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NOTAS CRÍTICAS

Logic with Trees

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Logic with Trees: An Introduction to Symbolic Logic, by COLIN HOWSON, LSE/ROUTLEDGE, LONDON AND NEW YORK, 1997, XIII + 197 pp., US$ 19.99 (paperback).

During the academic years 1993-94 and 1994-95 I had the opportunity of assisting Professor Howson in the teaching of the "Elements of Logic" course at the London School of Economics (LSE), a course currently taken by a mixture of students (mostly undergraduates) from the Economics, Mathematics, Computer Science and Philosophy Departments. The written Notes for that course have now appeared, in an enlarged version, as a textbook published under the joint imprint of LSE and Routledge. The result is an excellent self-contained introduction to symbolic logic up to an elementary-intermediate level, for readers or students of any background, with plenty of exercises and equally suitable for classroom use and independent study.

The most salient feature of this book, and the one which makes it so different from the majority of other textbooks on the subject, is its genuinely argumentative style: before introducing a new concept or technique, the author thoroughly explains why it is needed and how the definition must be constructed so that it fulfils its purpose; this happens, for instance, with the semantic motivation for propositional and first-order languages [pp. 5-15, 66-70 and 77-80] or the set-theoretic framework for first-order interpretations, and in particular the adoption of extensionality [pp. 89-93].

Similarly, before introducing a new result (a lemma or a theorem), the author carefully argues for its importance as well as for its plausibility, and it is only afterwards that he proceeds to state it in precise terms and then to prove it; clear examples of this are the soundness and completeness proofs for both propositional and first-order logic [pp. 53-60 and 102-14]. And sometimes the author reaches the point of splitting up the definition of a concept or technique into various stages, each more accurate than the preceding one, either according to the theoretical resources available at that particular point in the book, or according to what is required from the defined concept at that point; this happens with the definition of 'deductively valid inference' [pp. 3-4, 16-7, 63-6 and 97-9], with the rules of the tree method for both propositional and first-order logic as a systematic search for counterexamples to inferences [pp. 16-28, 53-6, 80-5 and 104-7], and with the extension of the tree method for the theory of identity, first with identity axioms [pp. 115-8] and later with identity rules [pp. 118-21].

In addition to this, there are a great number of methodological remarks and detailed discussions of philosophical problems: the sections on intuitionism [pp. 139-41], on the liar paradox [pp. 155-60], on counterfactual conditionals [pp. 161-65], on indicative conditionals [pp. 169-73] and the concluding section [pp. 173-4], are particularly attractive and illuminating. The author himself points out that the last two chapters "could be used as a supplementary text in philosophy of logic" [p. xii]. In this way the student is given not only a view of the 'finished façade' of first-order logic, but also of the 'foundations of the building': he is given not only the 'whats' and the 'hows' of elementary logic, but also the 'whys' and the 'why not otherwises'.

This is not to say, by the way, that the book is only addressed to philosophy students: it contains a good deal of genuine 'mathematical logic' topics, and a number of logic programming remarks scattered throughout, for the benefit of computer science students.

Another aspect of the book, somewhat related to the one just mentioned, is the fresh and lively way in which it has been written. The author himself emphasizes his intention "to show that the sorts of formal techniques exploited in proving even some 'deep' metalogical results are within the grasp of even determinedly non-mathematical students" [p. xii]; likewise, the publisher remarks in the presentation of the book that "preferring explanation and argument to intimidatingly rigorous development, Colin Howson presents the formal material in a clear and informal style [...]" [p. i]. Indeed, the book is readily accessible to undergraduate or postgraduate students of any background, and, with few exceptions, the material is very clearly presented and easy to follow.

It is likewise remarkable, for a book at such an introductory level, that the author does not avoid entering into polemics, defending his own opinions against those of others in connection with various problems for which we lack an 'established' view on the matter. These include the status of the theory of identity [pp. 129-31] and of first-order logic itself [pp. 173-4], the liar paradox [pp. 155-60] and the suitability of the material conditional for representing natural language indicative conditional statements [pp. 169-73]. His criticisms, however, are based on powerful but nonetheless elementary arguments, and presented in such a way that they are clearly understandable even for beginners. Moreover, in each case the author hastens to give an alternative bibliography and to urge the student to read it.

As for the contents, the book has 12 chapters, distributed in two parts: (I) 'Truth-functional logic', which includes chapters 1, 'The basics'; 2, 'Truth trees'; 3, 'Propositional languages'; and 4, 'Soundness and completeness'; and (II) 'First-order logic', which includes chapters 5, 'Introduction'; 6, 'First-order languages: syntax and two more rules'; 7, 'First-order languages: semantics'; 8, 'Soundness and completeness'; 9, 'Identity'; 10, 'Alternative deductive systems for first-order logic' (with an axiomatic system, a natural deduction one and a discussion of intuitionism); 11, 'First-order theories' (with 'advanced logic' topics, plus a long section on the liar paradox and Tarski's versus Kripke's theories of truth); and 12, 'Beyond the fringe' (about counterfactual and indicative conditionals, modal propositional logic and a concluding section on the scope and status of first-order logic as a whole).

A better division of the book would see chapters 10 to 12 included in a third part: these last three chapters cover topics of a higher level than the preceding ones, and they are somewhat harder to read and less intended for classroom work — e.g. they contain virtually no exercises. If the book is to be used as a basis for a teaching course in elementary logic, the obvious advice is to lecture on chapters 1 to 9 (with or without the starred sections) and leave the other three for the private study of those students who want to study the subject more deeply.

In addition to the chapters, the book contains a basic bibliography, separate Name and Subject indices, a table of symbols and a final section with the answers to 'selected exercises' (in practice, those with a concise or a 'technical' solution).

In short, the book sets out to achieve four challenging goals, by giving students the following:

> — a complete and clear account of the truth tree system for first-order logic;
> — an understanding of the importance of logic and of its relevance to other disciplines;
> — the skill to grasp sophisticated formal reasoning techniques that are necessary to explore complex metalogic;
> — and the ability to contest claims that 'ordinary' reasoning is well represented by formal first-order logic" [p. i].

As the title suggests, there is extensive use of several kinds of 'trees' throughout the book. Besides the tableau method (truth trees) the book contains what the author calls 'conjugate tree diagrams' (a prototype of the propositional truth tree rules), and what he calls 'ancestral trees' (trees which display the syntactical structure of sentences: more about them to come); finally, truth trees are also exploited for reading off counterexamples to both propositional and first-order invalid inferences.

The use of truth trees for developing first-order logic is no longer a novelty in a textbook — after the huge success of R. Jeffrey's (1967) *Formal Logic: Its Scope and Limits*, New York, McGraw-Hill, 3rd rev. ed., 1991 — but it continues to be possibly the best sensible option. As it happens, the present book is bound to be a close competitor of Jeffrey's in the bookshops. Those who compare both books will find Jeffrey's style more 'technical', so to speak, while Howson's is more 'philosophical' — although no less rigorous. Jeffrey's book includes more material on probability and recursion theory, while Howson's focuses more on the principle of induction and set theory.

Jeffrey's book includes, as is well-known, a complete proof of Gödel's first incompleteness theorem for second-order logic, but Howson's contains a detailed discussion — and sketchy proofs — of *both* Gödel's incompleteness theorems, in various versions (pp. 142-54: the first one, for first-order Peano Arithmetic and for second-order logic, and the second one, for first and second-order Peano Arithmetic), as well as of many other advanced topics such as: infinite cardinalities [p. 144], Schröder-Berstein's and Cantor's theorems [p. 145], the Continuum Hypothesis and Gödel-Cohen's independence proof [pp. 145-46], the notions of decidability [p. 17] and recursive enumerability [p. 149], Church's theorem [p. 149], the elimination of function symbols [p. 123-4], the compactness of first but not of second-order logic [pp. 112-4 and 154], the Löwenheim-Skolem downward and upward theorems [pp. 112 and 146-7], Skolem's paradox [p. 146], non-standard models of first-order Peano Arithmetic [p. 143-4] but categoricity of its second-order counterpart [p. 147], Tarski's theorem on the non-definability of truth [p. 156], definability of identity in second-order logic [p. 147] but not by a first-order theory [p. 131], and non-trivial mathematical content of second-order logic [pp. 154-55].

As the material is well-distributed and very clearly presented, the book can easily be used for its two main purposes, both as an elementary and an intermediate introduction to modern logic, and in each case the menu that can be extracted from it makes for a balanced diet: the book enables each individual reader to decide how much he is capable of 'digesting' at a particular time. Without doubt, the four objectives that the author lays down on page i are extensively fulfilled throughout.

A novelty of the book is what the author calls 'the Principle of Induction on Immediate Predecessors'. This is a pedagogical way of presenting induction as applied to proofs in propositional logic, which makes it remarkably easier for students to grasp, and constitutes an excellent introduction to the study of this mathematical principle.

The author defines the notion of the 'immediate predecessors' of a sentence (where, e.g. the immediate predecessors of $A \rightarrow B$ are A and B, and that of $\neg A$ is A), and then what he calls 'ancestral trees': upside down trees which display the structure of the 'predecessors' of a given sentence [pp. 33-4]. Thus for example:

```
(A ∧ B) → ¬C
      /    \
  A∧B     ¬C
  / \       |
 A   B      C
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Then he states the Principle of Induction on Immediate Predecessors as follows:

> If (1) all the sentence letters of [a propositional language] L have [a property] *P*, and (2) from the assumption that the immediate predecessors of any non-atomic sentence X in L have *P*, it follows that so too does X, then (3) every sentence in L has *P* [p. 34].

This principle is used in numerous proofs and exercises in chapters 3 and 4 of the book. Its relation to mathematical (weak) induction is also discussed [p. 38], and later, the principle of strong induction (on degrees of logical complexity of sentences) is introduced and used in proofs in both propositional and first-order logic [pp. 59-60 and 109-11].

The statement of strong induction, by the way, is slightly redundant:

> Let Δ be any set of L sentences and k any integer. Suppose that (1) all the sentences of degree $\leq k$ in Δ have some property *P*, and (2) where X is any sentence of degree $> k$ in Δ, if all sentences in Δ of lower degree have *P* so does X. Then all the sentences in Δ have *P* [pp. 59 and 109].

Indeed, the references to the integer k and condition (1) can be omitted altogether, obtaining something like:

> Let Δ be any set of sentences. Let X be any sentence in Δ, and suppose that if all sentences in Δ of lower degree than X have some property *P*, so does X. Then all the sentences in Δ have *P*.

This is obviously equivalent to the original version above: for degree $(X) = 0$ there are no sentences of degree smaller than X and the claim is trivially true.

However, it must conceded that the previous formulation might be clearer to the readers of the book.

Another interesting point is Howson's practical demonstration of how a classical first-order language can be used to represent natural language subtleties to which it is rarely exposed, particularly in an elementary logic textbook. For example, he formalizes 'Minerva is thinking deeply' as $\exists x\ (T(x) \wedge D(x) \wedge Q(x))$: 'there is a process (x) which is a thinking process (T(x)), which is deep (D(x)) and which is currently being undergone by Minerva (Q(x))'. In this way the formalization captures the fact that 'Minerva is thinking' is a consequence of the former sentence [p. 73]. The author employs a great deal of patience in carefully explaining to the student how to formalize this and other even more complicated examples [pp. 73-6].

Indeed, it is one of the author's concerns throughout the book to show that first-order languages are more powerful at formalizing natural language sentences than they are usually assumed to be, although he also makes clear to the reader that "there are other constructions in English that pose more of a challenge to first-order formalization", among which he mentions counterfactual conditionals, modal sentences and sentences involving probabilities [pp. 75-6]. I still wonder, however, whether examples of formalization such as 'Minerva is thinking deeply' are the best ones for a first year logic student, who has only seen a few others. And in any case, the reader should be warned to consider the position and the nature of the adverb in question, before proceeding to formalize the sentence: for example, from 'Minerva is deeply interested in philosophy' it follows 'Minerva is interested in philosophy', but from 'Minerva is *hardly* interested in philosophy' it does not.

The distinction between propositional and first-order logic, by the way, is beautifully described [pp. 64, 78 and 99-101], carrying it beyond the mere distinction between the languages — as is usually done —; in particular, the author explains, with numerous examples and exercises, how a sentence of a first-order predicate language can nevertheless be at the same time a (propositional) tautology [pp. 99-101].

However, the author allows an isolated sentence such as 'all Ps are Qs' to be formalized, in the absence of any other constraints, either as $\forall x\ (P(x) \rightarrow Q(x))$ or, taking the set of Ps as the domain, simply as $\forall x Q(x)$ [p. 72]. But considering that domains are later defined — as usual — to be *non-empty* sets [p. 91], the latter formalization implies that there exists at least one P, while the former does not: $\forall x\ (P(x) \rightarrow Q(x))$ will be true when the domain contains no Ps, just as the sentence 'all Ps are Qs' would be trivially true were the set of Ps to be empty.

Passing on now to other things, the book includes an interesting discussion about the status of the theory of identity. The arguments are very elementary but sharp — as is the point discussed —, and the author makes it clear that he is defending his personal opinion on the matter, in the absence of an established view. To sum up, it constitutes an excellent discussion for first year logic students to follow, and get an exciting taste of what is a 'debatable question' within the subject — one on which different logicians disagree.

Colin Howson argues that if identity theory appears to be at first sight a logical theory, that is only because we are used to presupposing hidden premises, "which we tend so unquestioningly to accept as true and we know that other people do too, that we do not bother to make them explicit " [p. 115]. However, he continues to argue, "the view taken here [in the book] is that = is *not* a logical item, on the ground that all other binary relations are regarded as extralogical and to make an exception of = requires a very strong justification. Yet all the justificatory arguments beg the question in one way or other" [p. 129; I always quote with the author's own italics].

Indeed, although the book devotes a whole chapter to the theory of identity, the technical treatment is that of a mathematical extension of first-order logic: the chapter comes immediately *after* the proofs of soundness and completeness of first-order logic, and therefore it is not covered by them; it begins with a discussion of those fundamental properties of identity which should be embedded in any first-order theory thereof; these properties are 'translated' into first-order axioms to be used with the tree method — as sentences which can be inserted at any time in the construction of a tree —, and later these axioms are transformed into tree rules; finally, the extent to which a 'good' characterization of identity is thus achieved, is also discussed at various points [pp. 117 and 131].

The strongest argument I know for considering identity a logical relation comes from the old definition of 'logic' as 'the study of correct reasoning independently of the subject': identity is surely the only relation which may appear on literally *any* subject, and the norms of correct reasoning using it are always the same.

Howson disputes this argument: "another popular reason for regarding '=' as logical is that its meaning is *domain independent*: it is always meaningful to say that a = a whatever domain the denotation of a might be in"; however, he continues: "there are two objections to this argument. One is that it is far from clear that the meaning of identity is domain independent: for example, in set theory an axiom (*Extensionality*) has to tell us the condition for two sets to be identical: they must have the same members. The second is that the set membership relation $\in$ is itself 'almost' domain independent, since '$x \in y$', i.e. 'x is an element of the set y' is true if and only if x is an element of the set y, whatever the nature of the individuals making up the sets: we could be discussing sets of numbers, trees, people or whatever" [p. 130].

I do not think these objections are too strong. Firstly, it can be argued that the reason why the identity criterion for sets must be laid down explicitly is simply as part of the definition of the concept of 'set': without this criterion the notion of set is unclear — as the paradoxes show. But once this criterion has been laid down, reasoning concerning the identity relation obeys exactly the same laws as anywhere else. Moreover, there are also entities for which the conditions of 'existence' must be laid down explicitly (for example 'depression' or 'homicide' in their technical senses), but this does not imply that the existential quantifier is not a logical operator. And secondly, the relation $\in$ of membership '$x \in y$' does only make sense when y is a *set*, that is, a very special kind of thing; hence it cannot be argued that it is 'almost' domain independent.

In any case, I agree completely with the author when he notes that this is largely a conventional matter, with little if any practical implications: "mathematics may be denied the status of logic, but that doesn't diminish its importance or authority. The same can be said of the accepted parts of science generally" [p. 131]. And this is particularly true of identity, as Howson points out as well, which is not uniquely definable by a first-order theory [p. 131].

Another philosophical discussion comes in a delightfully clear and attractive section on indicative conditionals [pp. 169-73], where Howson tries to show that "the truth-functional conditional is overall the best coherent model of inferences involving indicative conditionals" [p. 172]. However, at the end of it he remarks that "the foregoing discussion [...] is heavily coloured by the author's own views" and that "the reader is strongly encouraged to examine alternative accounts" [p. 172: reference to 5 books follow]. I am not an expert on this topic, but I cannot help making a brief comment on one of the arguments given and discussed by the author: Howson introduces the following alleged counterexample to the truth-functional reading of indicative conditionals: '*if* I add sugar to my coffee *then* it will taste sweet' appears to be true, while '*if* I add sugar to my coffee and I add diesel to my coffee *then* it will not taste sweet' is obviously true as well; but in this case the '*if-then*' certainly could not be the truth-functional conditional.

Howson argues that "it is quite extraordinary that this could ever have been regarded as a serious objection, yet it certainly has been. At any rate, it is no counterexample, merely a failure to be explicit. 'If I add sugar to my coffee it will taste sweet' is accepted as true only because 'and I add nothing else' is tacitly added to the antecedent. What is really being asserted is a statement of the form 'If A and D then B' " [p. 169].

It seems to me, however, that there are many other facts which could make the consequent fail if we put them in conjunction with the antecedent, beyond the addition of further ingredients to the coffee by its owner. For example, somebody *else* could add diesel, or dirty rain water could fall into it from a roof leak, or the temperature could suddenly fall down, cooling the coffee and preventing the sugar dissolving in it. Hence the sentence which, in conjunction with 'I add sugar to my coffee', would definitely force the consequent 'it will taste sweet' should be of the form 'other things being equal', that is: a *ceteris paribus* clause.

However, we could simply say that in the event of the coffee's owner adding diesel to it (or any other misfortune happening) then the original conditional 'if I add sugar to my coffee it will taste sweet' would have simply proven to be false. This analysis seems to square better with the interpretation of the conditional above as a concrete indicative claim, about *one particular coffee* at a *one particular time*.

Something which is difficult to find in other books at this level is Colin Howson's presentation of natural deduction: he first chooses an intuitionistic system [pp. 134-5], then proves the equivalence or interderivability, in the presence of the other rules, of the three principles: excluded middle, double negation and classical reductio — or 'indirect proof' — [pp. 137-8]; and finally explains that the addition of any one of these will convert the intuitionistic system into a classical one. In a short — but brilliant — section on intuitionism, he examines the intuitionistic interpretation of the logical particles, and concludes that "though the *formal* difference between classical and Intuitionistic logic can be represented in terms of the acceptance and rejection respectively of any one of the rules of Excluded Middle, Double Negation, and Classical Reductio (all principles, incidentally, that the Intuitionists themselves regard as illegitimate), semantically they are strongly divergent" [p. 140].

In the introduction to propositional logic the author points out a modal subtlety: "consider the statement 'You may have tea or you may have coffee' [...]: it actually says that you may have tea *and* you may have coffee (though it does not mean that you may have both)" [p. 9]. It would have been nice to resume the explanation in the section about modal propositional logic, by noting that the corresponding inference $\Diamond A \wedge \Diamond B \therefore \Diamond(A \wedge B)$ is invalid in S5 (cf. B. F. Chellas (1980), *Modal Logic: an Introduction*, Cambridge, Cambridge University Press, p. 11).

In the explanation of Gödel's first incompleteness theorem [pp. 151-2] there are two points which should be made explicit: one of them is the observation that, from the fact that the property of 'being the Gödel number of a proof from Peano's Axioms' is arithmetically definable, it easily follows that the related property of 'being the Gödel number of a *formula provable from* Peano's Axioms' is arithmetically definable too. And the other one is the distinction between 'arithmetically definable' and 'definable within first-order Peano Arithmetic', and in this connection, the remark that syntactic notions concerning first-order Peano Arithmetic are not only arithmetically definable, but also definable within the same theory (which, in turn, applies to the notion of 'being the Gödel number of a formula provable within first-order Peano Arithmetic', just mentioned).

The typographical presentation of the book is not everywhere perfect, and as for misprints, there are a number of them, some of which may cause confusion. This is a certain handicap for the classroom use of the book, which will surely be corrected in further editions. Before recommending the book for students to read, it would be advisable to give them a list of the 'non-trivial' misprints.

All in all, the present monograph makes for an excellent textbook, destined to reach a wide public, to appear in many editions and to live a long life. I strongly recommend that it be translated into Spanish as soon as possible, for the benefit of Spanish and Latin American universities. Indeed, I know first-hand that it is already being used in Spain by many lecturers to prepare their lecture notes, and so we must all collaborate to make further editions even better — and improve the use of the present one.

Colin Howson is a world-wide expert and contributor to Bayesianism. His book *Scientific Reasoning: the Bayesian Approach* (written in 1989 with Peter Urbach, 2nd rev. ed. in Chicago, Open Court, 1993) is a central reference for students of Bayesianism and of philosophy of science generally. With the present book on the elements of logic, Colin Howson has achieved the dream of many: to write two monographs, one about 'deductive' and the other about 'inductive' logic, thus covering the full scope of 'logic' in the wide sense.

He himself explains how the two things are closely related: "expounding first-order logic in isolation from the theory of probability is really only telling half the story, for the model of deductive reasoning provided by first-order logic interlocks with probability theory to provide a general account of both inductive and deductive reasoning" [p. 174].

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