regression model. Category Theory can still put together qualitative and quantitative analytical schemes, such as a common meta-language. This research project is helping to build up this vision.

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