A Note on Plural Logic

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Abstract: A distinction is introduced between *itemized* and *non-itemized* plural predication. It is argued that a full-fledged system of plural logic is not necessary in order to account for the validity of inferences concerning *itemized* collective predication. Instead, it is shown how this type of inferences can be adequately dealt with in a first-order logic system, after small modifications on the standard treatment. The proposed system, unlike plural logic, has the advantage of preserving completeness. And as a result, inferences such as 'Dick and Tony emptied the bottle, hence Tony and Dick emptied the bottle' are shown to be first-order.

Keywords: compound terms, itemized predication, singular logic, plural logic.

1 Introduction

(1) Dick and Tony emptied the bottle. Hence, Tony and Dick emptied the bottle.

Formalize and prove first-order validity.

It sounds like a routine exercise of a Logic course. But:

(a) It does not correspond to:

$E(d,b)$ & $E(t,b)$ $\therefore$ $E(t,b)$ & $E(d,b)$

since Dick and Tony's joint emptying of the bottle is not equivalent to a pair of emptyings, one by Dick and one by Tony.

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(b) Using a 3-place predicate letter F for 'x and y emptied z', we get

$F(d,t,b) \therefore F(t,d,b)$

which is not a valid inference of first-order logic.[1]

(c) If we then put e.g. G_1 for 'x is a emptying event', G_2 for 'y was emptied in event x' and G_3 for 'z and w were the ones who emptied in event x', we will obtain

$\exists x\,(\,G_1(x)\,\&\,G_2(b,x)\,\&\,G_3(d,t,x)\,) \therefore \exists x\,(\,G_1(x)\,\&\,G_2(b,x)\,\&\,G_3(t,d,x)\,)$

which is not a first-order inference either.

(d) If we use the set $\{d,t\}$ we will come to

$E(\{d,t\},b) \therefore E(\{t,d\},b)$

which is a correct inference of set theory, but again, not of pure first-oder logic.[2]

(e) And finally, using the so-called 'plural logic' we can render (1) as:

$$\exists xx\,(\,E(xx,b)\,\&\,d \prec xx\,\&\,t \prec xx\,\&\,\forall y\,(\,y \prec xx \rightarrow y = d \vee y = t\,)\,)$$
$$\therefore\ \exists xx\,(\,E(xx,b)\,\&\,t \prec xx\,\&\,d \prec xx\,\&\,\forall y\,(\,y \prec xx \rightarrow y = t \vee y = d\,)\,)$$

i.e.: 'There are some things *xx* which emptied the bottle, and only Dick and Tony are among them. Hence, only Tony and Dick are among them'. Plural logic is a recent system of non-classical logic which allows quantification over plural variables (like '*xx*'), intended to stand for various objects at the same time. The point of plural logic is to take plural expressions of natural language at face value, instead of singularizing them into sets, groups, or aggregates of any kind.

Within plural logic, inference (1) obtains as a consequence of the 'indiscernibility' (or 'extensionality') axiom for pluralities.[3] However, al-

[1] Besides obscuring the fact that the activity denoted by the 2-place letter E and the 3-place letter F is essentially the same.

[2] Besides the dubious implication that the bottle would have been emptied by a set (cf. Boolos 1984, 72, one of the pioneering works of plural logic).

lowing quantification over plural variables is not without cost. Plural logic is a much stronger theory than classical first-order logic, and in particular, it was soon discovered to be not recursively axiomatizable.[4]

So we are left again with our initial question: is (1) not a first-order inference, after all?

2 The Logical Analysis of Plural Predication

A plural predication is that in which a predicate is attributed to a plurality of subjects. It is called 'distributive' when the predicate applies to each of the subjects of the plurality taken one by one. For example, 'Dick and Tony are fair-haired', which is tantamount to saying that Dick is fair-haired and Tony is fair-haired. And a plural predication is called 'collective' when the predicate only applies to the subjects of the plurality taken collectively. For example, in the case of inference (1), just seen.[5]

[3] Burgess (2004, 197); McKay (2006, Ch. 6); Linnebo (2009, §1.2).

[4] Burgess (2004, 220); Oliver – Smiley (2006, 318); Yi (2006, 257).

[5] We often find the collective/distributive distinction formulated as a differentiation between the predicates themselves, as if each predicate carried within its meaning an indication about whether it should be taken collectively or distributively, when applied to a plural subject. According to such a view, a predicate *P* would be distributive when it is an analytic truth that *P* applies to a plurality of things if and only if *P* applies to each of them; and it would be collective otherwise (cf. Oliver – Smiley 2006, 322; and similarly in Linnebo – Nicolas 2008, 188; Linnebo 2009, §1.1). However, we have to take into account that many predicates admit both readings. For example, 'lifted the table' in the sentence 'Dick and Tony lifted the table', can be interpreted either collectively – they did it together – or distributively – they did it one after the other. In cases like this the predicate in question would have to be regarded as a pair of different predicates, which is very artificial. And on top of it all, such 'mixed' or ambiguous predicates turn out to be the most common ones in English (Winter 2002, 495; Yi 2005, 481).

Hence, it seems much more convenient to conceive the collective/distributive distinction, as depending on the way in which the predicate applies to the plural subject in order to form the statement in question. On this other view, the ambiguity of a sentence such as 'Dick and Tony lifted the table' is regarded as a purely structural ambiguity, just as a sentence of the form 'p and q or r' is ambiguous between its two possible readings, '$(p \,\&\, q) \vee r$' and '$p \,\&\, (q \vee r)$'. The collective/distributive distinction should therefore be seen as dwelling on the structure of the predication, rather than on the nature of the predicate used (a similar view is

A much lesser known distinction is that between *itemized* and *non-itemized* plural subjects. We shall say that the subject of a predication is an *itemized* (or *individualized*) plural subject, when each of the elements which make up the plurality is individually referred to, or at least the exact number of individuals which compose it is given. For example: 'Dick and Tony emptied the bottle', 'Dick and Tony are fair-haired', 'Dick and somebody else emptied the bottle', 'two people emptied the bottle', or 'two people are fair-haired'.

The opposite case is that of *non-itemized* (or *non-individualized*) plural subjects, in which there is no indication as to which or how many individual members make up the plurality in question. Examples of non-itemized plural subjects are: 'various people emptied the bottle', 'various people are fair-haired', 'Dick and some other people emptied the bottle', and 'Dick and some other people are fair-haired'.[6]

The difference between itemized and non-itemized plural predication suggests a more general distinction, between *singular logics* and *plural logics*. *A singular logic is that in which the individual* (*the singular object*) *is the only vehicle to convey reference, predication and quantification. A plural logic is one which admits genuine reference, predication and quantification over a plurality of objects, without reducing the plurality to a composite unit of any kind*. Of course, all plural logic proposals advanced so far are 'plural logics' in this sense. And standard first-order logic is, in this sense, a 'singular logic' beyond any doubt.

Moreover, it is clear that standard first-order logic does not enable us to account for the logical validity of inferences regarding itemized collective predication, as we have seen with the case of inference (1). But in fact only a few modifications are needed to patch this up. The resulting system will be a singular logic in the sense just given – in fact, almost a variant of first-order logic as standardly formulated. And it will preserve recursive axiomatizability.

defended in Moltmann 1997, 52; Cameron 1999, 129; Yi 2005, 481; McKay 2006, Ch. 1, Note 15; Fernández Díez 2006, 42).

6 This includes of course pluralities described by means of generalized quantifiers (such as 'most', 'nearly all', 'more than ten', etc), unless the description happens to be equivalent to stating an exact number of elements for the plurality in question (as in 'more than four and less than six', for instance, which can only come out to five).

Therefore, such a system will be a considerable simpler and weaker logical theory than plural logic. And consequently, if it does suffice for the logical analysis of itemized plurals, it will serve as evidence for the fact that the logical status of itemized plural predication is deep down different – essentially simpler – from that of its non-itemized counterpart.

This is by no means arguing against the pertinence of plural logic, which has its natural field of action in the treatment of non-itemized pluralities and non-itemized predication – for which it is the natural option. The only thing I would like to suggest is that *the introduction of plural logic systems should not be motivated by examples of itemized collective predication.*[7]

3 The Logical Status of Predication over Itemized Plurals

Inference (1) shows a typical feature of itemized plural predication: in the absence of further indication, the order in which the subjects are listed is immaterial. This occurs not only with collective predication, but with distributive predication as well. For example:

(2) Dick and Tony are fair-haired. Hence, Tony and Dick are fair-haired.

Another feature of plural predication, both collective and distributive, is the immateriality of repetitions within the subjects listed:

(3) Dick and Tony emptied the bottle. Hence, Dick, Dick, and Tony, emptied the bottle.

[7] As it often occurs, e.g.: McKay (2006, Ch. 1): "*Non-distributive predication*. Standard first-order logic does not provide adequate resources for properly representing many ordinary things that we say: (1) Arnie, Bob and Carlos are shipmates". Yi (2005, 460): "The Fregean systems, Frege's system and its descendants cannot deal with the logic of the *plural constructions* (in short, *plurals*) of natural languages, such as the following: [P1] *Venus and Serena* are tennis players, and *they* won a U. S. Open doubles title". Such examples are cases of itemized plural predication, and they are easily treatable by means of the system which I will describe here.

(4) Dick and Tony are fair-haired. Hence, Dick, Dick, and Tony, are fair-haired.[8]

Another feature of plural predication, both collective and distributive, is substitutivity *salva veritate*:

(5) Dick and Tony emptied the bottle. Tony is Iron Man. Hence, Dick and Iron Man emptied the bottle.

(6) Dick and Tony are fair-haired. Tony is Iron Man. Hence, Dick and Iron Man are fair-haired.

And another is existential quantification over some of the subjects of the plurality in question:

(7) Dick and Tony emptied the bottle. Hence, Dick and somebody else emptied the bottle.

(8) Dick and Tony are fair-haired. Hence, Dick and somebody else are fair-haired.

I claim all inferences (1) to (8) are logical inferences. As a matter of fact, the even ones correspond to cases of distributive predication, and can be easily accounted for in standard first-order logic by means of the usual propositional disjunctions. Indeed (using H for 'being fair-haired', and i for Iron Man):

(2a) $H(d) \,\&\, H(t) \therefore H(t) \,\&\, H(d)$
(4a) $H(d) \,\&\, H(t) \therefore H(d) \,\&\, H(d) \,\&\, H(t)$
(6a) $H(d) \,\&\, H(t)\ ;\ t = i \therefore H(d) \,\&\, H(i)$
(8a) $H(d) \,\&\, H(t) \therefore \exists x\ (\ H(d) \,\&\, H(x)\ )$

However, the odd inferences of this list ((1), (3), (5) and (7)) correspond to cases of collective predication, and cannot be adequately treated in standard first-order logic. To be sure, inferences (5) and (7) could be validated by the introduction of the 3-place predicate letter F mentioned in §1, but this does not work for (1) and (3).[9]

[8] Assuming Dick being the same all along.

[9] Besides the awkward implication that e.g. in 'It was Dick and Tony who emptied the bottle, and not Charles' we would have to use two different predicate letters for 'emptied', thus: $F(d,t,b) \,\&\, {\sim}E(c,b)$ (cf. Note 1 again).

I will now indicate how first-order logic can be adapted so as to cope with these and similar inferences in a natural way. The resulting system could be called 'first-order logic for itemized collections', or 'first-order logic for itemized collective predication'.

4 First-order Logic for Itemized Collections: Syntax

To the usual individual terms of standard first-order logic, we add *compound terms*. A compound term will be a discrete and linear list of terms between square brackets, which behaving like a unit, acts itself as a new term.[10] Hence we will have:

(a) an individual variable is a term;
(b) an individual constant is a term;
(c) if r_1 , …, r_n are terms, then $[r_1, \ldots, r_n]$ is a term.[11]

Terms formed by clauses (a) and (b) are the usual individual terms; terms formed by clause (c) will be called 'compound terms'.

The inductive structure of the definition enables us to accommodate nested structures of itemized collections. Like for example: 'Dick and Hannah, together with Tony and Grace, introduced John and Alice to swinging':

(9) $I\,(\,[[d,h],[t,g]]\,,\,[j,a]\,)$[12]

Next, the usual definition of formula does not need to be changed. In particular, an atomic formula $P(r_1, \ldots, r_n)$ can be simply defined as

[10] Just as in 'Dick and Tony emptied the bottle', the phrase 'Dick and Tony' takes the argument place of the one who empties, with respect to the 2-place relation '*x* emptied *y*'.

[11] I take a language without descriptions and function symbols, for simplicity.

[12] Where predicate letter *I* stands for the 2-place relation '*x* introduced to swinging to *y*'. In this example both places of the relation are filled up by compound terms, the first of which is made up itself of two compound terms. Such nested pluralities are not normally accepted in plural logic treatments, although some authors have advocated for them (Rayo 2006; Linnebo – Nicolas 2008). Our treatment covers them, but only in the context of itemized collections.

an n-place predicate letter P followed by n terms. And quantification will be allowed – as usual – over individual variables only.[13]

As for the deductive system, the usual quantification rules (or axioms) will enable us to validate inference (7): 'Dick and Tony emptied the bottle. Hence, Dick and someone else emptied the bottle':

$$E\ ([d,t]\ ,b\ )\ \therefore\ \exists x\ \ E\ ([d,x]\ ,b\ )$$

And in order to cover the other odd inferences of our list (that is, inferences (1), (3) and (5)), we introduce a *Rule for the Indiscernibility of Compound Terms* (*RICT*), according to which two pluralities are equal if and only if they have exactly the same elements:

$$\frac{[\ r_1,\ \ldots,\ r_n\ ] = [\ s_1,\ \ldots,\ s_m\ ]}{\overline{(r_1 = s_1 \vee \ldots \vee r_1 = s_m)\ \&\ \ldots\ \&\ (r_n = s_1 \vee \ldots \vee r_n = s_m)\ \&\ (s_1 = r_1 \vee \ldots \vee s_1 = r_n)\ \&\ \ldots\ \&\ (s_m = r_1 \vee \ldots \vee s_m = r_n)}}$$

the double horizontal line meaning that the rule can be used in both directions, downwards and upwards. In other words: that $[r_1, \ldots, r_n]$ equals $[s_1, \ldots, s_m]$ if and only if every one of $r_1, \ldots, r_n$ equals at least one of $s_1, \ldots, s_m$, and vice versa.[14,15]

Indeed, applying RICT (in the upward direction) we obtain:

$$\frac{\dfrac{d = d \qquad t = t}{(\ d = t \vee d = d\ )\ \&\ (\ t = t \vee t = d\ )\ \&\ (\ t = d \vee t = t\ )\ \&\ (\ d = d \vee d = t\ )}}{[d\ ,\ t\ ] = [\ t\ ,\ d\ ]}$$

13 Recall that our aim is that the proposed system constitutes a singular logic in the sense given in §2. Hence the basic vehicle to convey reference, predication and quantification must only be the individual object.

14 In the form of an axiom:

$$[r_1, \ldots, r_n] = [s_1, \ldots, s_m] \leftrightarrow ((r_1 = s_1 \vee \ldots \vee r_1 = s_m)\ \&\ \ldots\ \&\ (r_n = s_1 \vee \ldots \vee r_n = s_m)\ \&\ (s_1 = r_1 \vee \ldots \vee s_1 = r_n)\ \&\ \ldots\ \&\ (s_m = r_1 \vee \ldots \vee s_m = r_n))$$

15 Rule RICT is a sort of weaker counterpart of the indiscernibility axiom for full plural logic systems (which we mentioned in §1), adapted to the case of itemized pluralities.

from which inference (1) follows: E ($[d,t]$, b) $\therefore$ E ($[t,d]$, b).[16]
By a similar application of RICT we get:

$$\frac{\dfrac{d = d \qquad t = t}{(d = d \vee d = d \vee d = t) \ \& \ (t = d \vee t = d \vee t = t) \ \& \ (d = d \vee d = t) \ \& \ (d = d \vee d = t) \ \& \ (t = d \vee t = t)}}{[\, d \,,\, t \,] = [\, d \,,\, d \,,\, t \,]}$$

from which inference (3) follows: E ($[d,t]$, b) $\therefore$ E ($[d,d,t]$, b).
And finally, again by means of upwards RICT we obtain:

$$\frac{\dfrac{d = d \qquad t = i \qquad i = t}{(\, d = d \vee d = i \,) \ \& \ (\, t = d \vee t = i \,) \ \& \ (\, d = d \vee d = t \,) \ \& \ (\, i = d \vee i = t \,)}}{[d \,,\, t \,] = [\, d \,,\, i \,]}$$

from which inference (5) follows: E ($[d,t]$, b); $t = i$ $\therefore$ E ($[d,i]$, b).

5 First-order Logic for Itemized Collections: Semantics

Let $\Im$ be any interpretation of the formal language, including an assignment of values to the variables. Then we give the denotation of individual terms as usual. And next, we simply take the denotation of a compound term $[r_1, \ldots, r_n]$ under $\Im$ to be the *set* formed by the denotation under $\Im$ of each of its constituents. That is to say, we give the interpretation of compound terms by means of the inductive clause

$$[r_1, \ldots, r_n]^{\Im} = \{\, r_1^{\Im}, \ldots, r_n^{\Im} \,\}$$

Thus, itemized collections are represented as sets.[17]

16 So we have finally shown (1) to be a first-order inference, as we set out to do in §1.

17 Strictly speaking, the clause has to be spelt out by induction on the degree of complexity of the term. Thus $[r_1, \ldots, r_n]$ has complexity 1 when all $r_1, \ldots, r_n$ are individual terms (i.e., terms of complexity 0), and in that case the values $r_1^{\Im}, \ldots, r_n^{\Im}$ are already

This leads us to deal with two kinds of '*entities*' made up from the individual members of the interpretation $\mathfrak{I}$: on the one hand, *simple entities*, which will just be the individuals of U (where U is the universe of $\mathfrak{I}$); and on the other hand, *complex entities*, which will be finite sets of individuals of U, or finite sets of finite sets of individuals of U, or combinations of these, etc. etc. In more precise terms (by induction): every individual of U will be a *simple entity* (entity of complexity 0) of U; and if $u_1, \ldots, u_n$ are *entities* of U, the more complex of which has complexity k, then $\{u_1, \ldots, u_n\}$ will be a *complex entity* of U of complexity k+1. No need to say that $\mathfrak{I}$ will assign simple entities to individual terms, and complex entities to compound terms.[18]

Likewise, we shall consider that a *property* defined on U is a set of *entities* of U, and more in general, that an n-place *relation* defined on U is a set of n-tuples of *entities* of U. Then the interpretation of an n-place predicate letter P under $\mathfrak{I}$ (say $P^{\mathfrak{I}}$) will be an n-place relation on U, i.e., a set of n-tuples of *entities* of U. So that an n-tuple of entities of U will be said to *satisfy* P under $\mathfrak{I}$, if and only if it belongs to the relation (set) $P^{\mathfrak{I}}$. Thus, if P is for example a 2-place predicate letter, its interpretation $P^{\mathfrak{I}}$ will be a set of ordered pairs, pairs whose components do not necessarily have to be plain individuals of the universe U (as it occurs in the standard first-order logic treatment), but that can be complex entities, as long as they are built up starting from the individuals of U only.[19]

given. And $[r_1, \ldots, r_n]$ has complexity k+1 when the most complex term among $r_1, \ldots, r_n$ has complexity k, and in that case the values $r_1^{\mathfrak{I}}, \ldots, r_n^{\mathfrak{I}}$ are taken for granted by the inductive clause of the definition.

18 Complex entities are bound to be finite sets (of individuals of U, or of finite sets of individuals of U, etc.), in fair accord with the syntactic structure of compound terms in our language, which can only consist of finite lists of terms (cf. §4–(c)). It could not be otherwise in a singular logic system, in which the only vehicle to convey reference is the individual object (cf. §2); and this is so, of course, given the impossibility of denoting an infinite plurality by making individual reference to each of its elements. Restriction to finite sets, incidentally, eliminates any possibility of Russell's paradox style arguments affecting our system.

19 Sets which have among its elements single individuals as well as other sets of different complexities, are indeed familiar from set theory. The same can be said of n-tuples whose components are single individuals as well as sets and other n-tuples of different complexities. This involves no risk of ambiguity: letting u, v and w be

Then we can simply say that an atomic formula such $P(r_1, \ldots, r_n)$ is true under $\Im$ if and only if the n-tuple $(r_1^{\Im}, \ldots, r_n^{\Im})$ belongs to the relation $P^{\Im}$. And this will hold irrespectively of whether some of the terms $r_1, \ldots, r_n$ turn out to be compound terms or not, or what amounts to the same thing, irrespectively of whether some of the entities $r_1^{\Im}, \ldots, r_n^{\Im}$ turn out to be complex entities or not.

In our logical system a 2-place predicate such as '*x* emptied *y*' can be satisfied by an ordered pair whose two components are single individuals (as it happens for instance in 'Charles emptied the bottle'), but also by an ordered pair whose first component is not a single individual of the universe, but a complex entity composed of two of them (as it happens in 'Dick and Tony emptied the bottle'). Something which is in perfect match with our intuition that the 2-place predicate '*x* emptied *y*' (or, in other words, the relation corresponding to it) is essentially the same in the case where it was a single person who emptied the bottle, that in the case where it was two people together, hand in hand, who carried out the emptying.

Similarly, in our system, a 2-place predicate such as '*x* introduced to swinging to *y*' can be satisfied by an ordered pair whose first and second components are complex entities of different levels (as it happens for instance in 'Dick and Hannah, together with Tony and Grace, introduced John and Alice to swinging'). And a similar remark applies, mutatis mutandis, to all other cases of plural collective predication.

This settings make it very easy to obtain proofs of soundness and completeness for our proposed system. Indeed, soundness of rule RICT follows at once from the extensionality of sets.

And as for completeness, only a few minor modifications are needed on the customary Henkin-style proof.[20] Indeed, the usual proof proceeds by taking any consistent set of formulas Φ, and enlarging it into a maximal consistent and existentially saturated Ψ. Then an equi-

distinct objects, for instance, the set-theoretical definition of the ordered pair $([u,v],w)$ makes it clearly different from the pair $(u,[v,w])$:

$$([u,v],w) = (\{u,v\},w) = \{ \{\{u,v\}\} , \{\{u,v\},w\} \}$$
$$\neq \quad (u,[v,w]) = (u,\{v,w\}) = \{ \{u\} , \{u,\{v,w\}\} \}$$

And a similar remark applies to n-tuples in general (see e.g. Roitman 1990, 29 – 30).

20 As exposed e.g. in Chang – Keisler (1990, 61 – 66).

valence relation between terms is introduced, in such a way that $r \sim s$ if and only if the formula $r = s$ belongs to Ψ. A model is thereby defined that assigns to each term its own equivalence class, which makes it relatively easy to prove that such a model satisfies the whole of Ψ, hence rendering Ψ (and Φ) satisfiable. And from this satisfaction theorem (if Φ consistent, Φ satisfiable) the completeness theorem follows at once.

All this can be done just the same in our first-order logic for itemized collections. In particular, the equivalence relation will be established not only for individual terms, but for compound terms too. And the fact that the set Ψ is maximal consistent, immediately guarantees that it contains a formula of the form $[r_1, \ldots, r_n] = [s_1, \ldots, s_m]$ if and only if, for every r_i among $r_1, \ldots, r_n$ it contains at least a formula $r_i = s_j$ with s_j among $s_1, \ldots, s_m$, and vice versa.[21] From which satisfaction by the constructed model will be ensured.

Hence we can conclude: (a) that our proposed system is a singular logic in the sense given in §2, in that the individual is the essential vehicle to convey reference, predication and quantification; (b) that, unlike full-fledged plural logic formal systems, it preserves completeness; and (c) that it suffices to account for the logical validity of inference (1) and related inferences, showing them to be first-order.[22]

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21 This is of course in virtue of RICT.

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