

# Expected comparative utility theory: A new theory of instrumental rationality

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## Abstract

Starting from the premise that expected utility (EU) is the correct criterion of rational preference both in decision cases under certainty and decision cases under risk, I argue that EU theory is a false theory of instrumental rationality. In its place, I argue for a new theory of instrumental rationality, namely expected comparative utility (ECU) theory. I show that in some commonplace decisions under risk, ECU theory delivers different verdicts from those of EU theory.

## 1 | INTRODUCTION

This paper is a restatement of my previously published papers Robert (2018) and Robert (2021). I have revised and restructured the paper to present its main points more clearly and convincingly. Let me now briefly introduce the paper.

Standard decision theory, otherwise known as *expected utility (EU) theory*, counsels agents to rank their choice options (in order of preference) according to their EU and to choose whichever option carries the greatest EU (or one of them in the event that several alternatives are tied). The EU of an option is a probability-weighted sum of each of its possible utilities. EU theory has been the dominant normative theory of rational choice under risk since the 18th century (Bernoulli, 1738), and in more recent times (1920s–), has received foundational support from both economists and philosophers (Bolker, 1966; Jeffrey, 1983; Joyce, 1999; Ramsey, 1931; Savage, 1954; von Neumann & Morgenstern, 1947).

In this paper, I will argue for a new normative alternative to EU theory. Starting from the premise that EU is the correct criterion of *rational preference* in decision problems under certainty and decision problems under risk, and from the premise that we need a graded, quantitative measure of *choiceworthiness* for decisions under certainty and decisions under risk,<sup>1</sup> I will argue that we need a new normative theory of rational choice under risk, or, more specifically, a new theory of instrumental rationality, namely *expected comparative utility (ECU) theory*. In this paper, I will show that in some ordinary decisions under risk, ECU theory gives different verdicts from those of EU theory and that EU theory is therefore a false theory of rational choice under risk, or, more

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<sup>1</sup> A *decision under certainty* is a choice situation where an agent is subjectively certain about which state the world is in and where he or she assigns probability 1 to that state being actual, whereas a *decision under risk* is a choice situation where an agent is *not* subjectively certain about which state the world is in, but where he or she can nevertheless assign probabilities to the different possible states.

specifically, a false theory of instrumental rationality.

The core of this paper is divided into two sections: In Section 2, I will compare EU theory and ECU theory, revealing how and why they differ, and in Section 3, I will develop a step-by-step argument for ECU theory (and against EU theory).

## 2 | EU THEORY VS. ECU THEORY

This section will explicate and contrast EU theory and ECU theory.

In what follows, I will assume that for any agent,  $S$ , and for any choice option,  $a$ , for  $S$ ,  $a$ 's utility is a *cardinal* indicator of preference and is derived from  $S$ 's preferences as in standard decision theory, that is, via a *representation theorem*. This requires that  $S$ 's preferences obey a series of conditions or axioms of *rational preference*, one of which is the *Independence of irrelevant alternatives (IIA)* (for preferences): if an option,  $a$ , is preferred over some alternative option,  $b$ , then introducing a third option,  $c$ , in the choice situation will not change the preference ordering between  $a$  and  $b$  (see Gintis, 2018).

According to EU theory, the EU of an option,  $a$ , in a decision problem with  $n$  states is formally defined as:

$$EU(a) = \sum_{i=1}^n U(a, s_i)P(s_i)$$

where  $U(a, s_i)$  denotes the utility of option  $a$  when state  $s_i$  is actual, and  $P(s_i)$  denotes the probability assigned to state  $s_i$ . In other words, for any number of alternative options,  $a, b, c, d$ , and  $e$ , one calculates the EU of  $a$  as follows: for each state of the world, one calculates  $a$ 's utility and one multiplies the result by the probability that one assigns to that state; finally, one sums the totals for every state.<sup>2</sup>

According to EU theory, for any agent,  $S$ , faced with any decision under *certainty* or any decision under *risk* and for any number of alternative options,  $a, b, c, d$ , and  $e$ , for  $S$ , it is rational for  $S$  to prefer  $a$  to  $b$ , and  $a$  is more choiceworthy than  $b$  for  $S$ , if and only if  $a$ 's EU is greater than  $b$ 's; it is rational for  $S$  to be indifferent between  $a$  and  $b$ , and  $a$  is just as choiceworthy as  $b$  for  $S$ , if and only if  $a$ 's EU is equal to  $b$ 's; the extent to which  $S$  rationally prefers  $a$  to  $b$ , and the extent to which  $a$  is more choiceworthy than  $b$  for  $S$ , is the difference in EU between  $a$  and  $b$ ; finally, it is rational for  $S$  to weakly prefer<sup>3</sup>  $a$  over the alternative options available to  $S$ , and  $a$  is choiceworthy for  $S$ , if and only if  $a$  maximizes EU within the set of alternatives available to  $S$ .

As a first approximation, ECU theory says that for any agent,  $S$ , and for any choice option,  $a$ , for  $S$ , the measure of the choiceworthiness of  $a$  for  $S$  (relative to any explicitly given utility unit and zero point of utility) is the ECU of  $a$ . The *ECU* of an option,  $a$ , in a decision problem with  $n$  states is formally defined as:

$$ECU(a) = \sum_{i=1}^n (U(a, s_i) - U(bm(a), s_i))P(s_i)$$

where  $U(a, s_i)$  denotes the utility of option  $a$  when state  $s_i$  is actual,  $U(bm(a), s_i)$  denotes the utility of

<sup>2</sup> The version of EU theory employed above is of the sort developed by Savage (1954).

<sup>3</sup> For any agent,  $S$ , and for any two choice options,  $a$  and  $b$ , for  $S$ , if  $S$  *weakly prefers*  $a$  to  $b$ , then  $S$  either prefers  $a$  to  $b$  or is indifferent between  $a$  and  $b$ .

the benchmark for  $a$  when state  $s_i$  is actual (i.e., the utility in state  $s_i$  of whichever alternative(s) to  $a$  have the highest utility in state  $s_i$ ), and  $P(s_i)$  denotes the probability assigned to state  $s_i$ . In other words, for any number of alternative options,  $a$ ,  $b$ ,  $c$ ,  $d$ , and  $e$ , one calculates the ECU of  $a$  as follows: for each state of the world, one subtracts  $a$ 's utility from the utility of whichever alternative to  $a$  (i.e.,  $b$ ,  $c$ ,  $d$ , or  $e$ ) carries the greatest utility in that state (or, in the event that several alternatives to  $a$  are tied as best, one subtracts  $a$ 's utility from the utility of one of those alternatives), and one multiplies the result by the probability that one assigns to that state; finally, one sums the totals for every state.

More precisely, for any agent,  $S$ , faced with any decision under *certainty* and for any choice option,  $a$ , for  $S$ , the measure of the choiceworthiness of  $a$  for  $S$  (relative to any explicitly given utility unit and zero point of utility) is the *comparative utility* ( $CU$ ) of  $a$  in the state of the world to which  $S$  assigns probability 1. Let us call this principle the *CU principle*. For any choice option,  $a$ , and for any state of the world,  $G$ , the choiceworthiness or  $CU$  of  $a$ , in  $G$ , is the difference in utility, in  $G$ , between  $a$  and whichever alternative(s) to  $a$  carry the greatest utility in  $G$ .<sup>4</sup> (Henceforth, *c-utilities* are defined as units of  $CU$ .) For any agent,  $S$ , faced with any decision under *risk* and for any choice option,  $a$ , for  $S$ , the measure of the choiceworthiness of  $a$  for  $S$  (relative to any explicitly given utility unit and zero point of utility) is the *comparative expected comparative utility* ( $CECU$ ) of  $a$ , that is to say, the difference in ECU between  $a$  and whichever alternative(s) to  $a$  carry the greatest ECU. Let us call this principle the *CECU principle*. We are now in a position to precisely define ECU theory: ECU theory is the conjunction of the  $CU$  principle (for decisions under certainty) and the  $CECU$  principle (for decisions under risk). According to ECU theory, for any agent,  $S$ , faced with any decision under *certainty*, and for any option  $a$ , for  $S$ ,  $a$  is choiceworthy for  $S$  if and only if  $a$ 's  $CU$  is equal to or greater than zero, and for any agent,  $S$ , faced with any decision under *risk*, and for any option  $a$ , for  $S$ ,  $a$  is choiceworthy for  $S$  if and only if  $a$ 's  $CECU$  is equal to or greater than zero.<sup>5</sup>

The idea of calculating differences between the utility of an option under consideration and the utilities of its alternatives in the choice situation—idea which accords with ECU theory—has been explored in the philosophical literature (Colyvan, 2008; Colyvan & Hájek, 2016)<sup>6</sup> and the economic modeling literature (Zhang, 2015).

Let a *finite decision* be a decision problem where there are only finitely many states and no infinite utilities. In all finite decisions under risk requiring a choice between *only two* alternative options, ECU theory delivers the same verdicts as EU theory. However, in some finite decisions under risk requiring a choice between *more than two* alternative options, ECU theory gives different verdicts from those of EU theory.<sup>7</sup> Let us consider the following example:<sup>8</sup> Alice is going for a long walk. She knows

<sup>4</sup>  $CU$  should be distinguished from the purely descriptive economic concept of *opportunity cost*. For any agent,  $S$ , let  $a$  be the highest-valued choice option available to  $S$ . The  $CU$  of  $a$ , for  $S$ , is the value of whatever *additional benefit*  $S$  would enjoy by choosing  $a$  over the highest-valued alternative to  $a$ . By contrast, the opportunity cost of  $a$ , for  $S$ , is the value of whatever *cost*  $S$  would incur by choosing  $a$  over the highest-valued alternative to  $a$ , where this includes the *total value* of the highest-valued alternative to  $a$  (Henderson, 2008).

<sup>5</sup> ECU theory only applies to decision problems where the states of the world are probabilistically independent of the agent's choices. The theory therefore fails to give any verdicts in Newcomb decision problems, in which options and states are probabilistically dependent.

<sup>6</sup> Colyvan (2008) has argued for a new decision theory that gives the right verdicts in decision problems where there are an infinite number of states with only finite utilities attached, such as the St-Petersburg game, and where EU theory gives no verdicts whatsoever. Colyvan's new theory, i.e., *relative expectation theory*, states that rational agents rank their choice options on the basis of their *relative expected utility*: for any agent,  $S$ , and for any two options,  $a$  and  $b$ ,  $S$  prefers  $a$  to  $b$  if and only if the probability-weighted sum of the differences in utility between  $a$  and  $b$  for each possible state is positive, and  $S$  is indifferent between  $a$  and  $b$  if and only if the probability-weighted sum of the differences in utility between  $a$  and  $b$  for each possible state is zero. Relative expectation theory gives the same decision advice as EU theory in all decision cases where there are only a finite number of possible states and where the states are probabilistically independent of all choice options. See also Colyvan and Hájek (2016).

<sup>7</sup> In a number of decision cases where there are infinitely many states with only finite utilities attached (e.g., the St. Petersburg game), ECU theory inherits the advantages of Mark Colyvan's relative expectation theory over EU theory. More specifically, in such infinite decision cases, ECU theory delivers the intuitively correct verdicts, whereas EU theory delivers none (Colyvan, 2008; Colyvan & Hájek, 2016, pp. 838–839).

that within the next hour, there is a 50% chance of sunny skies (state *A*) and a 50% chance of rain (state *B*). She is faced with a choice between five options: bring a rain poncho and wear rain boots (option *a*), bring an umbrella and wear rain boots (option *b*), bring an umbrella and wear running shoes (option *c*), not bring an umbrella, nor a rain poncho, and wear rain boots (option *d*), and not bring an umbrella, nor a rain poncho, and wear running shoes (option *e*).<sup>9</sup> Each possible outcome of Alice's choice corresponds to the experience of taking a walk, and the utilities indicate Alice's preferences between those possible outcomes. Which option should Alice choose? Should she lug around a poncho or an umbrella and wear heavy rain boots in case it rains, should she forego the poncho and the umbrella and wear running shoes in case the skies are sunny, or should she go for the middle ground: bring an umbrella, but wear running shoes, or not bring an umbrella, nor a rain poncho, but wear rain boots?

The above decision problem can be stated more formally as follows: an agent, *S*, is faced with five choice options: *a*, *b*, *c*, *d*, and *e*. *S* assigns probability 0.5 to a state of the world, *A*, and 0.5 to a state of the world, *B*. If state *A* or state *B* were realized, then *S* would assign the following utilities to the set of options (see Table 1):

**TABLE 1** Decision matrix

	<i>A</i> (0.5)	<i>B</i> (0.5)	EU	CECU
<i>a</i>	2	10	6	0
<i>b</i>	4	8	6	-1
<i>c</i>	6	6	6	-1
<i>d</i>	8	4	6	-1
<i>e</i>	10	2	6	0

According to EU theory, all the options (i.e., *a*, *b*, *c*, *d*, and *e*) are choiceworthy since their EU is the same. By contrast, according to ECU theory, only options *a* and *e* are choiceworthy, since their ECU is equal to or greater than zero. ( $ECU(a) = -3$ ,  $ECU(b) = -4$ ,  $ECU(c) = -4$ ,  $ECU(d) = -4$ ,  $ECU(e) = -3$ ) For Alice's walk, ECU theory therefore recommends that Alice either bring a rain poncho and wear rain boots, or not bring an umbrella, nor a rain poncho, and wear running shoes. This verdict may seem *prima facie* counterintuitive, as ECU theory rules out as unchoiceworthy the middle ground (i.e., bring an umbrella, but wear running shoes, or not bring an umbrella, nor a rain poncho, but wear rain boots). However, this verdict is not counterintuitive in light of the arguments for ECU theory that I will develop in Section 3.

Here is a slightly revised decision matrix (see Table 2):

**TABLE 2** Decision matrix

	<i>A</i> (0.5)	<i>B</i> (0.5)	EU	CECU
<i>a</i>	2	10	6	0
<i>b</i>	5	8	6.5	-0.5
<i>c</i>	6	6	6	-1
<i>d</i>	8	4	6	-1
<i>e</i>	10	2	6	0

<sup>8</sup> This example is inspired from Briggs' (2019) umbrella example.

<sup>9</sup> From the outset, Alice rules out bringing a rain poncho and wearing running shoes because she believes that her feet will get too wet if it rains.

According to EU theory,  $b$  is more choiceworthy than  $a$ , for  $S$ , since the EU of  $b$  (6.5 utiles) is greater than that of  $a$  (6 utiles). In fact, according to EU theory,  $b$  is choiceworthy tout court since its EU is greater than that of every other option. By contrast, according to ECU theory,  $a$  is more choiceworthy than  $b$ , for  $S$ , since the CECU of  $a$  is greater than that of  $b$ . In fact, according to ECU theory,  $a$  (and also  $e$ ) is choiceworthy tout court, since its CECU is equal to or greater than zero. ( $ECU(a) = -3$ ,  $ECU(b) = -3.5$ ,  $ECU(c) = -4$ ,  $ECU(d) = -4$ ,  $ECU(e) = -3$ ) Therefore, for Alice’s walk, EU theory recommends that Alice bring an umbrella and wear rain boots, whereas ECU theory recommends that Alice either bring a rain poncho and wear rain boots, or not bring an umbrella, nor a rain poncho, and wear running shoes.

ECU theory gives different verdicts from those of EU theory because ECU theory, contrary to EU theory, violates the IIA (for choiceworthiness evaluations). According to this principle, for any decision situation,  $T$ , and for any choice option,  $a$ , in  $T$ , if  $a$  is choiceworthy in  $T$ , then  $a$  is also choiceworthy in  $T$  if some other option(s) are eliminated from the pool of options in  $T$ . Likewise, if  $a$  is not choiceworthy in  $T$ , then  $a$  is also not choiceworthy in  $T$  if some other option(s) are added to the pool of options in  $T$ .

Let us consider again the decision situation illustrated in Table 2. In that decision situation, ECU theory dictates that  $a$  is choiceworthy. However, if options  $c$ ,  $d$ , and  $e$  are eliminated from the pool of options, then  $b$  is choiceworthy according to ECU theory (and according to EU theory), as shown below (see Table 3):

**TABLE 3** Decision matrix

	A (0.5)	B (0.5)	EU	CECU
$a$	2	10	6	-1
$b$	5	8	6.5	1

According to ECU theory,  $b$  is choiceworthy tout court since its CECU is equal to or greater than zero. ( $ECU(a) = -0.5$ ,  $ECU(b) = 0.5$ )

Here is another example where ECU theory violates the IIA and gives different verdicts from those of EU theory<sup>10</sup>: An agent,  $S$ , is faced with two choice options:  $a$  and  $b$ .  $S$  assigns probability 0.001 to a state of the world,  $A$ , and 0.999 to a state of the world,  $B$ . If state  $A$  or state  $B$  were realized, then  $S$  would assign the following utilities to the set of options (see Table 4):

**TABLE 4** Decision matrix

	A (0.001)	B (0.999)	EU	CECU
$a$	1000	1	1.999	0.002
$b$	0	2	1.998	-0.002

According to ECU theory,  $a$  is choiceworthy tout court, since its CECU is equal to or greater than zero ( $ECU(a) = 0.001$ ,  $ECU(b) = -0.001$ ). And according to EU theory,  $a$  is also choiceworthy tout court, since  $a$  maximizes EU. Let us now introduce a third choice option ( $c$ ) in the decision situation, all else being the same (see Table 5):

<sup>10</sup> Thanks to an anonymous reviewer for giving this example.

**TABLE 5** Decision matrix

	<b>A (0.001)</b>	<b>B (0.999)</b>	<b>EU</b>	<b>CECU</b>
<i>a</i>	1000	1	1.999	-0.898
<i>b</i>	0	2	1.998	0.898
<i>c</i>	900	0	0.9	-2.097

In this new decision situation, *b* is choiceworthy tout court according to ECU theory, since *b*'s CECU is equal to or greater than zero. ( $ECU(a) = -0.899$ ,  $ECU(b) = -0.001$ ,  $ECU(c) = -2.098$ ) By contrast, according to EU theory, *a* is choiceworthy tout court, since *a* maximizes EU. This example is particularly telling because option *c* is statewise dominated by *a*. Whether state *A* or state *B* is actual, option *a* is strictly preferred to option *c*. Yet, introducing option *c* in the decision situation changes ECU theory's verdict: *b*, instead of *a*, is uniquely choiceworthy. ECU theory thus violates the *Irrelevance of statewise dominated alternatives (ISDA)* (Quiggin, 1994).

This gives rise to a worry. Without the IIA (and ISDA), it is possible to make up alternatives in any choice set and these manufactured alternatives would be altering the degrees of choiceworthiness of reasonable options.<sup>11</sup> This opens the door to strategic manipulation in the decision process. The worry can be overcome, however, if we accept Monton's (2019) *theory of rationally negligible probabilities*: For any given decision situation, any outcome with probability  $\leq p$ , where *p* is very close to 0, should be excluded from consideration in the decision process. As such, the *very improbable* outcomes of manufactured alternatives cannot alter the degrees of choiceworthiness of the other available options in the choice set.

### 3 | THE ARGUMENT FOR ECU THEORY

This section will argue for a new normative theory of rational choice under risk, or, more specifically, a new theory of instrumental rationality, namely ECU theory. The argument can be broken down into 15 steps, which are numbered below.

First, note that, in what follows, *what it is to be choiceworthy* and *choiceworthiness* will be given the following conceptual analyses: For any agent, *S*, faced with any decision under certainty or any decision under risk and for any option, *a*, for *S*, *a* is *choiceworthy* for *S* if and only if *a* is worthy of being chosen by *S* in light of *S*'s rational preferences within each of the various possible states of the world, where *S*'s *rational preferences* are preferences that obey the series of rationality conditions or axioms of standard decision theory, and the *degree* to which *a* is *choiceworthy* for *S*, or (i.e.) the *choiceworthiness* of *a* for *S* is the degree to which *a* is worthy of being chosen by *S* in light of *S*'s rational preferences within each of the various possible states of the world, where *S*'s *rational preferences* are preferences that obey the series of rationality conditions or axioms of standard decision theory.

The argument for ECU theory proceeds as follows:

1. For any agent, *S*, and for any option, *a*, for *S*, *a* is choiceworthy for *S* if and only if *a* is *maximally choiceworthy* for *S* over the space of all alternatives in the choice set.
2. For any agent, *S*, and for any option, *a*, for *S*, *a* is *maximally choiceworthy* for *S* over the space of all alternatives in the choice set if and only if *a* *maximizes choiceworthiness* for *S* over the space of all alternatives in the choice set.

<sup>11</sup> Thanks to Douglas Lackey for raising this point and for wording suggestions.

3. For any agent,  $S$ , and for any option,  $a$ , for  $S$ ,  $a$  is choiceworthy for  $S$  if and only if  $a$  maximizes choiceworthiness for  $S$  over the space of all alternatives in the choice set (i.e., the *choiceworthiness maximization (CM) principle*). (3 follows from 1 and 2.)
4. For any agent,  $S$ , faced with any decision under *risk* and for any option,  $a$ , for  $S$ , the measure of the choiceworthiness of  $a$  for  $S$  is its CECU (i.e., the CECU principle).
5. For any agent,  $S$ , faced with any decision under *risk* and for any option,  $a$ , for  $S$ ,  $a$  is choiceworthy for  $S$  if and only if  $a$  maximizes CECU. (5 follows from 3 and 4.)
6. For any agent,  $S$ , faced with any decision under *risk* and for any number of alternative options,  $a$ ,  $b$ ,  $c$ ,  $d$ , and  $e$ , for  $S$ , it is rational for  $S$  to prefer  $a$  to  $b$  if and only if  $a$ 's EU is greater than  $b$ 's, it is rational for  $S$  to be indifferent between  $a$  and  $b$  if and only if  $a$ 's EU is equal to  $b$ 's, and the extent to which  $S$  rationally prefers  $a$  to  $b$  is the difference in EU between  $a$  and  $b$ .
7. For any agent,  $S$ , faced with any decision under *risk* and for any option,  $a$ , for  $S$ , it is rational for  $S$  to weakly prefer  $a$  over the alternative options in the choice set if and only if  $a$  maximizes EU. (7 follows from 6.)
8. In decisions under *risk*, what option(s) maximize CECU sometimes differ from what option(s) maximize EU.
9. In decisions under *risk*, what option(s) are choiceworthy sometimes differ from what option(s) it is rational to weakly prefer over the alternative options in the choice set. (9 follows from 5, 7 and 8.)
10. For any agent,  $S$ , faced with any decision under *risk*, it is a requirement of instrumental rationality that  $S$  should measure and rank his or her options (for the purpose of choice) in terms of *how choiceworthy* they are for  $S$ .
11. It is *not* the case that for any agent,  $S$ , faced with any decision under *risk*, it is a requirement of instrumental rationality that  $S$  should measure and rank his or her options (for the purpose of choice) in order of rational preference. (11 follows from 3, 9 and 10.)
12. For any agent,  $S$ , faced with any decision under *risk*, it is a requirement of instrumental rationality that  $S$  should measure and rank his or her options (for the purpose of choice) in terms of how choiceworthy they are for  $S$ , that is, according to their CECU, rather than in order of rational preference, that is, according to their EU. (12 follows from 4, 6, 10 and 11, as well as from 3, 4, 6, 9 and 13.)
13. For any agent,  $S$ , faced with any decision under *risk*, it is a requirement of instrumental rationality that  $S$  should choose out of what option(s) are *choiceworthy* for  $S$ .
14. It is *not* the case that for any agent,  $S$ , faced with any decision under *risk*, it is a requirement of instrumental rationality that  $S$  should choose out of what option(s) it is rational for  $S$  to weakly prefer over the alternative options in the choice set. (14 follows from 9 and 13.)
15. For any agent,  $S$ , faced with any decision under *risk*, it is a requirement of instrumental rationality that  $S$  should choose out of what option(s) are choiceworthy for  $S$  (i.e., what option(s) maximize CECU), even in cases where what option(s) are choiceworthy for  $S$  (i.e., what option(s) maximize CECU) differ from what option(s) it is rational for  $S$  to weakly prefer over the alternative options in the choice set (i.e., what option(s) maximize EU). (15 follows from 5, 7, 9, 13 and 14, as well as from 3, 5, 7, 9 and 10.)

I will now discuss the different steps in the argument:

1. For any agent,  $S$ , and for any option,  $a$ , for  $S$ ,  $a$  is choiceworthy for  $S$  if and only if  $a$  is *maximally choiceworthy* for  $S$  over the space of all alternatives in the choice set.

The question whether a given option is more (or less) choiceworthy than (or just as choiceworthy as) another option within a set of alternatives is well-formed and meaningful. Therefore, the question whether a given option is *maximally choiceworthy* within a set of alternatives is also well-formed and

meaningful. A given option is *maximally choiceworthy* within a set of alternatives if and only if it is at least as choiceworthy as each of the other options within the set of alternatives. I will assume that Step 1 is true without further argument.

2. For any agent,  $S$ , and for any option,  $a$ , for  $S$ ,  $a$  is *maximally choiceworthy* for  $S$  over the space of all alternatives in the choice set if and only if  $a$  *maximizes choiceworthiness* for  $S$  over the space of all alternatives in the choice set.

For any number of alternative choice options,  $a$ ,  $b$ ,  $c$ ,  $d$ , and  $e$ , we want to say that  $a$  (utility: 100) is more choiceworthy than  $b$  (utility: 5) even if  $a$  is not choiceworthy tout court (i.e.,  $a$  does not maximize utility). We also want to say that the *extent* to which  $a$  is more choiceworthy than  $b$  is greater than the extent to which  $c$  (utility: 10) is more choiceworthy than  $b$ . In order to say that  $a$  is more choiceworthy than  $b$  (and to what extent), we cannot rely on a binary measure of choiceworthiness. Whether (and to what extent)  $a$  is more choiceworthy than  $b$ , and by implication, whether (and to what extent) any option is more choiceworthy than any other within a set of alternatives is necessarily a function of *how* choiceworthy each of the two options is within the set of alternatives (and not necessarily a function of one being choiceworthy tout court and the other unchoiceworthy tout court). To ask how choiceworthy an option is is to ask how desirable or worthy of being chosen that option is. Such a question is well-formed and meaningful. In order to answer the question, we require a graded, quantitative measure of how choiceworthy options are—i.e., we require a graded, quantitative measure of *choiceworthiness*.

3. For any agent,  $S$ , and for any option,  $a$ , for  $S$ ,  $a$  is choiceworthy for  $S$  if and only if  $a$  *maximizes choiceworthiness* for  $S$  over the space of all alternatives in the choice set (i.e., the CM principle). (3 follows from 1 and 2.)
4. For any agent,  $S$ , faced with any decision under *risk* and for any option,  $a$ , for  $S$ , the measure of the choiceworthiness of  $a$  for  $S$  is its CECU (i.e., the CECU principle).

### 3.1 | The CU principle

In order to establish the CECU principle, I first need to argue for a graded, quantitative measure of choiceworthiness for decisions under *certainty* (i.e. the *CU principle*). According to the CU principle, for any agent,  $S$ , faced with any decision under *certainty* and for any option,  $a$ , for  $S$ , the measure of the choiceworthiness of  $a$  for  $S$  is its *comparative utility* (*CU*). For any choice option,  $a$ , and for any state of the world,  $G$ ,  $a$ 's *CU* in  $G$  is the difference in utility, in  $G$ , between  $a$  and whichever alternative(s) to  $a$  carry the greatest utility in  $G$ . In what follows, I will provide three arguments for the CU principle.

To that end, I will assume that for any agent,  $S$ , faced with any decision under certainty and for any option,  $a$ , for  $S$ ,  $a$  is *choiceworthy* for  $S$  if and only if  $a$  maximizes utility over the space of all alternatives in the state of the world to which  $S$  assigns probability 1. I will refer to this principle as the *utility maximization* (*UM*) *principle*. The UM principle defines a binary measure of choiceworthiness for decisions under certainty (i.e., whether an option is choiceworthy tout court or unchoiceworthy tout court).

#### 3.1.1 | Argument 1

It is generally accepted that for any agent,  $S$ , faced with any decision under certainty and for any

number of alternative options,  $a$ ,  $b$ ,  $c$ ,  $d$ , and  $e$ , available to  $S$ , the extent to which  $a$  is more choiceworthy than  $b$ , for  $S$ , is the extent to which  $S$  (rationally) prefers  $a$  to  $b$ , or equivalently the extent to which  $S$  (rationally) prefers  $a$  to  $b$  *more* than  $S$  (rationally) prefers  $b$  to  $a$ . However, intuitively, that is a mistake. Even though we are comparing  $a$  to  $b$ , we want to see how  $a$  and  $b$  measure up to the *very best alternative options on offer*, in the following way: the extent to which  $a$  is more choiceworthy than  $b$ , for  $S$ , is the extent to which  $S$  (rationally) prefers  $a$  to the most (rationally) preferred alternative to  $a$  (either  $b$ ,  $c$ ,  $d$ , or  $e$ ) *more* than  $S$  (rationally) prefers  $b$  to the most (rationally) preferred alternative to  $b$  (either  $a$ ,  $c$ ,  $d$ , or  $e$ ).<sup>12</sup> After all, if  $S$  must choose an alternative to  $a$ , then  $S$  should choose the most (rationally) preferred alternative to  $a$  (either  $b$ ,  $c$ ,  $d$ , or  $e$ ) (or one of them in the event that several alternatives are tied), and not necessarily the option to which  $S$  is comparing  $a$  (i.e., option  $b$ ). The same goes for option  $b$ . Therefore, the extent to which  $a$  is more choiceworthy than  $b$ , for  $S$ , is the extent to which {the difference in utility between  $a$  and whichever alternative(s) to  $a$  carry the greatest utility (i.e.,  $b$ ,  $c$ ,  $d$ , or  $e$ )} is greater than {the difference in utility between  $b$  and whichever alternative(s) to  $b$  carry the greatest utility (i.e.,  $a$ ,  $c$ ,  $d$ , or  $e$ )}. It follows that the extent to which  $a$  is choiceworthy for  $S$  (or [i.e.] the measure of *how* choiceworthy  $a$  is for  $S$ ) is  $a$ 's *CU*, that is to say, the difference in utility between  $a$  and whichever alternative(s) to  $a$  carry the greatest utility (i.e.,  $b$ ,  $c$ ,  $d$ , or  $e$ ). The same goes for option  $b$ . This is what I have referred to as the *CU principle*.

An alternative approach is to say that the extent to which  $a$  is choiceworthy for  $S$  (or [i.e.] the measure of *how* choiceworthy  $a$  is for  $S$ ) is  $a$ 's *CU\**, i.e., the difference in utility between  $a$  and whichever option(s) carry the greatest utility (i.e.,  $a$ ,  $b$ ,  $c$ ,  $d$ , or  $e$ ). The same goes for option  $b$ . I will refer to this as the *CU\* principle*.<sup>13</sup> The *CU\** principle is however untenable, since it results in a double standard. It entails that the degrees of choiceworthiness of all the option(s) that do *not* carry the greatest utility depend on what other options are available in the choice set—those degrees of choiceworthiness may be different negative numbers, but never 0—whereas the degrees of choiceworthiness of all the option(s) that *do* carry the greatest utility do *not* depend on what other options are available in the choice set—those degrees of choiceworthiness are 0 no matter what the utilities of the other options are. Moreover, the latter standard is implausible. It's as if the degrees of choiceworthiness of all the option(s) that do *not* carry the greatest utility did *not* depend on what other options are available in the choice set—it's as if those degrees of choiceworthiness were the same negative number, e.g.,  $-1$ , no matter what the utilities of the other options are. Contrary to the *CU\** principle, the original *CU* principle does not suffer from these problems.

Let us now consider four choice situations involving decisions under certainty (see Table 6): Compared to the *difference in utility* and the *difference in CU\**, the *difference in CU* is a more plausible measure of the extent to which  $a$  is more choiceworthy than  $b$  in situations 1–4, as explained above. The differences in utility and *CU\** between  $a$  and  $b$  are the same in all four situations (4 units), whereas the differences in *CU* between  $a$  and  $b$  are as follows (in situations 1–4):

<sup>12</sup> More precisely, the extent to which  $a$  is more choiceworthy than  $b$ , for  $S$ , is the extent to which  $S$  (rationally) prefers  $a$  to the most (rationally) preferred alternative (or alternatives) to  $a$  (i.e.,  $b$ ,  $c$ ,  $d$ , or  $e$ ) *more* than  $S$  (rationally) prefers  $b$  to the most (rationally) preferred alternative (or alternatives) to  $b$  (i.e.,  $a$ ,  $c$ ,  $d$ , or  $e$ ).

<sup>13</sup> For any choice option,  $a$ , and for any state of the world,  $G$ ,  $a$ 's *CU\** in  $G$  is the difference in utility, in  $G$ , between  $a$  and whichever option(s) carry the greatest utility in  $G$  (i.e.,  $a$ ,  $b$ ,  $c$ ,  $d$ , or  $e$ ). The rule of maximizing *expected CU\** (or *ECU\**) counsels agents to choose whichever option in the choice set has the greatest *ECU\** (or one of them in the event that several alternatives are tied), where *ECU\** is a probability-weighted sum of an option's *CU\*s* across the various states of the world. The rule of maximizing *ECU\** is equivalent to the rule of maximizing *EU* (i.e., *EU theory*), which means that both rules deliver the same verdicts in all decision cases.

**TABLE 6** Decision matrix

	1	2	3	4
<i>a</i>	5	5	5	5
<i>b</i>	1	1	1	1
<i>c</i>	1	2	2	2
<i>d</i>	1	3	3	3
<i>e</i>	1	3	5	8

1.  $(5 - 1) - (1 - 5) = 8$  c-utiles
2.  $(5 - 3) - (1 - 5) = 6$  c-utiles
3.  $(5 - 5) - (1 - 5) = 4$  c-utiles
4.  $(5 - 8) - (1 - 8) = 4$  c-utiles

The CU principle is therefore well-supported.

### 3.1.2 | Argument 2

1. For any agent, *S*, faced with any decision under certainty and for any option, *a*, for *S*, *a* is choiceworthy for *S* if and only if *a* is worthy of being chosen by *S* over whichever alternative(s) to *a* are the most choiceworthy for *S*. (True by definition)
2. *a* is choiceworthy for *S* if and only if *a* maximizes choiceworthiness for *S* over the space of all alternatives in the choice set (i.e., the CM principle). (Assumption)
3. The extent to which *a* is choiceworthy for *S* (or [i.e.] the measure of *how* choiceworthy *a* is for *S*) is the extent to which *a* is worthy of being chosen by *S* over whichever alternative(s) to *a* are the most choiceworthy for *S*. (3 follows from 1 and 2.)
4. *a* is choiceworthy for *S* if and only if *a* maximizes utility over the space of all alternatives in the choice set (i.e., the UM principle). (Assumption)
5. *a* maximizes choiceworthiness for *S* over the space of all alternatives in the choice set if and only if *a* maximizes utility over the space of all alternatives in the choice set. (5 follows from 2 and 4.)
6. *a* maximizes choiceworthiness for *S* over the space of all alternatives in a subset of the choice set if and only if *a* maximizes utility over the space of all alternatives in that subset of the choice set. (6 follows from 5.)
7. Whichever alternative(s) to *a* are the most choiceworthy for *S* are whichever alternative(s) to *a* carry the greatest utility. (7 follows from 6.)
8. The extent to which *a* is worthy of being chosen by *S* over some alternative to *a* is the difference in utility between *a* and that alternative to *a*. (True by conceptual analysis)
9. Therefore, the extent to which *a* is choiceworthy for *S* (or [i.e.] the measure of *how* choiceworthy *a* is for *S*) is the difference in utility between *a* and whichever alternative(s) to *a* carry the greatest utility (i.e., the *CU principle*). (9 follows from 3, 7 and 8.)

### 3.1.3 | Argument 3

Let us consider a final argument for a graded, quantitative choiceworthiness measure for decisions under certainty. *Measures* of quantities that have an interval scale, for example 20°C for temperature, are meaningful (and *only* meaningful) relative to a given zero point and unit of

measurement. (Let us call this the *measurement principle*.) In accordance with the measurement principle, for any agent, *S*, faced with *any* decision situation under certainty and for any option, *a*, for *S*, the measure of the choiceworthiness of *a* for *S* depends on a unit of measurement of choiceworthiness as well as a zero point of choiceworthiness (or *benchmark*) in the following way: the measure of the choiceworthiness of *a* for *S* (relative to *any* explicitly given utility unit and zero point of utility) is the *difference in utility* between *a* and some *benchmark* for *a*, such that (i) *a* is choiceworthy for *S* if and only if the difference in utility between *a* and the benchmark for *a* is equal to or greater than zero (and not choiceworthy otherwise), and (ii) the degree of choiceworthiness of *a* for *S* is the difference in utility between *a* and the benchmark for *a*. In other words, the measure of the choiceworthiness of *a* for *S* is the degree to which *a* is worthy of being chosen over the benchmark for *a*. The benchmark for *a* can be, for example, some option in the set of available options, such as whichever option has the highest utility, whichever option has the lowest utility, or the status quo, or some average of the utilities of the available options.<sup>14</sup> As will become clear in what follows, the concept of choiceworthiness itself presupposes a given benchmark (or zero point of choiceworthiness).

If there are any alternatives to *a* which carry a greater utility than does *a*, then the benchmark for *a* is whichever alternative to *a* carries the greatest utility (or one of them in the event that several alternatives are tied). Indeed, if there are any alternatives to *a* with a greater utility than *a*, then, in accordance with the UM principle, *a* is not choiceworthy for *S*. But if *a* is not choiceworthy for *S*, then how choiceworthy *a* is for *S* is simply how *a* compares to whichever alternative(s) are choiceworthy for *S* (or, per the UM principle, whichever alternative(s) to *a* carry the greatest utility). I will now argue that if there are *not* any alternatives to *a* which carry a greater utility than does *a*, then the benchmark for *a* still has to be whichever alternative to *a* carries the greatest utility (or one of them in the event that several alternatives are tied).

Let us consider two decision situations: 1 and 2. In each situation, *S* is faced with the same three options: *a*, *b*, and *c*. What’s more, in each situation, *S* assigns probability 1 to a given state of the world (but not the same state for both situations). If that state of the world were realized, then *S* would assign the following utilities to the set of options (see Table 7):

**TABLE 7** Decision matrix

	1	2
<i>a</i>	100	100
<i>b</i>	-100	99
<i>c</i>	-100	99

Per the UM principle, *a* is choiceworthy for *S* in both situations 1 and 2. *a* is also more choiceworthy for *S* in 1 than in 2. In 2, *S* misses out on only 1 utile by not choosing *a*, but instead choosing the best alternative to *a* (i.e., *b* or *c*), whereas in 1, *S* misses out on 200 utiles by not choosing *a*, but instead choosing the best alternative to *a* (i.e., *b* or *c*). Another way of putting it is that *a* is more choiceworthy in 1 than in 2 because *a* is more worthy of being chosen over the best alternative to *a* in 1 than in 2.

<sup>14</sup> Wedgwood (2013) argues for *benchmark theory* (BT). The basic idea of BT is to rank choice options (in terms of how choiceworthy they are) according to their *expected comparative value*, where the comparative value of an option is its *value* (broadly construed) in some state of the world compared to a benchmark for that state of the world. Wedgwood identifies the benchmark as an average of the options’ values within a given state of the world. He emphasizes that all statewise dominated options and more generally, “all the options that *do not deserve to be taken seriously*” (p. 2664) should be excluded from consideration at the outset. Wedgwood explicitly rejects the idea that the value of an option is its utility. Nevertheless, if the value of options is understood to be their utility, BT\* is ruled out by the two examples presented below.

Here is a different example (see Table 8):

**TABLE 8** Decision matrix

	1	2
<i>a</i>	100	100
<i>b</i>	-100	-100
<i>c</i>	-100	100

Per the UM principle, *a* is choiceworthy for *S* in both situations 1 and 2. *a* is also more choiceworthy for *S* in 1 than in 2. In 2, *a* is merely optional—*S* misses out on zero utiles by not choosing *a*, but instead choosing the best alternative to *a* (i.e., *c*)—whereas in 1, *a* is not optional—*S* misses out on 200 utiles by not choosing *a*, but instead choosing the best alternative to *a* (i.e., *b* or *c*). Again, *a* is more choiceworthy in 1 than in 2 because *a* is more worthy of being chosen over the best alternative to *a* in 1 than in 2.

These two examples serve to illustrate that if there are *not* any alternatives to *a* with a greater utility than *a*, then how choiceworthy *a* is depends on how much utility *S* would miss out on by not choosing *a*, but instead choosing the best alternative to *a*. The greater the amount of utility *S* would miss out on by not choosing *a*, but instead choosing the best alternative to *a*, the more choiceworthy *a* becomes. Thus, the benchmark for *a* must be whichever alternative to *a* carries the highest utility (or one of them in the event that several alternatives are tied).

What follows is that whether or not there are any alternatives to *a* which carry a greater utility than does *a*, the benchmark for *a* has to be whichever alternative to *a* carries the greatest utility (or one of them in the event that several alternatives are tied). This means that there is no unique benchmark for a given choice situation. Instead, the benchmark is defined in relation to a specific choice option. The benchmark for *a* may be some alternative, *b*, and the benchmark for *b* may be *a*. Therefore, for any agent, *S*, faced with any decision under certainty and for any option, *a*, for *S*, the measure of the choiceworthiness of *a* for *S* (relative to any explicitly given utility unit and zero point of utility) is the *CU* of *a* (in the state of the world to which *S* assigns probability 1). The *CU* of *a* is the difference in utility between *a* and whichever alternative(s) to *a* carry the greatest utility. As previously indicated, I will refer to this principle as the *CU principle*. The *CU* principle entails the *UM* principle.

Now, according to Briggs (2019), “expected utility theory provides a way of ranking the acts according to how *choiceworthy* they are: the higher the expected utility, the better it is to choose the act. (It is therefore best to choose the act with the highest expected utility—or one of them, in the event that several acts are tied.)” If that is right, then the way of defining a graded, quantitative choiceworthiness measure for decisions under certainty is as follows: for any agent, *S*, faced with any decision under certainty and for any option, *a*, for *S*, the measure of the choiceworthiness of *a* for *S* is the utility of *a* in the state of the world to which *S* assigns probability 1. I will refer to this as the *utility principle*. The *UM* principle is true if (*but not* only if) the utility principle is true. However, in light of the *CU* principle, the utility principle can be falsified. If the utility principle were true, then in accordance with the measurement principle, it would be the case that for *any* given decision situation, there is at least one specification of a utility unit and zero point of utility such that it is possible to ascertain how choiceworthy any available option is (for *S*) by solely considering its utility value in relation to that specification of a utility unit and zero point of utility. In other words, it would be the case that for *any* given decision situation, there is at least one specification of a utility unit and zero point of utility such that (a) any available option is choiceworthy (for *S*) if and only if its utility value is equal to or greater than zero (and not choiceworthy otherwise) and (b) *the degree of choiceworthiness of any available option (for S) is its utility value*. As we will now see, that is not the case. Let us consider the following decision

setup:  $S$  is faced with three options:  $a$ ,  $b$ , and  $c$ . What's more,  $S$  assigns probability 1 to a given state of the world. If that state of the world were realized, then  $S$  would assign the following utilities to the available options:  $a$  (0),  $b$  (-100),  $c$  (-1000). Therefore, no matter what zero point of utility is selected,  $S$  assigns the following utility intervals between the available options: between  $a$  and  $b$ ,  $S$  assigns a positive interval of 100 utiles, between  $b$  and  $c$ ,  $S$  assigns a positive interval of 900 utiles and between  $a$  and  $c$ ,  $S$  assigns a positive interval of 1000 utiles. Per the CU principle, the degrees of choiceworthiness of the available options are as follows:  $a$  (100),  $b$  (-100),  $c$  (-1000). Therefore, the differences between the degrees of choiceworthiness of the available options are as follows: between  $a$  and  $b$ , the difference is 200 c-utiles, between  $b$  and  $c$ , the difference is 900 c-utiles and between  $a$  and  $c$ , the difference is 1100 c-utiles. Since the utility intervals and the differences in degrees of choiceworthiness are at variance, we have a decision situation where no matter what zero point of utility (and what utility unit) is selected, it is *not* the case that the degree of choiceworthiness of any available option is its utility value.

Since the utility principle is false, that is, since the measure of the choiceworthiness of  $a$  for  $S$  *cannot* be the utility of  $a$  in the state of the world to which  $S$  assigns probability 1, and since the expected utility of  $a$  equals the utility of  $a$  in the state of the world to which  $S$  assigns probability 1, it follows, pace Briggs (2019), that for any agent,  $S$ , faced with any decision under *certainty* or any decision under *risk* and for any option,  $a$ , for  $S$ , the measure of the choiceworthiness of  $a$  for  $S$  *cannot* be the expected utility of  $a$ .

### 3.2 | The CECU principle

As I argued in discussing Step 2, we require a graded, quantitative measure of how choiceworthy options are. When we move from decision-making under certainty to decision-making under risk, we can, in light of the CU principle, identify the measure of an option's choiceworthiness as expressing that option's *expected choiceworthiness*, or *ECU*, that is to say, the *expected value*, or the probability-weighted sum of all possible values, of that option's choiceworthiness, or CU, *in the actual state of the world*. That roughly encapsulates ECU theory.

ECU theory, as formulated above, is not quite right though. In accordance with the measurement principle, if the measure of the choiceworthiness of options is their ECU, then only options with ECU equal to or greater than zero can be choiceworthy. However, as I illustrated in Section 2, there will always be cases (regardless of what utility unit and zero point of utility are specified) where every option in a decision situation *under risk* has negative ECU. Since at least one option in a decision situation must be choiceworthy—the one with the highest degree of choiceworthiness (or one of them in the event that several alternatives are tied) (i.e., the CM principle)—ECU theory, as defined above, is false in decision cases *under risk*. By the same lines of reasoning as employed in Section 3.1.3, we reach the following conclusion: ECU theory is the conjunction of the CU principle (for decisions under certainty) and the CECU principle (for decisions under risk). Let us recall that according to the CECU principle, for any agent,  $S$ , faced with any decision under *risk* and for any choice option,  $a$ , for  $S$ , the measure of the choiceworthiness of  $a$  for  $S$  (relative to any explicitly given utility unit and zero point of utility) is the CECU of  $a$ , that is to say, the difference in ECU between  $a$  and whichever alternative(s) to  $a$  carry the greatest ECU.

One line of argument in support of ECU theory is that, contrary to EU theory, ECU theory entails Wedgwood's *Gandalf's principle*: the choiceworthiness of an option *in a given state of the world* should be measured only relative to the values of the other options *in that state*, and not to the values of the options *in other states*. According to Wedgwood (2013, p. 2654),

to make a rational choice in [cases involving risk], one *does not need to consider* whether one is in a nice state of nature or a nasty one. All that one needs to consider are the

*degrees* to which each of the available options is better (or worse) than the available alternatives *within* each of the relevant states of nature. Admittedly, when one is uncertain which state of nature one is in, one must make *some* comparisons across the states of nature. But since one does not even need to know whether one is in a nice state of nature or a nasty one, it seems that the only relevant comparisons are comparisons of the *differences* in levels of goodness between the various options *within* each state of nature with the *differences* between those options within each of the other states of nature—not any comparisons of *absolute* levels of goodness across different states of nature.

Although Wedgwood uses terms such as “better,” “worse,” and “levels of goodness” in his explication of Gandalf’s principle, the principle can be expressed equally well using replacement terms such as “preferred,” “dispreferred,” and “levels of utility.”

Gandalf’s principle is an eminently reasonable principle (see Wedgwood, 2013, pp. 2652–2655). In a paper critiquing Wedgwood’s BT, Robert Bassett (2015) concurs: “Gandalf’s principle strikes me as an eminently sensible principle to incorporate into rational decision-making.” There is, however, one alternative decision theory which entails both the CU principle and Gandalf’s principle and which has some *prima facie* plausibility—*maximum likelihood comparative utility (MLCU) theory*: for any agent, *S*, and for any option, *a*, for *S*, the measure of the choiceworthiness of *a* for *S* (relative to any explicitly given utility unit and zero point of utility) is the *most likely value* of *a*’s choiceworthiness (or CU) in the actual state of the world, and in cases where there is more than one maximally likely value of *a*’s choiceworthiness (or CU) in the actual state of the world, the measure of the choiceworthiness of *a* for *S* (relative to any explicitly given utility unit and zero point of utility) is *a*’s CECU across the maximally likely states of the world. We require a further argument to rule out MLCU theory.

This brings me to the following decision case: Let us suppose that an agent, *S*, is faced with three choice options: *a*, *b*, and *c*. *S* assigns probability 0.51 to a state of the world, *A*, and 0.49 to a state of the world, *B*. If state *A* or state *B* were realized, then *S* would assign the following utilities to the set of options (see Table 9):

**TABLE 9** Decision matrix

	<b>A (0.51)</b>	<b>B (0.49)</b>
<i>a</i>	110	−1000
<i>b</i>	80	110
<i>c</i>	100	100

According to MLCU theory, *a* is uniquely choiceworthy for *S*, since state *A* is more likely to obtain than state *B* and the CU of option *a* in state *A* is greater than that of any other available option. Yet, it is clear that choosing option *a* is a mistake, since state *B* is almost as likely to obtain as state *A* and the comparative *disutility* of option *a* in state *B* is very high (−1110 c-utiles). I take this to be a counterexample to MLCU theory.

A second line of argument in support of ECU theory is that for the same reasons as those given in Section 3.1.1 (except that we consider here rational preferences within various possible states of the world instead of rational preferences within a decision situation under certainty), compared to the difference in EU, the difference in CECU is a more plausible measure of the extent to which option *a* is more choiceworthy than option *b* in the following decision matrices (Tables 10–13). The differences in EU between *a* and *b* are the same in all four decision matrices (4 units), whereas the differences in CECU between *a* and *b* are as follows:

**TABLE 10** Decision matrix

	A (0.5)	B (0.5)	EU	CECU
<i>a</i>	5	5	5	8
<i>b</i>	1	1	1	-8
<i>c</i>	1	1	1	-8
<i>d</i>	1	1	1	-8
<i>e</i>	1	1	1	-8

Note: The difference in CECU between *a* and *b* = 16 units.

**TABLE 11** Decision matrix

	A (0.5)	B (0.5)	EU	CECU
<i>a</i>	5	5	5	4
<i>b</i>	1	1	1	-6
<i>c</i>	2	2	2	-5
<i>d</i>	3	3	3	-4
<i>e</i>	3	3	3	-4

Note: The difference in CECU between *a* and *b* = 10 units.

**TABLE 12** Decision matrix

	A (0.5)	B (0.5)	EU	CECU
<i>a</i>	5	5	5	0
<i>b</i>	1	1	1	-4
<i>c</i>	2	2	2	-3
<i>d</i>	3	3	3	-2
<i>e</i>	5	5	5	0

Note: The difference in CECU between *a* and *b* = 4 units.

**TABLE 13** Decision matrix

	A (0.5)	B (0.5)	EU	CECU
<i>a</i>	5	5	5	-6
<i>b</i>	1	1	1	-10
<i>c</i>	2	2	2	-9
<i>d</i>	3	3	3	-8
<i>e</i>	8	8	8	6

Note: The difference in CECU between *a* and *b* = 4 units.

5. For any agent, *S*, faced with any decision under *risk* and for any option, *a*, for *S*, *a* is choiceworthy for *S* if and only if *a* maximizes CECU. (5 follows from 3 and 4.)
6. For any agent, *S*, faced with any decision under *risk* and for any number of alternative options, *a*, *b*, *c*, *d*, and *e*, for *S*, it is rational for *S* to prefer *a* to *b* if and only if *a*'s EU is greater than *b*'s, it is rational for *S* to be indifferent between *a* and *b* if and only if *a*'s EU is equal to *b*'s, and the extent to which *S* rationally prefers *a* to *b* is the difference in EU between *a* and *b*.

Decision-theoretic representation theorems—such as those of von Neumann and Morgenstern (1947), Savage (1954), Bolker (1966) and Jeffrey (1983), and Joyce (1999)—show that if an agent fails to prefer choice options with higher EU, then that agent violates at least one of a series of axioms of *rational preference*,<sup>15</sup> one of which is the IIA. Besides the intuitive plausibility of the axioms of rational preference, a further justification for Step 6 comes from money-pump arguments for those axioms (see Gustafsson, 2022).

7. For any agent, *S*, faced with any decision under *risk* and for any option, *a*, for *S*, it is rational for *S* to weakly prefer *a* over the alternative options in the choice set if and only if *a* maximizes EU. (7 follows from 6.)
8. In decisions under *risk*, what option(s) maximize CECU sometimes differ from what option(s) maximize EU.

For examples, see Section 2.

9. In decisions under *risk*, what option(s) are choiceworthy sometimes differ from what option(s) it is rational to weakly prefer over the alternative options in the choice set. (9 follows from 5, 7 and 8.)

Rational preference is not a reliable measure of choiceworthiness. That is because whereas the criterion of rational preference (i.e., EU) satisfies the IIA, the criterion of choiceworthiness (i.e., CU/CECU) violates that principle (see Section 2 for examples). In other words, whereas the criterion of rational preference (i.e., EU) only takes into account the two choice options that are being compared within the set of alternatives, the criterion of choiceworthiness (i.e., CU/CECU) takes into account the entire set of alternatives when evaluating each option, as should any plausible criterion of choice. This gives us a reason to opt for the proposed criterion of choice (i.e., choiceworthiness) over the standard choice criterion (i.e., rational preference). At this point, it is important to emphasize that the proposed criterion of choice (i.e., choiceworthiness) is independent from the standard choice criterion (i.e., rational preference). The latter is not shown here to violate the assumptions, for example, the IIA, which are needed to derive utilities from preferences via a representation theorem.

10. For any agent, *S*, faced with any decision under *risk*, it is a requirement of instrumental rationality that *S* should measure and rank his or her options (for the purpose of choice) in terms of *how choiceworthy* they are for *S*.

Whether (and to what extent) any option is more choiceworthy than any other within a set of alternatives is necessarily a function of *how choiceworthy* each of the two options is within the set of alternatives. (See my discussion of Step 2.)

Let us now consider the *instrumental rationality argument (v1)*:

1. For any agent, *S*, faced with any decision under certainty or any decision under risk and for any option, *a*, for *S*, the *degree* to which *a* is *choiceworthy* for *S*, or (i.e.) the *choiceworthiness* of *a* for *S*, is the degree to which *a* is worthy of being chosen by *S* in light of *S*'s rational preferences within each of the various possible states of the world, where *S*'s *rational preferences* are preferences that obey the series of rationality conditions or axioms of standard decision theory.
2. The *degree* to which *a* is *instrumentally rational* for *S* is the degree to which *a* is a suitable means

<sup>15</sup> Following the formulation of Briggs (2019).

to  $S$ 's ends.<sup>16</sup>

3. The *degree* to which  $a$  is a *suitable means* to  $S$ 's ends is the degree to which  $a$  is worthy of being chosen by  $S$  in light of  $S$ 's rational preferences within each of the various possible states of the world.
4. Therefore, the *degree* to which  $a$  is *instrumentally rational* for  $S$  is the degree to which  $a$  is choiceworthy for  $S$ , or (i.e.) the choiceworthiness of  $a$  for  $S$ . (4 follows from 1 to 3.)

For any agent,  $S$ , faced with any decision under certainty or any decision under risk, it is a requirement of *instrumental rationality* that  $S$  should measure and rank his or her options (for the purpose of choice) in terms of the *degrees* to which they are *instrumentally rational* for  $S$ , or (i.e.) the degrees to which they are suitable means to  $S$ 's ends. Therefore, in accordance with the instrumental rationality argument (v1), it is a requirement of *instrumental rationality* that  $S$  should measure and rank his or her options (for the purpose of choice) in terms of the *degrees* to which they are *choiceworthy* for  $S$ , or (i.e.) the degrees to which they are worthy of being chosen by  $S$  in light of  $S$ 's rational preferences within each of the various possible states of the world.

11. It is *not* the case that for any agent,  $S$ , faced with any decision under *risk*, it is a requirement of instrumental rationality that  $S$  should measure and rank his or her options (for the purpose of choice) in order of rational preference. (11 follows from 3, 9 and 10.)

In accordance with my discussion of Step 10, for any agent,  $S$ , faced with any decision under *risk*, it is a requirement of *instrumental rationality* that  $S$  should measure and rank his or her options (for the purpose of choice) in terms of the degrees to which they are worthy of being chosen by  $S$  in light of  $S$ 's rational preferences within each of the various possible states of the world. It follows that for any agent,  $S$ , faced with any decision under *risk*, it is a *theory* of instrumental rationality that  $S$  should measure and rank his or her options (for the purpose of choice) in order of rational preference. However, it does *not* follow that it is true *by definition* that for any agent,  $S$ , faced with any decision under *risk*, it is an *actual requirement* of instrumental rationality that  $S$  should measure and rank his or her options (for the purpose of choice) in order of rational preference.

12. For any agent,  $S$ , faced with any decision under *risk*, it is a requirement of instrumental rationality that  $S$  should measure and rank his or her options (for the purpose of choice) in terms of how choiceworthy they are for  $S$ , that is, according to their CECU, rather than in order of rational preference, that is, according to their EU. (12 follows from 4, 6, 10 and 11, as well as from 3, 4, 6, 9 and 13.)

I can offer two further arguments for Step 12: First, there is Wedgwood's Gandalf's principle (see Section 3.2). Second, compared to the criterion of rational preference (i.e., EU), the criterion of choiceworthiness (i.e., CU/CECU) supplies a more plausible measure of the extent to which any given option is more choiceworthy than any other in any ranking of more than two choice options, both in decision situations involving certainty and decision situations involving risk. For decisions under certainty, see Section 3.1.1. For decisions under risk, consider the extent to which option  $a$  is more choiceworthy than option  $b$  in decision matrices 10–13.

13. For any agent,  $S$ , faced with any decision under *risk*, it is a requirement of instrumental rationality that  $S$  should choose out of what option(s) are *choiceworthy* for  $S$ .

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<sup>16</sup> According to Kolodny and Brunero (2020), "Someone displays instrumental rationality insofar as she adopts suitable means to her ends."

Step 13 a tautology. Therefore, it is necessarily true. Here is another way of arguing for Step 13. Let us consider the *instrumental rationality argument* (v2):

1. For any agent, *S*, faced with any decision under certainty or any decision under risk and for any option, *a*, for *S*, *a* is *choiceworthy* for *S* if and only if *a* is worthy of being chosen by *S* in light of *S*'s rational preferences within each of the various possible states of the world, where *S*'s *rational preferences* are preferences that obey the series of rationality conditions or axioms of standard decision theory.
2. *a* is *instrumentally rational* for *S* if and only if *a* is a suitable means to *S*'s ends.
3. *a* is a *suitable means* to *S*'s ends if and only if *a* is worthy of being chosen by *S* in light of *S*'s rational preferences within each of the various possible states of the world.
4. Therefore, *a* is *instrumentally rational* for *S* if and only if *a* is choiceworthy for *S*. (4 follows from 1 to 3.)

For any agent, *S*, faced with any decision under certainty or any decision under risk, it is a requirement of *instrumental rationality* that *S* should choose out of what option(s) are *instrumentally rational* for *S*, or (i.e.) what option(s) are suitable means to *S*'s ends. Therefore, in accordance with the instrumental rationality argument (v2), it is a requirement of *instrumental rationality* that *S* should choose out of what option(s) are *choiceworthy* for *S*, or (i.e.) what option(s) are worthy of being chosen by *S* in light of *S*'s rational preferences within each of the various possible states of the world.

14. It is *not* the case that for any agent, *S*, faced with any decision under *risk*, it is a requirement of instrumental rationality that *S* should choose out of what option(s) it is rational for *S* to weakly prefer over the alternative options in the choice set. (14 follows from 9 and 13.)

In accordance with my discussion of Step 13, for any agent, *S*, faced with any decision under *risk*, it is a requirement of *instrumental rationality* that *S* should choose out of what option(s) are worthy of being chosen by *S* in light of *S*'s rational preferences within each of the various possible states of the world. It follows that for any agent, *S*, faced with any decision under *risk*, it is a *theory* of instrumental rationality that *S* should choose out of what option(s) it is rational for *S* to weakly prefer over the alternative options in the choice set. However (and contrary to Step 13), it does *not* follow that it is true *by definition* that for any agent, *S*, faced with any decision under *risk*, it is an *actual requirement* of instrumental rationality that *S* should choose out of what option(s) it is rational for *S* to weakly prefer over the alternative options in the choice set.

15. For any agent, *S*, faced with any decision under *risk*, it is a requirement of instrumental rationality that *S* should choose out of what option(s) are choiceworthy for *S* (i.e., what option(s) maximize CECU), even in cases where what option(s) are choiceworthy for *S* (i.e., what option(s) maximize CECU) differ from what option(s) it is rational for *S* to weakly prefer over the alternative options in the choice set (i.e., what option(s) maximize EU). (15 follows from 5, 7, 9, 13 and 14, as well as from 3, 5, 7, 9 and 10.)

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p4vajs/a\_restatement\_of\_expected\_comparative\_utility/, [https://www.reddit.com/r/academic\\_economics/comments/tnnfd5/a\\_restatement\\_of\\_expected\\_comparative\\_utility/](https://www.reddit.com/r/academic_economics/comments/tnnfd5/a_restatement_of_expected_comparative_utility/),  
[https://www.reddit.com/r/philosophy/comments/ttrhhl/a\\_restatement\\_of\\_expected\\_comparative\\_utility/](https://www.reddit.com/r/philosophy/comments/ttrhhl/a_restatement_of_expected_comparative_utility/)).

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