

This Paper Might Change Your Mind*

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Abstract Linguistic intervention in rational decision-making is captured in terms of information change. Cases that *look like* changes in value functions are actually changes in information. This gives us no way to model interventions involving expressions that only have an attentional effect on conversational contexts. How do expressions with non-informational content – like epistemic modals – intervene in rational decision making? We show how to model rational decision change without information change: replace a standard conception of value (on which the value of a set of worlds reduces to values of individual worlds in the set) with one on which the value of a set of worlds is determined by a selection function that picks out a *generic* member world. We discuss some upshots of this view for theorizing in philosophy and formal semantics.

Introduction

Standard decision theory conceives of agents as making decisions on the basis of two psychological inputs: an information state, which is represented by a credence function that assigns probabilities to possibilities, and a motivation state, which is represented by a value function that assigns values to possible outcomes. Expected value calculation then provides static norms for decision making. Standard decision theory also provides a dynamics of rational decision making – learning can change what we ought, or are inclined, to do. But the standard dynamics of decision theory updates only one of the two psychological inputs. We have a dynamics of credence change: we update our beliefs through *conditionalization* on evidence. But there is no standard machinery for shifting our values.

On the face of it, this credal chauvanism leaves us without the tools to explain all dynamics in decision making. Learning things can not only change your credences for propositions but can also change your values for states of the

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world. Learning that the drinking water is tainted will not only change what you believe about the drinking water, it may also affect whether you want any water. If learning can change values as well as credences, it can affect rational decision making twice over, and we need a story about the value shifts as well as the credence shifts.¹

This lack of a dynamics for value change is connected to two explanatory lacunas in standard decision theory.

For one thing, there is not much place for the role that shifts in *attention* play in rational decision change.² When you give a particular problem your attention, this does not change your information state – it does not affect your credence function. Nevertheless, such attention shifts can have effects on the way we make decisions.

A natural first thought is that changing what features of a situation we are attending to has some effect on the way that we value particular outcomes. If I think of the choice that I'm facing as a decision between eating a sweet snack and a bland one, I might have different preferences than if I view the choice that I'm facing as a decision between eating an unhealthy snack and a healthy one.

This relates to another issue, which has to do with the way that language interfaces with practical rationality. Though we have a compelling story about how truth-conditional assertions affect the inputs to decision making, no such story is available for utterances that are conversationally active, but informationally inert. This includes epistemic modals, questions, imperatives, and expressives.³

1 Some may have the following thought: it's a mistake to think that the standard tools of decision theory allow credence shifting but don't allow value shifting. Rather (goes this line of thought) the standard tools of decision theory allow *information state updating*, as new evidence removes worlds from consideration. That information state updating then changes credences in propositions and values in outcomes, as some worlds are removed from both propositions and outcomes. We'll return to this picture in section 1.3 below, but for now we'll just note that our central interest will be on forms of learning that shift values without eliminating worlds from the information state.

2 Though see [van Rooy 2003](#), [Yalcin 2018](#) for some work connecting decision-making to questions which is in the spirit of this paper.

3 A terminological stipulation: we use 'learning' to label processes of cognitive uptake of the contents of declarative sentences, and 'information' for truth-conditional information, modelled as sets of possible worlds. Learning thus comes apart from acquiring information in the cases of non-truth-conditional declarative sentences. Upon uptake of an utterance of *Might p*, one learns something, but there is no information that one gains through that learning.

This paper seeks to close this explanatory gap, using epistemic ‘might’ claims as a test case. Epistemic ‘might’ claims bring possibilities to attention (Ciardelli et al. 2009). Working within an inquisitive framework, utterances of the form ‘Might p ’ create a partition among possibilities. This is the best way to represent attention shifts in decision-making (Yalcin 2018).

An agent’s credence in each partition element is determined in the standard way: by a probabilistic weighing of the worlds in that element. What is novel in our proposal is how we determine value assignments for each partition element: the value for each partition element is determined by a selection function that picks out the most value-unremarkable – or *generic* – world in that element. On the resulting picture, dynamic repartitioning of an information state, induced by non-truth-conditional, and hence non-informational, learning, can change values (while leaving credences unaltered), and thus change rational decisions.

In section 1, we introduce a sample decision problem in which learning something epistemically modalized (that the peach might be rotten) can change rational behavior. We set out the standard machinery for the dynamic effects of learning on decision-making, and show that this standard machinery won’t account for our sample problem. In section 2, we turn to the semantics of epistemically modalized sentences, setting out a dynamic semantic picture using the resources of inquisitive semantics that captures the attention-focusing effects of these sentences. Section 3 then develops our core proposal that values of partition elements are determined by values of representative worlds in those partition elements, and shows that the resulting dynamics can account for shifts in value and decision upon learning that (e.g.) the peach might be rotten. The proposal in section 3 rests on a schematic notion of a selected world; in section 4 we consider possible constraints on the selection function and illuminating connections between the notion of a selected world and the notion of a generic object. The paper then closes in section 5 with some considerations of upshots of the proposed dynamics for various philosophical issues.

1 Linguistic Interventions in Rational Decision Making

A coin is flipped and a fair die is rolled. James is then offered a choice between two actions:

- i. **Action A1:** James receives a peach if the coin lands heads, and a plum if the coin lands tails.
- ii. **Action A2:** James receives a pomegranate if the die lands 1-5 and a persimmon if the die lands 6.

He is initially inclined to prefer A1 to A2. However, Spiker then tells James that the peach might be rotten. In light of this, James comes to prefer A2 to A1. How are we to make sense of James' change in preferred action?

1.1 Low Hanging Fruit

Let's start with the off-the-shelf technology for rational action selection. James is an expected value maximizer. Thus his initial preference for A1 over A2 is driven by a higher expected value, in light of his credences and utilities, for A1 than for A2. Keeping things simple (we'll grapple with the subtleties later), for any action A we have:

Definition 1.1. Expected Value: $EV(A) = \sum_{w \in W} c(w) \cdot u(w, A)$

where W is the set of worlds, c is James' credence function, and u is James' utility function (assigning values to undertaking specific actions in specific worlds).⁴

For our purposes, we can work with a space of 12 worlds:

w_1 : coin lands H, die lands 1	w_2 : coin lands H, die lands 2	w_3 : coin lands H, die lands 3
w_4 : coin lands H, die lands 4	w_5 : coin lands H, die lands 5	w_6 : coin lands H, die lands 6
w_7 : coin lands T, die lands 1	w_8 : coin lands T, die lands 2	w_9 : coin lands T, die lands 3
w_{10} : coin lands T, die lands 4	w_{11} : coin lands T, die lands 5	w_{12} : coin lands T, die lands 6

We'll then make two assumptions about James' psychology:

- i. James' credence function c assigns equal likelihood for each of w_1 through w_{12} , so $c(w_i) = \frac{1}{12}$ for $1 \leq i \leq 12$.
- ii. James' utility function u derives from underlying preferences regarding fruit (equivalently, we can say that the only features James values

⁴ Here we treat credences as being assigned to worlds. If we want credences for propositions, we model propositions as sets of worlds and then sum the credences for the worlds in those sets. (Exploiting the simplifying assumption that there are only finitely many worlds under consideration. We'll return later to the question of what the objects of credence should be.

in a world are the fructuous features). In particular, James' preferences fix the following fruit values:

$f(\text{peach}) = 4$	$f(\text{plum}) = 1$
$f(\text{pomegranate}) = 2$	$f(\text{persimmon}) = 3$

Using these fruit values, we can specify James' utility function u :

$u(w_1, A1)$ $= f(\text{peach})$ $= 4$	$u(w_2, A1)$ $= f(\text{peach})$ $= 4$	$u(w_3, A1)$ $= f(\text{peach})$ $= 4$	$u(w_4, A1)$ $= f(\text{peach})$ $= 4$	$u(w_5, A1)$ $= f(\text{peach})$ $= 4$	$u(w_6, A1)$ $= f(\text{peach})$ $= 4$
$u(w_7, A1)$ $= f(\text{plum})$ $= 1$	$u(w_8, A1)$ $= f(\text{plum})$ $= 1$	$u(w_9, A1)$ $= f(\text{plum})$ $= 1$	$u(w_{10}, A1)$ $= f(\text{plum})$ $= 1$	$u(w_{11}, A1)$ $= f(\text{plum})$ $= 1$	$u(w_{12}, A1)$ $= f(\text{plum})$ $= 1$
$u(w_1, A2)$ $= f(\text{pomegranate})$ $= 2$	$u(w_2, A2)$ $= f(\text{pomegranate})$ $= 2$	$u(w_3, A2)$ $= f(\text{pomegranate})$ $= 2$	$u(w_4, A2)$ $= f(\text{pomegranate})$ $= 2$	$u(w_5, A2)$ $= f(\text{pomegranate})$ $= 2$	$u(w_6, A2)$ $= f(\text{persimmon})$ $= 3$
$u(w_7, A2)$ $= f(\text{pomegranate})$ $= 2$	$u(w_8, A2)$ $= f(\text{pomegranate})$ $= 2$	$u(w_9, A2)$ $= f(\text{pomegranate})$ $= 2$	$u(w_{10}, A2)$ $= f(\text{pomegranate})$ $= 2$	$u(w_{11}, A2)$ $= f(\text{pomegranate})$ $= 2$	$u(w_{12}, A2)$ $= f(\text{persimmon})$ $= 3$

Now we can do some quick expected value calculations for James:

- $$EV(A1) = c(w_1) \cdot u(w_1, A1) + c(w_2) \cdot u(w_2, A1) + c(w_3) \cdot u(w_3, A1) + c(w_4) \cdot u(w_4, A1) + c(w_5) \cdot u(w_5, A1) + c(w_6) \cdot u(w_6, A1) + c(w_7) \cdot u(w_7, A1) + c(w_8) \cdot u(w_8, A1) + c(w_9) \cdot u(w_9, A1) + c(w_{10}) \cdot u(w_{10}, A1) + c(w_{11}) \cdot u(w_{11}, A1) + c(w_{12}) \cdot u(w_{12}, A1)$$

$$= \frac{4}{12} + \frac{4}{12} + \frac{4}{12} + \frac{4}{12} + \frac{4}{12} + \frac{4}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12}$$

$$= \frac{30}{12} = 2\frac{1}{2}$$
- $$EV(A2) = c(w_1) \cdot u(w_1, A2) + c(w_2) \cdot u(w_2, A2) + c(w_3) \cdot u(w_3, A2) + c(w_4) \cdot u(w_4, A2) + c(w_5) \cdot u(w_5, A2) + c(w_6) \cdot u(w_6, A2) + c(w_7) \cdot u(w_7, A2) + c(w_8) \cdot u(w_8, A2) + c(w_9) \cdot u(w_9, A2) + c(w_{10}) \cdot u(w_{10}, A2) + c(w_{11}) \cdot u(w_{11}, A2) + c(w_{12}) \cdot u(w_{12}, A2)$$

$$= \frac{2}{12} + \frac{2}{12} + \frac{2}{12} + \frac{2}{12} + \frac{2}{12} + \frac{3}{12} + \frac{2}{12} + \frac{2}{12} + \frac{2}{12} + \frac{2}{12} + \frac{2}{12} + \frac{3}{12}$$

$$= \frac{26}{12} = 2\frac{1}{6}$$

The expected value of A1 is thus higher than the expected value of A2, so we have an explanation for James' initial preference for A1 over A2.

1.2 Certain Persimmon

So much for James' initial preference. Before tackling the problem of how learning that the peach *might* be rotten changes James' preference, let's work through two easier problems. First, suppose Spiker tells James that the die landed 6. James now updates his credences with the new information. This update occurs via probabilistic conditionalization, where James conditionalizes on the proposition *The die landed 6*, which we can model as $\{w_6, w_{12}\}$. We thus produce a new credence function for James, c_1 , where $c_1 = c(\cdot|\{w_6, w_{12}\})$, which has the following values:

- $c_1(w_6) = c_1(w_{12}) = \frac{1}{2}$
- $c_1(w_1) = c_1(w_2) = c_1(w_3) = c_1(w_4) = c_1(w_5) = c_1(w_7) = c_1(w_8) = c_1(w_9) = c_1(w_{10}) = c_1(w_{11}) = 0$

With new credences come new expected values. Thus:

- $$\begin{aligned} EV_1(A1) &= c_1(w_1) \cdot u(w_1, A1) + c_1(w_2) \cdot u(w_2, A1) + c_1(w_3) \cdot u(w_3, A1) + c_1(w_4) \cdot \\ &u(w_4, A1) + c_1(w_5) \cdot u(w_5, A1) + c_1(w_6) \cdot u(w_6, A1) + c_1(w_7) \cdot u(w_7, A1) + c_1(w_8) \cdot \\ &u(w_8, A1) + c_1(w_9) \cdot u(w_9, A1) + c_1(w_{10}) \cdot u(w_{10}, A1) + c_1(w_{11}) \cdot u(w_{11}, A1) + \\ &c_1(w_{12}) \cdot u(w_{12}, A1) \\ &= \frac{4}{2} + \frac{1}{2} = 2\frac{1}{2} \end{aligned}$$
- $$\begin{aligned} EV(A2) &= c_1(w_1) \cdot u(w_1, A2) + c_1(w_2) \cdot u(w_2, A2) + c_1(w_3) \cdot u(w_3, A2) + c_1(w_4) \cdot \\ &u(w_4, A2) + c_1(w_5) \cdot u(w_5, A2) + c_1(w_6) \cdot u(w_6, A2) + c_1(w_7) \cdot u(w_7, A2) + c_1(w_8) \cdot \\ &u(w_8, A2) + c_1(w_9) \cdot u(w_9, A2) + c_1(w_{10}) \cdot u(w_{10}, A2) + c_1(w_{11}) \cdot u(w_{11}, A2) + \\ &c_1(w_{12}) \cdot u(w_{12}, A2) \\ &= \frac{3}{2} + \frac{3}{2} = 3 \end{aligned}$$

$EV_1(A2) > EV_1(A1)$, so James prefers A2 after learning that the die landed 6. And the reasons for the change in preference make sense: knowing that the die landed six, James is certain that A2 gets him a persimmon, and James prefers a persimmon (his second favorite fruit) to a 50-50 chance at a peach or a plum.

In this first case, James' change in preferred action is entirely informationally driven in the most straightforward sense – he does not change his underlying fructuous values, but merely changes his credences in achieving those values.

1.3 Rotten Fruit

Now consider a second case that's not quite so straightforward. This time Spiker tells James that the peach is rotten (*is*, not *might be*; we'll come to the epistemically modalized case next). James now needs to update his credences in light of Spiker's new information.

But now we encounter a problem. Our space W of worlds doesn't even specify information about whether the peach is rotten. So \llbracket The peach is rotten \rrbracket is undefined with respect to W , and there is no way for James to update his credences via conditionalization on *the peach is rotten*.

(A tempting thought is that Spiker's assertion causes James' value for a peach to decline. But this is unrealistic. First of all, James' fruit values are fixed by an underlying ranking of possible worlds, and it is unrealistic that James' ranking of worlds changes. Furthermore, it is unrealistic to think that our assertions can directly (in a systematic, predictable way) affect on one another's preference functions.)

This just looks like a modelling error on our part, though. Worlds really assign truth values to all propositions, but for simplicity (and in particular to allow for modelling with a finite set of worlds), we ignore the role played by worlds in determining the truth values of propositions that look irrelevant to the problem under consideration. More formally: we work not directly with the set of worlds, but with a partitioning of the set of worlds under the equivalence relation of *material equivalence with respect to the propositions we don't care about*. That's all harmless as long as we made the right initial modelling assumption about which propositions mattered and which didn't matter. But when we learn that Spiker said that the peach is rotten, we learn that we didn't make the right initial modelling assumptions.⁵

We should instead have been working with a space of 24 worlds:

⁵ If this were a real betting situation it might be assumed that James' reward is going to be something desirable. If this were the case, then learning that the peach is rotten could actually *add* possible worlds (specifically, the rotten peach worlds) back into James' information space. We agree that there are some nuances here to be explored concerning the relationship between presupposition and decision making. If James were to win a rotten peach he might, for instance, be surprised – though he also might admit that he had not strictly speaking been lied to.

v_1 Coin: H Die: 1 Peach: Normal	v_2 Coin: H Die: 1 Peach: Rotten	v_3 Coin: H Die: 2 Peach: Normal	v_4 Coin: H Die: 2 Peach: Rotten
v_5 Coin: H Die: 3 Peach: Normal	v_6 Coin: H Die: 3 Peach: Rotten	v_7 Coin: H Die: 4 Peach: Normal	v_8 Coin: H Die: 4 Peach: Rotten
v_9 Coin: H Die: 5 Peach: Normal	v_{10} Coin: H Die: 5 Peach: Rotten	v_{11} Coin: H Die: 6 Peach: Normal	v_{12} Coin: H Die: 6 Peach: Rotten
v_{13} Coin: T Die: 1 Peach: Normal	v_{14} Coin: T Die: 1 Peach: Rotten	v_{15} Coin: T Die: 2 Peach: Normal	v_{16} Coin: T Die: 2 Peach: Rotten
v_{17} Coin: T Die: 3 Peach: Normal	v_{18} Coin: T Die: 3 Peach: Rotten	v_{19} Coin: T Die: 4 Peach: Normal	v_{20} Coin: T Die: 4 Peach: Rotten
v_{21} Coin: T Die: 5 Peach: Normal	v_{22} Coin: T Die: 5 Peach: Rotten	v_{23} Coin: T Die: 6 Peach: Normal	v_{24} Coin: T Die: 6 Peach: Rotten

Our original worlds w_1 through w_{12} can now each be thought of as an equivalence class of the u worlds, with $w_1 = \{u_1, u_2\}$, $w_2 = \{u_3, u_4\}$ and in general $w_n = \{u_{2n-1}, u_{2n}\}$. James' credence and utility functions need to be adapted to the new space U of worlds. To do this, we need to take stands on two new issues:

- i. James' credence that the peach is rotten. (We tacitly assume that James views the state of the peach as probabilistically independent of the coin flip and die roll outcomes.)
- ii. James' values for normal and rotten peaches. (Here we continue to assume that James' utilities are entirely fructuous, while allowing the *state* of the fruit to count as among the fructuous features of the world.)

Let's suppose (arbitrarily) that James takes the peach to be $\frac{1}{10}$ likely to be rotten and $\frac{9}{10}$ likely to be normal. Let's also suppose that James values a

normal peach at 5 and a rotten peach at -5.⁶ Then we get new credence and utility functions c_2 and u_2 for James:

v_1 $c_2(v_1) = \frac{3}{40}$ $u_2(v_1, A1) = 5$ $u_2(v_1, A2) = 2$	v_2 $c_2(v_2) = \frac{1}{120}$ $u_2(v_2, A1) = -5$ $u_2(v_2, A2) = 2$	v_3 $c_2(v_3) = \frac{3}{40}$ $u_2(v_3, A1) = 5$ $u_2(v_3, A2) = 2$	v_4 $c_2(v_4) = \frac{1}{120}$ $u_2(v_4, A1) = -5$ $u_2(v_4, A2) = 2$
v_5 $c_2(v_5) = \frac{3}{40}$ $u_2(v_5, A1) = 5$ $u_2(v_5, A2) = 2$	v_6 $c_2(v_6) = \frac{1}{120}$ $u_2(v_6, A1) = -5$ $u_2(v_6, A2) = 2$	v_7 $c_2(v_7) = \frac{3}{40}$ $u_2(v_7, A1) = 5$ $u_2(v_7, A2) = 2$	v_8 $c_2(v_8) = \frac{1}{120}$ $u_2(v_8, A1) = -5$ $u_2(v_8, A2) = 2$
v_9 $c_2(v_9) = \frac{3}{40}$ $u_2(v_9, A1) = 5$ $u_2(v_9, A2) = 2$	v_{10} $c_2(v_{10}) = \frac{1}{120}$ $u_2(v_{10}, A1) = -5$ $u_2(v_{10}, A2) = 2$	v_{11} $c_2(v_{11}) = \frac{3}{40}$ $u_2(v_{11}, A1) = 5$ $u_2(v_{11}, A2) = 3$	v_{12} $c_2(v_{12}) = \frac{1}{120}$ $u_2(v_{12}, A1) = -5$ $u_2(v_{12}, A2) = 3$
v_{13} $c_2(v_{13}) = \frac{3}{40}$ $u_2(v_{13}, A1) = 1$ $u_2(v_{13}, A2) = 2$	v_{14} $c_2(v_{14}) = \frac{1}{120}$ $u_2(v_{14}, A1) = 1$ $u_2(v_{14}, A2) = 2$	v_{15} $c_2(v_{15}) = \frac{3}{40}$ $u_2(v_{15}, A1) = 1$ $u_2(v_{15}, A2) = 2$	v_{16} $c_2(v_{16}) = \frac{1}{120}$ $u_2(v_{16}, A1) = 1$ $u_2(v_{16}, A2) = 2$
v_{17} $c_2(v_{17}) = \frac{3}{40}$ $u_2(v_{17}, A1) = 1$ $u_2(v_{17}, A2) = 2$	v_{18} $c_2(v_{18}) = \frac{1}{120}$ $u_2(v_{18}, A1) = 1$ $u_2(v_{18}, A2) = 2$	v_{19} $c_2(v_{19}) = \frac{3}{40}$ $u_2(v_{19}, A1) = 1$ $u_2(v_{19}, A2) = 2$	v_{20} $c_2(v_{20}) = \frac{1}{120}$ $u_2(v_{20}, A1) = 1$ $u_2(v_{20}, A2) = 2$
v_{21} $c_2(v_{21}) = \frac{3}{40}$ $u_2(v_{21}, A1) = 1$ $u_2(v_{21}, A2) = 2$	v_{22} $c_2(v_{22}) = \frac{1}{120}$ $u_2(v_{22}, A1) = 1$ $u_2(v_{22}, A2) = 2$	v_{23} $c_2(v_{23}) = \frac{3}{40}$ $u_2(v_{23}, A1) = 1$ $u_2(v_{23}, A2) = 3$	v_{24} $c_2(v_{24}) = \frac{1}{120}$ $u_2(v_{24}, A1) = 1$ $u_2(v_{24}, A2) = 3$

Notice that c_2 and u_2 are set up so that whenever we think of w_n as being the equivalence class $\{v_{2n-1}, v_{2n}\}$, we get that $c(w_n)$ is $\sum_{v \in w_n} c_2(v)$, and $u(w_n, Ai)$ is the probabilistically-weighted average of $u_2(v_{2n-1}, Ai)$ and $u_2(v_{2n}, Ai)$ (for $i = 1, 2$), where the probabilistically weighted average is:

$$\bullet \frac{u_2(v_{2n-1}, Ai) \cdot c_2(v_{2n-1}) + u_2(v_{2n}, Ai) \cdot c_2(v_{2n})}{c_2(v_{2n-1}) + c_2(v_{2n})}$$

So in particular consider $u(w_1, A1)$. James' value of w_1 when performing A1 (a situation in which James gets a peach) is determined by James' initial *value of a peach*, which is 4. We then think of w_1 as being the equivalence class $\{v_1, v_2\}$. James values v_1 when performing A1 at 5 (a normal peach) and values v_2 when performing A1 at -5 (a rotten peach). The probabilistically weighted average of these is:

⁶ Though James' credence that the peach is rotten is arbitrary, his values for normal and rotten peaches are not. They are the values we must assign, given James' arbitrarily assigned credences, in order for the value function on W to match the value function on V .

- $\frac{5 \cdot \frac{3}{40} + -5 \cdot \frac{1}{120}}{\frac{3}{40} + \frac{1}{120}} = 4$

So in retrospect we can think of James' value of 4 for a peach as being a value formed under uncertainty balancing the high possibility of getting a desirable normal peach against the low possibility of getting a very undesirable rotten peach.

We've now corrected our initial modelling error. With the better model in hand, we're ready to think about what happens when Spiker tells James that the peach is rotten. With our revised model, we can assign propositional content to *The peach is rotten*, just like we did with *The die landed 6*. We have $\llbracket \text{The peach is rotten} \rrbracket = \{v_2, v_4, v_6, v_8, v_{10}, v_{12}, v_{14}, v_{16}, v_{18}, v_{20}, v_{22}, v_{24}\}$. By conditionalizing on this content, James gets a posterior credence function c_3 on the reduced space of worlds:

- $c_3(v_2) = c_3(v_4) = \dots = c_3(v_{24}) = \frac{1}{12}$.

(By removing uncertainty about the state of the peach, Spiker has returned James to a state in which his only uncertainties are about the results of the coin flip and the die roll. Since he views both flip and roll as fair, he regards each of the 12 possible outcomes as equally likely.)

Using the informationally-updated c_3 and the unchanged utility function u_2 , we can calculate expected values for James for each of A1 and A2:

i. $EV(A1) = -5 \cdot \frac{6}{12} + 1 \cdot \frac{6}{12} = -2\frac{1}{2}$

ii. $EV(A2) = 2 \cdot \frac{10}{12} + 3 \cdot \frac{2}{12} = 2\frac{1}{6}$

After being told that the peach is rotten, James' expected value for A1 plunges from $2\frac{1}{2}$ to $-2\frac{1}{2}$, and he now prefers A2 to A1. (Note that, unsurprisingly, being told that the peach is rotten has no impact on James' expected value for A2, since A2 involves no prospect of receiving a fruit.)

The shift in James' action preferences in this case is entirely informational. Spiker's assertion that the peach is rotten changes his credences for receiving different kinds of outcomes, but it doesn't change his values for those outcomes. It can *look* as if Spiker's information changes James' value for a peach. But that's because of our initial modelling inadequacy. James' values for a normal peach and for a rotten peach both remain unchanged – it's just that neither of these values was represented in our initial model. What changes is his *expected value* for a peach, which is a credence-weighted average

of his values for normal and rotten peaches. When his credences shift, of course his credence-weighted expected values can also shift.

1.4 Intermediate Reflection

The standard dynamics for rational decision making are centered around three things: information, world-elimination, and credence change. Thus, our standard model for linguistic interventions changing behavior has those interventions affect credences by shrinking spaces of worlds via the proffering of information. (There isn't anything unique about language in this regard: all sorts of information-gathering activities will affect decision making in precisely this way.)

On the standard model, cases that look like changes in value functions are actually changes in spaces of worlds, resulting in changes in the induced value of intentionally specified sets of worlds. And that's all to be expected on a model of language that matches expressions with informational content.⁷

⁷ In fact, making the standard model fully precise requires a bit more subtlety than this. The objects of credence are propositions, which we are modelling as possible worlds. Similarly, the objects of value are outcomes, which we are modelling as proposition-action pairs, and hence as pairs of a set of worlds and an action. But if information change genuinely involves removing worlds from the model, then we no longer have the same propositions available for consideration. If learning that the peach is rotten removes worlds v_1, v_3, \dots, v_{23} , then the proposition that the coin lands heads, understood as the set $\{v_1, \dots, v_{12}\}$, is no longer an available object of credence. On this picture, rational decision dynamics wouldn't be a result of changing credences or values in propositions or outcomes – rather, the dynamic effects would be a result of propositions and outcomes going *out of consideration* by being removed from the modelling space and from the domains of the credence and value functions.

To avoid this way of putting things, we can think of information change, via conditionalization, as involving not *removing worlds* from our information state, but rather *setting worlds* (or regions of worlds, in the infinite case) to credence 0. Then we have genuine credence shift, but now there is not even the semblance of value shift – since the set of peach worlds remains the same throughout (with only shifting credences among those worlds), the values of the peach worlds also remain the same throughout. Thus although there's a tempting picture on which both credence and value shift is driven by underlying information space shift, there remains on that tempting picture an asymmetry between credence shift and value shift.

In any case, all of these subtleties are beside the point once we turn to cases, such as the epistemically modalized cases, in which what is learned, and what induces decision shift, is not informational.

Issues arise when we consider that we sometimes learn things without getting any new information, and thus without changing our space of worlds (either through world-elimination or shifting credences in particular worlds to 0). It is likely that some of this non-informational learning comes through language. Various theories – especially in dynamic semantics – associate some expressions with non-informational content, epistemic modals like ‘might’ among them (imperatives, questions, and expressives are among the other types of expressions that get associated with non-informational content).

Often non-informational content is modelled using updates to other aspects of the *conversational scoreboard* (Lewis 1979) besides what information is available. But the conversational scoreboard needs to earn its theoretical keep by being connected to consequences of linguistic interaction for rational action.⁸ So there’s a demand for modelling rational decision change without credence change.

It is not that we think that linguistic interventions are crucial for rational decision change – rather, it’s that we are interested in non-informational prompts for rational decision change, and our best theories of language give us our clearest pictures of learning that would be non-informational, whether received linguistically or not.

2 Epistemic Modals and Action Selection

In both of the examples we’ve just considered, Spiker changes James’ mind about what to do by *changing James’ mind* in a particularly straightforward way – Spiker’s assertion changes James’ doxastic state. (You can think of this as changing James’ partial beliefs, or changing his full beliefs and thereby shifting some of his credences, as you prefer.) This straightforward way is then, as we’ve seen, easily incorporated into our technology. So it would be nice if we could say the same thing about the change in James’ action preference on learning from Spiker that *the peach might be rotten*.

⁸ This motivation underwrites much of Stalnaker’s foundational work in formal pragmatics (Stalnaker 2002).

2.1 Contextualist Semantics for ‘Might’

To say the same thing requires a theory that assigns informational content to epistemic *might* claims. Of course, since our ultimate interest is in the learning effects of *non-informational* claims, for our purposes informational accounts of epistemic modals just miss the point. Even if (contrary to our earlier claims) an informational account of epistemic modals were correct, this would just show that we had picked the wrong test case, not that there was no need for an account of non-informational value shift. But it’s still worth seeing the challenges that informational accounts specifically of epistemic modals face here.

Consider a standard *contextualist* semantics for epistemic modals.⁹ In a contextualist semantics:

- We have some epistemic accessibility relation R_e among worlds. (Roughly: $wR_e v$ just in case v is compatible with some body of information determined at w – maybe the speaker’s evidence in w , maybe the reported agent’s evidence in w , maybe what is conversational common ground in w , and so on.)
- Then $\llbracket \text{Might } \phi \rrbracket = \{w : R_e[w] \subseteq \llbracket \phi \rrbracket\}$. (Roughly: the semantic value of *Might* ϕ is the subset of ϕ worlds compatible with some body of information determined at w .)

The contextualist semantics has the advantage that we assign possible-worlds propositional content to epistemic *might* claims, which opens up the possibility that receiving such a claim can change James’ mind in the manner we’ve explored above.

Of course, possibility is not actuality. For $\llbracket \text{The peach might be rotten} \rrbracket$ to change James’ mind, it needs to give him *new* information. If Spiker’s claim is in some sense old news to James, updating on the possible-worlds content won’t shift James’ information state and will thus leave unchanged his credences and his credence-weighted values.

For $\llbracket \text{The peach might be rotten} \rrbracket$ to be informative for James, it must at a minimum not take as content the set of all worlds.¹⁰ But if R_e is a total relation

⁹ See, for example, [Hacking 1967](#), [Teller 1972](#). For more contemporary work on contextualist semantics for epistemic vocabulary see [Dowell 2011](#), [von Fintel & Gillies 2007](#), [Moss 2015](#), and [Mandelkern 2019](#).

¹⁰ A bit stronger: let W^+ be the minimal subset of the total space W of worlds such that James assigns a credence of 1 to W^+ . (In the finite case, $W^+ = \{w \in W : c(w) > 0\}$. In the infinite case,

on the set of worlds, then $R_e[w] = R_e[v]$ for any w and v , so $w \in \llbracket \text{The peach might be rotten} \rrbracket$ iff $v \in \llbracket \text{The peach might be rotten} \rrbracket$, and thus $\llbracket \text{The peach might be rotten} \rrbracket$ is the set of all worlds.

But if the space of worlds in our model is representing James' open possibilities, and James is minimally self-reflective, we will get that R_e is total. If there are any rotten peach worlds that are open possibilities for James, then James is in a position to know that, for any open possibility about what his evidence is, that evidence is compatible with rotten peaches. To avoid this conclusion, we either need to deny self-reflection or read the epistemic modal exocentrically. If, for example, James takes Spiker's claim to be reporting on *her* evidence, then he might give non-zero credence to two worlds w and v such that w and v differ about which worlds are relevant to the truth of *The peach might be rotten*, because w and v differ on what evidence *Spiker* possesses.

But even if a contextualist semantics can account for some cases of learning of epistemically modalized claims via information-shifting, the resulting picture is insufficiently general. When Spiker tells James that the peach might be rotten, James can (i) reply 'Yes, of course', and (ii) nevertheless be brought to prefer A2 to A1. In other words, an epistemic *might* claim can change preferred actions even when the claim is in some important sense treated as *not news*.¹¹ Even when the epistemic modal is exocentrically linked to Spiker's information state, James might already know that rotten peaches

things aren't so simple, and when W is uncountable, W^+ may not be uniquely defined. These finer points won't matter for our purposes.) For *The peach might be rotten* to change James' behavior, we then need that $\llbracket \text{The peach might be rotten} \rrbracket \not\subseteq W^+$, so that conditionalizing on that information will change James' credence function.

11 The worry might be raised that this glosses over an important distinction between the informational content semantically associated with a speech act and the information we get through various secondary mechanisms from the performance of a speech act. When Spiker tells James that the peach is rotten, James does not simply update his beliefs on the content of Spiker's assertion. James may in addition conditionalize on the secondarily conveyed proposition *that Spiker asserted that the peach is rotten* or on the implicated proposition *that the peach's being rotten is relevant to the current decision problem*. Likewise, Spiker's assertion that the peach might be rotten – though not associated with straightforward informational content itself – can also be associated with secondary informational updates for James: James may conditionalize on the proposition *that Spiker asserted that the peach might be rotten* or *that the chance that the peach is rotten is high enough to be worth mentioning*. We do not dispute that assertions of epistemic modals might (even invariably) be associated with some informational update (one that may give evidence about the speaker's beliefs). Nevertheless, there can be additional dynamic effects that come directly from the core non-informational semantic content of the epistemic modal claim; it is modelling these effects that is our interest, so we set aside the various potential secondary informational effects.

were compatible with her information state (as well as his), but nevertheless be rationally inclined to change his values and hence his behavior upon learning Spiker’s news. The contextualist tools won’t help us explain that shift.

2.2 Partitioning Technology for Epistemic Modals

Let’s thus consider some more contemporary non-truth-conditional tools for giving semantics for epistemic modals. We’ll focus on techniques from inquisitive semantics.¹² Inquisitive semantics is an offshoot of dynamic semantics, so we start with the core dynamic idea. In the dynamic setting, sentences are associated with conversational scoreboard update rules, mapping prior to posterior scoreboards. A first step, then, is deciding what goes on the conversational scoreboard.

To keep things simple, we start by using scoreboards only to track information states. Thus a scoreboard σ can simply be associated with a set of possible worlds. (Thus, our conversational scoreboard will look a lot like the spaces of worlds we used to model James’ decisions.) In this simple setting, there is a simple account of the update rules for the Boolean non-modal fragment of the language:

- i. $\sigma[p] = \{w \in \sigma : V(w, p) = 1\}$
- ii. $\sigma[A \wedge B] = \sigma[A][B]$
- iii. $\sigma[\neg A] = \sigma - \sigma[A]$

For our purposes, of course, the crucial question is how *epistemic modals* get implemented in the dynamic setting. The Veltman-inspired technology treats modals as tests.¹³ *Might* modals, in particular, check whether their prejacent is compatible with the current information state (in the sense that updating that state with the prejacent does not trivialize the state by forcing it to become empty). If the test is passed, the information state is left unchanged; if the test is failed, the information state is shifted to the empty ‘absurd’ information state \perp :

$$\text{iv. } \sigma[\textit{Might } A] = \begin{cases} \sigma & \textit{if } \sigma[A] \neq \perp \\ \perp & \textit{if } \sigma[A] = \perp \end{cases}$$

¹² For other accounts of epistemic ‘might’ roughly in this spirit see [Willer 2013](#), [Yalcin 2007](#).

¹³ See [Veltman 1996](#).

The test update clause does a good job of respecting the thought that when Spiker tells James that the peach might be rotten, what she tells him is in a semantically important sense not news. Assume for simplicity that the information state on the conversational scoreboard is just the set of worlds compatible with James' evidence. If James has no prior evidence about the state of the peach, then some worlds in which the peach is rotten are compatible with James' evidence, and thus in the information state. Thus when we perform the *The peach might be rotten* test by hypothetically updating the information state with the atomic (and hence non-modal) *The peach is rotten*, the information state is not driven to absurdity – the rotten peach worlds in the state remain. The test is passed, and the information state is unchanged. Updating James' conversational scoreboard with *The peach might be rotten* doesn't do anything – the scoreboard remains unchanged, and it is as if nothing has been said.

If nothing has been said, there's been no news. But also if nothing has been said, nothing has been said to change James' preferences for action. The simple dynamic implementation of epistemic *might* claims goes *too far* in making those claims inert. They become *entirely conversationally inert*, but we want them to be only *informationally inert*. With our simple picture of the conversational scoreboard as nothing but an information state/set of worlds, though, there's no room for a difference between conversational and informational inertness.

Enter the new millenium, then, and a more sophisticated model of the conversational scoreboard.

We can use the scoreboard to track more than just information states. On our more sophisticated model, we add in some features that allow us to track distinctions between conversational and informational efficacy.

In conversations there is a direction to the flow of information. There is a structure on the information in a discourse. In addition to tracking information states, our scoreboard will track what information is *salient*. A scoreboard is a set of worlds along with a *partition* of that set of worlds (Roberts 2012).¹⁴ Sentences then update the scoreboard by changing partitions:

¹⁴ The dynamic rules for inquisitive semantics we give below guarantee only that the conversational score includes a *cover* of the space of worlds, not necessarily a *partition*. However, in the case of epistemic modals that is our current focus, the update rules always transition from partitions to partitions. If desired, the update rules can trivially be rewritten always to produce partitions by mapping any given cover to the partition consisting of all maximal non-empty intersections of cover elements.

- i. $\sigma[p] = \{\pi \cap \{w : V(w,p) = 1\} : \pi \in \sigma\}$
- ii. $\sigma[A \wedge B] = \sigma[A][B]$
- iii. $\sigma[A \vee B] = \sigma[A] \cup \sigma[B]$
- iv. $\sigma[\neg A] = \{U\sigma - U\sigma[A]$
- v. $\sigma[\textit{Might } A] = \sigma[A \vee \neg A]$

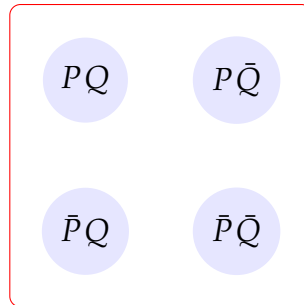
In inquisitive semantics, sentences express proposals for how the common ground should be updated. An assertion that p proposes that the common ground be updated in a way that eliminates all non-p indices. A question, on the other hand, is treated as a proposal of two or more alternative updates (partition elements) to the common ground. Since we seek to establish one of these updates, other participants in the conversation are expected to provide information (proposals that eliminate indices / partition elements) which establishes one of these updates.

This framework is well-suited to capture “a sentence’s potential to draw attention to certain possibilities” (Ciardelli et al. 2009)¹. Raising a possibility (represented by partition elements) to salience is something that we can represent as having an effect on the scoreboard without changing information (eliminating possible worlds).

Some examples will help here. We start with:

- i. Two propositions:
 - a. That the peach is rotten (P)
 - b. That the quince is smashed (Q)
- ii. A resulting space of four worlds
- iii. A coarse-grained initial ‘partition’ of that space into a single partition element (representing the initial lack of salience of any substantive propositions).

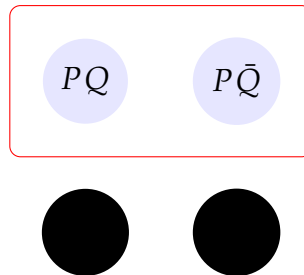
The initial scoreboard σ_0 is thus:



First Example: Made in σ_0 , Spiker's utterance of:

- (1) The peach is rotten.

intersects each partition element of σ_0 with the set of rotten peach worlds. Because there is only one partition element in σ_0 , the result is:

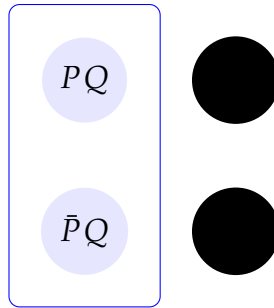


The new scoreboard changes *informationally* by eliminating the non-rotten-peach worlds and thus incorporating the information that the peach is rotten, and also changes *in focus* by raising the proposition that the peach is rotten to salience, unlike σ_0 .

Similarly, an utterance of:

- (2) The quince is smashed.

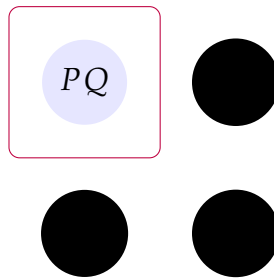
intersects the one partition element of σ_0 with the set of smashed quince worlds:



Second Example: In σ_0 , Spiker utters

- (3) The peach is rotten and the quince is smashed.

The conjunction successively updates the scoreboard with each conjunct. Thus we first intersect the sole partition element of σ_0 with the set of rotten peach worlds, and then intersect the resulting partition element with the set of smashed quince worlds. The result is:

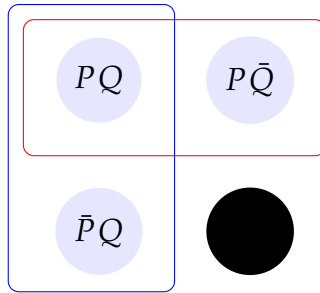


After updating we have gained information (eliminating three worlds) and shifted focus (raising the proposition consisting of the single world PQ to salience).

Third Example: In σ_0 , Spiker utters

- (4) The peach is rotten or the quince is smashed.

The resulting update is the union of the two updates considered in the previous example:

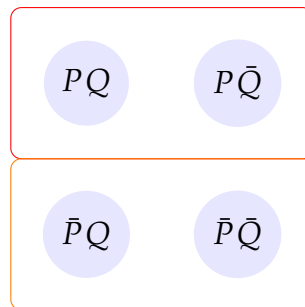


Again there is an informational and an attentional component of the update. Informationally, we eliminate one world ($\bar{P}\bar{Q}$) and retain three.¹⁵ Attentionally, we raise *two* propositions to salience: that the peach is rotten and also that the quince is smashed.¹⁶

Fourth Example: In σ_0 , Spiker utters

(5) The peach might be rotten.

The epistemic modal *Might p* has the same partition effect as an utterance of $p \vee \neg p$:



Might p thus has no informational content – no worlds are eliminated through the *Might p* update. But *Might p* does have an inquisitive effect, causing the prior state to be partitioned along the $p/\neg p$ line.

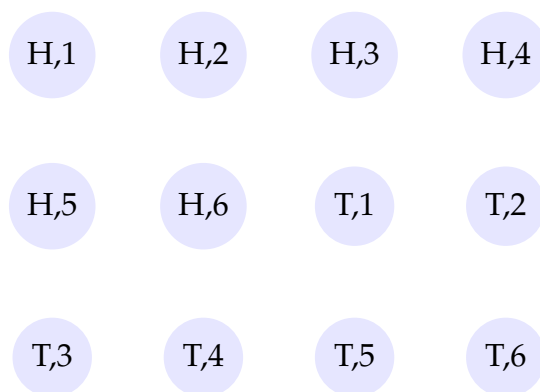
¹⁵ On the full compositional story, the update with the first disjunct eliminates the two worlds $\bar{P}Q$ and $\bar{P}\bar{Q}$, while the update with the second disjunct eliminates the two worlds $P\bar{Q}$ and $\bar{P}\bar{Q}$. But then unioning the two resulting states restores the worlds $P\bar{Q}$ and $\bar{P}Q$.

¹⁶ States with more than one partition element, and hence more than one proposition in attentional focus, can be thought of as states with inquisitive status. Placing multiple propositions in attentional focus amounts to raising the *question* of which of those propositions is true.

3 Interfacing with Practical Rationality

Might p does not eliminate worlds, but does change the partitioning of worlds. How can that affect decisions?

Our off-the-shelf decision-theoretic machinery ran credences and values on worlds. In the new inquisitive framework, it would be natural to run the machinery on partition elements rather than on worlds. And in fact it's not unnatural to construe ordinary uses of the off-the-shelf machinery as running on partitions. In our initial formulation of the Low-Hanging Fruit case (§1.1) we ran credence and value functions on the following space of worlds W :



But, as we saw in §1.3, updating the decision problem with certain kinds of information (such as the information that the peach is rotten) reveals that we should have really been thinking of these “worlds” as equivalence classes of more fine-grained worlds, which include information that is (putatively) irrelevant to the decision problem. Possible worlds, after all, are maximal specifications of how things could be. Even from an agent like James’ limited perspective, they include more propositions than just those having to do with what side a die or coin landed on. But as a practical matter, modelling decision situations by enumerating *all* of the fully-determinate worlds compatible with an agent’s information isn’t feasible. We always needed a strategy for representing things in a more coarse-grained way.¹⁷

¹⁷ Another perspective: possible worlds are specifications maximal *with respect to a given language*. As theorists, we adopt a minimal modelling language whose expressive resources are limited to matters relevant to the decision problem being modelled, and then consider a space of “small worlds” that are maximally specified with respect to that language. Low-Hanging Fruit, from this perspective, merely brings out the need to pick a modelling language adequate to the decision problem at hand. But on this way of thinking about

Given the introduction of an inquisitive framework, we can now recognize the w_i as partition elements. We can adapt the same strategy as above for assigning credences and values to these elements. The strategy we employed for assigning a credence and value function to W was one of probabilistic weighting. And so in a way the suggestion that we treat these as partition elements with worlds as members is nothing new: representing James' information space in terms of partition elements (which function like our coarse-grained 'small' worlds) rather than maximally-specified worlds is a notational preference, which helps us get around the difficulty of specifying every conceivable world in which James gets a peach.¹⁸

3.1 The Supervenience Picture

To take seriously the thought that partition elements, not worlds, are the genuine objects of credence and value we need an account of how partition elements come to have the credences and values that they do (for an agent at a time). A tempting story then derives these credences and values in a straightforward way from prior credences and values on the individual worlds.¹⁹ A crucial first step toward an adequate rational dynamics for

things, small worlds are entirely a theorist's artefact. This isn't adequate to our purposes, since we want to track ways in which the new-found salience of possibilities dynamically shifts the rational preferences of the deciding agent, so we need some version of "smallness" to be cognitively implemented in that agent. Partitions, and in particular the possibility of shifting partitioning on a scoreboard that the deciding agent is aware of, allows that implementation (see chapter 2 of Joyce 1999 – especially 2.6 – for related discussion).

18 Another formal option responsive to the same theoretical considerations would be to make the objects of credence and value be *partial possibilities*. Formally, there is little ground for choosing between these two options, given that partial possibilities can be modeled as sets of worlds. (Running the machinery in terms of partitions makes somewhat easier the formulation of the inquisitive dynamics, as it avoids the need for special procedures that would otherwise be required to make sure that competing partial possibilities described things at the same level of partiality.) However, we argue in section 3.2 below that philosophically, the notion of value has an essential connection to full worlds that makes the assignment of value to partial possibilities artificial, requiring first an abandonment of the partiality.

19 Our final story in section 4 also derives credences and values from prior credences and values on individual worlds, albeit not in so straightforward a way. This derivation perhaps sits ill at ease with our insistence that it's partition elements that are the genuine objects of credence and value. Perhaps a more thorough-going reworking of the conceptual foundations of decision theory would attempt to ground credences and values of partitions directly in the cognitive lives of agents, perhaps via a functionalist story using a version of Savage's representation theorem retuned to the partition-based decision theory technology (Savage 1954). We're not

values under non-informational updates is to see why that tempting picture nevertheless needs to be rejected.

Take a set of worlds W and a partition on that set of worlds Π . Our credence and value assignments for some partition element π supervene on the worlds in that partition element. We get the credence for π by summing the credences for the worlds in π , and we get the expected value of π by probabilistically weighting the expected value of (w, A) for each world w in π :

Definition 3.1. Supervenient Credence: $c(\pi) = \sum_{w \in \pi} c(w)$

Definition 3.2. Supervenient Value: $u(\pi, A) = \frac{\sum_{w \in \pi} u(w, A)c(w)}{c(\pi)}$

Definition 3.3. Expected Value*: $EV(A) = \sum_{\pi \in \Pi} c(\pi) \cdot u(\pi, A)$

The supervenience picture gets the relation between Rotten Fruit (§1.3) and Low-Hanging Fruit (§1.1) right. Rotten-Fruit is a more fine-grained partitioning of James' information space than Low-Hanging Fruit, such that every partition element of Rotten Fruit can be reached by partitioning some partition element in Low-Hanging Fruit. If we think of Low-Hanging Fruit as being the twelve-fold partitioning of the 24-element world space of Rotten Fruit using the underlying Rotten Fruit value function, we get the value function for Low-Hanging Fruit.

But the supervenience view also makes our decision-theoretic machinery indifferent to how things are partitioned. Supervenient Value entails that no matter how a space of worlds W is partitioned, the expected value for some action A will be the same, given some value function on W . Call this feature 'Stability':

Stability Given a space of worlds W with c and u on W , if Ξ and Π are partitions of W , then $EV^\Xi(A) = EV^\Pi(A)$ for all A .²⁰

averse to the more thorough-going approach, but small steps seem appropriate to a first move in that direction.

²⁰ **Proof:** $EV^\Xi(A) = \sum_{\pi \in \Xi} c(\pi) \cdot u(\pi, A) =$ (by Supervenient Value) $\sum_{\pi \in \Xi} c(\pi) \cdot \frac{\sum_{w \in \pi} u(w, A)c(w)}{c(\pi)} = \sum_{\pi \in \Xi} \sum_{w \in \pi} c(w) \cdot u(w, A) =$ (by Ξ being a partition) $\sum_{w \in W} c(w) \cdot u(w, A)$. Thus $EV^\Xi(A)$ equals the standard expected value of A . By analogous reasoning, so does $EV^\Pi(A)$, so the two are equal, proving Stability.

Stability is an unfortunate consequence, for our purposes.²¹ Changing the worlds in an information state would have had an effect on our expected value calculations for actions on the old, worlds-based model. Our expected value calculations for actions now run on partition elements rather than worlds, so it would be natural to think that changing the partition elements in an information state would have a similar effect. But because the credence and value assignments for partition elements supervene directly on the worlds in the information state, repartitioning a set of worlds W has a merely superficial effect.

What we set out to do was say something about how informationally-inert expressions, like epistemic modals, might have an effect on an agent's decision making. On the semantics side of things, this required us to find a picture of epistemic modals that captured their ability to change an information state without changing the information in that state (and thus capturing our intuition that epistemic modals are informationally inert, while at the same time conversationally efficacious).

But if epistemic modals only re-partition an information space without changing the underlying space of worlds, we need the re-partitioning to have some effect on our expected value calculations. Stability says that never happens; though we run credence and value functions on partitions, not worlds, only changes in information (worlds) can have an effect on the outputs of those functions.

Stability is a result of reducing the value of a partition element to the values of the individual worlds in that partition element. We want a way to assign a value to a set of worlds that doesn't reduce value of the set of worlds to the values of the individual worlds in the set.

3.2 An Aside: Alternative Motivations for Rejecting Supervenience

We always needed an alternative to the supervenience picture, before any issues about epistemic modals were in the picture. There are deep concerns

²¹ It is not clear that Stability will be entailed by corresponding supervenience rules for rank-dependent theories, like the Risk Weighted Expected Utility theory defended by [Buchak \(2013\)](#). Such theories take an agent's attitudes towards alternatives into account in determining a preference ordering on acts. Such views do not include a compelling story about why changing the way alternative outcomes are framed changes an agent's internal attitudes towards those outcomes. As we will see, once such a story is given, the standard decision theoretic tools can be employed unproblematically.

we might raise about differences between what information an agent is considering, and is aware of, and what information is part of their information state more generally.

As we noted basically at the outset, possible worlds are much more fine-grained than decision theory usually recognizes. Possible worlds are maximum inclusive situations; maximum sets of possibilities concerning the way the world could be. Because agents make decisions under a great deal of ignorance, there will typically be a very large (perhaps infinite) number of possible worlds associated with the outcome of any particular choice. Many of these worlds have features that are (a) relevant to the agent's decision-making, and (b) easily-accessed by that agent.²²

If James is faced with the decision of taking a peach or a plum, he might consider worlds in which the peach is rotten, and worlds in which the peach is ripe. But it is unlikely that he is going to consider worlds in which the peach is ripe *and* radioactive; worlds in which he gets a ripe peach and because of this spends the rest of the day doing good deeds; worlds in which he gets a rotten peach and in a far-away galaxy billions of innocent people are horribly tortured; worlds in which he gets a ripe peach, and in a far-away galaxy, billions of innocent people are wonderfully rewarded. But these are things that James is *capable* of considering; they are possibilities that are open to him, and they are possibilities that could have an effect on the decisions James makes.

These worlds can be thought of as among those contributing to James' overall credence in the proposition that *James gets a peach*. When James is told that his getting a peach might, through some virtually unpredictable causal chain, result in a stranger's death, there will be a sense in which this is not news to James. He is not being given information that was not already accessible to him.²³

But their contribution is one we are more than happy to ignore; credence in a proposition is something we are capable of recognizing from a top-down perspective. James might be certain of *p* without having any idea as to how *p* will be instantiated, or what effects we will be able to trace out from *p*. If James

22 Thus, the issue we raise here is distinct from the problem of *logical omniscience*, which is about agents not having access to propositions which are true across all possible worlds in their information space.

23 This is not to claim that these possibilities were ones that James had ever considered, or were actively represented by some part of James' cognitive system (see Audi 1994).

his certain of p he will likely nevertheless accept the possibility that many things he has no certainty of will be involved in p 's being instantiated.

But the same is not true for how James values p . This is something that seems to require him to trace out at least some of the consequences of p 's obtaining. The proposition that James gets a peach is not something that James values without consideration of any of the other features of the world in which he gets a peach (is it ripe, does he get to eat it, etc.). Further, the more detailed we get, the more precise James' estimation of value may become. This is contrary to the case of credence, where the more precise we get the less likely it is that an agent will be able to give a precise credence.

James' value in some proposition seems to be tied to specific features related to that proposition's being instantiated at a (particular) world. Other potentially relevant worlds are value-neutral with respect to a proposition like *James gets a peach*, but these worlds are or may be accessible to James given his consideration / endorsement of the proposition that he gets a peach.

It would thus be cognitively overdemanding for an agent to sum up the consequences across all of the possibilities that contribute to their credence in a proposition. It might be mathematically impossible as well, if there are infinitely many possible realizations, and therefore total values increase unboundedly (this will be inevitable if I believe that there are infinitely many ways that p could be true – see also [Pettigrew 2016](#), especially chapter 16).²⁴

3.3 The Generic Selection View

We want to avoid Stability, because we want partitioning to make a difference in value. However, we do not want to throw the baby out with the bathwater: we want a formal apparatus that still grounds credence and value in worlds.

²⁴ One possible response to all of this is that many of the possible worlds will just cancel one another out in value. Roughly, for each peach world that has some significant valuation not directly relating to the consumption of a peach, there will be a corresponding plum world, and all of the non-peach / non-plum stuff will just cancel out. This cancellation argument fails in cases where there are infinitely many possibilities to consider due to the failure of associativity for infinite sums including positive and negative values. Further, there can be unexpected lacks of correspondences where the presence of a peach really matters.

Here we present a novel formal apparatus that meets these needs: our alternative to the supervenience view given in §3.1. Our proposal is that there is a selection function f that maps each partition element to one of its members, such that the value of the partition element is given the value of that member world:

Definition 3.4. Selection Function $u(\pi, A) = (f(\pi), A)$

where $f(\pi)$ is a typical π world (more detail on this in the next section). That is, for each partition element an agent considers a world that represents the salient features of that partition element, rather than a probabilistic weighting of the worlds in that partition element (which is psychologically too demanding).

Even with a very simple conception of the view – one that makes no claims about the nature of typical π worlds – we can show how it earns its theoretical keep. Let's return to our initial example:

James and the Dubious Peach: A coin is flipped and a fair die is rolled. Spiker then offers James a choice between two actions:

- i. Action A1: James receives a peach if the coin lands heads, and a plum if the coin lands tails.
- ii. Action A2: James receives a pomegranate if the die lands 1-5 and a persimmon if the die lands 6.

James is initially inclined to perform A1, but when Spiker says "The peach might be rotten", he becomes instead inclined to perform A2.

We start with James' initial information space. Standard decision theoretic assumptions tell us that James' initial decision space will have many, many worlds (James is ignorant of many things). We begin with a large and unpartitioned space of worlds.

The initial decision problem that James faces partitions the worlds in James' initial information state. Specifically, we partition on the results of the coin toss and the results of the die roll:

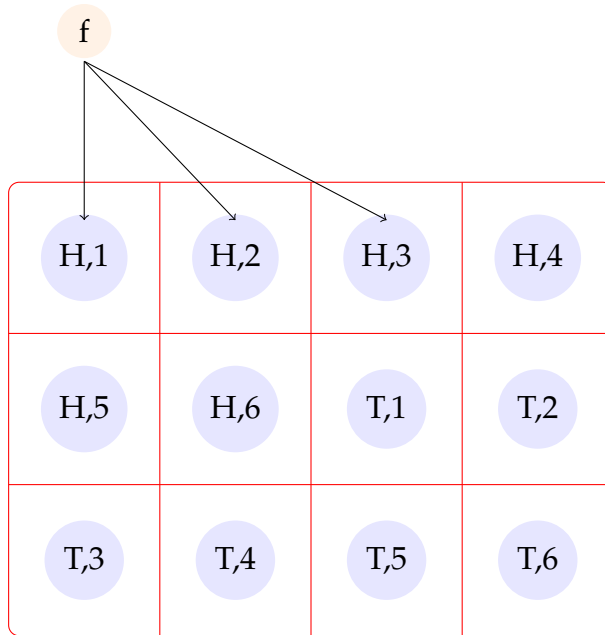
- Coin toss: H || T
- Die roll: 1 || 2 || 3 || 4 || 5 || 6

This gives us a model like the original model for Low-Hanging Fruit, but will cells now clearly conceived as partition elements rather than individual worlds.

H,1	H,2	H,3	H,4
H,5	H,6	T,1	T,2
T,3	T,4	T,5	T,6

Each partition element is then mapped to a typical exemplar world (i.e., the typical H,1 world, the typical H,2 world, etc.). For each partition element π , $f(\pi)$ selects a world in which the relevant fruit is ripe, and where there are no other interesting value features.²⁵

²⁵ There is a question of how to conceive of these partition elements once the possibility of performing some action has been raised. One way follows a standard decision-theoretic conception of the relationship between worlds and actions. That is, we can conceive of the choice between actions as a separate set of elements from the various partition elements, with values assigned to element-action pairs. Alternatively, we can plug the actions into the information space. This allows us to represent an agent's choice as a part of the partition space (this lines up nicely with some comments in [Roberts 2012](#) about the relationship between questions and commands). James' choice between A1 and A2 is thus conceived of as a choice between instantiating a property that reifies one set of partition elements or another (the A1 set or the A2 set).

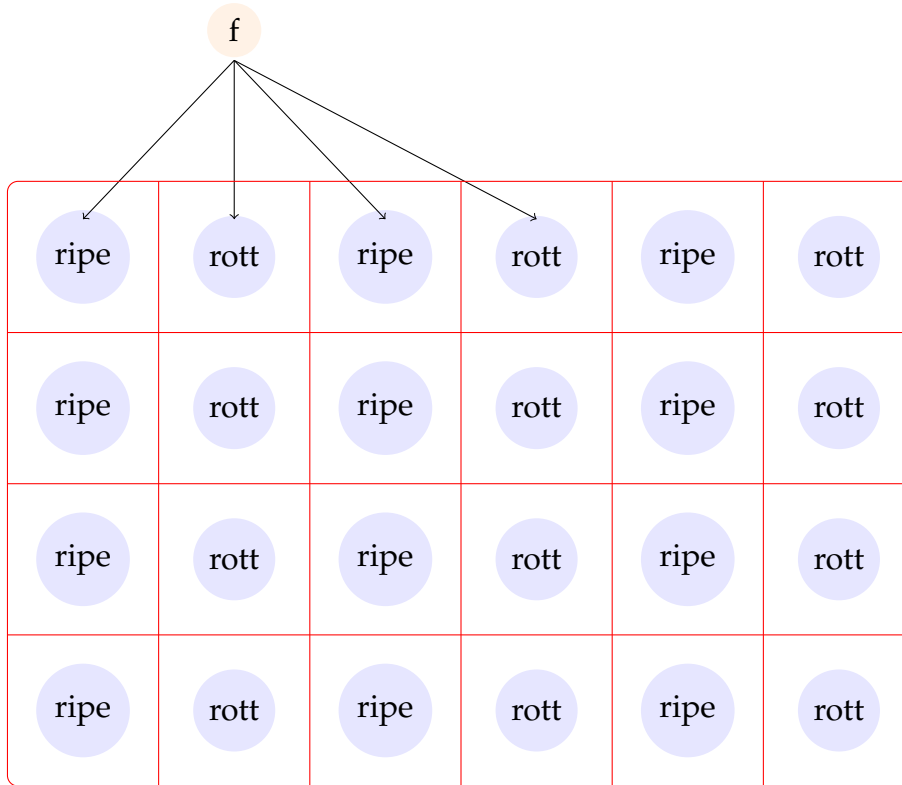


When Spiker says “The peach might be rotten” this induces a fine-graining of the partitions. It separates the ripe peach worlds from the rotten peach worlds within each existing partition element.

H,1,ripe	H,1,rot	H,2,ripe	H,2,rot	H,3,ripe	H,3,rot
H,4,ripe	H,4,rot	H,5,ripe	H,5,rot	H,6,ripe	H,6,rot
T,1,ripe	T,1,rot	T,2,ripe	T,2,rot	T,3,ripe	T,3,rot
T,4,ripe	T,4,rot	T,5,ripe	T,5,rot	T,6,ripe	T,6,rot

Our selection function then picks exemplar worlds from each of the new partition elements to be the value representatives for those partition elements.

Ripe peach partitions are represented by typical ripe peach worlds and rotten peach partitions are represented by typical rotten peach worlds.



James' value assignments to each partition element is thus decided by his valuation of the most typical worlds in which the peach should turn out to be ripe or rotten. This means that the rotten peaches have no impact on value until partitioning reveals partition elements that select rotten peach worlds. The rotten peach worlds were always in James' decision space. It's not news to him that there is a possibility of a rotten peach. But the modal, by partitioning, makes the rotten peach worlds salient in a way that allows them to contribute value to the decision problem.

With newly selected rotten peach worlds figuring in the decision problem, expected values change. Depending on the details of the model, this might be enough to change James' action preferences. Let us consider a model with such details. Perhaps James suppresses thoughts of rotten fruit because the idea of it is so vile to him. So, while he assigns a value of 4 to ripe peach worlds, he assigns a value of -20 to rotten peach worlds.

If we hold fixed the assumption that there is a one in ten chance of a rotten peach world, then the probabilistically weighted value of worlds in which James gets a peach (worlds in which he picks A1 and the coin lands heads) is 1.6. We can calculate expected values for James for each of A1 and A2:

$$\text{i. } EV(A1) = 1\frac{3}{5} \cdot \frac{12}{24} + 1 \cdot \frac{12}{24} = 2\frac{1}{10}$$

$$\text{ii. } EV(A2) = 2 \cdot \frac{20}{24} + 3 \cdot \frac{4}{24} = 2\frac{1}{6}$$

Thus, considering rotten peach worlds is enough to change James' mind about what to do.

4 Generic Worlds

The selection function picture given in 3.3 is very weak as it stands, because we have made no commitments to any particular property had by a 'typical' selected world. As such, our post-partitioning values may be wholly unconstrained by our pre-partitioning values, and so any new partitioning can change everything that it is reasonable for you to do. Here we will consider some formal and conceptual constraints on this framework.

What is the nature of the world selected by the selection function? We start with a minimal constraint of *success*:

- **Success:** $f(\pi) \in \pi$

The selected representative of a partition element needs minimally to be in the partition element. Beyond this minimal constraint, our guiding thought is that the selected world is a *generic representative of the partition element*. It should have no other interesting value features beyond those assured by membership in the partition element. So, if an information state is partitioned along the question whether p , then the relevant worlds will be the *most* value-unremarkable p -world and the most value-unremarkable $\neg p$ world.²⁶

The question remains whether this guiding generic thought can ground any further structure for the selection function. A tempting thought might then

²⁶ This raises some interesting questions about the intensional nature of partition spaces. For example, consider the partition created by the question: Did Batman kill the Joker? This is extensionally equivalent to the partition created by the question: Did Bruce Wayne kill the Joker? But is the most value-unremarkable world in which Batman killed the Joker the same world as the most value-unremarkable world in which Bruce Wayne killed the Joker?

be that worlds are linearly ordered by some form of normality or typicality. This thought would then give us:

Semantic Constraint There is a global well-ordering $<$ of worlds. $f(\pi)$ is then the $<$ -minimal element of π .

We might, for example, run our selection function on properties such as *likelihood* and *value*. Idealizing away the possibility of ties, both likelihood and value impose global well-orderings $<$ on an information space W . There will always be a globally most likely world, and there will always be a globally most desirable world (that is, a $<$ -minimal element). That world would then be selected by applying f to W . If we then impose a further partition on that information space, then the selection function will select the $<$ -minimal worlds of each partition element: that is, the most likely world of each partition element.

Crucially, the global $<$ -minimal world (the most likely world) must also be the $<$ -minimal world of some partition element. Since partitioning does not change the worlds in the information state, and since the selection function simply picks, from each partition element, the $<$ -minimal world in that partition element for some *global* well ordering of worlds, more fine-grained partitioning preserves the world selection(s) from the more coarse-grained partitionings. Thus, the Semantic Constraint entails the following Selection Constraint:

Selection Constraint If $\pi = \mu \cup \nu$ where $\mu \cap \nu = \emptyset$, then $f(\pi) = f(\mu)$ or $f(\pi) = f(\nu)$.

The selection constraint entails a certain constraint on rational action. What it entails is the following: if an epistemic might claim changes an agent's preference from $A1$ to $A2$, then they will still prefer $A1$ if the decision problem is restricted to one of the partition spaces created by the might claim. Call this *preservation*. More formally:

Preservation Suppose that an agent α initially prefers $A1$ to $A2$, but upon learning *Might* ϕ comes to prefer $A2$ to $A1$. Then there are two conditional decision problems:

- i. D^ϕ : The results of $A1$ and $A2$ are as in the original decision problem but with the further stipulation that the resulting situation is a ϕ situation.

- ii. $D^{\neg\phi}$: The results of A1 and A2 are as in the original decision problem but with the further stipulation that the resulting situation is a $\neg\phi$ situation.

such that α still prefers A1 to A2 in one of D^ϕ and $D^{\neg\phi}$. D^ϕ if the selection function picked a ϕ -world before a learned Might ϕ , and $D^{\neg\phi}$ if the selection function picked a $\neg\phi$ -world before a learned Might ϕ .

James prefers the coin toss to the die roll, but upon hearing that the peach might be rotten, comes to prefer the die roll to the coin toss. But James *still* prefers the coin toss to the die roll if it is then stipulated that the peach is not rotten.

Taking stock: we have just floated a plausible constraint (the Semantic Constraint) and traced some of its decision-theoretic consequences. We want our selection function to pick out – from any given partition element – the least-altered world in which the proposition governing that element is true. If we are partitioning on p, q, r then the selection function will pick out the minimally-altered p -world from the p partition element, etc.

However, we reject the Preservation/Selection Constraint/Semantic Constraint package. We start with a direct counterexample. Suppose that really desirable peaches are on the borderline between ripeness and rottenness, and our starting selection function picks those peaches, making A1 preferable to A2. But from the ripe peach worlds we pick normal ripe peaches, which are not overripe and thus are less desirable. And from the rotten peach worlds we pick normal rotten peaches, which are also less desirable. So after partitioning, we don't on either partition prefer A1 to A2.

Counterexample in hand, the further suggestion is that counterexamples of this sort are to be expected. Consideration of the logic and semantics of generics, thought of as the standard linguistic implementation of the notion of normality and typicality that we want our selection function to capture, already shows us that we shouldn't be expecting selection to run on a global well-ordering. We want to be able to endorse simultaneously 'Birds fly' and 'Penguins swim'. But we then can't link the truth of a generic to what happens in a world that is most normal *simpliciter*. In w_1 , Pen Pen swims; in w_2 , Pen Pen flies. Thus we don't want a simple ranking of w_1 and w_2 for overall normality. Rather, we want to say that w_1 is more penguin normal, and w_2 is more bird normal.

Running decision theory on partitions rather than worlds via the use of selection functions, and then linking the behavior of selection functions to the

notion of genericity then opens up a research project of further characterizing the behavior of the selection function by linking it to features of the logic of generics. For example, generics are often taken to obey a constraint of *Specificity*.²⁷ Specificity makes inconsistent the triad:

Generically, birds fly.
Generically, penguins don't fly.
Generically, birds are penguins.

Specificity is similar to, but logically weaker than, a generics-equivalent of Preservation. Generic-Preservation would yield the inconsistency of:

Generically, birds fly.
Generically, penguins don't fly.
Generically, non-penguin birds don't fly.

But generics plausibly violate the analog of Preservation. Perhaps the generic pet is a cat. But pets are either indoor pets or outdoor pets, and the generic indoor pet is a fish, and the generic outdoor pet is a dog.

Just as Preservation follows from a selection function constraint (that selection is achieved via a background global well-ordering), Specificity also follows from a selection function constraint.²⁸ Unfortunately, the selection function constraint that is linked to Specificity places no interesting constraints on the dynamic effects of repartitioning on decision problems, so we learn nothing here about the dynamics of decision-theoretic rationality from consideration of the logic of generics. But there is potentially fertile ground to be explored on the interaction between the two.

5 Further Issues

In this concluding section, we will explore some upshots that our project may have for other areas of philosophical theorizing.

5.1 Upshots for Practical Rationality

We often consider the different way things might go when making decisions. We don't *just* think about the ordinary worlds: it may be that, going into the

²⁷ See Asher & Morreau 1991.

²⁸ Namely, that if $X \subseteq Y$, then for any Z , if $f(X) \in Z$ and $f(Y) \notin Z$, then $f(Y) \notin X$.

bet, James is aware that the peach might be rotten. What happens, in such a case, when James is told that the peach might be rotten? In such cases, James' decision space will already have a partition fine-grained enough to represent such an assertion.

If James is *already* considering the possibility of the peach being rotten, then we should think that Spiker's utterance will have no real effect of James' decision making. And this is exactly how things stand on our model. If James is already considering the possibility of a rotten peach, then an utterance of "The peach might be rotten" has no partitioning to do. In addition to being informationally inert, it is psychologically inert for James (though not conversationally inert, as the utterance will still update the public record of the conversation).²⁹

Now we might raise the question of when such assertions – and thus such partitions – are *appropriate*. If James' rotten peach value assignments are low enough that even the possibility of a rotten peach is enough to change his mind about what to do, then is it the case that James *ought* to have been considering rotten peach partition elements to begin with? If we answer 'yes', we might endorse a norm on attention in decision-making. A norm that says something like: if partitioning (without making information changes) would change your mind about what to do, then you ought to partition. Slightly more formally: for any two partitions on an identical information space Q and R, such that every partition element of R is either identical to a partition element of Q, or a member of some partition element of Q, if Q and R assign different values to actions, then you ought to be in R, rather than Q.

Given such a norm, there will thus never be a case in which an agent who is thinking and behaving *rationally* has their mind changed about what to do by an epistemic modal: such an agent will already have considered the relevant partition. One role for epistemic modals in our decision theory is to make rational corrections to an agent's decision making.

Is such a norm realistic? We have our doubts. Often overthinking things (as in: raising more and more possibilities to salience) is more trouble than it's worth. Another possible norm would say to partition in a way that maximizes expected utility. When you take a heads-tails coin bet there is a reason you partition on heads and tails, and not on whether the coin was minted before

²⁹ The relationship between a conversation's information space and those of its participants is a complicated one, but we can assume – pace Stalnaker – that the public conversational record is whatever is *mutually accepted* by conversational participants.

or after the release of Star Wars. But such a norm comes dangerously close to one that simply says: *act rationally*.

Ordinary agents simply don't have the cognitive capacity to consider all the relevant possibilities. What we offer is a normative decision theory for non-ideal agents. Agents who, like all of us, exhibit various degrees of cognitive negligence.

Our program says something about ideal agents as well: to be an ideal agent is to consider all the partitions that would be relevant to making the decision that an agent is faced with. An ideal agent is one who partitions with maximum efficiency, attending to what is worth attending to, and not one who is panoptic, always attending to all distinctions.

5.2 Upshots for Semantic Theorizing

Work in formal semantics often posits complex structures as conversational scoreboards.³⁰ As scoreboard components proliferate, though, theoretical underdetermination worries increase. Consider question under discussion (QUD) structure in conversational scoreboards. It's plausible enough that conversations are often guided and shaped by a dynamically developing sequence of topical questions. But when, for example, we attempt to codify this plausible fact using a question stack as a component of the conversational scoreboard, and then to extract Gricean relevance facts as downstream consequences of the evolving QUD, there's a real worry that it's really the relevance facts that are fixing the QUD facts, or that the QUD facts are being ad hoc extracted from incomplete discourse descriptions to fit the relevance facts. In such cases it's not clear that the QUD is a genuinely explanatory component of the semantic theory, or that the proposed conversational scoreboard will be properly constrained by data. To avoid such worries, it's then helpful to tie specific scoreboard components to specific linguistic phenomena. QUDs can more clearly earn their theoretical keep, for example, by contributing to explanations of the acceptability of focus. The contrast in the minimal triad:

(6) Who did Brutus kill?

³⁰ See [Lewis 1979](#) for the basic scoreboard idea; various scoreboard components that have been proposed include a question under discussion ([Roberts 2012](#)), to-do lists ([Portner 2007](#)), entity representations ([Kamp 1981](#)), discourse referents ([Karttunen 1969](#)), probability thresholds [Lassiter \(2012\)](#), and many others.

- (7) a. Brutus killed [_F Caesar]
 b. # [_F Brutus] killed Caesar.
 c. # Brutus [_F killed] Caesar.

is usefully explained by linking or failing to link a QUD structure triggered by the question in (1) to sets of alternatives introduced by the focal stress in the variants of (2)

Our methodological suggestion, then, is that the interface with decision theory is another profitable area for helpful theoretical constraints. Worries about free-spinning theoretical wheels are especially pressing with non-truth-conditional portions of the language, whose scoreboard contributions lack the easy check of worldly information. Once we have a picture in hand of how non-truth-conditional, non-informational learning can influence rational decision dynamics, we are in a better position to determine whether the dynamic scoreboard contributions of non-truth-functional language is properly constraining and making sense of the decision-theoretic learning dynamics.

Consider a case in point. On our proposal, “The peach might be rotten” can change James’ preferred action because when the dynamics of the epistemic modal induce partition fine-graining, partition elements both of rotten peach worlds and of non-rotten peach worlds are revealed. Because the selected rotten peach world has substantially lower value than the selected merely-peach world, its uncovering changes James’ expected values.

But it then follows from the proposed semantic implementation of “might *A*” as “ $A \vee \neg A$ ” that learning *that the peach might not be rotten* will have the same effect on James’ decision problem. “The peach might not be rotten” is dynamically implemented as “Either the peach is not rotten or the peach is not not rotten”, which then (again) partitions the worlds into non-rotten peach worlds and rotten (that is, non-non-rotten) peach worlds. Same partition, same effect – a selected rotten peach world is revealed, and James’ expected value for peach actions goes down.

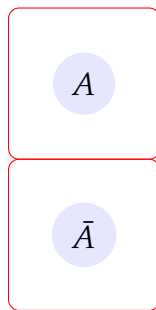
There’s now a decision-theoretic question: should learning that the peach might not be rotten have the same decision-making effect on James as learning that the peach might be rotten? We won’t try to settle that question here, but will simply note that *if* we think the answer is “no”, then we have decision-theoretic reason to modify the scoreboard dynamics for “might” in our inquisitive setting. Consider, for example, two possible dynamics for epistemic modals:

i. $\sigma[\textit{Might } A] = \sigma[A \vee \neg A]$

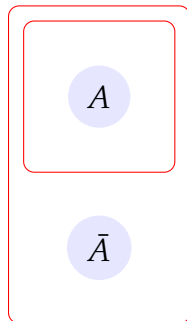
ii. $\sigma[\textit{Might } A] = \sigma[A \vee \top]$

Both of these dynamics make “Might A ” non-informative – in both cases, updating with “Might A ” leaves the space of worlds unchanged. And both dynamics make “Might A ” inquisitive, because “Might A ” guarantees that the scoreboard will contain two partitions.³¹ But they yield different inquisitive scoreboard structures:

i. $\sigma[\textit{Might } A] = \sigma[A \vee \neg A]$:



ii. $\sigma[\textit{Might } A] = \sigma[A \vee \top]$:



Because the second scoreboard does not contain a *rotten peach* partition element, there won’t be a selected rotten peach world contributing value – instead, the decision problem will run on the value of a generic peach world and a generic non-rotten-peach world. That difference allows for a difference in rational decision making upon learning “The peach might be rotten” versus “The peach might not be rotten”. The integration with decision

³¹ More carefully, the update rule $\sigma[\textit{Might } A] = \sigma[A \vee \top]$ gives us a *cover* with at least two elements, but that cover won’t be a partition, because the cover element induced by the \top disjunct will overlap with the cover element induced by the A disjunct. See the comments in footnote 15 on the cover/partition distinction. We continue to hide this issue under the hood.

theory thus allows evidential distinctions among semantic proposals that might otherwise be difficult to prize apart.

5.3 Upshots for Metaphilosophy

Our proposal uncovers real decision-theoretic consequences for possibilities being brought to an agent's attention. Without any new information being learned, there can still be a tractable value to something's being brought up. Epistemic modals, and also questions, have a real practical import that can be separated from their obvious role in information-gathering activities (cf Roberts 2012).

Often in philosophical theorizing it can feel as though very little progress is made (Chalmers 2015). But this lack of apparent progress can be located in a lack of eliminating possibilities; in the fact that we don't seem to be narrowing in on the truth.

But learning true things is important largely because of the success we enjoy in other normative domains when we seek (and obtain) the truth. What our paper has shined a light on is the fact that these gains can come from merely raising to salience certain possibilities. Putting a view like consequentialism on the table is putatively valuable even if in doing so we gain no new information.

This framework might thus help us explain something about the value of philosophy. If there is a practical import to asking questions and exposing possibilities, then philosophers are engaged in a project with practical import.

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