INTERSUBJECTIVE PROPOSITIONAL JUSTIFICATION

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0. Introduction

The distinction between propositional and doxastic justification is commonly accepted (if not uniformly understood) among epistemologists. It was introduced by Roderick Firth in 1978 in the context of a metaepistemological inquiry on whether epistemic concepts could be reduced to ethical concepts (which he concludes with a negative answer). Firth sets out to answer that question by focusing on one central epistemological concept, that of “epistemic justification” or, (in his terminology) equivalently, of “warrant.” He observes that although it is common to speak of justification (or warrant) *tout court*, this concept is ambiguous:

> Although I have referred in the singular to the epistemic concept expressed by the term 'warranted' there are in fact two such concepts. For there is an important respect in which a belief may be warranted although we are subject to epistemic criticism for having that belief. We may be criticized on the ground that our doxastic state is not psychologically based on or derived from the relevant evidence in a rational way. (Firth, 1978, p. 217)

These are cases in which a subject's evidence sufficiently supports a proposition, but the subject believes such a proposition for reasons other than the ones offered by her evidence. For instance, Sarah might have enough evidence for believing that Till is a thief, but she actually believes that Till is a thief because she is biased against the minority group to which Till belongs. She can therefore be critiqued for holding the belief that Till is a thief even if it is true and she has enough evidence for it.

According to Firth, and to what has become the most widely held view on the matter, *a subject has propositional justification to believe that p if and only if she has sufficient epistemic reasons to believe that p*. Propositional justification is thus detached from actual beliefs. In our example, Sarah has propositional justification to believe that Till is a thief even if she is subject to epistemic criticism for holding such a belief. Moreover, she would still have propositional justification to believe that Till is a thief even if she did not actually believe it.

What does attach to actual beliefs is another type of justification: *doxastic justification*. *A subject is doxastically justified in believing p if and only if (i) she has propositional justification for p, (ii) she believes p, and (iii) she bases her belief that p on her sufficient epistemic reasons to believe that p*. According to this widely held picture, propositional justification is fundamental and doxastic justification is a derivative notion. Olivera and Silva (2021) call it the *reasons-first picture*: "The reasons-first picture characterizes propositional justification in terms of epistemic reasons and doxastic justification in terms of propositional justification." This picture groups together a cluster of theories that cash out in different ways certain central notions, such as *sufficient epistemic reasons* and *basing relation*.[1]

[1] For a discussion of the epistemic basing relation see (Neta, 2019).

Doxastic justification is attached to existing psychological states whereas propositional justification is not. Nonetheless, propositional justification depends on the subject's evidence, and thus on "evidential psychological states" that is, those states that determine a subject's evidence:

> this assessment of propositional warrant is a judgment about the evidential relationship between certain psychological states and the proposition […]. With appropriate qualifications we might want to call this a 'logical' relationship. (Firth, 1978, pp. 218-219)

Among the qualifications, we should include the assumption that logical relationships are not only deductive relationships, but also inductive. Essentially, it must be in principle possible to rationally infer p from one's evidence. Crucially, however, it does not have to be *actually* possible for a particular subject to perform such an inference:

> an assessment of doxastic warrant requires psychological information that is irrelevant to assessments of propositional warrant. (*ibid.*, p. 219)

According to the reasons-first picture, whether a subject S has propositional justification for p depends exclusively on the evidential support relations between p and S's evidence. What's more, these evidential support relations are a priori knowable (Chisolm, 1977; Kornblith, 2017). Being not contaminated by our psychological idiosyncrasies, this conception of propositional justification is simple and clear. But is it perhaps too simple and clear to be of use in our epistemological theorizing? Hilary Kornblith (2017) thinks so. In (2017) and in a chapter of the present volume, Kornblith casts doubt on the adequacy of this conception of propositional justification to evaluate ordinary inferential beliefs. He argues that adopting this notion would lead to skepticism. He then proposes to break away from the reasons-first picture and to consider doxastic justification to be fundamental (more on this below).

In this essay, I focus on beliefs that derive from going through deductive arguments. In keeping with Kornblith, I suggest that even for these kinds of beliefs, the apsychological notion of propositional justification introduced by Firth can hardly be reconciled with the idea that justification is a central component of knowledge. In order to propose an alternative notion, I start with an analysis of doxastic justification. Like Kornblith, I thus argue for the fundamentality of doxastic justification. My strategy is, however, different insofar as it operates within the reasons-first picture. The fundamentality claim that I endorse is not a metaphysical claim, but rather a conceptual claim according to which it is only by starting with doxastic justification that we can formulate a notion of propositional justification that is adequate for epistemological theorizing.

One might think that a way to make this notion of propositional justification more useful in epistemology is to impose subjective constraints on it. As will become clear in the following, however, this poses the risk of ending up with a notion of justification that is too idiosyncratic and thus unable to perform its characteristic normative role. I will propose a notion of propositional justification, *intersubjective propositional justification*, that is neither entirely apsychological nor idiosyncratic. To do so, I will argue that in order to be able to attribute propositional justification to a subject, we will have to consider her social context as well as broad features of our human cognitive architecture.

The organization of the paper is as follows. In Section 1 I discuss the notion of propositional justification that arises naturally when taking propositional justification as fundamental, *objective propositional justification*. In Section 2 I describe Kornblith's take on ordinary inferential beliefs and his reason to think that doxastic justification is fundamental. In Section 3 I dive into the logical and mathematical case and show that the notion of objective propositional justification cannot play a substantial role in logical and mathematical knowledge. In Section 4 I argue that propositional justification should satisfy a specific doxastic constraint – namely, the *idealized capacity principle*. In Section 5 I introduce a new notion, that of *intersubjective propositional justification,* and suggest that this is the right one to use in an epistemological analysis of science and mathematics**.** I then explore how this notion can be honed for the case of mathematics.

1. Ordinary Inferential Beliefs

Alvin Goldman challenges the reasons-first picture and defines propositional justification in terms of doxastic justification (in his terminology, he defines *ex-ante justification* in terms of *ex-post justification*). In his seminal paper introducing reliabilism, he explains:

> the bulk of this essay was addressed to ex-post [doxastic] justifiedness. This is the appropriate analysandum if one is interested in the connection between justifiedness

> and knowledge, since what is crucial to whether a person knows a proposition is whether he has an actual belief in the proposition that is justified. (Goldman [1976], 2000, p. 345)

He then goes on to propose a way to define propositional justification in terms of doxastic justification:

> Person S is ex ante [propositionally] justified in believing p at t if and only if there is a reliable belief-forming operation available to S which is such that if S applied that operation to this total cognitive state at t, S would believe p at t-plus-delta (for a suitably small delta) and that belief would be ex post [doxastically] justified. (*ibid.*)

By appealing to the reliability of belief-forming operations, Goldman defines propositional justification in terms of doxastic justification. Therefore, he considers doxastic justification to be fundamental.

Kornblith (2017) takes Goldman's suggestion further. On a first pass, Kornblith's argumentative strategy is to show that if we conceive of propositional justification as fundamental, we end up with a theory of justification that is too demanding and would lead to skepticism (which is rejected from the get-go). His focus is on ordinary inferential beliefs. According to a large body of literature on reasoning heuristics in the tradition of Tversky and Kahneman,[2] such inferential beliefs are not formed, as one would think, by approximating the

[2] See (Kahneman, 2011).

laws of deductive or inductive logic. Instead, they are formed with the aid of various reasoning heuristics and rely on assumptions about our environment. This result can be interpreted (and has been interpreted) as a fault of our inferential practices. How can it be rational, for example, to use the law of small numbers rather than the law of large numbers?[3]

However, such heuristics are so widespread that, on pain of skepticism, they must produce justified beliefs, at least according to Kornblith. This thought is then at the basis of the project of articulating a model different from the one of inductive logic to evaluate our ordinary inferential beliefs. And this model can be developed only when focusing on what makes actual beliefs justified, that is, on doxastic justification. According to Kornblith, ordinary inferential beliefs should not be conceived and judged as based on arguments, but rather as originating from belief-producing mechanisms that can be, or fail to be, reliable.

Like Kornblith, I am not a skeptic. And, like Kornblith, I want to argue for the conceptual fundamentality of doxastic justification. I consider, however, highly reflective beliefs, beliefs formed by deductive reasoning rather than ordinary inferential beliefs. Thus, unlike Kornblith, I focus on justified beliefs that are formed on the basis of *good reasons*.[4]

The novelty of Kornblith's approach is to deny that justified ordinary inferential beliefs are based on good reasons. The novelty of my approach lies in proposing a novel conception of a *good reason*, one that is not entirely apsychological, but one that is itself tailored to our human cognitive capacities. To do so, it will be useful to first give a more detailed account of the apsychological notion of propositional justification as introduced by Firth.

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[3] See (Kahneman and Tversky, 1971) and (Kornblith, 2017, p. 73).

[4] In Kahneman's (2011) terminology, Kornblith centers his attention on the intuitive System 1 while I handle the case of slow System 2.

2. Objective Propositional Justification

According to Firth and other epistemologists that take it to be fundamental, propositional justification concerns the “objective degree of support that a subject’s evidence confers on a proposition” (Ichikawa and Jarvis, 2013, p. 163). Propositional justification, so conceived, is indexed to a subject’s evidence, but crucially it does not depend on the subject’s cognitive limitations and psychological make-up. Call this conception of propositional justification *OPJ*, *Objective Propositional Justification*.[5] Consider the following example.

> **POKER.** You, Ben, and I are playing poker. The fact that BF=<Ben is moving his feet>, which both you and I notice, provides strong evidential support for, and propositionally justifies believing that BB=<Ben is bluffing>. You believe BF and form an inferential belief in BB from it. I also believe that BF, but I fail to infer BB from it.

In this case, both you and I have (objective) propositional justification for BB. Our epistemic status, however, is different. This is possible because propositional justification does not entail belief. Consider another example:

[5] This terminology is introduced in (Melis, 2018).

CHESS. We are in Bryant Park in New York City, witnessing a game of speed chess. You realize that B3=<Black can mate in three moves>, I don't. We have the same evidence (i.e., the position of the chessboard at time *t*).

We both have OPJ for B3, but I fail to appreciate it. Thus, I do not form the relevant belief. Apart from the fact that, in CHESS, the evidence entails your conclusion, the two cases are analogous. In both of them, our epistemic status is different because, while we both have OPJ, you also have *doxastic justification*. As we have seen, according to the reasons-first picture, *doxastic justification* is obtained from propositional justification by forming the relevant belief and basing it on sufficient epistemic reasons.[6]

So construed, doxastic justification depends on propositional justification. The latter tracks objective support relations while the former is impinged upon by our psychological idiosyncrasies – for the cases at hand, your superior skill at poker and chess. It is for this reason that most epistemologists consider propositional justification to be fundamental. Nonetheless, doxastic justification cannot be disregarded since, without it, propositional justification could not perform its role in underwriting knowledge. Knowledge not only requires justification but *justified belief*. We can see the entailment relations between knowledge and these two types of justification schematically:

Knowledge → Doxastic Justification → Propositional Justification

[6] Note that most externalists allow for doxastically justified beliefs that are not based on reasons (e.g., proprioceptive beliefs, beliefs that were formed on evidence which is now forgotten, etc.). For a discussion of a range of such cases, see (Silva, forthcoming).

Let's take stock. According to the view articulated here, while doxastic justification is crucial to connect justification with knowledge, it is a derivative notion. Propositional justification is fundamental. This seems right since propositional justification, when contrasted with doxastic justification, is an objective notion, uncontaminated by the idiosyncrasies of our cognition, perspective, and abilities.

This view is endorsed by a number of epistemologists. In their classical defense of evidentialism, Feldman and Conee (1985) write:

> The doxastic attitude that a person is justified in having is the one that fits the person's evidence. More precisely: EJ [Epistemic Justification]: Doxastic attitude D toward proposition p is epistemically justified for S at t if and only if having D toward p fits the evidence S has at t. (Feldman and Conee, 1985, p. 15)

Propositional justification perfectly tracks evidential support relations between propositions. In their view, there are no constraints in terms of abilities to form beliefs. And in fact, they admit that "epistemic justification might have been normally unattainable" (*ibid*., p. 19). Smithies (2015) discusses the case of logical truths and endorses a similar position:

> If rationality requires one to be certain of all logical truths, then it follows that one has sufficient reason or justification to do so. (Smithies, 2015, p. 2776)

Again, the notion of epistemic justification is indexed to a subject's evidence but has nothing to do with cognitive limitation of the subject.

I now turn to considering inferential beliefs deriving from deductive reasoning. In some cases, these beliefs constitute knowledge. What will emerge is that OBJ is not the main ingredient of such knowledge. This in turn implies that this conception of propositional justification is not poised to play a role in an analysis of knowledge that arises from sophisticated deductions in domains such as logic and mathematics.

3. Mathematical Beliefs

If we apply it to the realm of deductive inquiries, OPJ offers the very best kind of epistemic likelihood: entailment. If propositional justification concerns the "objective degree of support that a subject's evidence confers on a proposition," then, for example, my proof that proposition p follows certain axioms, should license *prima facie* certainty of this conditional claim.

But what is the *epistemic* role of deductive reasoning? Consider CHESS. You correctly *see* that Black can mate in three moves (B3), and thus you form the relevant belief. Even if we share the same evidence (i.e., the configuration of the chessboard at a particular time), I don't form this belief. With enough patience, perhaps if you explain to me your reasoning step-by-step, I will reach the same conclusion and form the belief that B3 as well. The initial evidence is enough to justify your belief. The need for the explanation resulted only from my lack of competence. Similarly, in POKER, we both share the same evidence. When I challenge you or simply ask you for an explanation, you can then give me a story of how to put together the evidence *I already have*.

Logical and mathematical beliefs are similar. They are prototypical deductive beliefs. For the present purpose, it will be useful to consider the beliefs in theorems or other mathematical propositions as conditional on certain starting points (i.e., the axioms) in order to avoid controversial assumptions about our knowledge of such starting points. Your explanation in CHESS is then analogous to a mathematical proof. Suppose that, by way of explanation, you are extraordinarily talented and well-trained in mathematics. Further, suppose that you correctly *see* that a complicated number-theoretic conjecture, C, follows from the Peano Axioms (PA). You then form the doxastically justified belief that PA → C. Even if your reasoning is obscure to me, we both have OPJ for the conditional proposition PA → C, even in the absence of a proof. Actually, we also had it before you discovered that PA → C was the case.

In line with these considerations, Smithies (2015), claims that *all* subjects have OPJ for all logical truths:

> consider a case in which I believe some complicated logical theorem T on the basis of sheer guesswork. If T is true, then I have propositional justification to believe that T is true.[7] (2783)

[7] Smithies puts leverage on the asymmetry between the a priori and empirical domains to argue that logical omniscience is a requirement of rationality. He then argues that this fact implies that we have sufficient justification for all logical truths.

In this passage, to have reasons or (propositional) justification can be interpreted in two distinct ways: (i) A substantive claim about us: something along the lines of "we grasp the reasons or justification and its validity." (ii) A claim independent of us, simply stating that "there are reasons, or there is justification." Both alternatives are problematic for Smithies: (i) is not consistent with Smithies' view of propositional justification. Not all logical truths are sufficiently short and simple enough to have a justification we can grasp, yet for any logical truth T, "I have propositional justification to believe that T" is true; (ii) is not consistent with the ordinary meaning of "we have a reason or justification" which implies that it is something about us, since "we have it." In this case, it is even misleading to say, like Smithies, that *we have [objective] propositional justification*. We should instead say that *there is [objective] propositional justification.*

Basically, in the logical and mathematical domains (and the a priori domain more generally), what OPJ is picking out is not a notion of justification we can use to understand knowledge but the logical and mathematical facts themselves. These considerations suggest that, in some sense, notwithstanding the name, OPJ is not justification at all, *epistemic* justification, *bien entendu*. This is because it does not play a central role in knowledge.[8] Let me take this a bit more slowly. The conceptual role characteristic of justification is connected to knowledge. More precisely, propositional justification is the main component of knowledge. But OPJ is not the main component of logical and mathematical knowledge, at least if we

[8] One could argue, like Smithies, that all the difference is carried exclusively by doxastic justification. Actual beliefs are justified if and only if they co-vary reliably with truth. Pure reliability would then be the only criterion for doxastic justification. In particular, a reliable clairvoyant would be as justified as a reliable mathematician developing good reasons for her beliefs. This view, however, cannot lead to an adequate epistemology of mathematics. What we need for mathematical justification are good reasons, at least for first-hand justification. And good reasons, in mathematics, are good mathematical arguments.

consider, as I assume most do, knowledge as *human* (and not divine) knowledge. This implies that OPJ is not justification at all.[9] This does not mean that it is not useful in epistemology. Of course it is! As a matter of fact, in the case of mathematics OPJ perfectly tracks mathematical facts, and we need to refer to such facts to epistemically assess our beliefs. Besides being connected to knowledge, justification must be truth-conducive (albeit not truth-entailing). Therefore, it is only with respect to logical and mathematical facts that we can evaluate whether a putative notion of justification is at all appropriate.

One possible strategy to overcome this problem and articulate a different notion of propositional justification is to invert the order of explanation and start by considering doxastic justification first:

Knowledge → <u>Doxastic Justification</u> → Propositional Justification.

If the new notion of propositional justification that will be developed below will earn its keep, then it will support the fundamentality of doxastic justification. We already have indirect evidence. This is because it is by considering propositional justification to be fundamental that we arrived at a conception of propositional justification that won't do.

My contention is that the domain of OPJ is at the same time too large and too small. That is, that it presents two problems:

[9] Similarly, Kornblith (2017, p. 77) argues that it is exactly because propositional justification must play this central role in knowledge that there cannot be different types of it. That is, in the present terminology, why we cannot just be ecumenical and just add another notion of propositional justification along with OPJ.

PROBLEM ONE: According to the characterization of objective propositional justification above, we have objective propositional justification to believe mathematical propositions whose proofs are beyond our ability to understand.[10] But this can seem counterintuitive.

PROBLEM TWO: According to the characterization of objective propositional justification above, we cannot have objective propositional justification in virtue of a fallacious deductive argument. But this can seem counterintuitive.

I develop and address PROBLEM ONE and PROBLEM TWO in the following sections. I will present two cases to support the claim that these are indeed problems of OPJ.

4. Idealized Capacity Principle

In the previous section, we saw that PROBLEM ONE is indeed a problem. This is because if propositional justification is to be a central component of knowledge, it should not be reduced to logical and mathematical facts.

One way to address PROBLEM ONE, is to impose a doxastic constraint on propositional justification. Giacomo Melis (2018) contrasts OPJ with what he calls *ordinary propositional justification*, which restricts propositional justification to reasons that are within reach of the

[10] Moreover, it also implies that we have propositional justification to believe propositions that are too long and complex for us to understand.

subject.[11] A subject has "ordinary propositional justification only for the subset of propositions that her cognitive and doxastic abilities enable her to believe with justification" (371).[12] I re-name this notion *subjective propositional justification* (SPJ) because it is indexed to the doxastic abilities of the subject – still, SPJ is not subjective in the sense that what looks like a good justification for p to the subject necessarily subjectively propositionally justifies p. As a matter of fact, SPJ can be constructed from OPJ by imposing the following principle:

> **SUBJECTIVE CAPACITY PRINCIPLE**: X provides S with propositional justification for p only if S's cognitive abilities enable her to form a doxastically justified belief that p on the basis of X.[13]

The SUBJECTIVE CAPACITY PRINCIPLE has been motivated in various ways in the literature. One cluster of arguments relies on some version of the *ought-imply-can* principle coupled with a deontological conception of justification. If S has propositional justification X for p, S ought to believe p on the basis of X. But, by the ought-implies-can principle, this cannot be correct since S cannot believe p on the basis of X. Of course, this principle is rejected by friends of OPJ.[14] Moreover, the SUBJECTIVE CAPACITY PRINCIPLE seems to deliver counterintuitive verdicts. Here is a case in point.

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[11] Melis (2018) argues that we should just keep both notions of propositional justification. But as argued above, this is not a viable position if we take justification to be the main component of knowledge. See Footnote 9.

[12] To be sure, it is not clear how these abilities should be identified and how modally stable they should be – but let me gloss over this issue.

[13] Adapted from (Smithies, 2010, p. 14).

[14] See (Feldman and Conee, 1985). Smithies (2010) points at the problem of over-intellectualization – which, however, does not apply in this context because we are focusing on reflective beliefs.

JEALOUSY: Cate, a renowned mathematician, proves conjecture C. She sends her proof to her colleagues. All except one, Jason, grasp the proof and applaud Cate. In normal circumstances, Jason would recognize Cate's argument as a proof as well, but he secretly has the aspiration to prove the conjecture himself and is jealous of Cate. This jealousy activates a psychological defense mechanism that prevents him from grasping Cate's proof.

In this case, Jason cannot form a doxastically justified belief on the basis of Cate's proof.[15] But something seems to have gone wrong here. For there is a perfectly natural sense in which Jason *has* evidence that gives him justification to believe p, even though he lacks the ability to properly respond to that evidence and thereby form a justified belief. He has enough evidence but is unable to put it together correctly. This intuition is elicited by the fact that abstracting from a very idiosyncratic feature of his psychology (i.e., his jealousy), Jason would form a doxastically justified belief in the conjecture. This case demonstrates that SPJ is too idiosyncratic to play the role propositional justification should play in our epistemological theorizing. Thus, while OPJ can be rejected on the ground that it is entirely apsychological, SPJ should be rejected on the ground that it is overly psychological.

Kornblith (2017) thinks otherwise. Although he does not explicitly endorse SPJ, he envisages an alternative notion of OPJ that is sensitive to subjective differences:

[15] We can construe Jason's defense mechanism to be a more or less robust feature of his psychology.

> When the order of explanation is reversed, however, and propositional justification is defined in terms of doxastic justification, psychological matters become directly relevant to matters of propositional justification. If I have some psychological peculiarities which you lack, then even if you and I share all of the same beliefs, and all of the other nonbelief potential justifiers as well, assuming there are such justifiers, a different set of propositions may be propositionally justified for the two of us. (Kornblith, 2017, p. 68)

This position, however, seems to give the wrong verdict in cases such as JEALOUSY.

The challenge is then to find a middle level of idealization, that is, a notion of propositional justification that can be placed between OPJ and SPJ. According to Smithies (2015), however, it is fine to abstract away from human cognitive abilities completely:

> But if there are requirements of rationality that apply to individual human beings who are incapable of meeting them, then why can't there be requirements of rationality that are beyond human capacities in general? (Smithies, 2015, p. 2779)

Smithies' move is to go directly from a denial of the Subjective Capacity Principle to the endorsement of no capacity principle at all, and thus to OPJ. But this is a mistake. As we saw in the previous section, OPJ is not the main component of mathematical knowledge. In other words, claiming that logical omniscience is a requirement of rationality does not help understand the epistemology of human logic and mathematics.

Let me now tweak JEALOUSY into a case that shows that, *pace* Smithies, it is reasonable to think that the requirements of rationality are bounded by our human capacities.

> **ALIEN-CATE**: Jason is now not affected by jealousy, and Cate is an alien disguised as a human. Her cognitive abilities far outstrip ours. Alien-Cate proves the conjecture with a deduction that no human being would be able to grasp, and she tries to share it with Jason, who inevitably fails to form an appropriately doxastically justified belief.

In this case, it is plausible to think that Jason does *not* have propositional justification. This is because Alien-Cate's deduction is beyond our capacity. For example, it might have as many inferential steps as atoms in the universe. If we endorse OBJ as the right notion of propositional justification, then, like any other subject, he would have (objective) propositional justification since the conjecture is indeed true. But I gave reasons to think that OBJ is not the right notion to work with if we want to articulate an epistemological account of mathematics. So, while in JEALOUSY Jason has propositional justification, it is plausible that in ALIEN-CATE he does not. The following intermediate principle can help to make sense of this case:

> **IDEALIZED CAPACITY PRINCIPLE**: X provides S with propositional justification for p only if an idealized human agent with the appropriate training would likely be in a position to form a doxastically justified belief that p on the basis of X.

What is the type of idealization at play? The idealized human agent is not a logically omniscient subject. In the case such as the ones at issue here, the idealization will have to be indexed to a specific domain D. If D is mathematics, an idealized human agent is a person of ordinary cognitive abilities belonging to a legitimate mathematical community operating around the time of S's life.[16] But what is a *legitimate mathematical community*? It is one that constitutes what Sanford Goldberg (2017) calls a *legitimate practice*:

> it is an ongoing and recognized practice, its standards are widely acknowledged, there have been no serious questions as to the propriety of either the practice or its standards. (Goldberg, 2017, p. 2867)

In normal circumstances, S's community will be a legitimate mathematical community, but this is not necessarily the case because S could be part of a deficient epistemic community, similar to the flat-earthers of mathematics.[17] We should then conceive of our idealized mathematician only considering S's background knowledge and appropriate training.[18] Note that the IDEALIZED CAPACITY PRINCIPLE is indexed to a particular time. If in the future our cognitive abilities will be drastically enhanced, we will be able to form doxastically justified beliefs on the basis of a larger body of arguments, and the IDEALIZED CAPACITY PRINCIPLE is sensitive to this. Moreover, the IDEALIZED CAPACITY PRINCIPLE is modulated according to

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[16] In special cases, it could be that no such community is available. In such cases, the idealization would have to be characterized in terms of the capacities an average agent would have in S's broad circumstances.

[17] Thanks to Neil Barton for discussion on this issue.

[18] This accounts for cases in which S is ahead of her time and no member of her actual mathematical community can grasp her arguments due to lack of background knowledge.

general features of our human cognitive architecture. In this more moderate sense, I agree with Kornblith that the notion of propositional justification should not be entirely apsychological:

> what is propositionally justified for typical humans, given a certain body of potential justifiers, may be quite different from what is propositionally justified for typical members of a different species with the very same body of such justifiers. (Kornblith, 2017, p. 68)

To be sure, the IDEALIZED CAPACITY PRINCIPLE gives some leeway on how to interpret it, and it does not draw sharp lines. This is not a problem. What matters is that it is a tool we can use to eliminate requirements that are clearly beyond us and that are posed when adopting OPJ, like the one of logical omniscience. Further work is needed to spell out how the IDEALIZED CAPACITY PRINCIPLE operates in particular contexts.[19] In the context of this paper, the important point is that this principle inherits some of the advantages of the subjective capacity principles, but skirts some of its problems. In particular, it makes sense of the fact that Jason, and any other human, does not have propositional justification for all logical truths (in CATE-ALIEN), but it is not overly psychological and remains an idealized principle that can be used for normative theorizing in epistemology (in JEALOUSY). This provides us with an answer to PROBLEM ONE, according to which endorsing OPJ as the right notion of propositional

[19] For an articulation of acceptability criteria in different mathematical communities see (De Toffoli, 2020b).

justification implies that we have propositional justification to believe propositions on the basis of proofs that are beyond our ability to understand.[20]

Let us now turn to PROBLEM TWO, according to which endorsing OPJ as the right notion of propositional justification implies that we cannot have justification in virtue of a fallacious deductive argument.

5. Good Arguments That Are Not *Ideally* Good

In light of the previous discussion, we have as a working hypothesis that in order for a reason to provide propositional justification, it has to satisfy the IDEALIZED CAPACITY PRINCIPLE. In the case of mathematics, these reasons are good mathematical arguments, that is, proofs. The IDEALIZED CAPACITY PRINCIPLE roughly says that proofs must be within our ken if they are to provide us with propositional justification.

But there is another issue. Proofs entail their conclusions. This leads to an infallibilist notion of justification in the realm of logic and mathematics, which, I suggest, is at odds with compelling cases. It is widely accepted among epistemologists that justification (both propositional and doxastic) allows for cases in which well-functioning agents do everything right but fail to achieve knowledge. For instance, this happens when a justified belief is false or when some sort of epistemic luck is involved. In POKER, it may very well be that while you were justified in believing that Ben was bluffing, it turned out that he was not bluffing. In

[20] Once again this also implies that we have propositional justification to believe propositions that are too long and complex to understand.

CHESS, the situation is not so straightforward. In certain circumstances, it is plausible to think that you could count as justified in believing that Black could mate in three moves, even having made a mistake in reasoning. Suppose that due to the time constraint, the reliable process that forms your chess-related beliefs has missed a subtle defense available to White. Given the circumstances, you reasoned pretty well (most likely Black *is* going to mate in three moves since the defense was hard to spot) but not impeccably.

Also in logic and mathematics, it is plausible to think that a subject can be mathematically justified in believing a false proposition or in believing in a true proposition in virtue of a fallacious argument. This is, however, controversial. Justification in mathematics has been associated, since Frege, with genuine proof. Note that here I am considering mathematical justification to be (i) first-hand and inferential (and thus not testimonial), and (ii) *direct*, that is deriving from a mathematical argument.[21] Frege explains:

> It is not uncommonly that we first discover the content of a proposition, and only later give the rigorous proof of it, on other and more difficult lines; and often this same proof also reveals more precisely the conditions restricting the validity of the original proposition. In general, therefore, the question of how we arrive at the content of a judgement should be kept distinct from the other question, Whence do we derive the justification for its assertion? (Frege, 1960, p. 3)

[21] An *indirect* type of inferential justification for a mathematical claim could be something of the form. I have justification that my deduction p → q is correct because you told me so (r), I believe in p. Now I can infer q from r and p. Thanks to Paul Silva for suggesting me this clarification.

Frege considers arithmetical truths to be a priori in the sense that their justification is a priori. Moreover, he adds that "An a priori error is thus as complete a nonsense as, say, a blue concept" (3). This makes it clear that fallacious arguments cannot, according to Frege, justify mathematical claims. Deploying our terminology, it is safe to say that Frege's focus was propositional justification alone. He aimed at shunning any question related to psychology. Frege's position is then endorsed by Hempel (1945) and the logical positivists. Moreover, it remains common among philosophers of mathematics to this day. The distinction between the context of discovery and the context of justification to which Frege alluded is widely considered to be a sharp distinction and it is taken for granted that proofs alone are to provide justification.[22] Moreover, they provide indefeasible a priori justification. In this respect, it is worth noticing that Kitcher (1984) pushes an empiricist account of mathematics exactly because he works with a conception of the a priori according to which a priori justification is infallible and he realizes that such a conception cannot make sense of mathematical practice.[23]

Cases from mathematical practice seem to call for a fallible notion of mathematical justification, at least if we consider doxastic justification. Here is an example. It has to do with the famous 4-color conjecture. Roughly, the 4-color conjecture states that four colors suffice to color a planar map (such as a map of the world) in such a way as no two neighboring regions have the same color. It is simple to gather inductive evidence for this conjecture; it suffices to take different maps and start coloring them. The conjecture became of interest for the mathematical community in the middle of the 19th century, especially when it reached the

[22] See, for example, (Burgess, 2015, p. 8).

[23] However, nowadays many epistemologists accept that a priori justification is not infallible. See, for example, (BonJour, 1998; Casullo, 2003).

attention of University College London professor Augustus De Morgan. The first accepted putative proof was found by Kempe.

> **KEMPE.** Alfred B. Kempe published an argument for the 4-color conjecture in 1879. This was a careful argument that divided the problem into several subcases. In 1890, Percy Heawood found a counterexample to one of the subcases.

It took roughly a century to find a genuine proof – and it still used Kempe's ideas.[24] When Kempe published his argument, it was reasonable for him to believe (and take himself to know) the 4-color conjecture because of it.[25] After all, not only did he check it, but other members of the mathematical community, starting with his reviewers, checked it as well – with none dissenting until eleven years later. Kempe could have done more. We can always do more. In practice, however, it is crucial to divide our time between innovation and verification.[26]

Epistemologists favoring OBJ will claim that Kempe had OBJ – just because the 4-color conjecture is true. But the clear problem is that, in this case, OBJ is not a path to knowledge because it is disconnected from Kempe's actual argument and thus from doxastic justification (allowing that he was indeed justified). What matters for doxastic justification, is instead that Kempe had a *good argument even if it is not an ideally good argument*. That is, an argument

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[24] Said proof is generally considered to be the first computer-assisted proof. It was published in 1976 by Wolfgang Haken and Kenneth Appel. See (Sipka 2002).

[25] Note that when I claim that Kempe did not have a proof but was nevertheless justified, I am treating justification as a full-on notion. To be sure, this notion is naturally graded.

[26] It has been recently pointed out that incessant checking is epistemically problematic even if each check brings epistemic benefits (Friedman, 2019).

that looked good to him and to the legitimate mathematical community to which he belonged, even if it failed to be a genuine proof.

One epistemological position that endorses this intuition is *phenomenal conservatism*. According to Huemer, this philosophical position can be defined as follows:

> If it seems to S that p, then, in the absence of defeaters, S thereby has at least some degree of justification for believing that p. (Huemer 2007, 30)

A key factor is that Kempe's argument not only looked good *to him*, but to the mathematical community at large. That is, not only did he have no defeaters, but he also had testimonial evidence that his argument was correct.

We can better spell out Kempe's epistemic situation by distinguishing between different types of undermining defeaters.[27] Kempe's argument admits an undermining defeater that cannot be defeated in turn. After all, his argument was fallacious. Neither Kempe nor his contemporary mathematicians were, however, aware of it. Stephen Cohen (1987) appeals to undermining defeaters to argue that standards that determine whether we should attribute knowledge to a subject are social in nature. He observes that defeaters of an argument can be obvious or very subtle and hard to spot. For instance, some arguments contain blatant mistakes. Even if they are not spotted by the subjects themselves, average members of the relevant community will have no problem pointing them out. In Cohen's words, those

[27] It is customary to distinguish between two different types of defeaters for your belief that p: *undermining* and *rebutting* defeaters. The former are evidence that your reason for believing p is not a good reason, the latter are evidence for not-p.

defeaters are *subjectively opaque* but *intersubjectively evident*. The defeaters of Kempe's original argument are instead both subjectively *and* intersubjectively opaque because they are very hard to spot even for the idealized mathematician (of course they are easy to spot now, with hindsight). In short, Cohen's thought is that arguments are candidates for supporting knowledge, and thus are *good arguments* on which doxastically justified beliefs are based, only if they do not admit intersubjectively evident defeaters for which the subject does not have a counter-defeater. More precisely:

> If knowledge does not entail ideally good reasons, then the intersubjective standards determine the level of opacity up to which a possessed undermining defeater (that is itself undefeated) will undermine knowledge. (Cohen, 1987, p. 11)

I have been concerned with justification rather than knowledge. In logic and mathematics, I conjecture that knowledge does indeed require ideally good arguments, that is, truth-entailing arguments.[28] But I have given reasons to think that justification does not. Cohen's considerations, when applied to justification, can help us in making sense of Kempe's epistemic situation: there is an indefeasible defeater of his argument, but this defeater is both subjectively and inter-subjectively opaque, and thus it does not undermine justification. As

[28] I agree with Cohen that even in this case there would be a social component at play: "If knowledge does entail ideally good reasons, then intersubjective standards will determine the level of opacity up to which a possessed undermining defeater will undermine knowledge without the possession of a subjectively evident restoring defeater" (Cohen, 1987, p. 2).

discussed in the previous section, the social context should be generally identified with a legitimate mathematical community.[29]

With this terminology in place, we can characterize a new notion of propositional justification.

INTERSUBJECTIVE PROPOSITIONAL JUSTIFICATION (IPJ). X provides S with propositional justification for p if and only if:

1) X belongs to S's evidence,
2) an idealized human agent with the appropriate training would be in a position to form the belief that p on the basis of X, and
3) X does not admit intersubjectively evident defeaters that are subjectively undefeated.

Some clarifications are in order. We do not need to work with a particular conception of evidence.[30] In the case of logic and mathematics, X will be a logical or mathematical argument. The first condition is supposed to accommodate cases like JEALOSY. That is, cases in which a subject does not actually form the belief – but could have (because of condition 2). Condition 2 is the IDEALIZED CAPACITY PRINCIPLE we discussed before. Condition 3 is a social condition that restricts what can confer propositional justification to reasons that do not admit intersubjectively evident defeaters that are subjectively undefeated.

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[29] Cohen (1987) identifies with the community of the person who attributes knowledge.
[30] See (Kelly, 2016) for different options.

I grant that friends of OPJ will not be convinced that IPJ is the right notion to use. I do not claim to be able to convince them. My aim here is merely to suggest that IPJ gives rise to a more capacious notion of propositional justification, one that could be attractive to someone who is already skeptical of OPJ. For instance, Kornblith (2017) rejects OPJ and looks for a notion of propositional justification that is not entirely apsychological – he aims to find a notion that is sensitive to broad features of our cognitive architecture. I think IPJ is a contender at least for cases of believing for a reason.

In mathematics, IPJ is conferred by what I called elsewhere (2020a) *simil-proofs.* These are arguments that seems to be proofs but may or may not be proofs. With the terminology introduced here, we can define them as follows:

SIMIL-PROOFS (SP). Mathematical argument X for p provides S with propositional justification, that is, is a *simil-proof* for S if only if:

1) X is part of S's evidence,
2) An idealized human agent with the appropriate training would be in a position to form the belief that p on the basis of X, and
3) X does not admit intersubjectively evident defeaters that are subjectively undefeated.[31]

[31] One difference with my previous definition is the following. In (2020a), I defined SPs independently of a particular subject. In particular, an SP could be such only for a group of individuals. Here I am focusing exclusively on individual justification, and I have glossed over the existence of good arguments that only a group of individuals could grasp. A well-known mathematical case of this is the proof of the classification of finite simple groups. Another difference is that in (2020a), I focused on doxastic justification alone. And I took the expression "having an SP X for p" to be synonymous of "basing one's belief that p on the SP X."

Simil-proofs are arguments that look like proofs to the relevant audience but might fail to be. Kempe's proof was an SP in his time but ceased to be an SP when the gap was spotted. Being an SP is, therefore, a time-sensitive property. No doubt, there is much to be said on how to characterize the relevantly trained subjects, and more generally, the relationship between a mathematical argument and its audience. But this is a matter for a different paper.

From the practitioners' perspective, SPs are likely to be proofs. As a matter of fact, proofs do not wear their correctness on their sleeves. Whether an SP is a genuine proof is a matter of logic and not of our understanding. If an SP is fallacious, the self-checking activity of the mathematical community is likely to discover the fallacy, so belief would then be suspended or turned into disbelief. In (2020a), I have argued that SPs are the type of justification that is generally required in practice. According to my account, Kempe has a true justified belief that does not amount to knowledge. Although mathematical knowledge necessitates proof, well-functioning mathematicians can be justified and still mistaken about whether a given argument really is a proof. The criterion for justification is given by SPs.

6. Conclusion

To argue for the fundamentality of doxastic justification, Kornblith (2021, this volume) argues against the reasons-first picture.[32] I also argue for the fundamentality of doxastic justification. To do so, I endorse the reasons-first picture, at least in a restricted form, for highly reflective

[32] Kornblith calls a variant of the reasons-first picture the *Arguments on Paper Thesis*. According to this thesis, a subject S is propositionally justified in believing proposition p if and only if either p requires no argument, or there exists a good argument for p from hypotheses that S already believes.

beliefs which are formed by performing conscious inferences. I, however, proposed a specific interpretation of it. I suggested that in logic and mathematics, being ideally good (that is, being a correct deductive argument) is neither a necessary nor a sufficient condition for being a good argument – that is, correct deductive arguments may fail to confer propositional justification and in turn propositional justification may be in place without a correct deductive argument.

In logic and mathematics, good arguments are SPs. I arrived at the notion of SP by trying to characterize the arguments on the basis of which mathematicians form doxastically justified beliefs. This perspective led me to impose an idealized capacity principle on propositional justification. Simil-proofs are the sort of things that can be scrutinized by appropriately trained subjects in normal circumstances. Because of this, they guarantee the possibility of self-checking activity both by the subject as well as by the community at large. To be sure, restricting the domain of proofs to *humanly graspable* proofs would be overly restrictive from the point of view of logic and, in particular, of proof theory. But it is not overly restrictive from the point of view of epistemology. Likewise, considering fallacious arguments might not be fruitful in other contexts, but it is in epistemology.

I discussed the case of logic and mathematics in detail. My suggestions, however, generalize to reflective activities in general (and not only to deductive reasoning) – activities that are carried out by what Daniel Kahneman (2011) calls *System 2*. In particular, to beliefs that require good arguments. No doubt there are disputes on how to delimit these, but uncontroversially, they include the domain of empirical sciences (as well as philosophy). Mathematics is a good testing ground because it is the hardest and cleanest case. In mathematics, ideally good arguments can be characterized in terms of logic. The contrast between ideally good arguments and intersubjectively good arguments is therefore particularly clear. While in empirical sciences beliefs can be false even if based on an ideally good argument, this is not the case in mathematics (at least if we consider mathematical propositions to be conditional on their axioms).

Doxastic justification, then, is conceptually fundamental because it is only by considering it first that we can articulate an adequate notion of propositional justification, one that can, in good cases, underwrite knowledge.[33] It is precisely starting from doxastic justification that I developed an alternative to both objective and subjective propositional justification: intersubjective propositional justification.

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[33] This is compatible with a conception of propositional justification as *theoretically* fundamental. "[A] notion F has theoretical priority over a notion G when the role played by G in philosophical theorizing is, in some important sense, subordinate to the role played by F" (Melis, 2018, p. 368).

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