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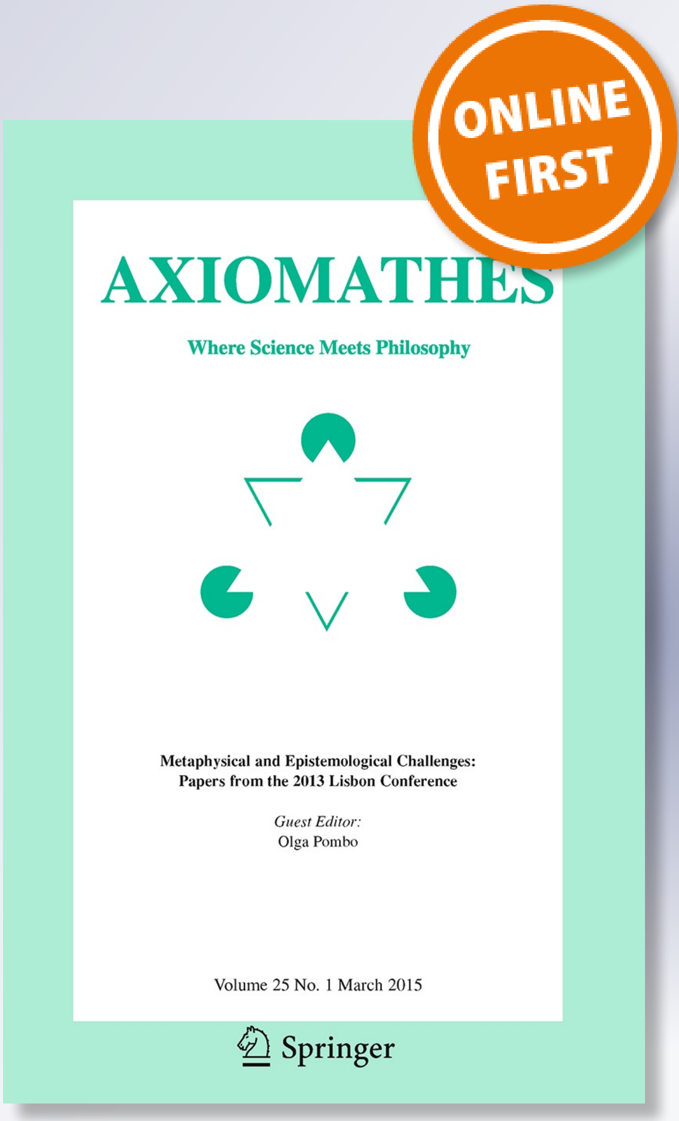

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ORIGINAL PAPER

Science Generates Limit Paradoxes

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Abstract The sciences occasionally generate discoveries that undermine their own assumptions. Two such discoveries are characterized here: the discovery of apophenia by cognitive psychology and the discovery that physical systems cannot be locally bounded within quantum theory. It is shown that such discoveries have a common structure and that this common structure is an instance of Priest's well-known Inclosure Schema. This demonstrates that science itself is dialetheic: it generates limit paradoxes. How science proceeds despite this fact is briefly discussed, as is the connection between our results and the realism-antirealism debate. We conclude by suggesting a position of epistemic modesty.

Keywords Apophenia · Dialetheism · Psychology · Quantum measurement · Realism · Scientific knowledge

1 Introduction

There is a well-known metaphor of using a ladder to gain some perch, and then jettisoning the ladder. This metaphor often carries the sense that the ladder is crucial early on, but ultimately takes one to a place where the ladder is or even must be jettisoned or destroyed. Wittgenstein provides a well-known example: he uses the metaphor to explain what understanding his *Tractatus* requires:

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> My propositions are elucidatory in this way: he who understands me finally recognizes them as senseless, when he has climbed out through them, on them, over them. (He must so to speak throw away the ladder, after he has climbed up on it.) He must surmount these propositions; then he sees the world rightly.
> *Tractatus*, 6.54

Here Wittgenstein is claiming that once the reader understands what he is saying in the *Tractatus*, the reader will realize that Wittgenstein has said nothing.[1]

This ladder-scrapping phenomenon shows up in widely diverse places. In Buddhism, the desire to relinquish all attachments ultimately takes one to a place where one sees that this prime desire is itself an attachment. One discovers, therefore, a desire to lose one's prime desire. So the desire to relinquish all desires must be relinquished: one must give up wanting to be a successful, enlightened Buddhist. Sometimes, as progress is made, the ladder self-destructs as required, but it still cannot be thrown away. Cases like this take us to science and mathematics. Scientists must always take into account the observer effect: all measurements affect the system being measured. Often this effect is negligible, but sometimes it is not. In these cases, the observer is not just measuring the system under investigation, but is rather measuring a meta-system comprising the observer's measuring process and the original system. The observer effect guarantees that the method of theorizing and measuring—regarded as an objective, public ladder to a deeper understanding of nature—eventually reaches a limit in which the measurements that are made measure an observer-system hybrid, not the intended system. While it was made famous by Heisenberg's uncertainty principle, this phenomenon stretches across science, from physics to psychology. As Gödel showed, a similar phenomenon occurs in mathematics. When outfitted with numbers and arithmetic, classical logical systems designed to produce all the provable number-theoretic sentences wind up generating sentences that are not provable. The method of proving theorems in such number systems, intended to be a ladder to more and more well-settled mathematical truths, winds up generating unsettled truths: sentences that cannot be proved in the system, but are provably true in a meta-system of the system. In both these cases, the ladder—theories and measurements or number systems and proofs—self-destructs due to being used, thus revealing that it was never the ladder we thought it was. Yet in neither case can we jettison the ladder. We have to make measurements to test our theories, and we have to use number systems and proofs to further understand the mathematical landscape. This all suggests paradox.

Here we explore this ladder-destruction phenomenon in some detail. We are especially interested in cases of the second type, the cases introduced above about science and mathematics. The next two sections describe two such cases, one in

[1] One could argue that the *Tractatus* does not fit the kind of "ladder paradoxes" we explore here because one can, in fact, jettison the ladder of the *Tractatus*, moving beyond its propositions to the correct understanding of the world (Wittgenstein himself appears to have thought so). However, Priest has argued (2002, pp. 191–192) that the Tractarian ladder cannot be used and then jettisoned, so Wittgenstein is saddled with paradox. Priest puts it thus: "… far from being the rungs of a real ladder that one can ascend, [the propositions of the *Tractatus*] are the like the rungs of a holographic ladder that will not support any weight put on them …" (p. 191).

psychology and one in physics. We argue in the fourth section that these cases are paradoxical, that they share a common structure, and that the emergence of paradoxes with this structure is a common feature of scientific revolutions as characterized by Kuhn (1962). We then show, in the fifth section, that this structure is an instance of Priest's (1994) Inclosure Schema, thus showing that they are limit paradoxes in Priest's sense. The sixth and seventh sections reflect, respectively, on how science copes with such paradoxes and their relevance to the realism-antirealism debate. We conclude that such paradoxes are unavoidable and must simply be lived with.

2 Our Apophenic Psychology

As psychological understanding of the human mind has increased, we have discovered that human minds impose patterns on input, rather than simply responding to patterns imposed on us (e.g. Bruner 1957; Shermer 2008; Love 2014). Historically, this essentially Kantian observation led to the downfall of behaviorism and the cognitive revolution in psychology; Fodor's (1975) radical nativism being one prominent outcome. Humans are, to use an apt metaphor, pattern hungry. Input limits what patterns we can see, but not by much.

Face perception provides a case in point. Human infants are able to recognize human-like faces from the earliest ages tested (e.g. Simion et al. 2011). From childhood onwards, many if not most people are able to see faces in the clouds, faces on trees or features of the landscape, the face of the "Man in the Moon." This face-perception ability has long been exploited by artists—Giuseppe Arcimboldo's *The Librarian* and *Vertumnus* are canonical examples—and is readily manipulated by designers of manufactured products and computer interfaces. The perception of agency provides a second case: most people not only can but unavoidably do see intentions and agency even in the animated motions of colored rectangles (e.g. Scholl and Tremoulet 2000; Scholl and Gao 2013). It is, moreover, clear *why* we see faces and agency when none are present: both are critical for distinguishing friend from foe, and the evolution of any biological system that detects threats can be expected to tolerate false positives but not false negatives (e.g. Dunbar and Shultz 2007).

This human tendency to find meaningful patterns in either meaningless information or in information clearly meaning something else entirely is commonly termed "apophenia."[2] Note that nothing prevents apophenic patterns from being *intersubjectively* perceptible: virtually all experimental subjects see agency in Scholl's animated displays, and people throughout history have seen the Man in the Moon. Either individual or intersubjective apophenia has been advanced to explain such phenomena as the ubiquity of religious "evidence" such as visions of the face of Jesus on a slice of toast, susceptibility to conspiracy theories, seeing UFOs, ghosts, and so forth, and experiences of "psychic" phenomena such as

[2] "Apophenia" has precedence over Shermer's (2008) term "patternicity" and is more general than "pareidolia."

communication with dead loved ones, telekinesis, and extra-sensory perception (e.g. Dennett 2006; Shermer 2005, 2008, 2011). Whether such phenomena are *in fact* cases of apophenia in any given instance is of course an empirical question; as often pointed out (e.g. Radin 2006; Luke 2011), patterns may simply be assumed to be apophenic if no theory is available to explain them or they contravene well-established assumptions.[3] As a theoretical construct, however, apophenia is powerful; for example, combining the well-supported idea that human religiousness is an evolutionary adaptation (e.g. Boyer and Bergstrom 2008) with the perceptual psychology of apophenia produces a powerful theory capable of explaining why religions persist, why there are so many of them, and why the evidence for each religion is robust and readily available, if only to believers (Dietrich 2015). It is, moreover, a construct with deep evolutionary roots: any nervous system capable of learning can be expected to over-generalize and hence to be apophenic. Fish that are fooled by lures and frogs that flick their tongues at anything that moves are displaying rudimentary forms of apophenia.

There is, however, a question to ask here: How do we know that we are apophenic, not just in this or that case, but in any case at all? In order to know this, we have to know that we sometimes find patterns that are not there, or that are there, in some sense, but are meaningless or do not mean what we intuitively think they mean. To answer this question, we must appeal to a division of perceptible patterns into the "real" and "meaningful" on the one hand and the "not real" or "meaningless" on the other. Common sense or, more recently, science is the arbiter of this division. Both common sense and science are, however, themselves both pattern recognizers that operate on an assumption that some patterns are meaningful while others are not.[4] Both common sense and science rely, moreover, on intersubjectivity as a key—and if pressed, perhaps the only available—criterion of objectivity. What if these pattern recognizers are themselves apophenic? In particular, what if science is apophenic, what if many or most of the patterns that science recognizes, names and theorizes about are in fact not "real" patterns at all? A well-funded and vocal minority in the United States, for example, argues that the patterns that science calls "evidence of global climate change" are not real patterns, but rather an instance of politically-motivated scientific apophenia.

We are here faced with a familiar slippery slope to scepticism: once science demonstrates apophenia as a phenomenon, the possibility of systematic apophenia is turned against science itself, and hence against the very idea of scientific

[3] For example, standard statistical meta-analysis techniques applied to multiple experimental studies regularly yield positive evidence for ESP-like phenomena (e.g. Radin 2006; Tressoldi 2011); meta-analysis similarly yields positive evidence for retrocausation (Mossbridge et al. 2012). However, no level of statistical significance beyond which such phenomena would be deemed to have been demonstrated has been widely agreed upon within the relevant research communities, and positive claims about either are typically met with suspicion or dismissal (e.g. Miller 2011).

[4] It is not often noted how important the assumption that some patterns are meaningless or "random" is for either science or common sense. Finding significance in every turn of a leaf can be overwhelming. While a full discussion of this issue is beyond the present scope, we return to it briefly in the section that follows.

knowledge.[5] However, the scepticism at the bottom of this slope is not merely speculative, and cannot be dismissed as merely an armchair worry. As in the case of scepticism about introspective self-knowledge induced by laboratory demonstrations of the confabulation of reasons for actions (e.g. Carruthers 2010; Scaife 2014), here we are faced with plentiful, well-established experimental evidence that human beings make fundamental errors in pattern recognition with considerable regularity.[6] Our worry about the potential apophenia of science is, in other words, motivated by science itself.

Sceptical worries about apophenia are, however, not what we wish to pursue and characterize here. Our interest is rather in the pattern—assuming it is not apophenic!—that this case illustrates. It is, in brief,

1. Science distinguishes real patterns from others that are not real.
2. Science shows that people sometimes cannot distinguish real from unreal patterns.
3. This demonstration by science naturally motivates a questioning of the distinction, and hence of science itself.
4. However, neither science nor its ability to draw the distinction can be abandoned.

This is precisely the phenomenon of the ladder that must be jettisoned but cannot be: the "ladder" is relying on the distinction between real and unreal patterns together with our theories of the real patterns to provide us deeper and deeper knowledge of our world. Except the distinction, and hence the ladder, self-destructs: apophenia, once discovered, destroys any blithe confidence we may have had in our ability to identify and correctly characterize the patterns that are real. Yet we cannot get rid of the ladder. We cannot live our lives without it. Certainly we cannot do science without it, and we cannot live our lives without science.

[5] Specifically, we are faced with a Pyrrhonic regress: an unmeetable demand for a further criterion that identifies "real" patterns. As is well known, such a regress is undefeatable by evidence. As Russell (1921) puts it,

> There is no logical impossibility in the hypothesis that the world sprang into being five minutes ago, exactly as it then was, with a population that "remembered" a wholly unreal past. There is no logically necessary connection between events at different times; therefore nothing that is happening now or will happen in the future can disprove the hypothesis that the world began five minutes ago.
> pp. 159–160

Pyrrhonic scepticism is, however, also unsustainable: It is impossible to spend any length of time believing that one knows nothing. To get through our day, we have to at least behave as if we know things. But even this is impossibly arduous for our minds, and we wind up implicitly adopting the position that we actually know things. We then, following Chisholm (1982), turn to a burden-of-proof argument, saddling the sceptic with the impossible task of proving that the world did begin just five minutes ago.

[6] It is worth noting that confabulation of motives for actions can also be intersubjective, as the effectiveness of crowd psychology and subliminal advertising demonstrate.

3 Our Boundaryless World

Science requires the ability to do experiments: repeatable public manipulations of the world that have repeatable, publicly-observable consequences. Repeated manipulations and observations require that the thing being manipulated and observed—the "system of interest" in the language of physics—be re-identifiable and re-manipulable over time. They also require that the system be not just distinct from, but also publicly and repeatably distinguishable from, the experimenter doing the manipulations and observations. Hence David Bohm, one of the foremost advocates of "holism" as an approach to theoretical physics, was still forced to claim that "the very idea of making an observation implies that what is observed is totally distinct from the person observing it" (1989, p. 585).

Publicly and repeatably distinguishing an observer from a system under observation—indeed, publicly and repeatably distinguishing any system A from any other system B—requires *drawing a boundary* in the world that separates the two systems. This boundary must be both public and time persistent; all relevant third parties must be able to agree that the boundary indeed distinguishes the two systems, and that it continues to distinguish the two systems for at least the period during which observations are being conducted. The possibility of doing experiments thus commits scientists not just to the persistence of systems through time, but to the persistence through time of boundaries that separate one system from another. If, *ceteris paribus*, any pair of competent observers, regardless of their locations, points of view, or other contingent facts about their situations would agree about the existence and time-persistence of a boundary separating two systems, the boundary and hence the separation can be called "objective" or "observer-independent." Ollivier et al. (2004, 2005), for example, define "objectivity" for properties of physical systems as follows:

"A property of a physical system is *objective* when it is:

1. simultaneously accessible to many observers,
2. who are able to find out what it is without prior knowledge about the system of interest, and
3. who can arrive at a consensus about it without prior agreement."

In our case, the property of being separated into two distinct parts by a time-persistent boundary is what is relevant, and "many observers" is strengthened to all competent observers, regardless of situation, a strengthening that is consistent with Ollivier, Poulin and Zurek's focus on "massive redundancy" in the encoding of observer-accessible information about the physical situation and the mutually-distant locations of the observers.[7] Bohm's claim that observer and observed must

[7] What counts as an "observer" in quantum theory has, in our opinion, received less attention than it deserves. Many theorists use the word "observer" merely to indicate a logically-possible point of view. There is, in particular, no requirement that "observers" be "like us" in any significant way. "Observer independent" is therefore considerably stronger than "intersubjective" if the latter term is used in a way that invokes or assumes any degree of psychological or experiential similarity. See Fields (2012a) for a discussion.

be *totally distinct* conveys this sense that all competent third parties, regardless of their location or perspective, would have to agree that observer and observed were separate entities, i.e. that a time-persistent boundary could be drawn between them.[8]

What, however, are these boundaries to which scientists are committed? The most obvious ones are spatial: one can make measurements with a voltmeter, for example, because one can locate it in space and distinguish it from the other objects around it. There are, however, also more subtle boundaries. Making measurements with a voltmeter also requires the ability to isolate the voltmeter's physical degrees of freedom—not just its location but its size, shape and ability to indicate a measured voltage by moving a needle or displaying a number—from the physical degrees of freedom of other things, for example, the physical degrees of freedom of the table it is resting on. This ability to isolate the degrees of freedom of the voltmeter involves an assumption, generally implicit, that the degrees of freedom of the voltmeter are in some significant sense causally independent of the degrees of freedom of its surroundings; for example, the ability of the voltmeter to indicate a voltage value is assumed to be causally independent of the mass or color of the table on which it is sitting, the phase of the moon, or what the people in the laboratory happen to be thinking about. Fuchs (2010), for example, following Einstein, expresses this assumption by asking, rhetorically, "What [does it mean] that A and B are spatially distant things but that they are causally independent?" The ordinary language of the laboratory reflects this assumption of observer-independent boundaries between spatially-distinct, causally-independent systems, as Bohr (1958), for example, argued that it must.

As is well known, quantum theory challenges this everyday assumption that physical systems can be bounded; the principle topic of Bohr (1958) is, indeed, this challenge. In quantum theory, the physical degrees of freedom of an object form the basis of its physical description: a quantum state is just a list of the current values of an object's (simultaneously measurable) physical degrees of freedom. To say that a system *has* a quantum state is to say that its (simultaneously measurable) physical degrees of freedom can, at least in principle, be isolated and measured and their values written down. Both the manipulations required to "prepare" a system for measurement and the measurements themselves require, however, that an experimenter interacts with the system. Quantum theory tells us that interaction

[8] Many scientists also insist that the possibility of doing experiments commits them to free will (e.g. Gisin 2012). This assumption creates yet another "ladder" phenomenon in any science that both claims applicability to scientists themselves and assumes some form of deterministic causation. This is particularly evident in quantum theory, which in standard formulations claims both universality (and hence applicability to physicists as components of the physical universe) and the unitary time evolution given by the Schrödinger equation. As discussed in more detail below, unitary time evolution entangles observer with observed. A physicist that is entangled with her apparatus may appear to *herself* to be manipulating its state and hence may appear to *herself* to be exercising free will, but to a third party who observes them interacting, neither she nor her apparatus can be even assigned a determinate individual state. That experimenter and apparatus are interacting and hence entangled is itself, moreover, not an observer-independent fact; yet a fourth observer could describe the world in a way that renders them mutually isolated and hence non-interacting (e.g. Zanardi 2001). Hence if unitary evolution is taken seriously, even the question of whether an experiment has been performed, let alone the question of whether it has been performed by an agent executing free will, does not have an objective, i.e. observer-independent, answer. We return to this point in Sect. 6.

inevitably leads to quantum entanglement. If systems A and B have become entangled, their states cannot be distinguished; indeed, there is no sense in which their individual physical states even exist. In formal terms, the entangled joint state $|A \otimes B\rangle$ cannot be separated into individual system states $|A\rangle$ and $|B\rangle$, i.e. $|A \otimes B\rangle \neq |A\rangle \otimes |B\rangle$. Any boundary drawn between two interacting systems—any boundary separating their physical degrees of freedom—is therefore at best an approximation, the approximation that treats quantum entanglement as negligible. How good is this approximation? Here's the rub: any experiment designed to determine whether non-entanglement is a good approximation must draw a boundary around one or both systems, and hence must *assume* that entanglement is negligible. As in the case of apophenia, we are off on another regress. The "ladder" of boundary-drawing and boundary-enabled experimental manipulations collapses.[9]

Again as in the case of apophenia, the reason the scientific ladder provided by boundary-drawing collapses is a deep one. In order to do science, we must assume that we can publicly and repeatably manipulate systems of interest, so we must assume that systems of interest can be bounded in an observer-independent way. We must, however, also assume that what we call things does not make a difference to their behavior. In particular, we have to assume that picking some particular thing out as the "system of interest" does not alter the laws of physics. We make this assumption, in both classical and quantum physics, by employing a mathematical

[9] An enormous literature attempts to show that the predictions of quantum theory can be made consistent with the existence of an objectively classical world, i.e. a world that is classical independently of any or all observers, and hence with the existence of objective, observer-independent boundaries around objectively-existing objects. While we cannot exhaustively review that literature here, our position is that all conceptually successful attempts to establish objective classicality are inconsistent with quantum theory and hence have long empirical odds against them, while weaker positions produce "emergent" classicality only by assuming it. The simplest gambit, of course, is to claim that quantum theory is false or "in need of modification"; this is the tactic of all "objective collapse" theories (a recent example is that of Weinberg 2012). As no experimental evidence supports the idea of an objective collapse, i.e. of an objectively non-unitary time evolution for any physical system, these theories remain largely curiousities (relevant experimental evidence as well as additional conceptual considerations are discussed in Schlosshauer 2006; Jordan 2010). Considerably more common is the claim that "collapse" and hence classicality are only apparent; this is the claim of standard decoherence theory (e.g. Zurek 1998; Schlosshauer 2007). *Merely* apparent, subjective and hence observer-*dependent* classicality is fully consistent with quantum theory, but provides no sense in which boundaries between systems—including boundaries between observers and their apparatus—can be ontological. It is often claimed, however, that decoherence provides a mechanism via which classicality can be *objectively* apparent, i.e. equally apparent to all observers. This claim, primarily advanced under the rubric of "quantum Darwinism" (e.g. Blume-Kohout and Zurek 2006; Zurek 2009) is inconsistent with the observer-relativity of entanglement (Zanardi 2001; Zanardi et al. 2004; Harshman and Ranade 2011) as has been argued (Dugić and Jeknić 2006; Dugić and Jeknić-Dugić 2008; Fields 2014). Either formulation of decoherence, moreover, assumes both a fixed system-environment boundary and a classical-statistics (i.e. "heat bath") description of the environment, either of which render any claim to "explain" classicality circular (Fields 2014; Kastner 2014). Assuming that the system of interest is quantum and its environment is classical is also, it should be noted, assuming that the laws of physics change from quantum to classical at the system-environment boundary, an assumption that makes the calculations simpler, but denies the universality of quantum theory and has neither empirical nor theoretical justification. Purely ontological approaches fare no better. Lam and Esfeld (2012), for example, claim that quantum theory can be supported by an ontic structural realist ontology in which well-defined "quantum objects" are held together by essential, and hence observer-independent, relational properties such as entanglement, an approach that is clearly inconsistent with entanglement being observer-relative.

formalism that allows arbitrary divisions and re-arrangements of "parts" of the world; we employ, in particular, the associative Cartesian (in classical physics) or tensor (in quantum physics) products to represent decompositions of state spaces and both commutative and associative addition to represent decompositions of forces or interactions. For example, physicists treat the coordinate systems ((x, y), z) and (x, (y, z)) as equivalent; both are simply (x, y, z).

These assumptions collide as soon as one realizes that a boundary between systems can be "objective" in the strong sense of observer-independent for *all* competent observers only if it is respected and hence held in place by the laws of physics themselves. Boundaries can be objective, in other words, only if they are *physically real*. By introducing entanglement as an inevitable physical consequence of dynamical interactions, quantum theory forecloses this possibility: a system could be *objectively* unentangled with all other systems—unentangled with all other systems from the point of view of any competent observer—and hence objectively bounded only if it was isolated outside of the universe, a situation inconsistent with the standard definition of "the universe" as "everything" as well as the assumption that quantum theory is complete. Hence quantum theory disallows the very assumption that makes the idealization of fully public, repeatable observations possible.

This inevitability of entanglement presents no problem if an "objective" boundary *merely* means a boundary that is "intersubjective" among typical human observers—the only observers that count in practice—but any stronger sense of objectivity for boundaries is problematic. We can safely assume that the states of macroscopic objects like cats, dust particles or even large molecules "decohere" (Zurek 1998; Schlosshauer 2007) when exposed to their surrounding environments as long as only *we humans* have to agree about this, but any significantly stronger assumption is either inconsistent with quantum theory or circular (Dugić and Jeknić 2006; Dugić and Jeknić-Dugić 2008; Fields 2014; Kastner 2014). Hence we can assume that the boundaries that we draw in the world are real "for all practical purposes," but we are prevented by our own best physical theory from assuming that they are real in any way that *physically matters*, i.e. that matters to physics itself and hence to any competent observer regardless of their situation (Fields 2014). If the boundaries we have drawn in the world are not *physically* real, though, the "systems" that we are manipulating and observing are in some sense just apophenic patterns, arrangements that we humans care about but which the world does not. Where does this leave science? As noted earlier, science cannot proceed in an atmosphere of total scepticism. While quantum theory does not entail scepticism about energy, angular momentum, electric charge, or even the existence of some kind of physical stuff, it does entail scepticism about boundaries between systems and hence about "things" in the ordinary sense.[10] All that we, as practicing scientists who must manipulate such things in the laboratory, can do is to assume that the boundary approximation is good enough and press ahead. Science is ironic because employing this boundary approximation to conduct experiments results in continual reminders that the approximation itself cannot be trusted.

[10] This is not to deny that many quantum theorists are idealists, but merely to emphasize that they *needn't be* idealists about those aspects of reality that quantum theory treats as objective.

4 The Ladder Schema

The apophenic nature of our perceptual psychology and the boundarylessness of nature revealed by our theoretical physics both suggest paradox. In both, the relevant science delivers a result that implies that the distinctions made by the science itself cannot be maintained and the manipulations required by the science itself cannot be performed. Science, in other words, shows us that it itself is in an important sense not possible. Yet, our knowledge of these results depends on doing the science. We have the results, so the science is obviously not only possible, but actual. Yet, the results destroy the science. This is clearly a paradox.

We generalize. If science is possible, it eventually produces a result that undermines its assumptions and methods (destroys the ladder, to keep the metaphor going). Hence, science is impossible. But the result is in our possession, and science is necessary for our knowledge of the result. Hence, science is actual. Whatever is actual is possible. Hence, science is possible.

The paradox also runs the other way. The science-destroying result must not be true, since the science clearly exists. But yet, there's the result as plain as day, produced by the science itself! Scientists still do the relevant science, but only by begging the relevant questions and then flatly ignoring the resulting contradiction: in the first case, assuming that not all the patterns our sciences discover and explain are due to apophenia, and in the second, assuming that there really are boundaries between things that allow their separate manipulation.

We now formalize this paradox in a general schema, the "Ladder Schema," and show how the schema captures the cases of apophenia and boundarylessness. We then show, in the next section, that this Ladder Schema is an instance of Priest's (2002) well-known Inclosure Schema. This shows that our science-generated paradoxes are in fact members of an important class, the limit paradoxes. Hence science generates limit paradoxes.

Let p be a proposition, and let κ be a defeasible knowledge operator: κp can be thought of as meaning that we would assert that we know p if asked. Hence κp is consistent with our not really knowing p, if, for example, our justificatory reasoning is flawed, or if p is not true. We also require an operator K for genuine knowledge: let Kp mean that p is known. As is standard, we assume that knowledge implies truth, i.e. Kp requires that p be true. The sort of knowledge we want K to range over here is scientific knowledge: knowledge produced by science, as opposed to, for example, perceptual knowledge (one knows that some thing is a tree because one sees it) and phenomenal knowledge (one knows what color green is because one is conscious of seeing green).[11]

[11] The semantic relationship between Kp and κp is clearly complicated (e.g. Koons 2013; Pollock 1987, 1995). One might insist that Kp implies κp, and deny the converse. The former inference could be based on the notion that p being only defeasibly known does not entail that it is actually able to be defeated. However, if one interprets "defeasibly known" as "defeatable in the actual world" or as "merely believed (even with high confidence)" then Kp cannot imply κp since Kp implies that p is true and hence not defeatable in the actual world. Still further, one might insist that the central notion of knowledge, at least in science, is defeasible knowledge: knowledge relative to assumptions (e.g. Moses and Shoham 1993). Except for footnote 13 below, we are going to ignore the complex epistemological and metaphysical issues surrounding the relation between Kp and κp, as addressing them is not required to demonstrate that the Ladder Schema is an instance of the Inclosure Schema.

We need a way to capture all of our scientific "knowledge," both genuine (i.e. true) and defeasible (possibly not true), including the data produced by scientific activity and phenomena characterized by science, as well as all of the presuppositions science makes (we are using "science" here metonymically). Let Ψ be the set of all current, perhaps defeasible, scientific knowledge, including data, descriptions of phenomena, explanations, and presuppositions, represented as a collection of propositions. Note that Ψ must be finite. Now let *Sci* be a predicate meaning "is a part of the current enterprise of science narrowly construed"; this predicate picks out the all of the propositions of science (data, phenomena, explanations, and presuppositions) regardless of their truth. In this case, we have $\Psi = \{p | Sci(\kappa p \vee Kp)\}$, where the disjunction captures the possibility—in all likelihood, the fact—that some of the propositions in Ψ are not true. We need the "narrowly construed" in the above to rule out, e.g., Einstein's violin playing, which undoubtedly helped him do his science, and in that expanded sense was a part of his science, but does not belong in Ψ.

We note that unearthing the presuppositions that make any science doable is difficult because they are most often implicit. Philosophy unearths some of them, and often a science itself unearths presuppositions it depends on. For example, to do any science at all, a scientist has to assume that there are boundaries: boundaries between parts of an experiment, boundaries between different cases of the phenomenon under study, and so forth. Other presuppositions include assuming that experiments are repeatable and hence that time objectively exists, that experiments reveal objective or at least inter-subjective truth, and that standard forms of anti-realism are false, e.g. that we are not living in the Matrix, even though there is arguably a genuine probability that we are (Bostrom 2003). We will assume in what follows that there is a finite set B of presuppositions needed to do all of our science; this B makes explicit all of the presuppositions that are usually implicit. Of course, insisting that B contain *all* relevant presuppositions smacks of a deus ex machina; Searle (1983), for example, has argued that the set of presuppositions underlying daily life and *ipso facto* science is unboundable. Still, at least a good approximation of such a set B must exist and could be populated with at least many of the missing presuppositions if a robust group effort was undertaken; we will just assume that such an effort has been undertaken and has been unusually successful. Clearly, $B \subset \Psi$; thus Ψ contains both genuine and defeasible knowledge produced by science and the (possibly defeasibly) known presuppositions that are required in order to do science.

We can now capture the ladder destroying nature of our paradoxes. A well-known and exciting occurrence in science is the discovery of some truth that renders an entire "paradigm" false, or even renders an entire world-view false. The discovery of evolution and its formulation by Darwin rendered false the world-view that placed humans at the center of a created world. James Hutton's formulation of the geological concept of deep time destroyed the world-view that held that Earth was created only a few thousand years ago. The development of quantum mechanics suggested—at least on one interpretation—that perhaps God does play dice with the world. Gödel's discovery and proof of the First and Second Incompleteness Theorems destroyed Hilbert's dream of finding an axiomatization of classical arithmetic that could be proved to be both consistent and complete by constructive means. Chomsky's linguistics and psychology of language theories undid behaviorism. The history of science and mathematics is so replete with such examples that Thomas Kuhn discerned a pattern—we assume not an apophenic one!—and wrote one of the most influential books of the late twentieth century about it: *The Structure of Scientific Revolutions*.

We need κ and K to range over sets of propositions, so we are also going to use a little syntactic sugar: if s is a set of propositions, then κs (similarly, Ks) will mean that every proposition in s would be claimed to be known upon questioning (similarly, is known). We will use the same syntactic sugar with the standard negation operator: "$\neg s$" will mean the negation of each and every proposition in s. Finally, let $\mathcal{P}(\Psi)$ be the power set of Ψ and let A be a binary function from $\Psi \times \mathcal{P}(\Psi)$ to $\mathcal{P}(\Psi)$ such that $A(Kp, \kappa s) = K(\neg s)$, with $p \in \Psi$ and $s \in \mathcal{P}(\Psi)$. Informally, if p is actually known, then the action of A uses p to make all of the propositions within the set s known to be false. With the proper domain elements, A can be used to capture what happens in scientific revolutions. Using the proposition, p (e.g. "Life evolves."), A annihilates s, a set of scientific propositions we thought we knew about life, but which p refutes.[12] We will use A as our ladder destruction function. To be explicit, the Ladder Schema which follows models scientific revolutions. We will show how the Schema naturally grows to capture our ladder phenomenon, which is now revealed to be a kind of "scientific revolution," but a revolution with a vengeance, a revolution that doesn't know when to stop.

Here then is the Ladder Schema:

1. $\Psi = \{p | Sci(\kappa p \vee Kp)\}$ exists.
2. Let $\sigma \subset \Psi$ contain the propositions of a given, specific science. Then $\sigma \in \mathcal{P}(\Psi)$, where σ is selected so as to comprise the truths of a given science.
3. Eventually, the science produces the "revolution" proposition $\omega \in \sigma$.
4. Let $I_\omega \subseteq \sigma$ be the set of propositions in σ incompatible with ω. That is, $I_\omega = \{p \in \sigma | p \text{ is true only if } \omega \text{ is false}\}$. Note that $\omega \notin I_\omega$.
5. And finally, the claim that revolutions are part of how science advances and that advancing is uncovering more truth is: $A(K\omega, \kappa I_\omega) = K(\neg I_\omega)$.[13]

[12] To more faithfully capture how A works in real scientific revolutions, we would need to add, at least, a time index, thus making A a 3-ary function of a time, a proposition p, and a set s of propositions. The time would be the *pivot time*, before which we would assert that we know the propositions in the set s (these constitute the alleged knowledge of the "old" paradigm), but after which we would assert that we know the new proposition p as well as the fact that p ushers in a revolution by rendering false the propositions of the old paradigm in s. However, since we assume here that scientific truths are timeless, we are going to ignore this complexity. Also, in scientific revolutions, A sometimes renders the propositions of the old paradigm false in the particular sense that they become approximations of propositions in the new paradigm. Relativity and quantum theory, for example, rendered Newtonian physics false, but it is still a "good enough" approximation to provide predictions that are accurate to within the relevant measurement resolutions for medium-sized objects moving at moderate velocities, and so is treated as "true for all practical purposes" in that circumscribed domain. We ignore this complication, too.

[13] The claim that ω is known is the tip of some interesting epistemology. The most important point for us is that A is part of a *knowledge engine*. The engine works something like this. At step 1, we have $A(\kappa\omega, \kappa I_\omega) = \kappa(\neg I_\omega)$. At step 2, the very falsity of I_ω, through a process only partially understood, elevates the defeasible knowledge of both ω and $\neg I_\omega$ to genuine knowledge, giving us

In normal scientific revolutions, I_ω is limited to a few propositions, so its loss is not devastating, at least not in the long run.[14] However, sometimes the subject matter of ω applies to everything: for example, boundaries are everywhere and essential to any knowledge whatsoever, so we're in serious trouble if our best boundary-using science says that there are no boundaries. Similarly, all human knowledge depends on finding patterns, basically everything is a pattern of some sort, so we're in trouble if our best science cannot distinguish between patterns that are real and those which are figments of our over-active pattern producing mechanism.[15] Whenever such universal application occurs, we get paradox. *Whether* it occurs depends essentially on the semantics of ω. Our two paradoxes obtain when ω applies to everything, for then I_ω expands to include all of Ψ. In this case ω is known and hence ω is true, but $A(K\omega, \kappa\Psi) = K(\neg\Psi)$, so since $\omega \in \Psi$, ω must be false. Whence, paradox. We see, therefore, that the Ladder Schema has an innocuous version in which I_ω is a proper subset of σ, but when this restriction fails due to the semantics of ω being all-inclusive, the Ladder Schema generates paradox.[16] This difference between an innocuous version and paradoxical version is important, for it allows us to relate our Ladder Schema to Priest's Inclosure Schema in the next section.

Consider the apophenia paradox. First, in the innocuous version, ω begins as the proposition "Those patterns are due to apophenia," where the referring expression "those" picks out patterns we mistakenly take as objectively real, but which in fact are not, like the Man in the Moon. Specifically, "those" picks out the patterns and explanations in I_ω, because I_ω is the set of pattern observations and explanations that are (going to be revealed to be) not objectively real. An actual example is the alleged face "carved" onto a mountain in the Cydonia region of Mars, with the accompanying explanation "space aliens made the face hoping that one day, we would find it."[17] The apophenia paradox results when the referring expression "those" expands—and cannot be prevented from expanding—to cover all patterns.

Footnote 13 continued
$A(K\omega, \kappa I_\omega) = K(\neg I_\omega)$. Indeed, demonstrating that the propositions in I_ω are false is sometimes regarded as a kind of aesthetic evidence that ω is true. Such aesthetic considerations are particularly significant in scientific revolutions in which the most dramatic confirmatory findings—the discoveries of the structure and replications mechanism of DNA, for example, or the demonstration of quantum entanglement across kilometer distances—come decades after the revolutionary theory is proposed or even accepted. This is part of what we mean by saying that scientific revolutions uncover (produce?) more truth. On the notion that defeasible knowledge aspires to genuine knowledge, see Williamson (2000), especially the introductory chapter.

[14] From the Introduction, in the case of the observer effect ω is "observing can strongly affect the system being observed" and in the case of number theory ω is Gödel's First Incompleteness Theorem. Both of these, though quite important, have limited application. The same is true for the scientific revolutions we discussed above.

[15] This is a good place to point out the relation between our two paradoxes. Basically, we are saying that apophenia running amok is an inability on science's part to draw a boundary between real patterns and merely "psychological" ones.

[16] Formally, of course, if I_ω simply engulfs σ, we get paradox because $\omega \in \sigma$, and hence is both true and false. But the case were interested in is when I_ω expands to Ψ for semantic reasons.

[17] See, e.g. http://science1.nasa.gov/science-news/science-at-nasa/2001/ast24may_1/ and the Wikipedia entry http://en.wikipedia.org/wiki/Cydonia_(region_of_Mars).

Now, I_ω expands to Ψ and the result is paradox. Only science can distinguish between real patterns and apophenic ones, but science itself depends on successfully drawing this very distinction, so the distinction cannot be objectively drawn, or it cannot be drawn without begging the question (or, it can only be subjectively drawn). So, we cannot tell which patterns, if any, are real and which are apophenic except pragmatically, which in turn means that there is no such thing as objective science. But there clearly is.

In the case of physics, an innocuous version of ω is a claim such as "electrons in a double-slit apparatus follow a superposition of classical trajectories." Electrons are invisible subatomic particles, so the early twentieth century expectation that electrons might follow classical, Newtonian trajectories was based more on faith than observation. The findings of early twentieth century experiments and the quantum theory that arose to explain their results thus invalidated classical physics only in a domain—the "quantum domain"—that could be easily circumscribed. Indeed the standard "Copenhagen" interpretation of quantum theory depends on just such a circumscription, as it insists that classical language can be used to describe both macroscopic apparatus and experimenters (e.g. Bohr 1958). The boundarylessness paradox arises when this circumscribed domain expands to include the entire universe, and one encounters an ω such as "The universe occupies an entangled quantum state" or "The time evolution of the universe obeys the Schrödinger equation." These claims reflect an assumption that quantum theory applies universally to all physical phenomena; here I_ω expands to include all of physics. A universe in an entangled quantum state, however, is a universe with no physically efficacious boundaries, so I_ω in fact expands to include all of science—all of Ψ—and the result is paradox: the evidence on which our knowledge of quantum theory rests is experimental evidence, and every experiment requires the ability to draw boundaries that can be regarded as physically efficacious. In both cases, however, the issue is not so much one of letting I_ω expand to all of Ψ, but rather one of preventing I_ω from expanding to all of Ψ. The problem is that, because of the universal applicability of ω, there seem to be no good reasons for not letting I_ω expand to all of Ψ; indeed any reason for not letting I_ω expand to all of Ψ seems arbitrary and hence question-begging.[18] Again, this is because of ω's semantics: its intrinsically universal applicability. How do we know that there is a real distinction between the objective patterns of most of science and the apophenic patterns revealed in a small branch of psychology? How do we know that the boundaries in, say, the Large Hadron Collider are real when our best physics says they are basically conventional or notional? This is the essence of the ladder phenomenon. Scientists still do psychology and physics, and all the rest of science, but only by begging the question and then flatly ignoring the resulting contradiction in the relevant science: in the first case, assuming that not all the patterns we perceive are due to

[18] One might ask at this point: what about emergent properties? Surely one can say, for example, that solidity is a real emergent property, and solid objects have real boundaries—try kicking one!—even if these boundaries are somehow dependent on emergence. If one carefully avoids the idea that "emergence" is a *physical* process, e.g. following Butterfield (2011) and defining emergence in terms of relationships between observables, this is all fine. As in the cases of "collapse" or "decoherence," the problems arise when one imagines that emergence is physical, and hence must postulate that physical laws change abruptly at some emergent boundary.

apophenia, and in the second, assuming that there really are boundaries between things.

5 The Ladder Paradoxes are Dialetheic

We have so far established that the ladder phenomenon engenders paradox. This is modeled by the Ladder Schema. The ladder paradoxes result when I_ω expands to cover all of Ψ. There is, therefore, a sense in which the ladder paradoxes result when ω is applied generally by taking it to the limit of all of knowledge produced by science (i.e. to the limit of all of Ψ). This suggests that the ladder paradoxes are in fact limit paradoxes: paradoxes that result from approaching a hard limit, and yet going beyond it. The standard way to prove that paradoxes of some kind are limit paradoxes is to show that they conform to the Inclosure Schema due to Priest (2002). To this, we now turn.

5.1 Dialetheism

Dialetheism is the thesis that some contradictions, sentences of the form *P* and Not-*P*, are true. They are false, of course, because all contradictions are false, but they are also true at the same time. Such a contradiction is called a *dialetheia*. A dialetheia is the locus of a *truth-value glut*: a given proposition has more than one truth value, namely it is both true and false. Most philosophers are not dialetheists. At least in our opinion, however, it is becoming increasingly difficult to dismiss or ignore dialetheism outright. What we show below is that psychology's apophenia and physics' boundarylessness result in dialetheias.

The canonical dialetheias are limit contradictions: they occur at some limit defined by some sort of thinking or other process such as enumerating. The limits could be limits on what is conceivable, knowable, sayable, iterable, etc. (Priest 2002, 2006). For an example, consider Cantor's paradox. Let *U* be the universal set, the set of all sets: by definition, *U* must be the largest set, and so must have the largest transfinite cardinality. Cantor's Theorem, however, requires that for every set *A*, the power set of *A* must have a strictly greater cardinality than *A* itself. Hence, the power set of *U* must have a larger cardinality than *U*. But that is impossible, since *U* is by definition the largest set. Hence the contradiction: *U* is the largest and not the largest set. Phrased another way, *U* is not a set (since, by Cantor's Theorem, there is no largest set) and yet *U* is a set. Put this way, Cantor's paradox is a limit contradiction, a dialetheia.[19]

5.2 The Inclosure Schema

Limit contradictions compose a family of paradoxes. This family contains the many well-known mathematical paradoxes such as Cantor's paradox, as well as those discovered by Burali-Forti (there is a largest ordinal number, which is also not the

[19] Classical, axiomatic set theory dodges this contradiction by insisting that the "set" of all sets is not itself a set, but is rather a "class" to which Cantor's Theorem does not apply.

largest) and Bertrand Russell (there is a set containing all and only sets that are not members of themselves, which both does and does not contain itself).

All members of this family of paradoxes share a similar structure: there is a collection—a totality—of all things having some property, and an operation—a 'diagonalizer"—that generates an object that is both outside of the totality and also inside of the totality. The first condition is called *transcendence* and the second is called *closure* (Priest 2002, pp. 3–4). The generation of the relevant object requires self-reference and negation; indeed, both of these are crucial to all limit contradictions. Russell's paradox provides a good example. Russell's set R contains all and only the sets that do not contain themselves as members. Now ask: "Is $R \in R$?" If it is not, then R does not contain itself, but since R contains all sets that do not contain themselves, it follows that R must contain itself: $R \notin R$ implies $R \in R$. Apparently then, R contains itself: $R \in R$. But since R contains only the sets that do not contain themselves, it follows that R must not contain itself: $R \in R$ implies $R \notin R$. But either $R \in R$ or $R \notin R$ (most dialetheists accept excluded middle). So, R both does not contain itself (transcendence), and does (closure). Contradiction. Not containing itself is the negation part, and referring to itself is the self-reference part.

Bertrand Russell was the first to formulate the structure of this argument in an explicitly general way (Russell 1906). Here, we follow Priest's reformulation of Russell's idea. Priest calls his version the *Inclosure Schema*. Let ψ and ϕ be properties, and let δ be a function, the diagonalizer. The Inclosure Schema is then:

1. $\Omega = \{y|\phi(y)\}$ exists and $\psi(\Omega)$ — Existence
2. if $x \subseteq \Omega$ and $\psi(x)$ then
   (a) $\delta(x) \notin x$ — Transcendence
   (b) $\delta(x) \in \Omega$ — Closure

When we let x expand to the entirety of Ω, giving us $x = \Omega$, then δ gives us our object that is both within and without Ω: $\delta(\Omega) \in \Omega$ (closure) and $\delta(\Omega) \notin \Omega$ (transcendence), as illustrated in Fig. 1a. As with the Ladder Schema, we see that the Inclosure Schema has an innocuous version when x is a proper subset of Ω, but when this restriction is lifted (and it seems impossible to keep it a proper subset), the Inclosure Schema generates paradox. Our task now is to show how our paradoxes fit Priest's schema. We demonstrate a more general result: we show that our Ladder Schema maps into the Inclosure Schema. Our result concerning our two paradoxes immediately follows.

5.3 Mapping the Ladder Schema to the Inclosure Schema

Clearly, Ψ in the Ladder Schema plays the role of Ω in the Inclosure Schema (Fig. 1b). The ω-dependent subset I_ω plays the role of the subset x since both expand to include all of the relevant totality (although, as we've noted, the reason I_ω expands to include all of Ψ is because of the universal applicability of ω: because of

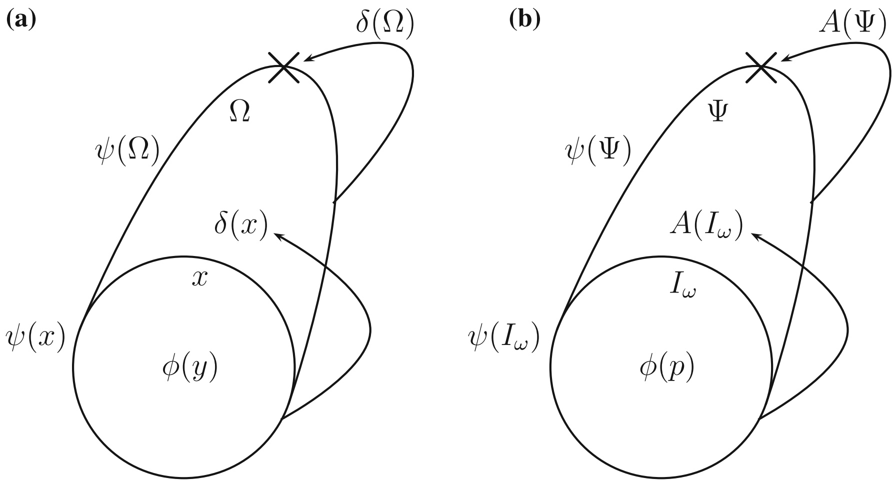

Fig. 1 **a** Inclosure Schema, adapted from Priest (2002) p. 156. **b** Mapping of the Ladder Schema to the Inclosure Schema. In both, '×' indicates contradiction

its semantic structure, ω applies to all of Ψ). The function A plays the role of δ. Recall that, in the apophenia case, we rendered ω as "Those patterns are due to apophenia." We presented a similar rendering when presenting the boundarylessness paradox. In the innocuous cases, "those" ranges over the benign elements of I_ω in its restricted interpretation. But in the paradoxical cases, "those" ranges over all of science (all of Ψ). To complete the Inclosure Schema, we have that ψ is the property of being definable (for example, I_ω is definable), while ϕ is the property of being a scientific proposition, i.e. of being in Ψ and therefore, given the Ψ–Ω correspondence above, being in Ω. Note that the action of the schema is to show that all propositions p such that $\phi(p)$ are only defeasibly known and and are in fact defeated by ω.

We now show that A functions as it should, producing both transcendence and closure. We first consider the innocuous versions for both apophenia and boundarylessness. Recall that $A(K\omega, \kappa I_\omega) = K(\neg I_\omega)$. In both the apophenic and boundarylessness paradoxes, we have $K(\neg I_\omega) \notin I_\omega$ (i.e. $\delta(x) \notin x$). This because $K(\neg I_\omega)$ says that every proposition in I_ω is false, yet $K(\neg I_\omega)$ is true because it is produced by a reliable and robust science. However, because it is a true scientific claim, we have $K(\neg I_\omega) \in \Psi$ (i.e. $\delta(x) \in \Omega$), as needed.

In more detail, in the apophenic case, recall that I_ω is the set of pattern observations that are going to be revealed by ω to be due solely to our human pattern-hunger; the accompanying folk explanations are going to be revealed to be false. So, ω makes false both that there is a face on Mars photographed by Viking I, and that space aliens made the face hoping that one day we would find it and know that we are not alone. Similarly, ω makes false both that there is an image of Thor on my piece of toast, and Thor made this happen because he wants me to know that he along with the other Norse gods are real. The knowledge that all the apophenic patterns and folk explanations are false is not in I_ω, clearly, but it is in psychology and hence in science. In the boundaryless case, as we noted, the various claims that atoms and subatomic particles follow Newton's laws of motion make up I_ω. We discover ω, which is essentially quantum mechanics, invalidating classical physics for the quantum domain. Again, the knowledge that classical physics is false in the quantum domain cannot be a part of Newtonian physics as applied to the quantum domain, but it is clearly a part of modern physics and hence part of science.

But what of $A(K\omega, \kappa\Psi) = K(\neg\Psi)$? Here our conclusions are easier to see if we proceed at a general level. $K(\neg\Psi)$ says that we know that all the propositions of Ψ are false due to our application of ω to all of Ψ. And as we argued above, we don't willfully and peevishly apply ω to all of Ψ, rather, we cannot prevent ω from applying to all of Ψ. We know that we are unable to prevent universal application, so we know that we know $\neg\Psi$. That is, $K(K\neg\Psi)$. Hence, $K(\neg\Psi)$ must be true (by the definition of K). But if $K(\neg\Psi)$ is true, then $K(\neg\Psi) \notin \Psi$ because all propositions in Ψ are false. This is transcendence.

But $K(\neg\Psi)$, as perverse as it is, is nevertheless a part of science. To see this, note that this paper, after all, is a bit of science. It is philosophy also, but it is a theoretical, scientific paper.[20] So our result, $K(\neg\Psi)$ is a bit of science; perhaps it is science out towards the fringe, but not so far out as to be not science at all. Whence, $K(\neg\Psi) \in \Psi$. This is closure.[21] Hence, A functions as our diagonalizer, as needed.[22] We therefore have what we wanted: The Ladder Schema is an instance of the Inclosure Schema. Ladder paradoxes are inclosure and therefore limit paradoxes.

This completes our demonstration that our Ladder Schema maps into the Inclosure Schema. We conclude that our ladder paradoxes are genuine limit paradoxes. We cannot logically prevent apophenia from expanding to cover all patterns in science. When it does, that apophenia exists is both true and false. We cannot prevent boundarylessness from expanding to all of science. When it does, that there are no boundaries becomes both true and false.

6 Preserving Our Sanity

Let us focus on the boundarylessness implied by quantum theory (the same conclusions below can be drawn, *mutatis mutandis*, for apophenia). As noted earlier, boundarylessness is forced onto physics by quantum entanglement, a phenomenon robustly supported by decades of experimentation (e.g. Schlosshauer 2006). Quantum entanglement is rendered ubiquitous by the assumption that the physical

[20] Two other of the many examples of science cum philosophy are Kelly 2008 (a philosopher does science) and, going the other way, Conway and Kochen 2006 (scientists do philosophy).

[21] Our case of closure has a property common to many other cases of closure: "[it is] established by reflecting on the conceptual practice in question; [in] a polemical context, this can appear as an *ad hominem* argument" (Priest 2002, p. 4).

[22] There is another way to get this conclusion. If $K(\neg\Psi)$ is in Ψ (via closure, as we've shown), then Ψ contains its own refutation (since $K(\neg\Psi)$ implies $\neg\Psi$). We have then that science contains its own refutation. Science emerges as one big Liar Paradox (provided we construe science as one big proposition, as we can easily do). But as Priest has shown (1994, 2002), the Liar can be rendered as an inclosure paradox, giving us both transcendence and closure (though the latter is now redundant).

universe **U** has a well-defined quantum state—whether we can write it down at any greater level of detail than '$|\mathbf{U}\rangle$' or not—an assumption that is inescapable if quantum theory is to be considered to be a complete physical theory. The roots of boundarylessness, however, extend even into classical physics. Consider a description of the world entirely in terms of classical atoms: unobservably tiny hard spheres that interact by forming classical "chemical bonds." Ordinary objects exchange such atoms with their environments constantly; your own breathing is an example. The laws that govern how atoms interact are, moreover, entirely local to the atoms themselves; whether an atom is part of you or part of the surrounding air, for example, is irrelevant. But if the physical dynamics of the world is given by the laws of atomic interaction, the irrelevance of macroscopic boundaries to atoms renders such boundaries irrelevant to physical dynamics. This boundarylessness is enshrined in the fundamental equations of classical mechanics and electrodynamics. The quantum entanglement introduced by the Schrödinger equation only renders this classical boundarylessness more virulent, by making it impossible to talk about the physical state of a system that is not fully isolated from all interactions and hence unobservable, even in principle.

Working physicists cope with boundarylessness by making "isolation" a good approximation in the laboratory. A particle flying through an accelerator at nearly the speed of light, or one trapped in a magnetic field at a few thousandths of a degree above absolute zero do not interact significantly with their environments (at least this appears to be true and is always assumed to be), and hence can be treated, to quite high accuracy, as occupying well-defined quantum states. The trouble begins outside of the laboratory, in the minds of theorists concerned with mathematical consistency. Here one encounters the notorious "measurement problem"—itself essentially a problem of defining boundaries (e.g. Fields 2012b)—and the competing "interpretations" of quantum theory devised to solve or dissolve it. These all deny something of importance: either the universality of quantum theory (the Copenhagen interpretation), the reality of quantum states and even of physical laws (quantum Bayesianism), or the reality of the ordinary world (the Everett or "multiple worlds" interpretation).

Isolating the trouble caused by boundarylessness in an "interpretation" brackets it so that it will not interfere with the practice of physics. This bracketing reflects an item of faith: that experiments will indicate any local effects of insufficient isolation of an experimental system, either as comprehensible experimental artifacts or as noise. This in turn reflects an even deeper item of faith: that the notion of "physical possibility" is coherent, that an experiment could turn out differently. It is precisely this item of faith that entanglement challenges, by holding up the possibility that the experience of the observer and the behavior of the observed system are both merely samples of the overall, global behavior of the world. Physicists call this possibility "conspiracy" and almost unanimously reject it, but see't Hooft (2013) for a Nobel laureate's dissenting view.

Why do we reject "conspiracy" or in older language, Descartes' evil demon? We reject it because a world in which our experiences are just part of what's happening is not a world with which we are cognitively equipped to cope. It is, in particular, not a world where the ideas of "action" or "behavior" make sense, or where "fitness" or even "survival" are applicable concepts. It is a world of Boltzmann brains,[23] not a world of agents capable of meaningful action. It is a world, in other words, that we can imagine living in even less readily than we can imagine absolute scepticism.

The boundarylessness implied by quantum theory reveals, then, an interesting asymmetry in our knowledge. That the world must be in a significant sense boundaryless is something that we have discovered: it is a surprising conclusion that follows, by straightforward inferences, from a formal model of the world that not only explains but successfully predicts the results of detailed, high-precision experiments. As we pointed out in step 5 of the Ladder Schema, we know ω. If $\omega =$ "There are no physically efficacious boundaries" then we know this. What ω directly contradicts, however, is not something that we have discovered; it is a presupposition of science, an element of *B*. We could not empirically discover that there are boundaries in the world. Think of an infant making its very first attempts to manipulate objects. An infant does not infer that a wooden block, for example, is bounded by a hard surface; she directly senses the hard surface. She sees the boundary of a toy hanging over the crib, or the boundary of her mother who picks her up. The functional architecture of a normally-developing human brain automatically divides the perceived world into bounded objects that are capable of motion and maintain their identities over time. This automatic structuring of the phenomenal world provides us with our own individual identities among much else; it is a prerequisite to our sense of ourselves as actors in and on the world. It is a prerequisite, in other words, of science itself. It is by contradicting this very basic prerequisite that ω brings the edifice of science—the edifice on which it itself depends—crashing down.

Under these conditions, "bracketing" the offending proposition is all we can do, not just to preserve science, but to preserve sanity itself. Our ω becomes a theoretical curiosity, like Gödel's theorem or even Bell's theorem, which we can employ in circumscribed theoretical contexts but must not let get out of control. We must not, because ω threatens not just our knowledge, but our architecture, our very structure. It is for this reason that we cannot, despite our best and most abstract attempts, really imagine a world in which ω is true.

7 Anti-realism and Realism

Unrestricted apophenia and boundarylessness suggest a kind of anti-realism; in particular, they suggest anti-realism about the supposedly separate, causally-independent, bounded "things" of ordinary, day-to-day experience.[24] Some patterns are genuine, while others are not. Boundaries do exist. But neither is really true in the sense of being true independently of our minds. Indeed if the universe is in an

[23] A "Boltzmann brain" is a statistical fluctuation with the apparent perceptions, thoughts, and memories of a human or other sentient being. With plausible assumptions, standard inflationary cosmology predicts that virtually all observers are Boltzmann brains in otherwise empty universes, not evolved systems in universes with material structure (Bousso and Freivogel 2007).

[24] Note that this position almost diametrically opposes the constructive empiricism of van Fraassen (1980, 2001), which is realist about "observables" but anti-realist or at least agnostic about theoretical entities.

entangled quantum state, and our minds are physically-implemented components of that state, then no statements about our observations are true independently of our minds. In this case, "contingent" facts can only be relational, which is just to say that there are no objective, observer-independent contingent facts. So from our point of view, it is we, via our minds, that supply the distinctions between real patterns and merely subjective ones, and the distinctions—i.e. the boundaries—between what we regard as objects and the undifferentiated "background" of the world. Dependence on mind is a hallmark of anti-realism, with realism often defined using the concept of "mind-independence." In the case of boundarylessness, however, our seeing boundaries is also ineluctable. Anti-realism of the type we suggest is something we can reason to via our physics but cannot accept; indeed, it itself undermines the very evidence on which such acceptance would be based. That boundaries are not real is something we can perhaps contemplate, after a fashion, while safe in our studies, but cannot take seriously once we leave our studies (our studies, of course, do not really exist as such if there are no boundaries). We cannot grasp boundarylessness in any substantial way. But because it is empirical, apophenia is different. The images are often ineluctable, of course, but their provenance is not. We do, at least in principle, have to accept the evidence that some apparent patterns are not real.[25] In this case, anti-realism is easier to come by. "All the patterns we see throughout our lives are mind-based, none exist independently of mind." That sentence is a paradigm of anti-realism, and is at least graspable, especially if one seasons it with thoughts of the Matrix or Descartes' demon while sitting safely in one's study. Of course, the minute one leaves one's study, unrestricted apophenia is impossible to believe.

We therefore have, at least, that under some circumstances, we can see, however dimly, that a form of anti-realism might be true. We supply the boundaries; we make real certain patterns. Our minds turn a world of undifferentiated "stuff" into a world of things. But in other, much more common circumstances, realism is true. Realism about boundaries and patterns and hence about things is something we can barely resist. The trouble is that both circumstances have about them a sense of the absolute. As we mentioned, in both cases, any restrictions seem artificial, indeed, question-begging. Perhaps then, both realism and anti-realism are true; perhaps they also form a dialetheic pair. This works especially well with boundarylessness. Boundaries really do exist. It would be insane to deny this. Yet, they also really do not exist: our best science, which is genuinely robust, says so.

8 Conclusions

The history of science suggests that apophenia and boundarylessness are not the end of the story: empirical discoveries that stress the ladders of our presuppositions to the breaking point will continue to arise. Because these ladders are in some cases not mere assumptions, but rather features of our human cognitive architecture, we

[25] But many do not, e.g. the religious are rarely dissuaded that an image of some deity or revered person does not bespeak a communicating god, and to this day, some believe the "face" in the Cydonia region of Mars was built by space aliens, and that NASA's denial of this is a conspiratorial cover-up.

will continue to be unable to jettison them. How dissonant from our automated, architectural inferences our science can become is an open question; our continuing technological success suggests that any limit has yet to be reached. We must be prepared to admit, however, that a cognitive architecture finely tuned by the needs of social survival in a harsh environment is not necessarily an architecture well adapted or even capable of learning the truth. Practical knowledge may be the best that we, as a species, can hope for, at least outside of the safety of our studies, out in the world.

In summary, it appears that what our science is telling us, with increasing urgency, is that the universe is not fully open to our comprehension. We are not just faced with a nagging scepticism that is easily remedied by a dose of pragmatism; we are faced instead by dialetheic paradoxes produced by our best and most careful sciences. In this predicament, even what is to count as "pragmatism" becomes doubtful. A key assumption of Enlightenment science was that human beings can, with sufficient inspiration and effort, achieve a God-like understanding. It is perhaps time for this assumption to be recognized as overly optimistic, and then abandoned.

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