#### POSSIBLE WORLDS AS PROPOSITIONS

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### 1. Introduction

Recently, a number of philosophers have argued that higher-order quantification – for example, quantification into predicate position, as in the sentence ' $\exists x \exists F(Fx)$ ' – should be treated as a genuine form of quantification in its own right, and so in particular, should not be analysed in terms of first-order quantification over e.g. abstract objects. Call this view (for want of a better name) *higher-orderism*. Advocates of higher-orderism argue that the view helps to solve (or dissolve) certain traditional philosophical puzzles, as well as open up new areas of enquiry which can be addressed in a systematic way by the application of higher-order logic. In this paper, I provide a further example of how higher-orderism can help to solve (or dissolve) philosophical problems: in this case, the problem of explaining how possible worlds represent possibilities.

The paper is structured as follows: first (§2), to set the stage, I provide a brief introduction to higher-orderism. Next (§3), I describe a non-Lewisian form of realism about possible worlds according to which possible worlds are propositions. Then (§4) I describe Lewis's (1986) well-known objection that there is no way to explain how possible worlds-aspropositions represent possibilities. Finally (§5), I argue that higher-orderism provides a response to Lewis's objection for those who identify possible worlds with propositions.

<sup>&</sup>lt;sup>1</sup> This sentence can be read informally as saying that *something is some way*.

<sup>&</sup>lt;sup>2</sup> Recent advocates of higher-orderism include Rayo & Yablo (2001); Bostock (2004); Williamson (2013); Krämer (2014); Dorr (2016); Fritz (2017); Goodman (2017); Jones (2018, 2019); and Trueman (2021, 2022). The view is typically traced to the work of Frege and Prior: see especially Frege (1879) and Prior (1971).

<sup>&</sup>lt;sup>3</sup> See e.g. Skiba (2021a) and Fritz & Jones (forthcoming a, b).

<sup>&</sup>lt;sup>4</sup> I address the question of whether higher-orderism should be thought of as providing a solution to, or a dissolution of, this problem in §6.

# 2. Higher-orderism

Consider the following sentences:

- (1)  $\exists x (Property(x) \land Instantiates(me, x) \land Instantiates(you, x)).$
- (2)  $\exists F(F(me) \land F(you))$ .
- (3)  $\exists x (Proposition(x) \land True(x) \land \Diamond (False(x))).$
- (4)  $\exists p(p \land \Diamond \neg p)$ .

Sentences (1) and (2) can both be read as saying that there is a way we both are. However, each sentence expresses this thought differently. Sentence (1) uses the familiar 'objectual' (or 'nominal') quantifier ' $\exists x$ ' of first-order logic, and is intended to be read as saying that there is some thing or object<sup>5</sup> – namely, a property – which is instantiated by me and by you. It does not tell us what kind of things properties are: they may be linguistic items, mental items, or abstract objects like sets or inhabitants of the Platonic realm. But it does tell us that there are such things as properties. Sentence (2), on the other hand, uses the perhaps less familiar 'predicate' quantifier ' $\exists F$ ' of second-order logic, which quantifies into predicate rather than name position. How is (2) intended to be read? The best way to read it is as simply saying that there is a way we both are (e.g. human). What is important to notice is that (2), unlike (1), does not tell us (at least, in the absence of some interpretation of the second-order

<sup>&</sup>lt;sup>5</sup> I use 'thing' and 'object' interchangeably in what follows.

<sup>&</sup>lt;sup>6</sup> Second-order logic was first developed by Frege (1879). See Trueman (2021) for a good introduction to Frege's thought and Bostock (2004) for an introduction to second-order logic.

quantifier) that there are such things as properties. It simply generalises over truths such as that we are both human.

Similarly, sentences (3) and (4) can both be read as saying that there is a contingent truth. However, each expresses this thought differently. Sentence (3) uses the familiar 'objectual' quantifier ' $\exists x$ ' of first-order logic, and is intended to be read as saying that there is some thing or object – namely, a proposition – which is true but could be false. It does not tell us what propositions are: they may be linguistic items, psychological items, or abstract objects like sets or inhabitants of the Platonic realm. But it does tell us that there are such things as propositions. Sentence (4), on the other hand, uses the perhaps less familiar 'sentential' quantifier ' $\exists p$ ' of higher-order logic, which quantifies into sentence rather than name or predicate position. How is (4) intended to be read? The best way to read it is as simply saying that there is a contingent truth (e.g. that I have some coins in my pocket). What is important is that, unlike (3), (4) does not tell us (at least, in the absence of some interpretation of the higher-order quantifier) that there are such things as propositions. It simply generalises over truths such as that I have some coins in my pocket but could have failed to do so.

So how should we interpret the higher-order quantifiers ' $\exists F$ ' and ' $\exists p$ ' that appear in sentences like (2) and (4) above? Many philosophers have defended analyses or interpretations of higher-order quantification in first-order terms. For example, Quine (1986: 66-8) analyses higher-order quantification in terms of first-order quantification over sets of objects.<sup>8</sup> In a similar spirit, some philosophers interpret higher-order quantification in terms of first-order quantification over properties and propositions conceived as abstract objects. Another well-known strategy due to Boolos (1985) is to analyse higher-order quantification

<sup>&</sup>lt;sup>7</sup> See especially Prior (1971).

<sup>&</sup>lt;sup>8</sup> This analysis is encouraged by the standard semantics for higher-order quantifiers: see e.g. Bacon (forthcoming a).

in terms of plural first-order quantification, i.e. in terms of quantifiers such as ' $\forall xx$ ' ('For any things') and ' $\exists xx$ ' ('For some things'). And finally, on the 'substitutional' interpretation of the quantifiers due to Marcus (1961, 1962), higher-order quantification is interpreted in terms of first-order quantification over 'substitutions' of linguistic items of the appropriate syntactic category (i.e. predicates or sentences) for variables.<sup>9</sup>

Higher-orderists reject the analysis of higher-order quantifiers in first-order terms (or, indeed, in any other terms). Rather, they treat higher-order quantifiers such as ' $\exists F$ ' ('There is an F') and ' $\exists p$ ' ('There is a p') as genuine quantifiers in their own right, distinct from – but just as 'quantificational' as – the familiar quantifiers over things found in natural language and in first-order predicate logic. 10 Thus for higher-orderists, the most metaphysically perspicuous way to explain the truth of e.g. sentence (2) above is not that there is some set to which we both belong, or some abstract object to which we both bear the instantiation relation, but that  $\exists F(F(me) \land F(you))$  (which can be expressed in English as the claim that there is, quantifying at the second order, a way we both are). Similarly, the most metaphysically perspicuous way to explain the truth of e.g. sentence (4) above is not that there is a set of possible worlds that has the actual world but not some other possible world as members, or that there is an abstract object that does but could fail to possess the property of being true, but that  $\exists p(p \land \Diamond \neg p)$  (which can be expressed in English as the claim that there is, quantifying at a higher-order, a contingent truth). More generally, it is natural to think of higher-orderism as implying that there aren't just things or objects, but also (quantifying at a higher-order) ways things are, and truths and falsehoods about those things and those ways

<sup>&</sup>lt;sup>9</sup> Marcus, like many who defend substitutional quantification, does not regard facts about 'substitutions' as providing an *analysis* of substitutional quantification. Rather, the relevant substitutional facts are regarded as merely providing truth-conditions for the relevant quantificational sentences. See van Inwagen (1981) on this point.

<sup>&</sup>lt;sup>10</sup> Should higher-order quantifiers like ' $\exists F$ ' and ' $\exists p$ ' be regarded as 'existential' quantifiers? Skiba (2021a) argues that higher-orderists should conceive themselves as 'engineering' a higher-order concept of existence distinct from, but analogous to, the familiar first-order concept. See also Krämer (2014) and Jones (2018).

<sup>11</sup> I discuss the matter of expressing higher-order quantification in natural language below.

(as well as ways ways are, and ways truths are, and truths about the ways ways and truths are, and so on *ad infinitum*).<sup>12</sup> And this commitment is intended to be taken at face value: it is intended not merely as a view about how we do or ought to speak and think, but as a view about the nature of reality.

Higher-orderism strikes some philosophers as objectionably mysterious, or as a mere façon de parler. One plausible reason for this is that it is at best difficult to express higherorder quantification in natural language. 13 For example, on its intended interpretation, the higher-order sentence ' $\exists p(p)$ ' is naturally rendered in English as 'There is a truth' or 'Something is true'. But these renderings of the sentence are liable to make it sound as if it involves first-order quantification (e.g. over 'truths') rather than higher-order quantification into sentence position. <sup>14</sup> Similarly, the question 'What does the sentence " $\exists p(p)$ " quantify over?' has no easy answer in English given higher-orderism: philosophers who are sceptical about the legitimacy of *sui generis* higher-order quantification are unlikely to be satisfied by answers such as 'propositions', 'states of affairs', or 'higher-order entities'. Finally, I wrote above that it is natural to think of higher-orderism as implying that there aren't just things, but also (quantifying at a higher-order) ways things are. However, as natural as this might be, there is in fact no meaningful way to express the claim that ways things are are not things in a sui generis higher-order language. 15 This makes it difficult for higher-orderists to express the philosophical difference between their view and that of reductionists about higher-order quantification such as Quine.

<sup>&</sup>lt;sup>12</sup> The best way to get to grips with what is strictly intended here is via *type theory*, which provides a formal specification of the infinite syntactic categories to which the expressions of higher-order logic can belong; informally, the kinds of higher-order entities over which the different higher-order quantifiers quantify. See Bell (2022) and Fritz & Jones (forthcoming b).

<sup>&</sup>lt;sup>13</sup> Higher-orderists such as Prior (1971: Chapter 3), Jones (2018), and Trueman (2021) seem to allow that English is sufficiently flexible to express higher-order quantification. However, even if it is, it remains the case that only a tiny fraction of higher-order sentences can be sensibly rendered in English or any other natural language.

<sup>&</sup>lt;sup>14</sup> 'Things are somehow' may be a little better: see Prior (1971: Chapter 3).

<sup>&</sup>lt;sup>15</sup> As Button & Trueman (forthcoming: §2.1) point out, this is at the heart of Frege's famous 'concept *horse*' paradox.

Higher-orderists have a developed a number of strategies for responding to philosophical concerns arising from the relative inexpressibility in natural language of higher-order quantification and of higher-orderism itself. For example, Williamson (2003: 259-60) argues that since higher-order quantification is meaningful despite being irreducible to any natural language, the best way to understand it is by learning the relevant higher-order language (or a portion thereof). Similarly, Trueman (2021: §9.4) argues that a helpful way to answer the question of what the higher-order quantifiers quantify over is in the spirit of Wittgenstein (1921), by showing rather than telling. For example, from the fact that Ramona is wise (Wr), it follows by a higher-order analogue of existential generalisation that there is a way Ramona is  $(\exists F(F(Ramona)))$ . This inference can be said to provide a demonstrative answer to the question of what the higher-order quantifiers quantify over. However, the focus of most higher-orderists has not been on defending the cogency of their view, but on developing its consequences: for example, for questions about the nature of representation and truth; 16 the metaphysics of properties and propositions; 17 identity, existence, and the structure of reality;<sup>18</sup> and possibility and necessity.<sup>19</sup> The goal of this paper is to make a modest contribution to that effort.

# 3. Realism About Possible Worlds

Some philosophers believe that there are possible worlds: call them *realists about possible* worlds, or 'realists' for short.<sup>20</sup> For many philosophers, realism will bring to mind Lewis' (1986) infamous view that there is a plenitude of possible worlds and possible worlds are

<sup>&</sup>lt;sup>16</sup> See Bacon & Russell (2019) and Trueman (2021).

<sup>&</sup>lt;sup>17</sup> See Jones (2018, 2019) and Button & Trueman (forthcoming).

<sup>&</sup>lt;sup>18</sup> See Uzqiano (2015); Dorr (2016); and Goodman (2017).

<sup>&</sup>lt;sup>19</sup> See Williamson (2013); Dorr and Hawthorne with Yli-Vakkuri (2021); and Bacon & Dorr (forthcoming).
<sup>20</sup> Since I argue below that realists should consider adopting an irreducibly higher-order conception of possible

worlds, the quantifier 'there are' in the characterisation of realism here should be read as permitting either a first-order or a higher-order interpretation (and similarly for the form of realism described below as the view that 'possible worlds are propositions').

maximally-interrelated spacetime systems ('cosmoi' from now on). However, the dominant form of realism has traditionally been one according to which possible worlds are abstract objects, such as sets of sentences (Carnap 1947); propositions (Chisholm 1981); collections of propositions or states of affairs (Adams 1974; Plantinga 1974); or properties (Stalnaker 1976, 1986). For ease of exposition, in what follows, I focus on the sort of realism according to which possible worlds are propositions, as per Chisholm (1981); but much of what follows applies equally to other non-Lewisian forms of realism, for instance, those that identify possible worlds with properties or states of affairs.

There are different ways of characterising possible worlds in terms of propositions. Here is one way:<sup>21</sup> say that a *world* (whether it is possible or impossible) is a *maximal* proposition, i.e. a proposition p such that for that for any proposition q, either p necessitates q or it necessitates not-q (or both, in which case it is impossible):

WORLD: For any propositions p and q, p is a world := either p necessitates q or p necessitates not-q.<sup>22</sup>

Given this definition of 'world', realists can then define what it is for something to be the case *at a* world; what it is for a world to be *actual*; and what it is for a world to be *possible*, as follows:

AT-W: For any proposition p and world w, p is true at world w := w necessitates p.

<sup>&</sup>lt;sup>21</sup> This way of characterising possible worlds as propositions is due to Dorr & Hawthorne with Yli-Vakkuri (2021: 49, fn.65), who credit Prior & Fine (1977); Fine (1977); and Chisholm (1981: 129) with inspiration for the approach. Note that Dorr & Hawthorne with Yli-Vakkuri present the characterisation in the context of a particular higher-order logic they call 'Hkr', which combines (ancestrally necessitated) higher-order versions of the principles of necessitation and the duality of ' $\diamond$ ' and ' $\Box$ ' with the modal K axiom. I discuss the higher-order

version of the characterisation in more detail in §4.

22 Read 'necessitates' here and throughout as 'metaphysically necessitates'. In what follows, I use ':=' to indicate the giving of an analysis.

ACTUAL WORLD: For any world w, w is actual := for any proposition p, if p is true at w then p.

POSSIBLE WORLD: For any world w, w is possible := w could be actual.

In short: something is the case at a world w just in case it is necessitated by w; the actual world is the world such that everything it necessitates is the case; and a possible world is a world that could be actual, i.e. a world such that possibly, everything it necessitates is the case.

The above definitions do not provide realists with a non-modal analysis of 'possible world', since in the definitions, 'world' is analysed in terms of metaphysical necessity.<sup>23</sup> This won't trouble most realists, who, unlike Lewis (1986), are content to treat at least some of the properties of propositions expressed by modal operators as fundamental.<sup>24</sup> But what's the point of being a realist about possible worlds if one accepts fundamental modality? Perhaps the most obvious is that it allows one to 'take seriously' the widespread object-language quantification over possible worlds found in contemporary modal metaphysics, and in particular, the analysis or explanation in terms of possible worlds of e.g. counterfactuals, causation, physical determinism, supervenience, semantic content, and rigid designation.<sup>25</sup> That is not to say that such 'possible worlds' analyses or explanations are always successful, or indeed simpler than a purely modal understanding of the relevant notions;<sup>26</sup> nevertheless, it is hard to deny that 'possible worlds' are a relatively useful philosophical tool.

<sup>&</sup>lt;sup>23</sup> Although see Bacon (forthcoming a: Chapter 9) for an argument that the 'broadest' (most demanding) necessity can be analysed in purely (higher-order) logical terms.

<sup>&</sup>lt;sup>24</sup> See Adams (1974); Plantinga (1974); Prior & Fine (1977); Fine (1977); and Chisholm (1981: 129).

<sup>&</sup>lt;sup>25</sup> See especially Lewis (1986: §§1.2-1.5).

<sup>&</sup>lt;sup>26</sup> See e.g. Dorr (2012) on rigid designation.

# 4. Lewis's Objection

Lewis (1986: Chapter 3) distinguishes three versions of what he calls 'ersatz modal realism': 'linguistic ersatzism', according to which possible worlds are sets of sentences, and represent possible worlds by describing them in an interpreted language;<sup>27</sup> 'pictorial ersatzism', according to which possible worlds are abstract 'scale models', and represent possibilities by being isomorphic with them; and 'magical ersatzism', according to which possible worlds are propositions or properties, and represent possibilities neither linguistically nor pictorially, but simply in virtue of the sorts of things they are. Lewis then argues that, given that both linguistic and pictorial ersatzism are ultimately constrained in their construction of possible worlds by our actual representational resources, both views suffer from what Sider (2002: 283) calls 'the problem of descriptive power': neither view can account for certain genuine possibilities – for example, those involving the instantiation of non-actual or 'alien' fundamental properties – and neither view can distinguish possibilities which are qualitatively identical but haecceistically distinct (i.e. distinct in virtue of the qualitative roles played by particular individuals). <sup>28</sup> Finally, Lewis argues that magical ersatzism should be rejected on the grounds that it fails to provide an explanation of how ersatz possible worlds represent possibilities. Since the version of realism I am interested in here is that according to which possible worlds are propositions, in what follows, I treat Lewis's objection to magical ersatzism as an objection to realism.

Lewis's objection as applied to realism is that realists cannot explain how the propositions they identify with possible worlds represent possibilities.<sup>29</sup> In contrast, linguistic ersatzers can explain how possible worlds *qua* sets of sentences represent possibilities, in

<sup>&</sup>lt;sup>27</sup> Linguistic ersatzism is defended by Melia (2001). See Divers (2002: 278-4) for discussion of Melia's view.

<sup>&</sup>lt;sup>28</sup> For discussion of the problem of descriptive power for linguistic ersatzism, see Nolan (2002: Chapter 5).

<sup>&</sup>lt;sup>29</sup> As Lewis (1986: 183) makes clear, the objection also applies to versions of realism which identify possible worlds with properties or states of affairs.

virtue of having as members sentences with certain meanings; pictorial ersatzers can explain how possible worlds *qua* abstract 'scale models' can represent possibilities, in virtue of a structural isomorphism between their parts and the relevant possibilities; and 'modal realists' such as Lewis himself can explain how possible worlds *qua* cosmoi can represent possibilities, in virtue of being, in effect, concrete instantiations of those possibilities.

According to Lewis, it is a significant cost of realism that it cannot provide an analogous explanation.<sup>30</sup>

As Lewis (1986: 175) points out, a natural realist response to this objection would be to try to explain propositional representation in modal terms.<sup>31</sup> For instance, a realist might argue that what it is for a proposition p to represent that q is for it to be the case that necessarily, p is true just in case q. Call this view *propositional representation*, or 'PR' for short:

PR: For any proposition p, p represents that q := necessarily, p is true if and only if q.

For example, given PR, what it is for the proposition that there are blue swans to represent that there are blue swans is for it to be the case that necessarily, the proposition that there are blue swans is true just in case there are blue swans.

However, Lewis argues that this sort of response fails to meet his objection, since it simply raises the question of why it is necessary that a proposition is true just in case what it represents is the case. Lewis (1986: 176) assumes that in answering this question, realists will

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<sup>&</sup>lt;sup>30</sup> It seems clear that Lewis' objection to magical ersatzism can be treated as a more general objection to the view that propositions are abstract objects (or at least, mereologically simple abstract objects). On Lewis's (1986: 185) own view, (part of) the proposition role is played by sets of cosmoi or by structured sets of inhabitants of cosmoi.

<sup>&</sup>lt;sup>31</sup> Lewis (1986: 175) frames this imagined response in terms of 'elements' being 'selected': he asserts that all actualist realists accept that 'an element E [i.e. proposition p] represents that so-and-so... iff, necessarily, if E is selected [i.e. true], then so-and-so.'

appeal to either an 'internal' or an 'external' relation between propositions and what they represent, where an 'internal' relation is one that depends only on the intrinsic characters of the relata – such as the relation of *being the same colour as* – and an 'external' relation is one that does not depend on the intrinsic characters of the relata, such as the distance relation between spacetime points.<sup>32</sup> For instance, a realist might argue that the proposition that there are blue swans possesses the fundamental intrinsic property of *representing that there are blue swans*, and it is the fact that it possesses this fundamental intrinsic property – along with the fact that there being blue swans has the fundamental intrinsic character that *it* has – that explains why the proposition that there are blue swans is such that there being blue swans entails and is entailed by its truth.

Lewis (1986: 178-9) raises two objections to this sort of explanation. First, he argues that such explanations fail to increase our understanding of the relevant internal relations between propositions and what they represent, since 'we have not the slightest idea what the "representational properties" [of propositions] are... never mind a general understanding... because the most we can say about the nature of an element [i.e. a proposition] is: it is of a nature to be selected [i.e. true] iff...'. Second, Lewis objects that it is a mystery how anyone could come to know about the relevant internal relations between propositions and what they represent, since given that propositions are abstract objects and therefore causally isolated from any concrete objects, the fundamental representational properties of propositions to which the explanation appeals must also be causally isolated from concrete objects, and therefore cognitively inaccessible. It follows, according to Lewis, that 'if the ersatzer [realist] has understood his own primitive, he must have done it by magic'.

<sup>&</sup>lt;sup>32</sup> As Lewis (1986: 182) points out, a relation can also be neither internal nor external ('mixed') if it depends partly but not only on the intrinsic characters of the relata.

Alternatively, a realist might argue that propositions do not represent what they do because they possess certain fundamental intrinsic properties, but rather because they bear certain fundamental relations to what they represent. Lewis (1986: 179-80) objects to this response that if the representational properties of propositions depend on fundamental relations, it should be possible – by a principle analogous to the 'Humean' principle that there are no necessary connections between distinct existences – for a proposition to be such that the relevant fundamental relation fails to hold between it and what it actually represents, or to hold between a proposition and that which some other proposition actually represents. For example, it should be possible for the relevant fundamental relation to hold between the proposition that there are blue swans and there being *red* swans, so that the proposition that there are blue swans is true just in case there are red swans. Insofar as realists reject this possibility, says Lewis, 'the relation itself... is magical: what spell constrains it to correspond rigidly to goings on in the concrete world?'. Lewis concludes that realists simply cannot explain how the propositions they identify with possible worlds represent possibilities.

How should realists respond to Lewis's objection? A potential response is to argue that what Lewis's objection shows is that realists should reject the assumption that propositions are mereologically simple.<sup>33</sup> Instead, they should prefer a structured or 'neo-Russellian' view of propositions, according to which propositions are mereologically complex abstract objects with a quasi-syntactic structure that mirrors the structure of sentences of a language, and have e.g. objects and (one or many-placed) properties as literal constituents.<sup>34</sup> This might allow realists to respond to Lewis's challenge to explain how possible worlds-as-propositions represent possibilities in a similar way to pictorial ersatzists,

<sup>&</sup>lt;sup>33</sup> Lewis (1986: 174) takes it to be definitive of magical ersatzism that propositions are mereologically simple. However, there doesn't seem to be any good reason to take it to be definitive of the view I am calling 'realism'. <sup>34</sup> See King (2007) and Soames (2010).

by appealing to structural isomorphisms between structured propositions-as-possible worlds and that which they represent.

However, this approach faces a significant problem: a number of philosophers have recently argued that the view that propositions have a quasi-syntactic structure that mirrors the structure of sentences – so that, e.g., the proposition that there are blacks swans is distinct from the proposition that there are black or non-self-identical swans – generates a version of the Russell-Myhill paradox.<sup>35</sup> This result is typically taken to show that the structured theory of propositions encodes the error of thinking that reality is as fine-grained as our ways of speaking and thinking about it, or in other words, of projecting our representations onto reality. It may be that realists who favour a structured view of propositions can avoid the inconsistency associated with the Russell-Myhill paradox;<sup>36</sup> but even if so, it would be nice to be able to offer realists a response to Lewis's objection that does not rely on the assumption that propositions are mereologically complex.<sup>37</sup> In the next section, I describe such a response. The key to that response is not to reject the assumption that propositions are mereologically simple objects, but to reject the assumption that they are *objects* at all.

### 5. The Higher-orderist Response

Let us call the combination of realism about possible worlds of the kind described in §3 with higher-orderism *HO-realism*.<sup>38</sup> Given higher-orderism, HO-realists can interpret the definition of 'world' in §3 in *sui generis* higher-order terms:

<sup>&</sup>lt;sup>35</sup> The discovery of the paradox is typically credited to Russell (1903) and Myhill (1958). See Uzquiano (2015), Dorr (2016), and Goodman (2017) for versions of the Russell-Myhill argument that target the structured view of propositions.

<sup>&</sup>lt;sup>36</sup> For example, see Bacon (forthcoming b) for a theory of structured propositions intended to avoid the paradox. <sup>37</sup> Realists who identify possible worlds with structured propositions will also plausibly face the problem of descriptive power, unless they accept the necessary existence of objects and properties. See Williamson (2003) for a defence of first- and higher-order necessitism. (I am grateful to an anonymous referee for drawing my attention to this point).

<sup>&</sup>lt;sup>38</sup> HO-realism (or something very similar) is defended by Prior & Fine (1977) and Fine (1977), and developed in detail by Fritz (forthcoming). An analogous irreducibly higher-order conception of times is defended by Prior (1968: 79). A different kind of higher-orderist realism about possible worlds, in which possible worlds are

HO-WORLDS: 
$$\forall p(\text{World}(p) := \forall q(\Box(p \to q) \lor \Box(p \to \neg q))).$$

And likewise for the subsequent definitions of what is for something to be the case at a world, what it is for a world to be actual, and what is for a world to be possible:

HO-AT-W: 
$$\forall p \forall w (At(p, w) := \Box(w \rightarrow p)).$$

HO-ACTUAL WORLD: 
$$\forall w (Actual(w) := \forall p (At(p, w) \rightarrow p)).$$

HO-POSSIBLE WORLD: 
$$\forall w (Possible(w) := \Diamond Actual(w))$$
.

Dorr & Hawthorne with Yli-Vakkuri (2021: 50-1) point out that this way of characterising possible worlds as propositions does not by itself guarantee the truth of the 'Leibnizian' principles that for something to be (metaphysically) possible is for it to be true at *some* possible world, and for something to be (metaphysically) necessary is for it to be true at *every* possible world:

LEIBNIZIAN POSSIBILITY: 
$$\Diamond p \leftrightarrow \exists w (\text{World}(w) \land \text{Possible}(w) \land \text{at } w(p)).$$

LEIBNIZIAN NECESSITY: 
$$\Box p \leftrightarrow \forall w (\text{World}(w) \to \text{Possible}(w) \to \text{at } w(p)).$$

defined using *sui generis* quantification into predicate position, is developed by Dunaway (2013). The idea that 'the world' – what HO-realists call 'the actual world' – is the maximal, true (higher-order) proposition is defended by Trueman (2021, 2022).

In particular, given the characterisation of possible worlds as propositions above, the truth of the Leibnizian principles requires the truth of the principle that for any p, if p is possible then p is necessitated by some q such that q is atomic, that is, possible and necessitates every  $p^*$  or its negation:

ATOMICITY: 
$$\Diamond p \to \exists q (\Diamond q \land \Box (q \to p) \land \forall p^* (\Box (q \to p^*) \lor \Box (q \to \neg p^*))).$$

Atomicity is guaranteed given a higher-order modal logic that includes the principles '5'  $(\forall p(\Diamond p \to \Box \Diamond p))$  and 'BF'  $(\forall Y(\forall x\Box Yx \to \Box \forall xYx))$  along with the necessitation of the principle that (ancestrally) necessarily, there is a truth that (ancestrally) necessitates all truths  $(\Box \exists p(p \land \forall q(q \to \Box (p \to q))))$ . And 5 and BF are not uncontroversial.<sup>40</sup>

However, it would be wrong to think that this puts HO-realists at a relative disadvantage compared with other kinds of realists. First, although is often taken for granted that realists should accept the Leibnizian principles, it is not obvious on reflection that they should. The truth of the Leibnizian principles is certainly important in the context of forms of realism such as Lewis's (1986), which aim for a complete reduction of the modal to the non-modal via a non-modal analysis of 'possible world'. But realists who identify possible worlds with propositions typically reject such a reduction of the modal to the non-modal, and are content to define 'possible world' in modal terms. In the context of such forms of realism — which as noted in §3 are the dominant kind — it is far from obvious why every truth expressed using ordinary modal operators should be equivalent to some truth about possible worlds. For

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<sup>&</sup>lt;sup>39</sup> 'BF' is a higher-order version of the Barcan Formula, due to Barcan (1946). For a proof of atomicity from 5, BF, and the necessitation of actuality, see Dorr & Hawthorne with Yli-Vakkuri (2021: 51, fn. 70). 'It is ancestral necessary that p' implies p, and  $\Box p$ , and  $\Box p$ , and so on: see Dorr & Hawthorne with Yli-Vakkuri (2021: 35, fn.

<sup>&</sup>lt;sup>40</sup> For a well-known argument against 5, see Salmon (1989). For a history of BF and a thorough guide to its controversial implications, see Williamson (2013).

<sup>&</sup>lt;sup>41</sup> See Adams (1974), Plantinga (1974), Prior & Fine (1977), Fine (1977), and Chisholm (1981: 129).

instance, the theoretical utility of being able to 'take seriously' the widespread objectlanguage quantification over possible worlds in contemporary metaphysics does not depend on the truth of the Leibnizian principles.

Second, HO-realists who are nevertheless attracted to the Leibnizian principles are free to endorse atomicity, even if the truth of that principle is not guaranteed by their analysis of possible worlds as propositions and favoured higher-order modal logic (i.e. even if they reject 5 or BF). In doing so, they would be in much the same position as realists such as Lewis (1986), whose non-modal analysis of possible worlds similarly fails to guarantee the truth of the Leibnizian principles, and who must arguably treat the principles as a presupposition rather than as derived from more fundamental principles.<sup>42</sup>

HO-realists define 'possible world' in irreducibly higher-order terms. What exactly are possible worlds according to HO-realists? An acceptable but potentially misleading answer is that they are irreducibly higher-order entities: specifically, entities of the type that the irreducibly higher-order quantifier '\$\frac{1}{2}p'\$ quantifies over.\frac{43}{3}\$ A better answer, as long as it is understood in the right (i.e. higher-order) spirit, is that they are ways things could be. But they are not just any ways things could be: that I have coins in my pocket is a way things could be, but it is not a possible world.\frac{44}{3}\$ Rather, they are ways things maximally could be, and in the case of the actual world, the way things maximally are; or, to paraphrase Wittgenstein (1921: \§1), they are totalities of what could be the case, and in the case of the actual world, all that is the case.\frac{45}{3}\$

<sup>&</sup>lt;sup>42</sup> On this point, see Menzel (2023: §2.1.4).

<sup>&</sup>lt;sup>43</sup> Trueman (2021, 2022) calls them 'states of affairs'; Button & Trueman (forthcoming) call them 't-propositions'; and other higher-orderists simply call them 'propositions', adding the proviso that they are not things or objects.

<sup>&</sup>lt;sup>44</sup> I return to this point below.

<sup>&</sup>lt;sup>45</sup> That the HO-realist conception of the actual world aligns closely with Wittgenstein's (1921) conception of 'the world' as 'the totality of facts, not of things' is noted both by Trueman (2021: 197) and Dorr & Hawthorne with Yli-Vakkuri (2021: 48).

HO-realism brings with it a number of advantages as compared to the traditional version of realism according to which possible worlds-as-propositions are abstract objects. For instance, traditional realists face the objection that their view has the implausible consequence that the actual world is abstract, and therefore non-concrete. The standard traditional realist response to this objection is to distinguish two senses of the expression the actual world: one that applies to a certain abstract object – namely, the proposition p such that for any proposition q, either p necessitates q or p necessitates not-q and everything p necessitates is true – and one that applies to the sum total of concrete reality. However, the objection cannot even be raised against HO-realism. Even if HO-realists allow that there is a sense of the actual world on which it is true that the actual world is concrete, given HO-realism, there is no sense of the actual world on which it is true that the actual world is non-concrete. Indeed, some higher-orderists argue that predicates like 'concrete' and 'non-concrete' cannot be meaningfully applied to higher-order entities; in that case, given HO-realism, there is no meaningful question to be asked as to whether the actual world qua irreducibly higher-order entity is concrete or not. He

More importantly, however, HO-realism provides realists with a response to Lewis's objection described in the previous section. The challenge that Lewis's objection poses to HO-realists is to explain how possible worlds-as-propositions represent possibilities given higher-orderism. The best way for HO-realists to address this question is to begin with the more general question of how propositions represent anything at all given higher-orderism. For instance, how does p, where p = there are blue swans, represent that there are blue swans

<sup>&</sup>lt;sup>46</sup> See e.g. Lewis (1973: 86).

<sup>&</sup>lt;sup>47</sup> See Divers (2002: 228-9).

<sup>&</sup>lt;sup>48</sup> For example, here is Jones (2018: 822) in the context of his defence of the view that properties are irreducibly second-order entities: 'The view I've been describing... rejects as contentless both attributions of location to properties, and the negations of such attributions.' The reason is that in a higher-order setting, second-order terms 'play a fundamentally different semantic role from the singular terms that can meaningfully occupy the argument positions of "L" ["is located at"]'.

given higher-orderism? The question effectively contains the answer: given higher-orderism, p represents that there are blue swans because p = there are blue swans.

A natural 'Lewisian' objection to this answer is that it cannot be that p represents that there are blue swans by being identical to there being blue swans since *there are no blue swans*, and therefore there is nothing for p to be identical to. However, this objection represents a (perhaps excusable) slip into 'first-order' thinking, according to which for any p, p represents that q by being identical to some *thing*, such as a concrete state of affairs. But propositions do not cease to be themselves just because they are false: for all p, p = p whether p or not-p. Returning to our example above, p = there are blue swans – and therefore p represents that there are blue swans – even though p is false, that is, even though there are no blue swans. Moreover, even though p is false, p could be true – i.e. there could be blue swans. Suppose q = there could be blue swans. How, given higher-orderism, does q represent that there could be blue swans? The answer, again, is that q represents that there could be blue swans because q = there could be blue swans. In short, given higher-orderism, the relation between propositions and what they represent is identity: (higher-order) propositions q are what they represent.

We are now in a good position to answer the question of how possible worlds-aspropositions represent possibilities given HO-realism. For example, consider the HO-realist definition of an *actual world* above as a (higher-order) proposition p such that for any q, either p necessitates q or p necessitates not-q and everything p necessitates is true. Call the actual world so defined 'A'. Given the above definition, it follows that A necessitates exactly what is the case. Given that for any p and q, p is identical to q just in case p and q are necessarily equivalent, it follows that A is identical to exactly what is the case.<sup>50</sup> And since as

<sup>&</sup>lt;sup>49</sup> Trueman (2021: 184-5) makes a similar point.

<sup>&</sup>lt;sup>50</sup> The view that for any p and q, p is identical to q just in case p and q are necessarily equivalent is not uncontroversial (more on this below). However, it has been defended by Stalnaker (1984), Lewis (1996) (as a

we saw above that given higher-orderism, the relation between propositions and what they represent is identity, it follows that *A represents* exactly what is the case. Thus we have an answer to the question of how the actual world represents a possibility given HO-realism: it does so by *being* that possibility. (And similarly for possible worlds: given HO-realism, a possible world *w* represents a certain possibility by being that possibility.)

There is another sense in which possible worlds-as-propositions 'represent possibilities' given HO-realism. All realists hold that possible worlds 'represent possibilities' in the sense that things are the case *at* possible worlds. For example, all realists agree that at the actual world, there are white swans. On Lewis's view, for it to be the case that at the actual world (i.e. at this cosmos) there are white swans is for it to be the case that there are white swans located at the actual world. What about HO-realists? What is it for there to be white swans at *A* given HO-realism? (It cannot be that *A* is identical to there being white swans, since as we saw above, *A* is identical to exactly what is the case, and there being white swans is not exactly what is the case.)

According to HO-realists, for it to be the case that at A, there are white swans is for it to be the case that A necessitates that there are white swans; and more generally, for it to be the case that at a possible world w, p is for it to be the case that w necessitates that p. To see that A necessitates that there are white swans, consider again the HO-realist definition of A. Given that for any q, A either necessitates q or necessitates not-q, A either necessitates that there are white swans or necessitates that there are no white swans. And since everything A necessitates is true and there are white swans, it follows that A necessitates that there are white swans. That is how A represents the possibility (in the relevant sense) that there are white swans given HO-realism: by necessitating that there are white swans. (And similarly

consequence of his identification of propositions with sets of possible worlds), Trueman (2021), and Williamson (2022).

for possible worlds: as we saw above, given HO-realism, a *possible world* is a (higher-order) proposition p such that such that for any q, either p necessitates q or p necessitates not-q and possibly, everything p necessitates is true. Given that there could be blue swans, it follows that there is a possible world w such that w necessitates that – and therefore in the relevant sense, represents that – there are blue swans.)

We saw in the previous section that Lewis assumes realists will define representation by propositions in terms of something like necessary equivalence (here stated in higher-order terms):

PR: 
$$\forall p \forall q (\text{Represents}(p, q) := \Box(p \leftrightarrow q)).$$

But as we have just seen, HO-realists will naturally define representation by propositions in terms of identity:

HO-PR: 
$$\forall p \forall q (\text{Represents}(p, q) := (p = q)).$$

And PR and HO-PR jointly entail *intensionalism*, the view that for any p and q, p is identical q just in case p and q are necessarily equivalent:

INTENSIONALISM: 
$$\forall p \forall q ((p = q) \leftrightarrow \Box(p \leftrightarrow q)).$$

Intensionalism is a controversial thesis, since it implies that there is exactly one necessary proposition and exactly one impossible proposition.<sup>51</sup> However, although intensionalism is compatible with HO-realism, it is not a view that HO-realists are forced to accept: they are

<sup>&</sup>lt;sup>51</sup> See Williamson (2022) for a response to this objection.

free to reject PR in favour of whatever definition of representation by propositions is compatible with their view of the 'fineness of grain' of reality. All that is essential to HO-realism is HO-PR, and HO-PR is compatible with many different views about what it is for two propositions to be identical, including views which carve propositions more finely than intensionalism.<sup>52</sup>

We also saw in the previous section that Lewis argues that realists must take the representation relation between propositions and what they represent to be either 'internal' or 'external'. It is clear from what we have seen that for HO-realists, the relation is internal: it is the identity relation between propositions.

Interestingly, this means that the HO-realist's view of how possible worlds represent is, in one sense, more like Lewis's own 'modal realist' view than that of realists who identify possible worlds with objects like sets of sentences or abstract scale models. Consider a possible world w at which there are blue swans. On Lewis's view, w represents that there are blue swans by having certain things – blue swans – as literal constituents. As we saw above, given HO-realism, w represents that there are blue swans by necessitating that there are blue swans. In each case, representation is u where p = there are blue swans. (Given higher-orderism, the way in which p where p = there are blue swans represents that there are blue swans is analogous to the way in which you represent yourself: namely, by being yourself.) Contrast this with, for example, the view of the realist according to whom possible worlds are sets of sentences of an interpreted language. On that view, w represents that there are blue swans by having a particular sentence s of interpreted language L as a member. In that case, the

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<sup>&</sup>lt;sup>52</sup> See especially Dorr & Bacon (forthcoming: §§2.4-2.6). Note that Dorr & Bacon characterise intentionalism not in terms of *metaphysical* necessity as I have done here, but in terms of *the broadest* necessity (essentially, the most demanding form of necessity), which is given a purely (higher-order) logical definition; and as Bacon (2018) argues, one is not forced to identify metaphysical necessity with the broadest necessity. This makes an important difference to questions of propositional identity and distinctness.

representation by w that there are blue swans is mediated by the relevant interpretation of s: s represents that there are blue swans because relative to the interpretation of L, s means that there are blue swans. Of course, there is also an important difference between the Lewisian and HO-realist views of how possible worlds represent: according to Lewis's view, possible worlds are objects, namely cosmoi, whereas according to HO-realism, they are not. Nevertheless, on both views, representation is done by the relevant 'entities' – first- or higher-order – themselves.

Finally, we saw above that Lewis raises two objections to the realist view that the representation relation between propositions and what they represent is internal: first, that since the relation must be explained in terms of the fundamental intrinsic properties of the relata, it is inexplicable; and second, that since the relation holds between abstract and concrete objects, it is a mystery how we should come to know of it. HO-realists are in a good position to respond to both of these objections. As to the first, we have already seen that HO-realists have a straightforward explanation as to why e.g. p where p = there are blue swans represents that there are blue swans: because it is identical to there being blue swans. More generally, HO-realists can explain the 'internal' representation relation between propositions and what they represent in terms of the fundamental relation of propositional identity, which is no more mysterious than the identity relation between objects.

As to the second objection, given higher-orderism, the representation relation (i.e. propositional identity) does not hold between abstract and concrete objects, because it does not hold between objects. Nevertheless, one may still wonder how we come to know about it. The answer must be that we come to know about it in much the same way we come to know about the identity relation between objects (including abstract objects such as sets, if there are such things), and it is clear that Lewis is not concerned about how we come to have knowledge of *that* relation. But this raises a further question: given that according to higher-

orderism, propositions are not objects, how do we come to know about *them*? Defenders of higher-orderism can answer: by the usual routes to knowledge. For example, how do I know that p, where p = there are black swans? In my case, by testimony; but some may know it by direct perceptual experience, or by inference from true premises. For defenders of higher-orderism, there is no special mystery here.

#### 6. Conclusion

Lewis's (1986) challenge to realists about possible worlds is to explain how possible worlds-as-propositions represent possibilities. The challenge is significant for traditional realists, who conceive of propositions (and therefore possible worlds) as abstract objects: if possible worlds are propositions, and propositions are abstract objects, how do they represent possibilities? Some traditional realists might answer that there is no answer: that is just what propositions (and therefore possible worlds) *do*, it is the philosophical role they play. But that is not a very satisfying response. Others might answer that possible worlds are mereologically structured abstract objects, and it is the fact that they are mereologically structured which explains how they represent possibilities. But as noted above, this response brings its own difficulties, especially in the form of the Russell-Myhill paradox.

In this paper, I have described an alternative response on behalf of realists to Lewis's challenge which relies on an irreducibly higher-order conception of possible worlds-aspropositions. Should this response, if it is successful, be regarded as providing a *solution to* or a *dissolution of* Lewis's challenge?<sup>53</sup> That seems to depends on how Lewis's challenge is conceived. If the challenge is simply to identify the relation between possible world-as-

<sup>&</sup>lt;sup>53</sup> Following Button & Trueman (forthcoming), we can think of a 'dissolution' of a challenge as a response such that, given that response, the challenge cannot be meaningfully posed. For instance, Jones (2018) characterises the higher-orderist response to the challenge of explaining whether properties have locations or not as a dissolution, on the grounds that, given higher-orderism, the question of whether properties have locations cannot be meaningfully posed. See also Skiba (2021b).

propositions and what they represent, HO-realism should be regarded as providing a solution: the relevant relation is identity. On the other hand, if the challenge is to identify the relation between possible worlds-as-propositions-as-objects and what they represent, HO-realism may be regarded as providing a dissolution: as mentioned above, if there is no meaningful way to express the view that propositions (or any other higher-order entities) are or are not objects in a *sui generis* higher-order language, there is no meaningful way to pose the challenge of identifying the relation between possible worlds-as-propositions-as-objects and what they represent. Either way, higher-orderism seems to provides realists with an attractive response to Lewis's challenge, and for that reason, I think it deserves serious consideration by realists.

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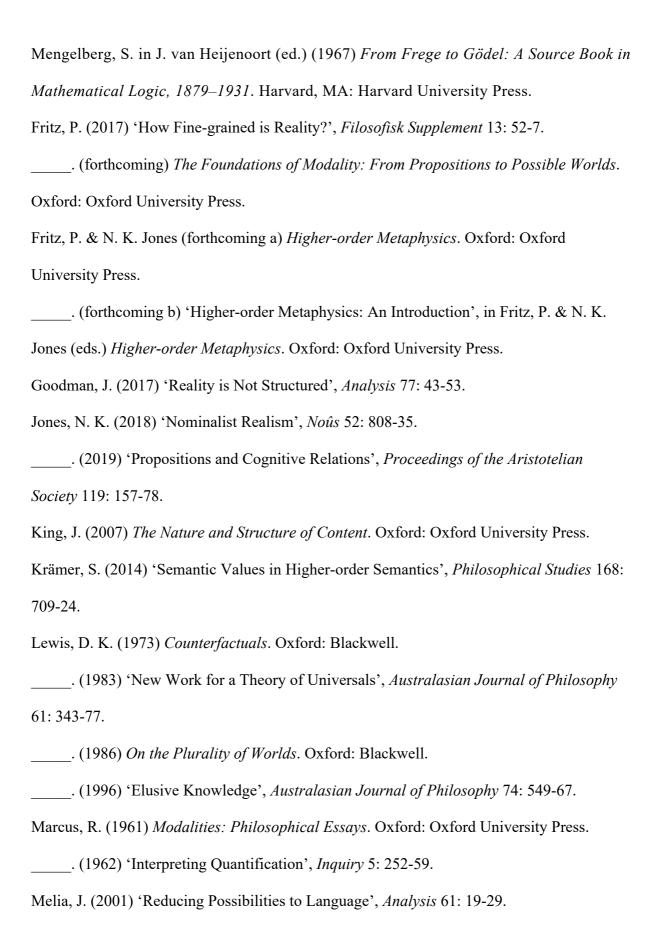
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