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Why Metaphysics Needs Logic and Mathematics Doesn't: Mathematics, Logic, and Metaphysics in Peirce's Classification of the Sciences

The view I defend here,[1] and which I take to be Peirce's, is that while metaphysics needs to be grounded in logic, mathematics does not. The metaphysician who is ignorant of the rules of logic is bound to go astray, as he is too easily charmed by the splendor of his speculations. Interestingly, it is precisely the need to *retain* this splendor of speculations that motivated Peirce to take the exact opposite view for mathematics. Mathematics should not be grounded in logic, as doing so would unduly restrict the mathematician and hence block the road of inquiry. Admittedly, logic provides the mathematician with interesting material to work with, but so do the rules of chess, quantum mechanics, and the seven bridges of Königsberg, but mathematics does not *rely* on logic to make sure that its inferences are correct. As will be shown, the difference between mathematics and metaphysics regarding their relation to logic is a direct result of a difference in the kind of mistakes that are made in either discipline, which is in turn caused by the different nature of the disciplines in question. Metaphysics is a positive science, as is logic, but mathematics is not.

To substantiate and further explicate this, a wider exploration of the relations between mathematics, metaphysics, and logic is needed. To get a good sense of how Peirce understood the relation between metaphysics, logic, and mathematics, and how this differs from the received view, I will begin by briefly examining Auguste Comte's classification of the sciences, which can be considered representative of the standard view in Peirce's day, if not its paradigm. Next, I run quickly through Peirce's own classification, pointing in very general terms to some of the main differences with Comte. Having thus set the scene, I give a more detailed account of mathematics, logic, and metaphysics as they were understood by Peirce, together with their interrelations. This will show why metaphysics needs logic whereas

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mathematics does not, and will also show the different ways in which mathematics and logic each relate to metaphysics.

Comte's Classification

Peirce's classification of the sciences is in part a reaction to a division developed by Comte in his six-volume *Cours de philosophie positive* (1830–42). Weary of explanations in terms of unobserved and unverifiable causes, Comte restricted positive philosophy (or positive science) strictly to general descriptions of phenomena. As he put it in the *Cours,* "In the final, the positive state, the mind has given over the vain search after Absolute notions, the origin and destination of the universe, and the causes of phenomena, and applies itself to the study of their laws, — that is their invariable relations of succession and resemblance."[2]

Having thus defined positive philosophy, Comte next divides it into the abstract and the concrete sciences. Abstract sciences aim to discover the regularities (or laws) in the phenomena we encounter; concrete sciences inquire how these regularities can be applied to special cases. In Comte's scheme, the abstract sciences are mathematics, astronomy, physics, chemistry, biology, and sociology, with each subsequent science relying on the principles of those preceding it. Sociology, since it concerns relationships among biological entities, relies on the findings of biology; biology, since its objects are physical objects, relies on the findings of physics; and physics, dealing with objects that can be counted, sequenced, and measured, relies on the findings of mathematics.

By taking this approach, mathematics becomes a positive science for Comte. Like physics and biology, it too concerns itself purely with the description of phenomena. Geometry and mechanics, Comte writes, must "be regarded as true natural Sciences, founded, like all others, on observation, though, by the extreme simplicity of their phenomena, they can be systematized to much greater perfection" (PP 1:33). The phenomena studied by geometry and mechanics, Comte continues, "are the most general, the most simple, the most abstract of all, — the most irreducible to others, the most independent of them" (PP 1:33). Hence, geometry and mechanics should be placed first in the division of the sciences.

Comte's positioning of mathematics is due in part to his positivistic outlook and in part to his idea about what mathematics is essentially about. Mathematics, for Comte, is the science of measurement, or, since every measurement involves a translation into numbers, mathematics is the science of numbers. The object of mathematics, Comte explained in the *Cours,* is "the *indirect measurement of magnitudes, and it proposes to determine magnitudes by each other, according to the precise relations which exist between them*" (PP 1:38). Since mathematics is the most general theory of measurement, any study dealing with phenomena that can be measured depends on mathematics. As

Comte also put it, "There is no inquiry which is not finally reducible to a question of Numbers" (PP 1:42).

Comte divided mathematics into a concrete and an abstract branch. For Comte, the conclusions drawn in concrete mathematics still depend on the character of the objects examined, so that its conclusions vary when its objects vary. In contrast, in abstract mathematics the conclusions are wholly independent of the nature of the objects they apply to. For example, in concrete mathematics adding two raindrops together gives again a single raindrop, albeit one that is bigger. In contrast, in abstract mathematics one plus one always makes two, no matter whether we are adding raincoats or raindrops.

Comte's approach makes it crystal clear that for him mathematics is a positive or empirical science that considers in a most abstract manner the numerical relationships between phenomena. Given that the positive sciences have as their sole object the relations of succession and resemblance of phenomena, mathematics, which can precisely quantify these relations, becomes, as Comte put it, "the true basis of the whole of natural philosophy" (PP 1:32).

Peirce's Classification

For Peirce, and here he differs from Comte, mathematics is not a positive science. In fact, Peirce begins his own classification of the sciences by sharply distinguishing mathematics from the positive sciences. As with Comte, for Peirce the positive sciences are those that make pronouncements of matters of fact; that is to say, they bring to light results that no reasoning could have foreseen all by itself. Mathematics is not a positive science, for Peirce, because it is not at all concerned with saying anything about positive facts. Instead, it confines itself wholly to drawing necessary conclusions from entirely hypothetical constructions, without caring in the least whether these constructions apply to anything real.

The most basic of the positive sciences is philosophy, which Peirce separated from the specialized sciences. Whereas special sciences, like quantum physics or molecular biology, require specialized background knowledge or dedicated equipment, philosophy studies those aspects of reality that are readily available to everyone. Philosophy, Peirce wrote, "contents itself with a more attentive scrutiny and comparison of the facts of everyday life, such as present themselves to every adult and sane person, and for the most part in every day and hour of his waking life" (EP2:146). Philosophy requires no special equipment, techniques, or background knowledge.

Philosophy is subdivided into phenomenology (or phaneroscopy as Peirce preferred to call it), the normative sciences (aesthetics, ethics, and logic), and metaphysics. Phenomenology studies whatever comes before the mind when we perceive, reason, daydream, etc. The normative sciences relate what thus appears to an ideal. For aesthetics this ideal is beauty, for ethics it is goodness, and for logic it is truth. Metaphysics, finally, seeks to develop a general conception of the universe, which can then become a basis for the special sciences. Metaphysics, for Peirce, comes thus close to a *Weltanshauung*. Hence, for Peirce, *pace* Comte, metaphysics *is* a positive science. After metaphysics follow the special sciences, which Peirce generally divided into the physical sciences and the psychical sciences.

A question which then emerges is how mathematics relates to the positive sciences, including logic and metaphysics. Over the years, several attempts have been made to ground mathematics in logic, metaphysics, phenomenology, or even psychology. To capture the general gist of such attempts, let me briefly touch on some of them.

1. The idea that mathematics should be grounded in logic is the central thesis of logicism. In its strict interpretation, logicism purports that the axioms of mathematics can be deduced from a primitive set of purely logical axioms, so that mathematics is essentially a logical extension of logic. Sometimes a more modest claim is made, for instance, that only the theory of numbers is grounded in logic.[3] Also at an intuitive level, that is without having given the issue much thought, there seems to be a natural affinity with the idea that mathematics must be grounded in logic, as any mathematical demonstration appears to rely on logic for its validity.

2. Others believe that mathematics should be grounded in metaphysics, if only because *everything* is grounded in metaphysics. Metaphysics is often considered the study of first principles, or alternatively, the most abstract study of being. On the first interpretation mathematical principles are either *first* principles, and thus part of metaphysics, or they are *derived* principles, so that they are grounded in metaphysics. On the second interpretation, metaphysics is taken to be foundational to mathematics on the ground that the objects of mathematics are beings of a certain kind.

3. Still others favor grounding mathematics in phenomenology. If phenomenology is supposed to deal with whatever comes before the mind when we think, independently of whether its objects are real (as dream images are phenomena too), then it is hard to deny that the objects of mathematics are phenomenological objects of a certain kind.

4. Finally it has been argued that mathematics, which is a product of the mind, should be grounded in the special science of psychology, which studies how the mind works with all its capabilities and limitations. However, it could be argued that most who maintain psychologism in mathematics do so because they (tacitly) combine a psychologism in logic with the view that logic is foundational to mathematics, in which case this view would collapse into the first.

To examine how Peirce saw the relation between mathematics, logic, and metaphysics, I will first describe in some more detail what mathematics, logic, and metaphysics stand for in Peirce's view. However, as they all qualify as sciences for Peirce, a few words should be said about what science means for Peirce.

Science, for Peirce, is not defined in terms of a systematized body of knowledge, nor is it defined in terms of specific method called "the scientific method." Instead, it is defined in terms of a certain attitude, which is "the devoted, well-considered, life pursuit of knowledge; devotion to truth, not the truth as the man sees it, but the truth he is not yet able to see" (R 1126:08).[4] As Peirce put it elsewhere, science should not be defined as "a systematized collection of ascertained truths," but as "the scientific activities of its promotors" (R 17:06). Consequently, for Peirce, the word "science" refers to *any* activity one engages in from a genuine desire to find true answers to the questions one asks. In its more restricted sense, science refers only to those activities where truth is sought for its own sake. Thus, solving a murder can be considered a science, but it is not a science in the restricted sense, as its search for truth is subsumed to an ulterior aim, namely, to ensure that justice is done by discovering who committed the crime.

Mathematics

Peirce's starting point in defining mathematics is the definition given by his father, who defined mathematics as "the science which draws necessary conclusions."[5] Around 1895, Peirce said of the relation between this definition and his own, "The definition I here propose differs from that of my father only in making mathematics to comprehend the framing of the hypotheses as well as the deduction from them" (R 18:02). And it need hardly be said that framing hypotheses is a quite different affair than proving theorems.

Peirce's extended definition of mathematics fits in with his general conception of science as the activities of those who are guided by a desire to find true answers to the questions they ask. This means that, in a fundamental way, mathematics is what mathematicians do, and the activity of mathematicians is by no means confined to drawing necessary conclusions. Drawing necessary conclusions is at best only part of mathematics, and Peirce is keen to observe that there have been brilliant and influential mathematicians who were particularly poor in this area, and that powerful mathematical ideas remained long unproven or were based on proofs that later turned out fallacious or that were even known to be fallacious at the time.

In determining what mathematics is for Peirce, it may be advantageous to jump slightly ahead and see how it relates to the positive sciences. For Peirce, mathematics as a theoretical science trails behind mathematics as a practical science. Peirce phrases it thus in what appears an early draft of the *New Elements of Mathematics:* "The business of the mathematician lies with exact ideas, or hypotheses, which he first frames, upon the suggestion of some practical problem, then traces out their consequences, and ultimately generalizes" (R 188:02). Thus when the physicist, the meteorologist, or the economist is confronted with a complicated issue, the mathematician may be called in for help. It is then the task of the mathematician, Peirce argues, "to imagine a state of things different from the real state of things, and much simpler, yet clearly not differing from it enough to affect the *practical* answer to the question proposed" (R 165a:67; emphasis added). Mathematics thus furnishes the scientist with a skeleton model that can be considered representative for the phenomenon being studied, and instead of studying the phenomenon with all its fortuitous detail, it suffices to study the model instead.

By defining science in terms of the activities of its promoters, Peirce's division of the sciences largely comes down to a division of labor. This attitude toward science enables Peirce to argue that it is the mathematician who is best equipped to translate the more loosely constructed theories about groups of positive facts generated by empirical research into tight mathematical models:

> The results of experience have to be simplified, generalized, and severed from fact so as to be perfect ideas before they are suited to mathematical use. They have, in short, to be adapted to the powers of mathematics and of the mathematician. It is only the mathematician who knows what these powers are; and consequently the framing of the mathematical hypotheses must be performed by the mathematician. (R 17:06f)

Now what constitutes a well-equipped mathematician? The three mental qualities that in Peirce's view come into play are imagination, concentration, and generalization. The first is, as Peirce put it, "the power of distinctly picturing to ourselves intricate configurations"; the second is "the ability to take up a problem, bring it to a convenient shape for study, make out the gist of it, and ascertain without mistake just what it does and does not involve"; the third is what allows us "to see that what seems at first a snarl of intricate circumstances is but a fragment of a harmonious and comprehensible whole" (R 252:20).[6] In particular the power of generalization, which Peirce believes "chiefly constitutes a mathematician" (R 278a:91), is a difficult skill to attain. Peirce's emphasis on imagination, concentration, and generalization draws the attention away from the notion that it is the premier business of mathematics to provide proofs.

Having thus delineated how mathematical models come to be and given some insight into the mathematical mind-set, we can characterize pure mathematics, as does Peirce, as "the exact study of ideal states of things" (R 165a:68).[7] That is to say, the practical motives that spurred the inquiry have been removed and all energy is directed to a study of the models themselves, irrespective of any relation they might have to anything external to them, and irrespective of any motives the inquirer might have other than studying the models entirely for their own sake. Generally, pure mathematics favors those models from which a great body of deductions can be drawn (R 14:29). In particular the models inspired by the acts of counting money and measuring land blossomed into large bodies of pure mathematics.

As the discussion of the relation between mathematics and the positive sciences shows, phenomena encountered in the sciences are an important source for mathematical notions and theories. More generally, we can say that, for Peirce, it is experience that furnishes mathematicians with their ideas. Take the mathematical conceptions of "surface," "line," "point," "right line" (which Peirce also terms "ray"), and "plane." According to Peirce,

> A geometrical *surface* is a *place* thinner than any goldleaf. It is a mere cleft as between a submerged stone and the water about it. A geometrical *line* is a place finer than any spiderline. It is a mere crack as between a partially submerged stone, the air, and the water. A geometrical *point* is a place smaller than any needle point, as between four bodies which fit perfectly together. . . . A line which viewed from the end is foreshortened so as to cover but one point in the sky is a *right line* or *ray*. . . . A surface which fits upon itself however much it be slid or turned about, and even when reversed, is called a *plane*. (R 94:56)

Hence, for Peirce, mathematical objects are derived from experience and precisely defined. At the same time, and this in contrast to the positive sciences, mathematics doesn't concern itself with positive facts. Instead the mathematician sets up as it were imaginary worlds, and shows how within these worlds certain inferences can be made, using rules that hold within them. In the end the mathematical hypothesis "is a pure mental creation involving no assertion about any thing but the mathematician's idea, — his dream, as it might be called, except for its precision, clearness, and consistency" (R 17:07).

Logic

Logic, for Peirce, deals with factual issues, most prominently with the question whether, as a matter of fact, the premises can be true and the conclusion false at the same time. Within the context of scientific inquiry — that is, when we want to find something out — this gets a normative ring to it. We approve of those theories where it is impossible, or unlikely, that the premises are true and the conclusion false, and we disapprove of those theories where there is a significant likelihood that true premises coexist with a false conclusion. Since how we inquire is a product of voluntary choice, this approval or disapproval is a *moral* approval or disapproval.[8] In fact, Peirce considers it the great advantage of reason, as opposed to instinct, that reason is self-critical and falls within the scope of self-control. Put very briefly, we have no instincts that we instinctively distrust, but there is reasoning that reasoning itself condemns (R 832:02). Since logic distinguishes between what it approves and what it condemns it divides propositions essentially by dichotomy into those that are good and those that are bad. Given that the purpose of logic is to represent something, this translates into a division between the true (good representation) and the false (bad representation).

Because logic studies good methods for discovering positive truth by means of reasoning, it does not confine itself to the study of deduction only, but also includes the study of induction and abduction. In fact, deduction, though often heralded as the core of logic or the paradigm of inference because it generates conclusions that are certain, cannot by itself generate any *positive* knowledge, making the study of induction and abduction of crucial importance for any logic that purports to be foundational to either metaphysics or the special sciences.

To conclude, for Peirce, logic is a *normative* science. It is, as Peirce called it, "the science of the principles of how thought ought to be controlled, so far as it may be subject to self-control, in the interest of truth" (R 655:26, 1910). The clause "in the interest of truth" is crucial, as it indicates that, in contrast to mathematics, logic is subservient to reality, and it is precisely because of this that it can be a normative science. Moreover, qua normative science, logic is not fundamentally different from, say, the science of archery; i.e., the study of how to hold a bow and arrow to maximize the chance of hitting a target. Whereas the science of archery studies the art of archery, the science of logic studies the art of reasoning. Of course, the *study* of the art of reasoning should not be confused with the art of reasoning itself. And what is more, the *study* of the art of reasoning may not even improve one's ability to reason. In fact, Peirce is quite skeptical about the virtues of studying logic. He put it bluntly in his 1894 logic book "How to Reason": "No science can compare with logic for the smallness of the minds it has produced" (R 413, p. 239).

The Relation between Mathematics and Logic

Given the views of mathematics and logic described above, the question arises how the two are related. Prima facie three viable options present themselves: First, the laws of logic are derived from mathematics. Second, and the reverse of the first, all mathematics is ultimately derived from the laws of logic. Third, mathematics relates to logic in the same way as mathematics relates to the other positive sciences.

The view that the laws of logic can be derived from mathematics is difficult to defend, as mathematics and logic — at least on Peirce's interpretation — are disciplines of a radically different nature. Whereas logic studies how we should reason to make our thoughts conform with positive truth, mathematics is very decidedly not interested in positive truth. This makes it difficult to see how mathematics can ever become the foundation of logic. One could retort that logical arguments must at least be a subset of mathematical arguments, namely those arguments that concern themselves with real facts rather than mere possibilities, but this requires that one shows how this subset can be differentiated from the wider field of mathematics, and this can be done only by introducing *ad hoc* precisely what distinguishes logic from mathematics. It cannot be *derived* from any mathematical principles, as of its very nature mathematics cannot differentiate between the positive and the possible. Moreover, such an approach would go against the grain of how Peirce conceived of mathematics in the first place, namely as an idealization of certain aspects and relations of the phenomena we encounter. Hence, as far as positive facts go — and note that it is only through its pronouncements that have a bearing upon *positive* facts that mathematics could be foundational to the *positive* science of logic — mathematics is a derivative science rather than a foundational one.

Can we then say the reverse, that all mathematics can be derived from logical principles? This is the logicist view. Peirce rejects this second option as well. It would make mathematics a positive science, and there is no pressing reason why we should restrict mathematics to what can be derived from principles that are designed to guarantee as good as possible that our ideas represent *positive* truth. This second option is especially problematic since Peirce repeatedly insists that positive truth is not what mathematics is after.

This brings us to the third option, which I take to be Peirce's view. Here the relation between logic and mathematics is no different from how the other positive sciences relate to mathematics. As I noted above, the special sciences turn to mathematics to replace complicated issues with mathematical models that are simpler but still representative. It is in this manner that the mathematical branches of sciences like physics, chemistry, and economics arose. Hence, like the physicist and the economist, also the logician goes to the mathematician for help. The mathematician takes up the material the logician brings to him and seeks to transform it into an ideal state of things, removing all that is accidental and replacing complicated relations with simpler ones that, though false, are adequate to the issue in question. Next, the mathematician studies the ideal state that thus ensued to see what is true in it. The mathematician goes even beyond this by tweaking certain features of this ideal state to see where it leads. A classic case of such tweaking, albeit not in logic but in arithmetic, is the bold idea that the square root of minus one does have a definite solution, which led to the fascinating and also fruitful conception of imaginary numbers.

Peirce's approach to mathematics clearly shows where logicism goes in the wrong. Defenders of logicism typically refer to formal, or mathematical, logic as the source of the most basic principles of mathematics. But, and this might sound odd at first, mathematical logic isn't really logic, just as mathematical economics isn't really economics; it's a branch of applied mathematics. Mathematical economics doesn't concern economic phenomena, but it concerns a set of axioms and permissible rules of inference that form a clearly false but highly advantageous representation of specific issues faced by the economist. For instance, traditional economic models happily assume a state where a large number of narrowly selfish atomic individuals act in markets that are wholly transparent even though they contain such a large number of actors that none of them can deliberately influence the price of any good. As everyone knows, economies don't really work like this. Something similar holds for mathematical logic. Material implication, to give one clear example, is about as far removed from how people *should* use conditionals as frictionless surfaces are from actual surfaces, and as transparent markets are from actual situations of supply and demand. Mathematical logic, like mathematical physics and mathematical economics, deals with ideal states that are helpful to logicians, just as the mathematical contributions to physics and economics are helpful to physicists and economists. The study of the models thus arrived at for its own sake, experimenting with them and generalizing them without any concern for positive fact, subsequently brings us into the realm of pure mathematics. In brief, mathematical logic is a *mathematical* theory. Hence, grounding mathematics in mathematical logic, far from grounding mathematics in logic, grounds mathematics in nothing other than itself. Put briefly, logicism is a fallacy caused by a confusion of logic with mathematics, or of an equivocation of logic with mathematical logic.

This account of the relation between logic and mathematics points at the interesting question whether mathematicians can do without logic. We clearly reason in mathematics and the demonstration of theorems is an integral part of mathematics, albeit by no means the whole of it, hence it seems natural that mathematics is grounded in logic. Before addressing this issue it is useful to distinguish between *logica utens* and *logica docens*. The first refers to our natural ability to reason; the second, which is also called acquired logic, refers to the study of correct rules of inference. Mathematics clearly depends on our natural ability to reason, as we do reason in mathematics. The true question is whether mathematics can do without a *logica docens* — that is, without a separate theory on correct rules of inference.

Peirce's answer runs roughly as follows. *Logica docens* comes in where our reasoning might be mistaken, as this gives us a motive for inquiring how the mistake could have been made. We can use the result of this inquiry to determine how to correct the mistake or prevent it from happening in the future. Consequently, we must ask how mistakes are made in mathematics and see whether they call for a *logica docens.*

Though mathematicians are in the business of drawing necessary conclusions, this doesn't mean no mistakes are made in mathematics. When two people are asked to total a long list of large numbers, it is not just possible but it is all but certain that mistakes creep in and that you get two different answers, both probably wrong. It is important to observe, however, what type of mistakes these are. They are mistakes for which the calculators themselves will readily admit that they are wrong the moment these mistakes are brought to their attention. What is more, in cases like this it is the very rules of arithmetic and not the discovery that some principle of logic has been violated, that convinces the calculator he has made a mistake and shows how to correct it. We can even go a step further and say that bringing in logic in cases like this is both unenlightening and counterproductive. It is counterproductive for the same reason that burdening the mind of the archer with the laws and theories of analytical mechanics is detrimental to his aim. It distracts without adding much. Because mistakes like the above are the type of mistakes one encounters in mathematics, mathematicians need no *logica docens.* Mistakes in mathematics are always confined to the models where they occur, and the way to correct them is to do more mathematics. Mathematicians do not need an independent science of logic to ensure that their demonstrations are correct. As we shall see, the situation is quite different for metaphysics.

Scientific Metaphysics

While astutely aware that metaphysics is in a dismal shape, Peirce rejected Comte's radical denial of it. For Peirce, the issue is not whether we should have a metaphysics — as everyone has a metaphysics whether they want to or not — but whether we want to keep our metaphysics unconscious or bring it out in the open where it can be subjected to the same scrutiny as our scientific work. For instance, instead of blindly assuming that there are individuals — as so many metaphysics bashers do — we must ask the very basic question whether there can be any strictly individual existence, and treat this question with the same care as, say, the question whether two bodies with a different mass will accelerate at a different rate when dropped.

Peirce agreed with Comte that in its current state metaphysics is detrimental to science, but, for Peirce, the main culprit is not *doing* metaphysics, as Comte had argued, but doing metaphysics unscientifically or not doing metaphysics at all. It is bad metaphysics that is bad for science, and it is only by *doing* metaphysics that bad metaphysics can be avoided. As Peirce keenly observed,

> It is when [scientists] promise themselves that they will not make any metaphysical assumptions that they are

> most in danger of slipping too deep into the metaphysical slough for deliverance, precisely because one cannot exercise control and criticism of what one does unconsciously. (CP 2.121)

As Peirce conceived it, metaphysics studies "the most general features of reality and real objects" (CP 6.6). Consequently, metaphysics deals with positive facts. It is, moreover, an observational science — one that differs from the special sciences only in that it studies the "kinds of phenomena with which every man's experience is so saturated that he usually pays no particular attention to them" (CP 6.2). The aim of metaphysics is a *Weltanschauung* that can become a roadmap for the special sciences by developing a general system in which all possible facts can be given a place.

Metaphysics, Logic, and Mathematics

Peirce's conception of metaphysics shows why he put it after phenomenology and logic in his classification of the sciences. First, as an observational science that deals with positive facts, metaphysics must be grounded in phenomenology. Second, as a science that requires the exercise of self-control and self-criticism while dealing with the positive facts that constitute its field of study, metaphysics must be grounded in logic. This reliance on logic comes down largely to the view that, like physics or renaissance studies, metaphysics should be studied with the scientific attitude. For instance, in doing metaphysics we should obey the important principle never to settle a priori what can conceivably be settled by experience. The routine violation of this principle has made metaphysics a discipline that hampers science rather than helps it, and has given metaphysicians such a bad rap among scientists.

Before elaborating upon the relation between metaphysics and logic, something should be said about its relation to mathematics, keeping the above distinction between logic and mathematics in mind. Historically there has been a strong connection between the two, in that new conceptions in metaphysics often follow new discoveries in mathematics or are adaptations from mathematical concepts. As Peirce put it concisely and repeatedly, "metaphysics is the ape of mathematics." For instance, metaphysics was long modeled after the first book of Euclid, by taking the view that all its results could be attained seamlessly from a handful of simple axioms. Spinoza's *Ethics* is a strong case in point and Peirce's evaluation of it clearly reveals Peirce's differing conception of mathematics and its relation to metaphysics:

> His *Ethics* [is] drawn up in theorems, with demonstrations which have always furnished a laughing-stock to mathematicians. But you must

> penetrate beneath these if you would enter the living stream of Spinoza's thinking. You then find that he is engaged in a somewhat mathematical style in developing a conception of the absolute, strikingly analogous to the metrical absolute of the mathematicians. He thus appears as a mathematical thinker, not in the really futile, formal way in which he and his followers conceived him to be, but intrinsically, in a lofty, living, and valuable sense.[9]

For Peirce, metaphysics, the conclusions of which in contrast to the special sciences remain largely unchecked by experience, relies heavily on necessary inferences to prevent it from going astray. Consequently, metaphysics relies heavily on mathematics, as mathematics is precisely the science specializing in drawing necessary inferences. As Peirce's discussion of Spinoza reveals, however, mathematics should be understood broadly. As shown before, on this broad interpretation the mathematician's main business is not so much the production of proofs, but the simplification of complicated sets of facts by reducing them to a shape that facilitates their study while still being representative. For this one needs recourse to the three key mental qualities Peirce associated with doing mathematics: imagination, concentration, and generalization. Demonstration, often heralded as the whole intent of mathematics, is for Peirce "but the pavement on which the chariot of the mathematician rolls."[10] In Peirce's view, Spinoza belonged to those philosophers who believe that mathematics is all about rigid demonstration, and this is why he charged that Spinoza failed to rightly interpret his own philosophy.[11] For Peirce, the relation between mathematics and metaphysics is no different from how other positive sciences relate to mathematics. Metaphysicians go to mathematicians for help just as logicians, economists, and astrophysicists do.

Having discussed the relation between metaphysics and mathematics, we can now address its relation to logic. That metaphysics purports to speak of the real world is of crucial importance. A moment ago it was argued that metaphysicians benefit from mathematics in that it helps them keep their theories on the right course by utilizing the mathematician's expertise in modeling facts in such a manner that optimal use is being made of necessary reasoning. However, precisely because metaphysics is a positive science and mathematics is not, this is not enough. Often the mistakes made in metaphysics are different from the kind found in mathematics, in that they are not such that they can be rectified simply by examining whether the rules that came with the model were applied correctly. Often the issue is whether certain theories or models correctly apply to the positive facts under consideration. The rules that come with the model are of little use here as they concern valid moves within the model, not its relation to external fact. Moreover, our instinctive *logica utens,* which is generally such a good guide in practical affairs, is too far removed here from its natural environment to be trustworthy. Hence, we cannot rely on our *logica utens* when trying to decide whether a metaphysical theory adequately represents the phenomena the metaphysician is interested in. Consequently, and this in contrast to mathematics, metaphysics needs a *logica docens* — i.e., a separate science on right reasoning — to keep it in check, and it needs it desperately. Without it, the metaphysician is, as Peirce puts it, "like a ship in the open sea, with no one on board who understands the rules of navigation" (CP 5.368). Metaphysics, Peirce concludes, must consist in "the interpretation of the facts of common experience in the light of a scientific logic"; that is, a *logica docens* (R 472:04, 1903). In short, whereas mathematics does not need logic when it is doing its business, metaphysics cannot do without it.

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NOTES

1. An earlier version of this paper was read at "'Peirce-pectives' on Metaphysics and the Sciences," Virginia Tech, April 23–25, 2004, and I would like to thank the participants in this conference, as well as Cassiano Rodrigues, for their most valuable comments.

2. *The Positive Philosophy of Auguste Comte.* 2 vols., Harriet Martineau (trans.), (London: John Chapman, 1853), pp. 1:2. Hereafter referred to as PP.

3. See e.g. Susan Haack, "Peirce and Logicism: Notes Towards an Exposition" *(Transactions of the Charles S. Peirce Society* 29 [1993]: 33–56), who ascribes to Peirce a moderate logicism, as well as Nathan Houser's reply, "On 'Peirce and Logicism' A Response to Haack" *(op. cit.,* 57–67*).*

4. All references to Peirce's Harvard manuscripts are by Robin catalogue number followed by a page number assigned by the Institute for Studies in Pragmaticism at Texas Tech University in Lubbock. The number reflects fairly accurately the order of the manuscripts as they can be found on *The Charles S. Peirce Papers,* microfilm, 33 reels (Cambridge, Mass.: Harvard University Library, 1963–70). See also Richard S. Robin, *Annotated Catalogue of the Papers of Charles S. Peirce* (Amherst, University of Massachusetts Press, 1967).

5. Benjamin Peirce, *Linear Associative Algebra* (Washington, D.C.: 1870), §1.

6. The three mental powers roughly conform to how Peirce's categories work out with respect to ideas. As Peirce put it in "The Law of Mind": "Three elements go to make up an idea. The first is its intrinsic quality as a feeling. The second is the energy with which it affects other ideas . . . The third element is the tendency of an idea to bring along other ideas with it" (CP 6.135, 1892).

7. Peirce incidentally believed that this conception of mathematics agrees with his father's definition: "In 1870, Benjamin Peirce defined mathematics as

'the science which draws necessary conclusions.' Since it is impossible to draw necessary conclusions except from perfect knowledge, and no knowledge of the real world can be perfect, it follows that, according to this definition mathematics must exclusively relate to the substance of hypotheses" (R 15:11f).

8. Charles S. Peirce, *Collected Papers,* 8 vols., Charles Hartshorne, Paul Weiss, and Arthur Burks (eds.), (Cambridge, Mass., Harvard University Press, 1931–1958), 5.130. Hereafter referred to as CP.

9. Charles S. Peirce, *Contributions to* The Nation, 4 vols., compiled and annotated by Kenneth L. Ketner and James E. Cook (Lubbock: Texas Tech Press, 1975–81), 2:86, 1894.

10. As is reported by secretary Thomas S. Fiske, Peirce dramatically made this point during a presentation at the 24 November 1894 meeting of the American Mathematical Society:

> At a meeting of the Society in November 1894 in an eloquent oration on the nature of mathematics, C.S. Peirce proclaimed that the intellectual powers essential to the mathematician are "Concentration, imagination, and generalization." Then, after a dramatic pause, he cried "Did I hear some one say demonstration?" Then, after a dramatic pause, he cried "Did I hear some one say demonstration?" "Why, my friends," he added, "demonstration is but the pavement on which the chariot of the mathematician rolls."

Quoted in Raymond Clare Archibald, *A Semicentennial History of the American Mathematical Society, 1888–1938* (New York, 1938), p. 7.

11. Cf. *Contributions to* The Nation, *op. cit,* 1:164, 1892.