

A Probabilistic Analysis of Cross-examination Using Bayesian Networks

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Abstract The legal scholar Henry Wigmore asserted that cross-examination is ‘the greatest legal engine ever invented for the discovery of truth.’ Was Wigmore right? Instead of addressing this question upfront, this paper offers a conceptual ground clearing. It is difficult to say whether Wigmore was right or wrong without becoming clear about what we mean by cross-examination; how it operates at trial; what it is intended to accomplish. Despite the growing importance of legal epistemology, there is virtually no philosophical work that discusses cross-examination, its scope and function at trial. This paper makes a first attempt at clearing the ground by articulating an analysis of cross-examination using probability theory and Bayesian networks. This analysis relies on the distinction between undercutting and rebutting evidence. A preliminary assessment of the truth-seeking function of cross-examination is offered at the end of the paper.

1 Introduction

Big Bank on Main Street was robbed on January 10th 2021. A team of investigators, police officers and forensic experts collected evidence, examined traces, interviewed people, watched surveillance videos. They singled out Joe Steele as the perpetrator. Joe was charged with robbery and faced trial. The best lawyer in town, Eddy Smith, defended Joe. Eddy argued that the police investigation was riddled with errors. The witnesses had a flaky recollection of what happened. The forensic laboratory that conducted the analyses did not follow protocols. The video evidence was confusing and unclear. For each piece of incriminating evidence presented at trial, Eddy was able to show on cross-examination that there was a reason to question its credibility. Joe was ultimately acquitted.

Was the outcome of the trial the correct one? In a formal sense, the answer should be affirmative. There was a reasonable doubt about Joe’s guilt. The correct course of action was to acquit. But, factually, was Joe the perpetrator? This question is less easy to answer.

The legal scholar Henry Wigmore (1940) asserted that cross-examination is ‘the greatest legal engine ever invented for the discovery of truth.’ The U.S. Supreme Court repeated the assertion on many occasions. For example, in *Davis v. Alaska* 415 U.S. 308 (1974), the Court wrote that ‘cross examination is the principal means by which the believability of a witness and the truth of his testimony are tested’ (316). But there are also reasons to question Wigmore’s optimism. Cross-examination saved Joe Steel from a conviction. But had his lawyer not been so skilled, Joe’s fate would probably have been different. Cross-examination seems to reflect the abilities of someone’s lawyer rather than factual guilt or innocence. Manuals often recommend that during cross-examination lawyers not ask more questions than necessary since the answers they will receive could lead to the discovery of information that weakens their case (Wellman, 1903; Clark *et al.*, 2010). From a lawyer’s standpoint, the goal of cross-examination is to win cases, not to discover the truth.

Was Wigmore wrong, then? It is difficult to say without becoming clear about what we mean by cross-examination; how it operates at trial; what it is intended to accomplish. Unfortunately, cross-examination is not widely studied in the philosophical or legal literature.

Despite the growing importance of legal epistemology (Goldman, 1999; Gardiner, 2019), there is virtually no philosophical work that discusses cross-examination, its scope and function. The contributions of legal scholars are also limited. Some have focused more generally on the differences between adversarial and inquisitorial adjudication.¹ More squarely on cross-examination, some legal scholars have argued that it does not promote truth discovery (Epstein, 2007). They have based this claim on the empirical evidence that our memories can be manipulated and our perceptions are unreliable (Loftus, 1996; Wells *et al.*, 2006; Simons & Chabris, 1999). Others have argued that, precisely because of our cognitive limitations, a false testimony is more cognitively burdensome on a witness than a truthful one. Thus, a truthful testimony is more likely to survive cross-examination than a false one (Sanchirico, 2009). On balance, the lessons from the legal scholarship are mixed.²

This paper makes a first attempt at clearing the ground by articulating an analysis of cross-examination using concepts from probability theory. This analysis will inevitably be abstract and devoid of the rich details of legal practice. My hope is that it will still capture certain fundamental aspects of cross-examination. The examples I use are drawn from criminal trials, but the discussion that follows can also be applied to civil trials. The conceptual ground clearing about the nature of cross-examination that I am going to offer should provide a firmer footing for examining the tenability of Wigmore's claim. I briefly discuss the truth-seeking function of cross-examination and outline some preliminary thoughts towards the end, but a fuller discussion is left for another time.

The conceptual ground clearing is important for another reason. Cross-examination is an integral part of the right to confront one's accusers, as defined by the US Supreme Court in landmark cases such as *Crawford v. Washington* 541 U.S. 36 (2004) and *Michigan v. Bryant* 562 U.S. 344 (2011). The right to confrontation—or one in its vicinity—is also enshrined in the constitutions of many countries around the world and international law.³ But until we know what cross-examination consists in, the substance of this right remains elusive. As one legal scholar put it, '[a]ttention to the undertheorized question of what cross examination actually accomplishes at trial could also ensure that the right to confrontation has some force when it applies' (Griffin, 2011, 52).

The plan is as follows. I begin with an informal analysis of cross-examination. This informal analysis relies on the distinction between undercutting and rebutting evidence. Next, I give a probabilistic analysis of the rebutting/undercutting distinction and apply it to cross-examination. The formalization of the distinction and its application take up the bulk of the paper. The final section returns to Wigmore's claim and sketches a few thoughts about the truth-seeking function of cross-examination.

2 Testing the evidence

Cross-examination is a method for 'testing' the evidence presented at trial—especially the live testimony of a witness—by asking probing questions. As Justice Scalia wrote in *Craw-*

¹See the exchange between John Langbein (1985) and Ronald Allen (1987) about the merits and demerits of the adversarial method of judicial fact-finding.

²The empirical evidence is not univocal either. Although it is clear we are overconfident about the power of our cognitive abilities, there is still a positive correlation between the accuracy of our judgments and our degree of confidence in them (Wixted & Wells, 2017).

³Defendants' rights are protected by the 6th Amendment to the U.S. Constitution; by article 11 of the Universal Declaration of Human Rights (1948); by article 67 of the Rome Statute of the International Criminal Court (1998); by article 111 of the Italian Constitution (1947); by article 24 of the Spanish Constitution (1978); by article 5.XXXVIII and 5.LV of the Brazilian Constitution (1988); and the examples could continue.

ford v. Washington, 541 U.S. 36 (2004), ‘the common-law tradition is one of live testimony in court subject to adversarial testing’ (43). Both parties at trial may take advantage of the opportunity to test the evidence presented by the other party. The defense lawyer may cross-examine a witness for the prosecution (or the plaintiff in a civil trial). The prosecution (or the plaintiff) may cross-examine a witness for the defense. It would be odd, however, to subject a document or physical evidence to cross-examination since they would be unable to answer questions. A document or physical evidence could still be indirectly cross-examined by asking questions to a witness who has knowledge of them.⁴

The intended outcome of cross-examination is the retrieval of supplementary information, not previously known to judges or jurors, with the purpose of painting a fuller picture of the facts. In this sense, cross-examination has an interpretative role. It supplies additional information that helps to better understand, contextualize and assess the existing evidence. But this interpretative role is closely tied with a polemical one. The party who cross-examines a witness and in this way elicits additional information has usually a partisan objective in mind, namely to weaken the other party’s case. This objective may not always be achieved successfully. The new information might inadvertently end up strengthening, not weakening, the evidence under cross-examination. Lawyers, however, are trained to only ask questions that—predictably enough—should elicit answers that weaken, not strengthen, the other party’s case.

The elicitation of additional information during cross-examination can take different forms. First, cross-examination can elicit information that is meant to attack someone’s credibility, reputation or expertise. Say a witness testifies about a DNA match, but cross-examination shows that the witness lacks the appropriate academic credentials. Second, without attacking the witness directly, cross-examination can elicit information that challenges the credibility of a particular statement made by the witness. Say a witness identifies the defendant as the perpetrator, but during cross-examination it turns out that the witness saw the perpetrator from very far away. Third, another way to attack a witness’ testimony, while also indirectly attacking the credibility of the witness, is to elicit further information from the witness that conflicts with what the witness previously asserted. Inconsistencies between assertions made by the same witness cast doubt on the credibility of the witness.

The examples of cross-examination mentioned so far targeted the credibility of a witness or statements made by the witness. But cross-examination can operate in a more indirect manner. Say a witness for the prosecution testifies to have seen the defendant run away from the crime scene at the relevant time. Under cross-examination, the witness admits to have seen another person, besides the defendant, run away from the scene at the same time. This additional information does not necessarily discredit what the witness asserted previously nor does it discredit the witness as such.⁵ The additional information simply provides a fuller picture about what the witness saw. Still, this fuller picture weakens the case against the defendant. If the defendant and another person were in the vicinity of the crime scene, the odds that the defendant was the perpetrator should be lower than previously thought (at least, assuming only one person committed the crime).

To summarize, cross-examination consists in asking probing questions to a witness with the intent of obtaining additional information that should weaken the other party’s

⁴Whether this focus on the live testimony of witnesses—which is common in the Anglo-american trial—is justified can be debated. For a different perspective, see Cheng & Nunn (2019).

⁵There are different reasons why the witness did not provide the additional information upfront. For example, if the prosecution did not ask the witness during direct examination whether they saw another person, the witness should not be blamed for not proving that information upfront.

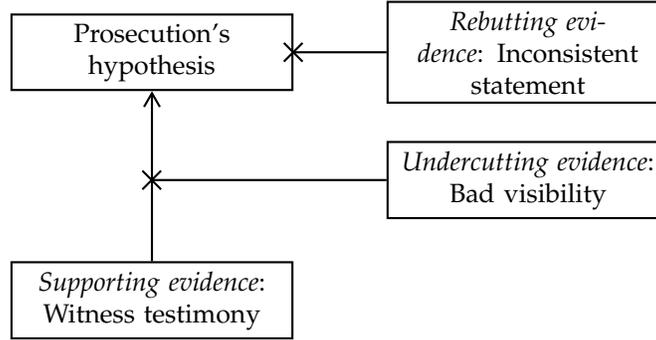


Figure 1: Undercutting and rebutting in a visual form

case. This additional information can attack the credibility of the witness under cross-examination or one of their statements. The additional information can also provide a fuller picture about the facts under dispute and thus prompt a reassessment of the relative balance of the evidence for and against the defendant.

3 Undercutting and rebutting

The task now is to offer a conceptual framework to understand the different forms of cross-examination just identified. To this end, I will rely on the distinction between *undercutting evidence* and *rebutting evidence* (Pollock, 1987). Suppose e is prima facie evidence in favor of hypothesis h . Additional evidence e' counts as rebutting evidence if it supports hypothesis h' that is incompatible with h . Instead, additional evidence e' counts as undercutting evidence if it weakens the nexus of evidential support between e and h without supporting any incompatible hypothesis.⁶ As will become clear (see Figure 1), the distinction between undercutting and rebutting provides a helpful conceptual framework for analyzing cross-examination.

Consider rebutting first. Suppose that after the prosecution laid out its incriminating case, the defense presents evidence of an alibi. A witness testifies that the defendant was with them when the crime was committed. The retrieval of rebutting evidence from another witness such as an alibi testimony is not an example of cross-examination since no witness was cross-examined in the first place. But the retrieval of rebutting evidence from the *same* witness is an instance of cross-examination. It is helpful here to distinguish the source of the evidence, say a witness, from the evidence itself, say the fact that a witness made a statement. Under cross-examination, a witness could make a statement that is inconsistent with a statement made by the same witness earlier during direct-examination. For example, suppose that under direct-examination the witness for the prosecution asserts 'We left the bar early that night and I dropped my friend off near the park because he had to meet someone.' This statement supports the hypothesis that the defendant was near the crime scene that night (assuming the crime was committed in the vicinity of the park). Under cross-examination, however, the witness admits 'I was with the defendant

⁶In the literature on reasons for actions, an attenuator weakens the force of a reason (Dancy, 2006). The analogy with undercutting evidence is evident. The extreme case is one in which an attenuator entirely disables a reason, what Joseph Raz (1999) calls 'exclusionary reason'.

at the bar all night long'. This statement supports the hypothesis that the witness was not near the crime scene that night (assuming the bar in question and the crime scene are far away). In this case, cross-examination eventuated in the retrieval of rebutting evidence in support of an alternative hypothesis to the prosecution's hypothesis.

Consider now undercutting. Suppose the prosecution's case against the defendant rests on the testimony of a key eyewitness. The defense lawyer cross-examines the witness by asking some probing questions. What were the lighting conditions? Could you see the perpetrator's face? The witness admits it was dark and visibility was limited. This additional information does not support any alternative hypothesis. But, taken at face value, the information elicited during cross-examination does weaken the credibility of the testimony. Information about bad visibility undercuts the support that the testimony lends to the claim that the defendant was the perpetrator.

Undercutting evidence can also attack the credibility of a witness as such, not just specific assertions. Suppose cross-examination shows that the key eyewitness for the prosecution had a strong incentive to testify against the defendant. If so, the eyewitness probably testified not because they saw the defendant commit the crime, but because of another reason unrelated to the crime. If this possibility is substantiated, it would cast doubt on the witness as a whole, not just a circumscribed set of assertions.⁷

Besides attacking the credibility of a witness or some of their assertions, undercutting evidence can also play a more indirect role. The additional information elicited during cross-examination can provide a fuller picture about what a witness saw (or what happened more generally) without undermining credibility. Recall the example used in the previous section. A witness initially testifies to have seen the defendant run away from the crime scene at the relevant time. But the same witness, under cross-examination, admits to have seen another person besides the defendant run away from the scene. This would still count as an example of undercutting evidence but in a more subtle way. We should distinguish here the hypothesis that the witness was *near* the crime scene from the hypothesis that the defendant was *present* at the scene. The first testimony ('I saw the defendant run away from the scene') supports the hypothesis that the witness was near the scene, which in turn supports the hypothesis that the defendant was present at the scene. The additional information elicited during cross-examination ('I saw a second person run away from the scene') does not undercut the relationships of evidential support from the first testimony to the hypothesis that the defendant was near the scene. But the additional information does undercut the relationship of evidential support from the hypothesis that the defendant was near the scene to the hypothesis that the defendant was present at the scene (assuming only one person committed the crime).

We can now provide a more general account. Cross-examination supplies information that is meant to weaken a chain of inferences. The chain starts with an undisputed fact (what we call 'evidence'), say a witness asserts they saw the defendant run away from the scene. The undisputed fact supports—at least *prima facie*—a hypothesis, say that the defendant ran away from the scene. This hypothesis, in turn, supports another hypothesis, say that the defendant was present at the scene. And so on. This chain of inferences from evidence to hypothesis to another hypothesis can be attacked by means of undercutting or rebutting evidence. For one thing, undercutting evidence can weaken one of the relationships of evidential support from evidence to hypothesis or from hypothesis to another

⁷This argument should be applied with circumspection. Expert witnesses are often paid, or in civil cases, plaintiff and defendant may have an incentive to testify against one another. The presence of an incentive, financial or otherwise, need not always be a reason to disbelieve a witness.

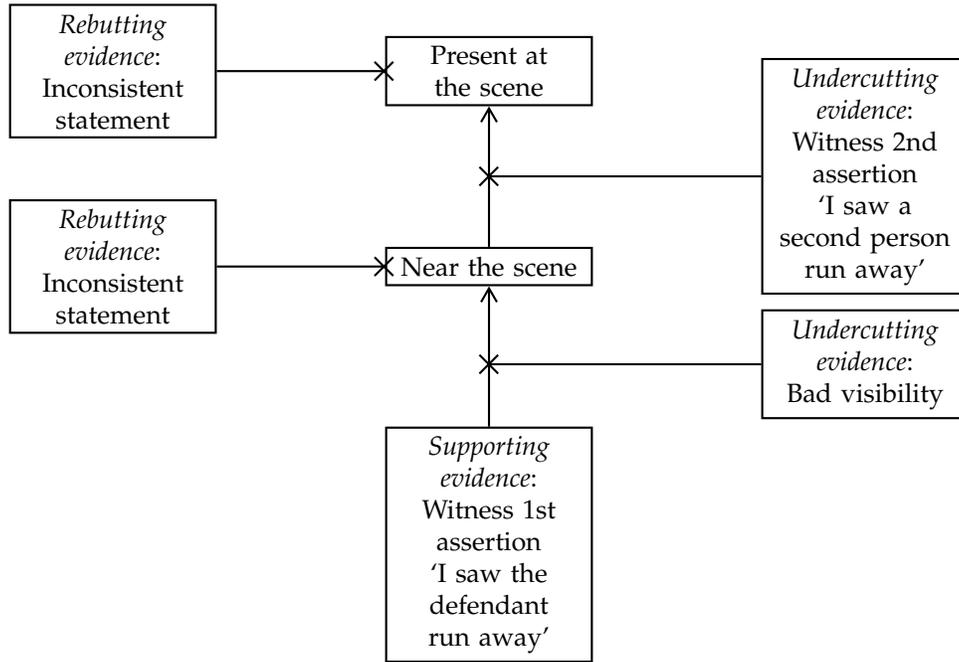


Figure 2: Rebutting and undercutting at different levels.

hypothesis. For another, rebutting evidence can directly attack one of the hypotheses along the chain by supporting an incompatible hypothesis, such as that the defendant was not near the crime scene or that the defendant was not present at the scene (see Figure 2 for an illustration).

4 Defeasibility and Probability

The previous section showed that the distinction between undercutting and rebutting is helpful for theorizing about cross-examination. We should now make the distinction more precise.

The natural place to start is the literature in defeasible logic. Suppose p is prima facie, defeasible evidence (or reason) for q , or in symbols, $p \Rightarrow q$ where the symbol ' \Rightarrow ' stands for a defeasible implication. Then u is evidence undercutting $p \Rightarrow q$ if and only if $p \Rightarrow q$ but $\neg((p \wedge u) \Rightarrow q)$. In words, while p is prima facie evidence for q , this no longer holds if u is added as an antecedent. By contrast, r is evidence rebutting p if and only if $p \Rightarrow q$ but $r \Rightarrow q'$ where q and q' cannot be both true. In words, p and q give prima facie support in favor of incompatible claims, q and q' .

This analysis is plausible but faces two challenges. The first is that the semantics of the connective \Rightarrow needs to be clearly defined. But, unlike the material implication \rightarrow , there is no agreed upon semantics for \Rightarrow . The proliferation of semantics in the field of defeasible and non-monotonic logic attests to this fact (Koons, 2017). Argument graphs can convey the difference between undercutting and rebutting evidence pictorially (see Figure 1 and Figure 2 in Section 3), but they also do not have a clear underlying semantics (see, how-

ever, Dung, 1995; Prakken, 2010). And even if this first challenge can be met, a second remains. The distinction between rebutting and undercutting evidence consists of a family of more fine-grained distinctions. Suppose p is prima facie evidence for q , while r is prima facie evidence for q' , incompatible with q . Do p and r cancel each other out? If a witness claims that the defendant was at the park that night, but another witness claims the defendant was at their house, we should not believe either testimony before assessing which is more credible. Presumably, p and q cancel each other out provided they are equally strong evidence in favor of incompatible claims, q and q' . If they have different strengths, however, the stronger evidence would still prevail on the other, although its strength would be weakened. Similar complications loom in the analysis of undercutting evidence. So, a good theory of undercutting and rebutting evidence should be able to express degrees of evidential strength.

The task I will undertake in the following sections is to deploy probability theory to articulate a plausible formalization of the notions of rebutting and undercutting.⁸ The reason for choosing probability theory as the underlying framework is twofold. First, it rests on a well-defined mathematical theory widely used to model uncertainty in many domains of applications. Although there are different interpretations of the concept of probability, there is wide agreement about the mathematical axioms that probability should satisfy. Second, probability theory is naturally suited to express relationships of evidential strength. In short, probability theory promises to overcome the challenges described above.⁹

Before getting started, a preliminary step is to formulate a probabilistic analysis of evidential support, that is, what it means for a piece of evidence e to favor (or oppose) a hypothesis h . I will rely on an analysis of evidential support that is common in the literature in confirmation theory (Crupi, 2015) and among probability-minded scholars of evidence law (Lempert, 1977; Koehler, 1996; Kaye, 2017; Sullivan, 2019). This analysis is not entirely uncontroversial, but it is good enough for my purposes.¹⁰

The evidential support—also called, probative value or evidential value—of an item of evidence e in favor of a hypothesis h can be measured by comparing two conditional probabilities, $P(e|h)$ and $P(e|\bar{h})$ (see, for example, Royall, 1997; Buckleton, 2005). If e is more probable on the assumption that h is true as opposed to \bar{h} , or in other words, if $P(e|h)$ is greater than $P(e|\bar{h})$, evidence e positively supports h rather than its negation. Evidential support (or lack thereof) is commonly expressed with likelihood ratios. More precisely, the degree of evidential support of e toward h is proportional to the ratio $P(e|h)/P(e|\bar{h})$. If $P(e|h)/P(e|\bar{h}) > 1$, evidence e positively supports h rather than its negation \bar{h} . The higher the ratio (for values above 1), the stronger the support in favor of h rather than \bar{h} . If the ratio is below 1, e supports \bar{h} rather than h . The lower the ratio (for values below 1), the stronger the support in favor of \bar{h} rather than h (or the greater the opposition against h). If the ratio equals one, e counts as irrelevant evidence, neither supporting nor opposing h .¹¹

⁸Kotzen (2019) has provided the most complete probabilistic account to date of undercutting and rebutting evidence. My formalization agrees, to some extent, with Kotzen's. The main difference is that Kotzen does not rely on Bayesian networks. A comparison between my approach and Kotzen's is beyond the scope of this paper.

⁹Weisberg (2009) has argued that probability theory—in particular, Bayesian conditionalization and Jeffrey conditionalization—is incompatible with the notion of undercutting evidence. Others—Christensen (1992) and Pryor (2013)—have expressed similar skepticism. For lack of space, I am not able to engage with this literature.

¹⁰On different probabilistic measures of confirmation, see Fitelson (1999). For why probabilistic accounts—in particular, the likelihood ratio—miss the mark, see Mayo (2018) and Allen & Pardo (2007).

¹¹Evidential support thus understood is relative to a pair of competing hypotheses. I will use exclusive and exhaustive hypotheses, such as h and its negation \bar{h} . Competing hypothesis need not be exhaustive of the entire space of possibilities and can be more circumscribed. But when they are not exhaustive, picking the right pair

With this account of evidential support at hand, let us return to our task: to make the distinction between undercutting and rebutting formally precise. As it turns out, likelihood ratios alone are not expressive enough. Say a piece of evidence e positively supports a hypothesis h , that is, $\frac{P(e|h)}{P(e|\bar{h})}$ is above one. The addition of undercutting or rebutting evidence, call it e' , to the body of evidence available should have the effect of weakening the evidential support in favor of h . This phenomenon of evidential weakening can be expressed formally by stating that the combined likelihood ratio $\frac{P(e \wedge e'|h)}{P(e \wedge e'|\bar{h})}$ is lower than the individual likelihood ratio $\frac{P(e|h)}{P(e|\bar{h})}$.¹² This formalization captures the commonality, but fails to capture the fact that rebutting evidence weakens evidential support in a different way from how undercutting evidence weakens evidential support. So an adequate account should capture the different mechanics of evidential weakening.

We will see that the difference between rebutting and undercutting evidence can be represented in a perspicuous manner in the language of likelihood ratios by means of Bayesian networks. The next section offers a crash course on Bayesian networks for the reader unfamiliar with the formalism. The reader familiar with the formalism can skip ahead.

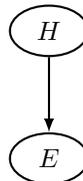
5 Bayesian Networks

A Bayesian network is a formal model that consists of a graphical part and a numerical part. The graphical part is a directed acyclic graph (DAG) whose nodes represent random variables that can take different values. For ease of exposition, I will sometimes use ‘nodes’ and ‘variables’ interchangeably. The nodes are connected by directed edges (arrows). No loops are allowed. Here is an example:



Since there is an arrow from A to B , node A is the parent of B , and conversely, B is the child of A . Since there is a directed path from A to another node C , through the intermediary node B , node C is a descendant of A . Besides the graphical part, the numerical part of a Bayesian network consists of conditional probability tables that specify, for each node, the probabilities of the values of the node conditional on the possible values of the parent nodes. If a node has no parent, its probability table should specify the prior probabilities of the possible values of the node.

Suppose we want to represent the simplest evidential relation, one of evidence bearing on a hypothesis of interest, either by favoring the hypothesis or by opposing it. We could use the following directed graph:



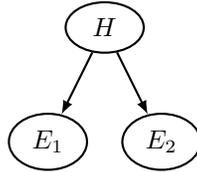
of competing hypotheses is not straightforward. The choice of different competing hypotheses can change the likelihood ratio significantly even holding fixed the same evidence (on this point, see Fenton *et al.*, 2014).

¹²This claim is established formally in later sections.

The arrow goes from H to E , loosely suggesting a causal influence of the world (as described by the hypothesis) onto the evidence. The arrow need not have a causal interpretation, however. The direction of the arrow indicates which conditional probabilities should be supplied in constructing the network. Since the arrow goes from H to E , we should specify the probabilities of the different values of E conditional on the different values of H , that is (for binary nodes), $P(e|h)$, $P(e|\bar{h})$, $P(\bar{e}|h)$, and $P(\bar{e}|\bar{h})$. In addition, since the hypothesis node has no parents, we should simply specify the prior probabilities of the different values of H , namely $P(h)$ and $P(\bar{h})$.¹³ For example, think of the evidence as the report of an instrument and the hypothesis as a medical condition. The conditional probabilities would then correspond to the rate of true positives, false positives, false negatives and true negatives. The prior probabilities would correspond to the prevalence rate in a given population. To calculate the posterior probability $P(h|e)$, it suffices to apply Bayes' theorem: $P(h|e) = \frac{P(e|h) \times P(h)}{P(e|h) \times P(h) + P(e|\bar{h}) \times P(\bar{h})}$.

The graph structure $H \rightarrow E$ can be used to represent evidence that positively supports a hypothesis, say e positively supports h , provided the probability tables satisfy the constraint that $\frac{P(e|h)}{P(e|\bar{h})}$ is greater than one. The same graph can also be used to represent evidence that negatively supports—opposes—the hypothesis provided the probability tables satisfy the constraint that $\frac{P(e|h)}{P(e|\bar{h})}$ is less than one.

A notable feature of Bayesian networks is that they provide a visually compact way to express relationships of probabilistic dependence and probabilistic independence among the variables in the network which would be very laborious to specify otherwise. In particular, Bayesian networks satisfy the so-called Markov condition. This condition (roughly) states that any node, conditional on its parents, is probabilistically independent of all the other nodes with the exception of its descendants.¹⁴ To acquire familiarity with this idea, consider a graph in which two items of evidence bear on the same hypothesis:



The graph above encodes a particular kind of probabilistic independence. In agreement with the Markov condition, variables E_1 and E_2 are independent of one another conditional on H , so $P(e_1 \wedge e_2|h) = P(e_1|h) \times P(e_2|h)$. Therefore, the following holds:

$$\frac{P(e_1 \wedge e_2|h)}{P(e_1 \wedge e_2|\bar{h})} = \frac{P(e_1|h)}{P(e_1|\bar{h})} \times \frac{P(e_2|h)}{P(e_2|\bar{h})}$$

The combined evidential support of the two items of evidence toward h —the combined likelihood ratio $\frac{P(e_1 \wedge e_2|h)}{P(e_1 \wedge e_2|\bar{h})}$ —is the result of multiplying the individual likelihood ratios.

¹³I am using the convention that upper case letters stand for random variables, while lower case letters stand for their values. I will mostly use binary variables X whose values are x and \bar{x} , even though random variable can take more than two values or be continuous. For any variable X and value x , I will write $P(x)$ to mean $P(X = x)$ and $P(\bar{x})$ to mean $P(X = \bar{x})$. I will also assume throughout out that $P(x) = 1 - P(\bar{x})$.

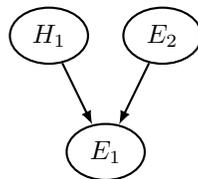
¹⁴A more precise statement of the Markov condition is given in any textbook on Bayesian networks (see for example, Darwiche, 2009).

Even though the two variables E_1 and E_2 are probabilistically independent conditional on H , they are not unconditionally probabilistically independent. It is not the case that $P(e_1 \wedge e_2) = P(e_1) \times P(e_2)$ even though $P(e_1 \wedge e_2|h) = P(e_1|h) \times P(e_2|h)$. The fact that they are probabilistically independent (conditional on H) and also probabilistically dependent (unconditionally) can be illustrated with an example. Suppose the same phenomenon (say blood pressure) is measured by two instruments. The reading of the two instruments (say 'high' blood pressure) should be probabilistically dependent of one another. In fact, if the instruments are both infallible and they are measuring the same phenomenon, they should give the exact same reading. On the other hand, the two instruments measuring the same phenomenon should count as *independent lines of evidence*. This fact is rendered in probabilistic terms by means of probabilistic independence conditional on the hypothesis.¹⁵

The same graph structure $E_1 \leftarrow H \rightarrow E_2$ can be used to represent converging as well as diverging evidence. Suppose e_1 and e_2 both favor h —that is, their likelihood ratios relative to h as opposed to \bar{h} are both greater than one. Then, the combined evidential support in favor of h must be greater than the individual support provided by the single pieces. Recall that the combined likelihood ratio is simply the multiplication of the individual likelihood ratios. This scenario captures the idea of *converging evidence*. By contrast, suppose e_1 favors h but e_2 opposes h —that is, their individual likelihood ratios relative to h as opposed to \bar{h} are greater than one and below one, respectively. This scenario captures the fact that the two pieces of evidence *diverge* in the support they provide toward the hypothesis.¹⁶

6 Undercutting in Bayesian Networks

With the proper background in place, this section formalizes undercutting evidence in the language of probability theory using Bayesian networks. The analysis will comprise two components: a graphical part along with numerical constraints on the assignments of conditional probabilities. The first part of the analysis consists of the following graph:



The idea (roughly) is that E_2 represents the evidence undercutting (or reinforcing) the relationship of evidential support between E_1 and H_1 . The arrow goes from E_2 to E_1 , but there is no arrow connecting E_2 to the hypothesis node.¹⁷ This is how it should be. Only E_1 bears directly on the hypothesis of interest, whereas E_2 provides additional information

¹⁵These ideas can be worked out more systematically in the language of Bayesian networks. Roughly, two variables are probabilistically dependent if there is an open path between them. There is an open path between E_1 and E_2 because information can travel from E_1 to H and from H to E_2 . Thus, variables E_1 and E_2 are probabilistically dependent. On the other hand, an open path can be closed by conditioning on one of the variables along the path. By conditioning on H , the path between E_1 and E_2 is closed, and the probabilistic dependence is eliminated. For a more rigorous exposition of the notions of open and closed paths, see Darwiche (2009).

¹⁶For a more detailed discussion of converging and diverging evidence in terms of Bayesian networks, see Schum & Starace (2001); Taroni *et al.* (2014).

¹⁷This graphical structure is not new in the forensic science literature (see, for example, Fenton *et al.*, 2013; Taroni *et al.*, 2014) or in the literature in philosophy of science (see, for example, Bovens & Hartmann, 2002).

about E_1 , but no direct information about the hypothesis. Information about bad visibility undercuts the support of an eyewitness testimony in favor of, say, the hypothesis that the defendant assaulted the victim, but does not directly bear on the hypothesis itself.

The graph $H_1 \rightarrow E_1 \leftarrow E_2$ is called a ‘collider’ because two incoming arrows collide at the evidence node E_1 . Since E_1 is a collider node, the path between H_1 and E_2 is closed. This means—in the theory of Bayesian networks—that H_1 and E_2 are probabilistically independent.¹⁸ So $P(h_1) = P(h_1|e_2)$, where e_2 is the undercutting (or reinforcing) evidence.¹⁹ This independence is plausible. Since the visibility conditions do not directly bear on the hypothesis, knowing about them should not change the probability of the hypothesis. At the same time, the two are not entirely unrelated. Knowing about the visibility conditions can help to determine what happened, at least in conjunction with evidence about what happened. Formally, conditioning on E_1 opens the path between H_1 and E_2 and introduces a probabilistic dependence between them. This probabilistic dependence is also plausible. If the defendant assaulted the victim, the fact that a witness claims to have seen the defendant commit the assault should make it less likely that the lighting conditions were bad. If the defendant did not assault the victim, the fact that the witness claims to have seen the defendant commit the assault should make it more likely that the lighting conditions were bad.

If the defendant assaulted the victim, the fact that a witness claims to have seen the assault, the fact that the lighting conditions were bad. If the defendant did not assault the victim and the witness still claims to have seen the assault, it should be more likely that the lighting conditions were bad.

To complete the analysis of undercutting evidence, the graph must be accompanied by assignments of conditional probabilities. Since node E_1 has two parents, H_1 and E_2 , the conditional probabilities that define this node should consider all possible values of the parents. Hence, there are four possible combinations to consider, at least with binary nodes: $P(e_1|h_1 \wedge e_2)$; $P(e_1|h_1 \wedge \bar{e}_2)$; $P(e_1|\bar{h}_1 \wedge e_2)$; and $P(e_1|\bar{h}_1 \wedge \bar{e}_2)$.

The constraints to impose on these conditional probabilities should capture what it means for a piece of evidence e_2 to undercut the support of another piece of evidence e_1 in favor of a hypothesis of interest h_1 . A natural constraint to impose is this:

$$\frac{P(e_1|h_1 \wedge e_2)}{P(e_1|\bar{h}_1 \wedge e_2)} < \frac{P(e_1|h_1 \wedge \bar{e}_2)}{P(e_1|\bar{h}_1 \wedge \bar{e}_2)} \quad (\text{UNDERCUT})$$

In words, the evidential support of e_1 in favor of hypothesis h_1 should be smaller if undercutting evidence e_2 holds than if e_2 does not hold. For example, the evidential support of the eyewitness testimony t in favor of the guilty hypothesis g should be smaller if the visibility conditions are bad ($bvis$) than if they are good (\bar{bvis}). That is, $\frac{P(t|g \wedge bvis)}{P(t|\bar{g} \wedge bvis)} < \frac{P(t|g \wedge \bar{bvis})}{P(t|\bar{g} \wedge \bar{bvis})}$. Presumably, the presence of bad visibility conditions reduces the ability of the testimony to discriminate between cases in which the defendant is the perpetrator (g) from cases in which he is not (\bar{g}). The extreme case would be $\frac{P(e_1|h_1 \wedge e_2)}{P(e_1|\bar{h}_1 \wedge e_2)} = 1$. Think, for example, of information elicited during cross-examination showing that the eyewitness was paid to testify against the defendant. If payment was the sole reason why the eyewitness testified, the testimony should be regarded as irrelevant.

(UNDERCUT) conveys in the language of probability what cross-examination is meant to accomplish. When an eyewitness testifies against the defendant, judges and jurors

¹⁸See Chapter 4 of Darwiche (2009) for a more formal argument.

¹⁹As noted before, I am using the convention that lower case letters x stand for values of random variables X .

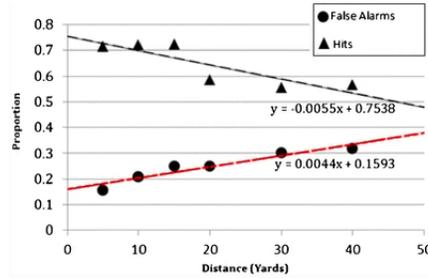


Figure 3: Hits and false alarms in eyewitness identification as distance increases.

should treat the testimony with caution, but absent any evidence of unreliability, they should assign to it a likelihood ratio greater than one. However, if the defense cross-examines the eyewitness and it turns out the lighting conditions were bad, judges and jurors should revise downwards their assessment of the likelihood ratio of the testimony.²⁰

A lingering problem here is how the relevant likelihood ratios should be assessed. Admittedly, a general problem in using Bayesian networks in the evaluation of evidence is to find the right numbers.²¹ Fortunately, a growing body of research in psychology has begun to quantify the effects of estimator variables, such as lighting, distance or duration of exposure, on people’s ability of recognizing faces.²² According to a recent study (see Figure 3), the ratio of hits to false alarms is (roughly) 75% to 15% at 0 yard distance; 70% to 20% at 10 yard distance; 65% to 25% at 20 yard distance; 60% to 30% at 30 yard distance; 55% to 35% at 40 yard distance (Lampinen *et al.*, 2014). These numbers can help to fill in the conditional probabilities needed to assess likelihood ratios such as $\frac{P(t|g \wedge bvis)}{P(t|\bar{g} \wedge bvis)}$ and $\frac{P(t|g \wedge \bar{bvis})}{P(t|\bar{g} \wedge \bar{bvis})}$.²³

This framework for thinking about undercutting is not only consistent with empirical research in psychology. It also allows us to establish formally a number of intuitive corollaries about the effects of undercutting evidence. For example, we expect that evidence e_1 supports a hypothesis less strongly when e_1 occurs in conjunction with undercutting

²⁰Against this analysis, one might argue that undercutting evidence should increase the known risk of a false positive. Judges and jurors should revise upwards their estimated risk that a witness would misidentify an innocent person as the perpetrator of the crime. I defended a similar position in earlier work; see Di Bello (2019). This alternative analysis, however, is controversial. Under bad visibility conditions, the risk that a witness would identify—and also misidentify—someone might diminish. If so, the probability of a misidentification might actually decrease. A less controversial approach should compare the risk of misidentification and the risk of correct identification under good and bad visibility conditions. (UNDERCUT) makes this very comparison.

²¹For a more radical critique of probabilistic methods to assess the value of evidence, see Allen & Pardo (2019).

²²Empirical research has identified a few canonical factors that affect the ability of a witness to correctly identify faces. The ability of a witness to make correct identifications is impaired by brief exposure, poor visibility (bad lighting or long distance) and a long interval between the first exposure and the moment of recollection. Other factors include the race of the alleged perpetrator (cross-racial identifications tend to be less reliable), stress (high stress can lead to worse memory), and weapon focus (the presence of a weapon leads to an impaired ability to recognize the perpetrator). Exposure, visibility, interval, race, stress and weapon focus are *estimator variables*, to be distinguished from *system variables* which instead describes the conditions imposed by the legal system for lineup identification. A point of controversy concerns the relationship between eyewitness accuracy and the subjective certainty of an eyewitness. Recent findings indicate a positive correlation between subjective certainty and accuracy. On this point and estimator and system variables, see Wixted & Wells (2017).

²³I have assumed that undercutting evidence takes only two values, say ‘bad visibility’ and ‘good visibility’. This is a simplification. The variable that stands for the visibility condition could take multiple values.

evidence e_2 . Formally, the combined likelihood ratio $\frac{P(e_1 \wedge e_2 | h_1)}{P(e_1 \wedge e_2 | \bar{h}_1)}$ should be lower than the individual likelihood ratio $\frac{P(e_1 | h_1)}{P(e_1 | \bar{h}_1)}$:

$$\frac{P(e_1 \wedge e_2 | h_1)}{P(e_1 \wedge e_2 | \bar{h}_1)} < \frac{P(e_1 | h_1)}{P(e_1 | \bar{h}_1)} \quad (\text{WEAKENING})$$

This is indeed the case. It is easy to show that (UNDERCUT) implies (WEAKENING).²⁴

To summarize, the formalization of undercutting evidence I have proposed consists of two parts: first, the graph $E_2 \rightarrow E_1 \leftarrow H_1$ that encodes relationships of probabilistic (in)dependence between variables; second, the condition (UNDERCUT) that places a restriction on the numerical probabilities to assign to the variables in the graph.

The same graph could represent a phenomenon opposite of undercutting, call it *reinforcing*. During cross-examination, a witness could say: ‘I was close to the perpetrator and looked at him right in the eyes’. The following can formalize reinforcing evidence:

$$\frac{P(e_1 | h_1 \wedge e_2)}{P(e_1 | \bar{h}_1 \wedge e_2)} > \frac{P(e_1 | h_1 \wedge \bar{e}_2)}{P(e_1 | \bar{h}_1 \wedge \bar{e}_2)} \quad (\text{REINFORCE})$$

Unlike undercutting evidence, the presence of reinforcing e_2 —say, good visibility—increases the ability of evidence e_1 to distinguish between cases in which h_1 holds from cases in which it does not hold. It is easy to show that (REINFORCE) implies

$$\frac{P(e_1 \wedge e_2 | h_1)}{P(e_1 \wedge e_2 | \bar{h}_1)} > \frac{P(e_1 | h_1)}{P(e_1 | \bar{h}_1)} \quad (\text{STRENGTHENING})$$

The evidential support provided by e_1 alone in favor of h_1 is strengthened when information provided by reinforcing evidence e_2 is added to the stock of evidence taken into account for assessing h_1 .²⁵

²⁴The computations are as follows:

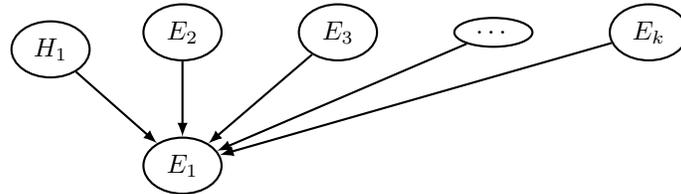
$$\begin{aligned} \frac{P(e_1 | h_1 \wedge e_2)}{P(e_1 | \bar{h}_1 \wedge e_2)} &< \frac{P(e_1 | h_1 \wedge \bar{e}_2)}{P(e_1 | \bar{h}_1 \wedge \bar{e}_2)} \text{ iff} \\ \frac{P(e_2) \times P(e_1 | h_1 \wedge e_2)}{P(e_2) \times P(e_1 | \bar{h}_1 \wedge e_2)} &< \frac{P(\bar{e}_2) \times P(e_1 | h_1 \wedge \bar{e}_2)}{P(\bar{e}_2) \times P(e_1 | \bar{h}_1 \wedge \bar{e}_2)} \text{ iff (*)} \\ \frac{P(e_2 | h_1) \times P(e_1 | h_1 \wedge e_2)}{P(e_2 | \bar{h}_1) \times P(e_1 | \bar{h}_1 \wedge e_2)} &< \frac{P(\bar{e}_2 | h_1) \times P(e_1 | h_1 \wedge \bar{e}_2)}{P(\bar{e}_2 | \bar{h}_1) \times P(e_1 | \bar{h}_1 \wedge \bar{e}_2)} \text{ iff} \\ \frac{P(e_1 \wedge e_2 | h_1)}{P(e_1 \wedge e_2 | \bar{h}_1)} &< \frac{P(e_1 \wedge \bar{e}_2 | h_1)}{P(e_1 \wedge \bar{e}_2 | \bar{h}_1)} \text{ iff} \\ \frac{a}{b} &< \frac{c}{d} \text{ iff (**)} \\ \frac{a}{b} &< \frac{a+c}{b+d} \text{ iff} \\ \frac{P(e_1 \wedge e_2 | h_1)}{P(e_1 \wedge e_2 | \bar{h}_1)} &< \frac{P(e_1 \wedge e_2 | h_1) + P(e_1 \wedge \bar{e}_2 | h_1)}{P(e_1 \wedge e_2 | \bar{h}_1) + P(e_1 \wedge \bar{e}_2 | \bar{h}_1)} \text{ iff} \\ \frac{P(e_1 \wedge e_2 | h_1)}{P(e_1 \wedge e_2 | \bar{h}_1)} &< \frac{P(e_1 | h_1)}{P(e_1 | \bar{h}_1)} \end{aligned}$$

(*) relies on the probabilistic independence of H_1 and E_2 . This independence is at the core of the probabilistic model of undercutting and is codified in the graph $E_2 \rightarrow E_1 \leftarrow H_1$. Further, (**) holds because by algebra $\frac{a}{b} < \frac{c}{d} \Leftrightarrow ad < cb \Leftrightarrow add < cbd \Leftrightarrow abd + add < abd + cbd \Leftrightarrow ad(b+d) < bd(a+c) \Leftrightarrow \frac{ad}{bd} < \frac{a+c}{b+d} \Leftrightarrow \frac{a}{b} < \frac{a+c}{b+d}$.

²⁵See the computations in footnote 24 and change the inequality sign.

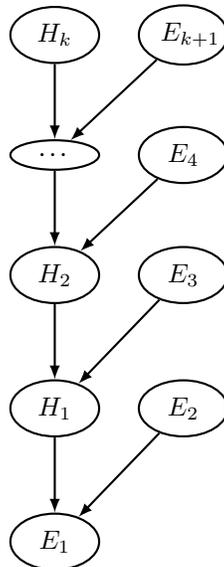
7 Complex Undercutting

The analysis of undercutting evidence (and its correlate, reinforcing evidence) presented in the previous section has several limitations and could be further extended in various directions. First, the graph $H_1 \rightarrow E_1 \leftarrow E_2$ represents a single item of undercutting or reinforcing evidence. But there could be multiple such items, some undercutting and others reinforcing. A more general graph could look as follows:



The conditional probability table associated with node E_1 will quickly grow in size.²⁶ This complexity may pose a problem in practice as it might be difficult to specify many values.²⁷

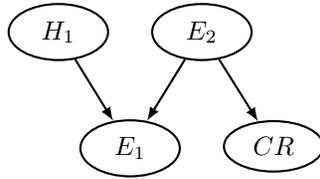
Second, the graph $H_1 \rightarrow E_1 \leftarrow E_2$ does not represent the fact that undercutting evidence can attack evidential relations at multiple steps along a chain of inferences. Suppose evidence e_1 supports hypothesis h_1 which in turn supports another hypothesis h_2 and so on. Undercutting evidence can attack the evidential support of e_1 toward h_1 , of h_1 toward h_2 and so on along the chain of inferences. A more general graph would be:



²⁶For binary nodes, the number of cells of the conditional probability table is 2^n where n is the number of parent nodes. With 3 parent nodes, the table will contain $2^3 = 8$ cells; with 6 parent nodes, the table will contain $2^6 = 64$; with 10 parent nodes, the table will contain $2^{10} = 1,024$ cells; and so on.

²⁷Computer scientists have devised workarounds that limit these practical difficulties, but whether they can be applied to the legal setting remains to be seen. See Chapter 5 of Darwiche (2009).

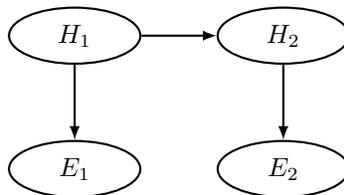
Third, the graph $H_1 \rightarrow E_1 \leftarrow E_2$ —or its expanded version with multiple items—neglects the fact that rebutting or reinforcing evidence is discovered via cross-examination. Cross-examination can fail to identify undercutting evidence—and if so, the first-order evidence would be regarded as stronger than it actually is. Cross-examination can also fail to identify reinforcing evidence—and if so, the first-order evidence would be regarded as weaker than it actually is. To account for this further level of uncertainty, the graph should be expanded as follows:



Whether the visibility conditions represented by undercutting evidence node E_2 were good or bad—strictly speaking—affects how the victim’s testimony should be assessed. Cross-examination represented by the new node CR may correctly or incorrectly track visibility. This graphical structure suggests that a change in terminology is in order. So far I have spoken of ‘undercutting evidence’ or ‘reinforcing evidence’ while referring to good or bad visibility. A more precise terminology would be ‘undercutting hypothesis’ (say bad visibility) and ‘reinforcing hypothesis’ (say good visibility). Evidence about these hypotheses—information retrieved during cross-examination about the visibility conditions—would count as undercutting or reinforcing evidence.

8 Rebutting in Bayesian Networks

Turning now from undercutting to rebutting, I will first outline a formalization of rebutting evidence (this section) and then apply it to cross-examination (next section). As in the case of undercutting, the formalization will include a graphical part accompanied by constraints on the probability assignments. The graph below can be used to represent pieces of evidence rebutting one another as they support incompatible hypotheses:



The arrows from hypothesis nodes, H_1 and H_2 , to evidence nodes, E_1 and E_2 , indicate that the evidence, e_1 or e_2 , bears directly on its own hypothesis, h_1 or h_2 , by supporting or opposing it. To say that each piece of evidence supports its own hypothesis, the graph should be supplemented by the following:

$$\frac{P(e_1|h_1)}{P(e_1|\bar{h}_1)} > 1 \text{ (that is, } e_1 \text{ supports } h_1) \tag{SUPPORT-E1}$$

$$\frac{P(e_2|h_2)}{P(e_2|\bar{h}_2)} > 1 \text{ (that is, } e_2 \text{ supports } h_2) \quad (\text{SUPPORT-E2})$$

To express the fact that the two hypotheses are incompatible—they cannot be both true—the following condition should be added:

$$P(h_2|h_1) = 0 \quad (\text{INCOMP})$$

The graph above supplemented by (SUPPORT-E1), (SUPPORT-E2) and (INCOMP) is the proposed formalization of what it takes for a piece of evidence, say e_1 , to rebut another, say e_2 .²⁸ A number of corollaries, some more intuitive than others, can now be derived.

The first intuitive corollary is that, if e_2 supports h_2 but h_2 is incompatible with h_1 , then e_2 will oppose, not support, h_1 . This claim can be established formally.²⁹ Further, as Figure 4 shows, the more strongly e_2 supports h_2 , the more strongly e_2 opposes the incompatible hypothesis h_1 . This result agrees with our intuitions. The stronger the evidence that the defendant spent the night with a friend away from the crime scene, the stronger the evidence *against* the hypothesis that the defendant was the perpetrator.

Another intuition is that, when two items of evidence rebut one another, the combined evidential support they provide to one of the hypotheses should be weaker than the support provided by one of them. That is,

$$\frac{P(e_1 \wedge e_2|h_1)}{P(e_1 \wedge e_2|\bar{h}_1)} < \frac{P(e_1|h_1)}{P(e_1|\bar{h}_1)}. \quad (\text{WEAKENING})$$

This claim can be established formally.³⁰ Interestingly, as shown before, undercutting evidence satisfies (WEAKENING) as well. This is what rebutting and undercutting evidence have in common. At the same time, rebutting and undercutting are different forms of

²⁸If the arrow went from H_2 to H_1 , the incompatibility constraint would be $P(h_1|h_2) = 0$.

²⁹The support of e_2 relative to h_1 (not h_2) satisfies the equality:

$$\begin{aligned} \frac{P(e_2|h_1)}{P(e_2|\bar{h}_1)} &= \frac{P(e_2 \wedge h_2|h_1) + P(e_2 \wedge \bar{h}_2|h_1)}{P(e_2 \wedge h_2|\bar{h}_1) + P(e_2 \wedge \bar{h}_2|\bar{h}_1)} \\ &= \frac{P(h_2|h_1) \times P(e_2|h_1 \wedge h_2) + P(\bar{h}_2|h_1) \times P(e_2|h_1 \wedge \bar{h}_2)}{P(h_2|\bar{h}_1) \times P(e_2|\bar{h}_1 \wedge h_2) + P(\bar{h}_2|\bar{h}_1) \times P(e_2|\bar{h}_1 \wedge \bar{h}_2)} \\ &=^* \frac{P(h_2|h_1) \times P(e_2|h_2) + P(\bar{h}_2|h_1) \times P(e_2|\bar{h}_2)}{P(h_2|\bar{h}_1) \times P(e_2|h_2) + P(\bar{h}_2|\bar{h}_1) \times P(e_2|\bar{h}_2)} \\ &= \frac{P(e_2|\bar{h}_2) \times [P(h_2|h_1) \times \frac{P(e_2|h_2)}{P(e_2|\bar{h}_2)} + (1 - P(h_2|h_1))]}{P(e_2|\bar{h}_2) \times [P(h_2|\bar{h}_1) \times \frac{P(e_2|h_2)}{P(e_2|\bar{h}_2)} + (1 - P(h_2|\bar{h}_1))]} \\ &= \frac{P(h_2|h_1) \times \frac{P(e_2|h_2)}{P(e_2|\bar{h}_2)} + (1 - P(h_2|h_1))}{P(h_2|\bar{h}_1) \times \frac{P(e_2|h_2)}{P(e_2|\bar{h}_2)} + (1 - P(h_2|\bar{h}_1))} \end{aligned}$$

The equality marked by the asterisk holds because of the independencies encoded in the graph $E_1 \leftarrow H_1 \rightarrow H_2 \rightarrow E_2$, specifically, E_2 is independent of H_1 conditional on H_2 . It is easy to see that the right hand side is always smaller than one so long as $P(h_2|h_1) = 0$ (that is, the two hypotheses are incompatible) and $\frac{P(e_2|h_2)}{P(e_2|\bar{h}_2)} > 1$ (that is, e_2 supports h_2). So, as expected, $\frac{P(e_2|h_1)}{P(e_2|\bar{h}_1)} < 1$.

³⁰To establish this claim, note that $\frac{P(e_1 \wedge e_2|h_1)}{P(e_1 \wedge e_2|\bar{h}_1)} = \frac{P(e_1|h_1)}{P(e_1|\bar{h}_1)} \times \frac{P(e_2|h_1)}{P(e_2|\bar{h}_1)}$ because E_1 and E_2 are probabilistically conditionally independent on H_1 . By the earlier argument, the second factor must be less than one; see the previous footnote. This suffices to establish (WEAKENING).

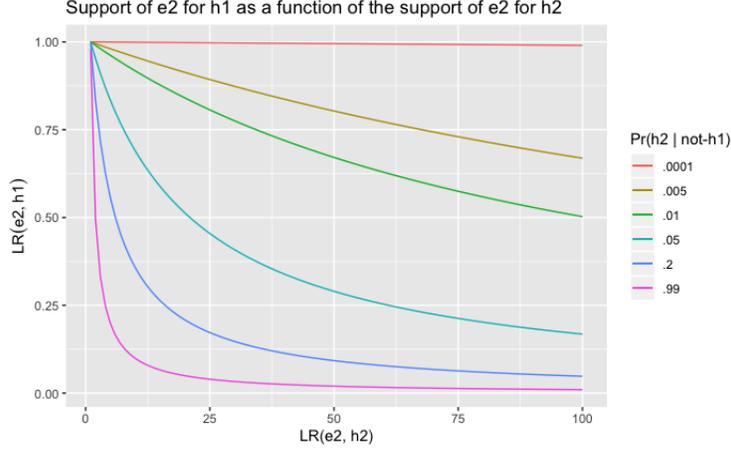


Figure 4: For different values of $P(h_2|\bar{h}_1)$, the stronger the support of e_2 in favor of h_2 , the greater the opposition of e_2 against the incompatible hypothesis h_1 . Crucially, $LR(e_2, h_1)$ is always below 1 so long as $LR(e_2, h_2)$ is above one, where $LR(e_2, h_1)$ is short for $\frac{P(e_2|h_1)}{P(e_2|\bar{h}_1)}$ and $LR(e_2, h_2)$ short for $\frac{P(e_2|h_2)}{P(e_2|\bar{h}_2)}$. The other noteworthy trend is that the lower $P(h_2|\bar{h}_1)$, the weaker the opposition of e_2 against h_1 even with the same support of e_2 in favor of h_2 .

attack. Their peculiarity is captured by their own Bayesian network along with the relationships of probabilistic (in)dependence the network encodes.

Consider now a corollary of the formalization that is less obvious. Start with the special case in which not only are hypotheses incompatible but also jointly exhaustive so that the falsity of one implies the truth of the other, that is, $P(h_2|\bar{h}_1) = 1$. In this case, it can be shown that

$$\frac{P(e_1 \wedge e_2|h_1)}{P(e_1 \wedge e_2|\bar{h}_1)} = \frac{P(e_1|h_1)}{P(e_1|\bar{h}_1)} \times \frac{P(e_2|\bar{h}_2)}{P(e_2|h_2)}.$$

So long as h_1 and h_2 are incompatible and jointly exhaustive, the combination of two items of evidence that rebut one another (with equal force) forms a body of evidence that offers no support to hypothesis h_1 in the sense that $\frac{P(e_1 \wedge e_2|h_1)}{P(e_1 \wedge e_2|\bar{h}_1)} = 1$. This results adds precision to the conjecture that two conflicting pieces of evidence cancel each other out (see earlier discussion in Section 4). For two items of evidence to cancel each other out, (a) they must support their respective hypotheses with equal strength, and (b) the hypotheses they support must be incompatible and exhaustive. If either of these conditions is not met, the two items of evidence will not cancel each other out.

Consider, for example, the situation in which the hypotheses are not exhaustive and thus $P(h_2|\bar{h}_1)$ is smaller than one. As Figure 4 shows, the lower $P(h_2|\bar{h}_1)$, the weaker the

³¹Recall that $\frac{P(e_1 \wedge e_2|h_1)}{P(e_1 \wedge e_2|\bar{h}_1)} = \frac{P(e_1|h_1)}{P(e_1|\bar{h}_1)} \times \frac{P(e_2|h_1)}{P(e_2|\bar{h}_1)}$ because of conditional independence of e_1 and e_2 . Further, from footnote 29, recall that $\frac{P(e_2|h_1)}{P(e_2|\bar{h}_1)} = \frac{P(h_2|h_1) \times \frac{P(e_2|h_2)}{P(e_2|\bar{h}_2)} + (1 - P(h_2|h_1))}{P(h_2|\bar{h}_1) \times \frac{P(e_2|h_2)}{P(e_2|\bar{h}_2)} + (1 - P(h_2|\bar{h}_1))}$. If we set $P(h_2|h_1) = 0$ (incompatibility) and $P(h_2|\bar{h}_1) = 1$ (exhaustivity), then $\frac{P(e_2|h_1)}{P(e_2|\bar{h}_1)} = \frac{P(e_2|\bar{h}_2)}{P(e_2|h_2)}$.

opposition of e_2 against h_1 for the same degree of support of e_2 in favor of h_2 . How should this result be interpreted? Say the prosecution puts forward hypothesis h_1 that the defendant assaulted the victim and offers good supporting evidence. The defense lawyer in response offers evidence for the incompatible hypothesis h_2 which is, however, highly improbable even assuming that h_1 is false. Even if the defense offers strong evidence for this alternative hypothesis, this evidence would mount a weak attack against the prosecutor's hypothesis, at least weaker than if the defense had offered rebutting evidence for an alternative hypothesis whose prior probability was higher. So, as they aim to successfully attack the prosecution's case by offering rebutting evidence for an alternative hypothesis, defense lawyers should be aware of the following: the more (a priori) improbable the alternative hypothesis, the stronger the evidence in its favor needs to be; the more (a priori) probable the alternative hypothesis, the less strong the evidence in its favor needs to be.³²

9 Rebutting evidence from the same source

The discussion of rebutting evidence so far does not obviously apply to cross-examination. When the defense presents independent evidence for a hypothesis incompatible with the prosecutor's hypothesis, such as evidence of an alibi, this is not an instance of cross-examination. Rebutting evidence, however, can be elicited during cross-examination. For suppose a witness claims to have given a ride to the defendant and dropped him off at the park at night (where we know the crime took place). Under cross-examination, the same witness admits to have spent the night with the defendant at the bar. The two statements made by the witness are inconsistent in the sense that they support inconsistent hypotheses.³³ When inconsistent statements are elicited from the same witness by asking probing questions, this is an example of rebutting evidence obtained during cross-examination.

The formalization of rebutting evidence in the previous section was developed without assuming that the inconsistencies would come from the same source. The question now is whether the Bayesian network for rebutting evidence should be modified—and if so, how—to capture the fact that one witness is the source of inconsistent statements.

Let's start with the conjecture that there is a difference in probative value between inconsistent statements made by different witnesses and inconsistent statements made by the same witness. What could this difference be? For one thing, when inconsistent statements are made by two independent witnesses, the probative value of each statement relative to a hypothesis of interest remains the same whether or not the statement is preceded by an inconsistent statement.³⁴ On the other hand, let e_1 represent the fact that a witness made a statement and e_2 represents the fact that the *same* witness subsequently made a statement that is inconsistent with the earlier statement. Suppose e_1 supports h_1 while e_2 supports the alternative hypothesis h_2 . Since the two items of evidence have a common source—the same witness—the conjecture is that e_2 should support h_2 more strongly—or be more

³²This point has implications for the standard of proof. Even if defense lawyers can offer extremely strong evidence for an alternative hypothesis, they might not manage to tip the balance of the evidence in their favor so long as the alternative hypothesis they have selected has an extremely low prior probability. Lawyers can dismiss this advice since it is unlikely that judges or jurors would assess the evidence by means of the probability calculus. But, normatively speaking, the advice stands.

³³The statement that the defendant was in the park and the statement that the defendant was at the bar support inconsistent hypotheses given certain assumptions, for example, that the bar and the park are in two locations.

³⁴In the Bayesian network for rebutting evidence, E_1 and E_2 are independent conditionally on H_1 or H_2 .

revealing of the truth of h_2 —in case it is accompanied by e_1 than otherwise. That is,

$$\frac{P(e_2|h_2)}{P(e_2|\bar{h}_2)} < \frac{P(e_2|h_2 \wedge e_1)}{P(e_2|\bar{h}_2 \wedge e_1)}. \tag{REVEAL}$$

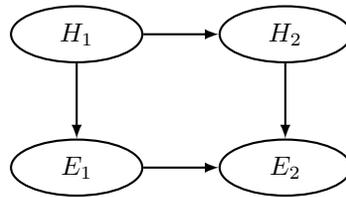
In words, when the same witness is the source, the probative value of evidence e_2 in favor of h_2 receives a boost if it is preceded by evidence e_1 which conflicts with e_2 . Given the boost, when an inconsistency emerges between statements made by the same witness, the inconsistency should weaken the prosecution’s case for h_1 more effectively than if the inconsistency had two independent witnesses as its source.³⁵

So does the conjecture in (REVEAL) hold? An intuitive justification for it is that witnesses strive to keep their assertions as consistent as possible. If a witness makes an assertion, they will be biased against making subsequent assertions that conflict with earlier assertions they made. If, despite what we might call a ‘consistency bias’, witnesses still contradict themselves, the assertion that contradicts an earlier assertion should count as especially revealing of the truth, as (REVEAL) indicates.³⁶ This informal argument is intuitively plausible. Nevertheless, we will see—perhaps surprisingly—that such consistency bias actually entails that (REVEAL) is false.

We should first formalize the consistency bias in the language of probability. It can be formalized by positing that e_2 is less likely assuming e_1 has been offered as evidence, no matter the truth or falsity of h_2 :

$$P(e_2|h_2 \wedge e_1) < P(e_2|h_2) \text{ and } P(e_2|\bar{h}_2 \wedge e_1) < P(e_2|\bar{h}_2). \tag{BIAS}$$

The thought is that the same witness is less willing to offer e_2 as evidence after offering e_1 as evidence whenever the contents of the two are inconsistent. As a sanity check, note that (BIAS) is false in the earlier graph for rebutting evidence $E_1 \leftarrow H_1 \rightarrow H_2 \rightarrow E_2$. This is as it should be. The graph for rebutting was not intended to formalize the consistency bias of witnesses. The two pieces of evidence were assumed to be probabilistically independent of one another conditional on either hypothesis. The graph could, however, be modified:



The additional arrow between E_1 and E_2 indicates that the values of the second piece of evidence depend on the values of the other evidence. But despite the change, it is easy to see that under plausible assumptions (BIAS) entails that $\frac{P(e_2|h_2)}{P(e_2|\bar{h}_2)} = \frac{P(e_2|h_1 \wedge e_1)}{P(e_2|h_2 \wedge e_2)}$, contra

³⁵Recall the result in the previous section that the stronger the support of e_2 in favor of h_2 , the stronger the opposition of e_2 against the incompatible hypothesis h_1 . If, assuming (REVEAL), e_2 given e_1 supports h_2 more strongly than e_2 alone does, then e_2 given e_1 opposes h_1 more strongly than e_2 alone does.

³⁶Another explanation posits a ‘loyalty bias.’ Witnesses are likely to have a bias for the party in whose favor they were called to testify. If they made an assertion against the defendant (or in favor of the defendant), they would presumably be unwilling to make assertions that went in the opposite direction. If they still make an assertion against the party they should be loyal to, that assertion should count as more revealing of the truth.

(REVEAL).³⁷ So a statement is no more revealing of the truth when it is inconsistent with a previous statement made by the same witness who has a consistency bias.³⁸

There is no simple way to justify the claim that inconsistencies attributable to the same witness are more truth-revealing than inconsistencies across statements made by different witnesses. So the same graph $E_1 \leftarrow H_1 \rightarrow H_2 \rightarrow E_2$ can be deployed as a model for rebutting evidence whether the evidence comes from different witnesses or the same witness. In modeling rebutting evidence coming from the same witness, an arrow could still be added between the evidence nodes to highlight a possible extra layer of dependence (say, capturing the consistency bias). But, as seen here, the dependence between the evidence nodes should not eventuate in a difference in the assessment of the probative value of the evidence. This conclusion is surprising, as there seems to be a difference between the two cases, though not one easily discernible in Bayesian networks.³⁹

10 Is Cross-examination truth conductive?

It is time to conclude and offer some thoughts about whether cross-examination, as Wigmore put it, is the greatest legal engine ever invented for the discovery of truth.

I have shown that cross-examination is a retrieval process best described as a refinement of the existing evidence by the addition of subsidiary information. This subsidiary information can be in the form of rebutting or undercutting evidence. It serves to ‘test’ the existing evidence to see whether it should be given more or less weight than it was initially assumed. My analysis of cross-examination deployed probability theory and Bayesian networks. The analysis showed that many intuitions we have about how evidence should be weighed in light of the supplementary information elicited during cross-examination are correct. There is no doubt that bad visibility conditions should reduce the probative value we assign to eyewitness identifications. The extent of such a reduction is less clear. I have shown how data from empirical research in psychology about eyewitness identifications can be integrated in my probabilistic analysis. Other intuitions that we commonly have are less easily justifiable. For example, we might be overly impressed when witnesses contradict themselves. My probabilistic analysis did not provide any compelling reason why we

³⁷It is enough to assume that $P(e_2|h_2 \wedge e_1) = P(e_2|h_2) \times 1/k$ and $P(e_2|\overline{h_2} \wedge e_1) = P(e_2|h_2) \times 1/k$. The assumption here is that the bias reduces (by a fixed, arbitrary factor $1/k$) the probability that the witness would offer the conflicting evidence e_2 . The reduction factor $1/k$ is the same since presumably the bias of the witness is equally strong regardless of the truth or falsity of h_2 .

³⁸But what if the witness is calculating what to say not only in light of previous assertions but also in light of what they know about h_2 ? Suppose the witness knows that h_2 is true (for example, the witness knows the defendant is innocent), but nevertheless testified in favor of h_1 (say the defendant is the perpetrator). The witness lied in offering incriminating evidence e_1 for the prosecutor’s hypothesis. Given the situation, the witness presumably does not want the truth to come to light and wants to avoid at all costs to offer e_2 as evidence for h_2 . This is a situation in which the witness is unwilling to offer e_2 as evidence, and the unwillingness is more pronounced assuming h_2 is true than in case h_2 is false. Nevertheless, against all these machinations, suppose the witness does still offer e_2 as evidence. Should this be particularly revealing of the truth? It should, but it has nothing to do with the fact that the witness contradicted themselves. This scenario amounts to the stipulation that the incriminating evidence was worthless because the witness purposefully lied.

³⁹Bovens & Hartmann (2002) rely on Bayesian networks to show that there is a difference between the following: (i) multiple reports about a phenomenon of interest coming from the same instrument and (ii) multiple reports coming from different instruments. However, given the difference between witnesses and instruments, their analysis might not readily apply to cross-examination. I leave this for future investigation. Another difference between inconsistent statements made by one single witness and those made by two distinct witnesses might be this: one witness would retract an earlier statement if an inconsistency is brought to their attention, while two witnesses would hold on to their statements. Such retraction may happen in some cases, but not always.

should treat internal inconsistencies as especially revealing of the truth. Besides solidifying some of our intuitions and challenging others, the probabilistic analysis also revealed trends that might have otherwise gone unnoticed. For example, when two pieces of evidence support incompatible hypotheses, it is important to consider whether or not the hypotheses are exhaustive. This difference affects the extent to which the two conflicting pieces of evidence cancel each other out.

Let me now sketch some thoughts about the truth-seeking function of cross-examination. My remarks will primarily cover cross-examination as the retrieval of subsidiary information about undercutting and reinforcing facts. To clarify the terminology, recall the distinction in Section 7 between undercutting (and reinforcing) evidence and undercutting (and reinforcing) hypotheses. Undercutting evidence (such as, the witness says the lighting conditions were bad) supports an undercutting hypothesis (such as, the lighting condition were bad). The same distinction applies to reinforcing evidence and hypothesis. A true undercutting (or reinforcing) hypothesis is an undercutting (or reinforcing) fact.

The claim that cross-examination as the retrieval of undercutting and reinforcing facts is truth-conducive rests on two premises:

- (P1) The assessment of the probative value of the evidence presented at trial—via the identifications of the relevant undercutting and reinforcing facts—is truth-conducive.
- (P2) Cross-examination is the best method for identifying all the relevant undercutting and reinforcing facts.

The first premise is very plausible. Some might even find it trivial. I will therefore take it for granted. The second premise is more controversial. Those who doubt that cross-examination is truth-conducive might doubt the second premise while accepting the first.

What would it mean to identify all undercutting and reinforcing facts relevant for assessing the probative value of a piece of evidence? This varies on the type of evidence under consideration. If the evidence consists in an eyewitness testimony, information about estimator variables such as visibility, stress, exposure, as well as system variables (say how the lineup identification was conducted) should be gathered. Information about the character of the witness, motives the witness might have to testify, relationships to the defendant—these should all be relevant factors to consider. If the evidence consists in testimonies given by an expert witness, there are other factors to consider: the academic credentials of the witness, the reliability of the method used, etc. Ideally, for each type of evidence there should be a repertoire of errors or ways the evidence could go wrong. If not all possibilities of errors are investigated or if they are investigated but misjudged, this could lead to one of two consequences: an item of evidence is judged to be more probative than it should be, or is judged less probative than it should be. In the long run, if incriminating evidence is given more weight than it should be given, a larger number of factually innocent people will be convicted. If it is given less weight than it should be given, a larger number of factually guilty people will be acquitted.

Failures to identify the relevant undercutting and reinforcing facts could cause an increase in false positive and false negatives. The question is to what extent, or under what conditions, these failures can be attributed to cross-examination. There are circumstances in which these failures cannot be attributed to cross-examination. For example, that the parties involved—prosecution or defense—do not know all the relevant undercutting or reinforcing facts is not a failure attributable to cross-examination.

But prosecution and defense will often willfully avoid to mention relevant undercutting or reinforcing facts. They will cherry pick information and thus offer an incomplete picture.

Cross-examination is likely to encourage—almost inevitably—such willfully neglect. For consider this exchange:

Lawyer: You said you saw the defendant at the park that night.

Witness: I did.

Lawyer: Can you tell us what time it was?

Witness: I believe it was sometime in the evening.

Lawyer: Would you say it was dark?

Witness: Well, I guess...

Lawyer: So you confirm it was dark?

Witness: I do.

Any textbook on cross-examination would recommend that the lawyer stop here. They made their point. They weakened the testimony. But suppose instead the lawyer went on.

Lawyer: If it was dark, how could you *see* the defendant there?

Witness: He was running and accidentally bumped against me. He seemed in a big hurry. I helped him get up. I could look at him straight in the eyes.

From the point of view of truth-seeking, this cross-examination is a success. It revealed an undercutting fact (bad visibility) accompanied by a reinforcing fact (the witness could get a close look at the defendant). But from a lawyer's standpoint, this is a disaster. Cross-examination discourages lawyers to seek all the relevant undercutting and reinforcing facts. Not only will prosecution and defense cherry pick the undercutting and reinforcing facts, they will also fabricate them by presenting misleading undercutting and reinforcing evidence. In this sense—so the worry goes—cross-examination is far from truth-conducive.

But this argument fails to acknowledge that both parties will seek to present undercutting and reinforcing facts. Naturally they will present the facts that best support their case. And they will do their best to present all such facts. So no relevant supplementary facts should be left out from the contributions of both parties together. Prosecution and defense will have an incentive to fabricate undercutting and reinforcing facts. There is no denying that. There will inevitably be noisy information presented by the parties. But both parties will have an incentive to identify and discard the noisy information presented by the other party. Each party will have an incentive—during cross-examination—to present noisy information that advances their case. Conversely, the other party will have an incentive in blocking—again, during cross-examination—the noisy information from their opponent. This adversarial game could succeed in maintaining the overall level of noisy information fairly low.

Perhaps the concern here is with the uneven allocation of resources between prosecution and defense. The uneven allocation of resources is not a feature of cross-examination, but no doubt has deleterious effects. If the prosecution has more resources, the reinforcing and undercutting facts that favor the prosecutor's side will be more readily identified compared to the reinforcing and undercutting facts that favor the defense. This will be detrimental to truth-seeking. Another concern might be with the uneven allocation of resources across defense lawyers for different defendants. This uneven allocation of resources across defense lawyers is primarily a concern for fairness and equal treatment of

defendants. When resources are unevenly allocated, some defendants will benefit from the right to confrontation more than others.

All in all, cross-examination understood as the retrieval of undercutting and reinforcing facts does not obviously discourage the pursuit of truth. If resources are allocated unevenly, however, truth and fairness will suffer. The case of rebutting is more complicated. As seen in the last section, the danger here is that inconsistencies between statements made the same witness could be given more weight than they should. So here is a conjecture for future research. If the subsidiary information elicited during cross-examination is given its appropriate weight and if resources are evenly allocated between prosecution and defense, cross-examination might well be the greatest legal engine ever invented for the discovery of truth.

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