Fuzzy Time & NP Hardness

(P*=BPP*, P*≠NP*)

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Abstract. We have shown the plausibility of considering time as a Fuzzy concept instead of classical time [7], [8].

By considering time as a fuzzy concept, we will have new classes of Complexity.

Here, we show that how some famous problems will be solved in this new picture.

Keywords: P*=BPP*, P*≠NP*, $MA^* = MA$, Random generator, Pseudo random Generator.

Introduction

"Unexpected Hanging Paradox" was discovered by Swedish Mathematician Lennart Ekbom [24] and then introduced by British Philosopher Daniel John O' Connor in his paper [1]. There is relatively a vast literature about solving this problem and some great logicians like Willard Quine [25] and Saul Kripke [23] works on that. You can find more details in [14], [16], [24].

In [7], [8], [10] by employing a new version of "Unexpected Hanging Paradox" we provide an argument based on that we are somewhat forced to see Time as a fuzzy concept.

The first attempts in this way was before 2010 and was presented in a Conference and later in Arxiv [4], [13]. The author had a similar lecture and seminars about in Sharif University and IPM. Later, more complete versions are presented in [5], [11].

Indeed, people usually answer this paradox in two major approaches. Logical and epistemological approaches. In the new attempt and the new version of this paradox [10], two different aspects are considered to be instead, the Physical and Logical approaches.

But two major problems arise in this approach, actually around the concept of Time:

First, time is a central concept in Physics. Whether our understanding of time as a fuzzy concept adopts to the existing Theories in Physics and Physical evidences?

Second, if the instants of time are operators (Fuzzy Numbers), how can we calculate them and

Whether the result of the calculation resolves the difficulties arise by the paradox?

In [3], [12] by introducing a new interpretation of Quantum Mechanics (Fuzzy time- Particle interpretation of Quantum Mechanics) we answer the above problems positively. Although it is shown that this new interpretation answers many usual and common problems in Quantum

Mechanics interpretations satisfactory [3], [12], nevertheless we try to make much progress in some details in a way it satisfies more evidences in Physics. The above attempts are partly presented in Conferences [5], [9], [11].

In [6], we see by considering time as a fuzzy concept the definition of Complexity Classes will change. What about NP hard and NP Complete problems?

How do we depict the difference between P problems and NP hard problems here, as probably the most important subjects in Complexity Theory and even in Theory of Algorithms?

In chapter 1, we show the known Probabilistic Classes could help us in this regard.

In chapter 2, we remind the existence of Random Generator and the impact of that on Theory of Computation when we assume time as a fuzzy concept. We show how some problems in Complexity Theory will be solved in this regard.

1. NP problems in the new frame work

In [6], by defining the conceptS P, BPP [20], [21] in the new framework we have P^*, BPP^*. It is shown that the new classes P^*, BPP^* are both equivalent to BPP. In contrast, what about the substitution of class of NP in this new framework. To represent NP problems in the Theory of Algorithm, it is required to define a new class for that. Possibly the best choice in probabilistic classes in this purpose is MA [15], [22] (introduced by Laszlo Babai, Shafi Goldwasser, Micheal Sipser).

The complexity class MA is known as the candidate of NP problems in probabilistic classes, also we have a theorem states [19]

$$P = BPP \rightarrow MA = NP$$

This point besides $P^* = BPP^*$ strengthen our choice. So, we try to define the NP concept in fuzzy time by applying the definition of MA.

Here, we define MA in Two sided version definition [20].

Definition: *The Complexity class **MA** is the set of decision problems like D such that there are*

Deterministic Polynomial Time Turing Machine M_D and p_D, q_D such that for every input x with length x' ($l(x)$=x')

1. *x belongs to D implies there exists string z with length $q_D(x')$ such that for all string y with length $p_D(x')$ $Pr(M_D(x,y,z) = 1) \geq 2/3$)*

2. *x belongs to D implies for all string z with length $q_D(x')$ such that for all string y with length $p_D(x')$ $Pr(M_D(x,y,z)=0) \geq 2/3$ (The definition is Quoted [20])*

As a conclusion, by changing and transforming the literature of Theory of Computation from Classical Time to Fuzzy time the classes of Complexity Theory changes to new classes. Likewise,

We have new problems.

The list of new possible classes are

P^*,BPP^* and MA^*,AM^*

Instead of $P=NP$ problem we have the following problems

$$BPP^*=MA^*$$

$$BPP^*=AM^*$$

$$MA^*=AM^*$$

The two last questions remained unproved.

It is easy to see:

1. $P^*=BPP^*$
2. $NP^*=MA^*$ (Considering certificate definition of NP)
3. $MA^*=MA$

Chapter 2.Pseudorandom generator & NP^+

Pseudo random generators play a major role in Theory of computation. The existence of pseudo random generator by applying classical time leads us to P≠NP. What about theory of computation when we consider time as a fuzzy concept (TC^*)?

First, we should redefine pseudo random generator in TC^*. One of the central concept here is indistinguishability. Here, it might be in some branches of our specific computation we have turning back in time, consequently probably in these branches of computation we have not the desired sequence.

So, we should speak about indistinquishability in a high probability (for instance bigger than 2 over 3).

By considering the above, we modify the definition of pseudo random generator in TC^*, as follows

Definition *(Pseudorandom Generator). A deterministic polynomial time algorithm G is called a pseudorandom Generator if there exists a stretching function* $l: N \to N$*, such that the following two probability ensembles, denoted* $\{G^n\}$ *and* $\{U^n\}$*, are computationally indistinguishable Distribution* $\{G^n\}$ *is defined as the out put of G whose length is* $l(n)$ *on a uniformly selected seed in* $\{0,1\}^n$*,*

Distribution $U_{l(n)}$ *is defined as the uniform distribution on* $\{0,1\}^{l(n)}$ *, where* $l(n) > n$.

1. *That is, we require that for any probabilistic polynomial-time algorithm* A *, for any positive polynomial* $p: N \to N$ *and for all sufficiently large* n*s, It holds that*

   $$|PR\left[A\left(G(U^n)\right) = 1\right] - PR[A\left(U^{l(n)}\right) = 1]\ | < 1/p(n)$$

   $seed$: *(The definition is*

   Quoted: $notes8\ great\ ideas\ in\ theoretical\ computer\ science\ saarland\ university, summer\ 2014$)

In contrast, we define Pseudorandom generator* as follows

Definition *(Pseudorandom Generator*). A deterministic polynomial time algorithm G is called a pseudorandom Generator if there exists a stretching function* $l: N \to N$*, such that the following two probability ensembles, denoted* $\{G^n\}$ *and* $\{U^n\}$*, are computationally indistinguishable in a high probability:*

2. *Distribution $\{G^n\}$ is defined as the out put of G whose length is $l(n)$ on a uniformly selected seed in $\{0,1\}^n$, for majority of seeds more than 2 over 3*
3. *Distribution $U_{l(n)}$ is defined as the uniform distribution on $\{0,1\}^{l(n)}$, where $l(n) > n$.*
4. *That is, we require that for any probabilistic polynomial-time algorithm A, for any positive polynomial $p: N \to N$ and for all sufficiently large ns, It holds that*

   $$|PR\big[A\big(G(U^n)\big) = 1\big] - PR[A\big(U^{l(n)}\big) = 1]| < 1/p(n)$$

As we see in [6], we have the following lemma,

Lemma 1: There is a pseudorandom generator in $\mathrm{TC}*$.

Proof. Let see [6].

To obtain our main result in Theorem*, we define NP+.

Definition (NP+) Non deterministically guess the input for deterministic Turing machine M, we call this new machine $M+$.

NP+ are the set of languages which accept by some M+.

When we consider time as a fuzzy concept in above, we have NP+*.

NP+ and NP and NP+* are subsets of NP*.

Theorem*: NP*=P* & the existence of random generator leads us to a contradiction, moreover by lemma1 we have NP*≠P*.

(Hint of proof: NP=P* implies NP+* is a subset of P*. First, we select all the seeds non deterministically, in a high probability we generate all random numbers. Since NP*=P* so the generator is not pseudo random*.But by lemma 1, we have a random generator.)*

Conclusion. In [6], we show that assuming time as a fuzzy concept is plausible, since we put aside a contradiction arise by the paradox. Moreover, possibly it is the unique way if we neglect employing Non Classical Logic like Paraconsistent Logic [18].

Also, we show that the concept of "Fuzzy Time" is plausible and acceptable in Physics too. To do it, we introduce Fuzzy time-Particle interpretation of Quantum Mechanics [3], [8], [12]. The great Philosophers and Mathematician like Husserl and Brower intuition about time were similar and in general on the same track [17].
Here, we show that considering time as a fuzzy concept, have some major results in solving famous problems in Complexity Theory in a way that it adopts to the intuition and expectation of people in Theory of Algorithm. In brief, $P^* \neq NP^*$, $P^* = BPP^*$.
This could be considered as the fourth reason to make a shift in the existing Theories about Time, equivalently to assume time as a fuzzy concept. Therefore, considering Fuzzy time is not only a style or taste as in some attempts had done. In these attempts, usually people consider both time and space fuzzy concepts. Since in these approaches there is no point to differentiate between the two concepts, time and space. Seemingly, they are in the line of fuzzifying all possible concepts. Respectful attempts, but here we have a specific reason to consider time as a fuzzy concept [7],[10]. In [2] we are able to find somewhat a similar sense about time from a different approach. But it is indirect and related to our interpretation, and it didn't provide a system of computation to calculate the instants of time as fuzzy numbers.

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