Directionalism and Relations of Arbitrary Symmetry

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ABSTRACT: Maureen Donnelly has recently argued that directionalism, the view that relations have a direction, applying to their relata in an order, is unable to properly treat certain symmetric relations. She alleges that it must count the application of such a relation to an appropriate number of objects in a given order as distinct from its application to those objects in any other ordering of them. I reply by showing how the directionalist can link the application conditions of any fixed arity relation, no matter its arity or symmetry, and its converse(s) in such a way that directionalism will yield the correct ways in which it can apply. I thus establish that directionalism possesses the same advantage Donnelly's own account of relations, relative positionalism, has over traditional positionalist accounts of relations, which do not properly treat symmetric relations. I then note some advantages that directionalism has over its closest competitors. This includes Donnelly's relative positionalism, since directionalism is not, like relative positionalism, committed to the involvement of relative properties in every irreducibly relational claim. I close by conceding that, as Donnelly notes, directionalism is committed to the primitive relation of order-sensitive relational application. But I don't find this notion as mysterious as Donnelly does. I conclude that, even if one construes this feature of directionalism as a drawback, the two views are at worst at a draw, other things being equal, since this drawback is mitigated by the advantage directionalism has over relative positionalism.

1. Introduction

Since Timothy Williamson's (1985) and Kit Fine's (2000) critiques of Bertrand Russell's (1903) view about the nature of relations, directionalism, according to which relations are understood as having a direction, applying to their relata in an order, philosophers have largely turned away from it.[1] They have turned toward views according to which relations are adirectional, or *neutral*. One popular sort of theory of neutral relations is absolute positionalism, according to which relations have positions or roles associated with them which their relata occupy or have, respectively (see Gilmore 2013 and 2014, Orilia 2011 and 2014, and Dixon 2018).[2] As Fine (2000) argues, however, absolute positionalist views face the *problem of symmetric relations*; they are unable to properly treat relations with certain symmetries. That is, they are unable to deliver the correct possible *completions* of such a relation, where a completion of a relation is anything which results from that

[1]The view is also known as 'the standard view' and 'the standard account'.

[2]Following Donnelly (2016), I qualify these forms of positionalism as *absolute* to distinguish them from her positionalist view, which she qualifies as *relative*.

relation applying to some things in a certain way, e.g., a fact, a state of affairs, or a proposition.[3] I will characterize a way a relation can apply formally in what follows, but for now, a couple of examples will serve to elucidate the idea. The binary relation *loving*, for example, seems able to apply to two objects in two ways. Goethe's loving Charlotte Buff is a different state of affairs from Buff's loving Goethe. The binary relation *being next to*, on the other hand, seems able to apply to two objects in only one way. Goethe's being next to Buff is the same state of affairs as Buff's being next to Goethe.

The difficulty absolute positionalism has with symmetric relations has led to the development of other, neutral, views of relations which can solve this problem, including Fine's (2000) antipositionalism, Fraser MacBride's (2014) relational primitivism, and Maureen Donnelly's (2016, 2021) relative positionalism. These views properly treat any fixed arity relation, no matter its particular symmetry structure. Donnelly has recently argued that directionalism is, like absolute positionalism, also unable to properly treat symmetric relations. I begin, in the remainder of this section, by explaining the difficulty Donnelly alleges directionalism has with symmetric relations, which emerges clearly even in the case of binary relations, and state how my reply on behalf of directionalism goes in that case. I then remind the reader of fixed arity relations of arity greater than two, which can have more complex symmetries, and which any account of relations, including directionalism, ought to be able to to treat properly.

In section 2, I develop a way of formally representing the symmetry structure of any fixed arity relation, similar to Donnelly's (2016), and a way of formally modeling the ways a fixed arity relation can apply. Along the way, I discuss several relations with various symmetry structures, some of which are known to cause problems for absolute positionalism. In section 3, I explain how Donnelly takes her objection to generalize to n-ary relations for all $n \geq 2$, and I develop my reply to this generalized criticism by showing how the directionalist can link the application conditions of any fixed arity relation, no matter its arity or symmetry structure, and its converse(s) in such a way that directionalism yields the correct manners in which it can apply.[4] I thus establish that directionalism

[3] Fine (2000: 17–18, including fn. 10) first articulates this problem, and (2000: 4–5) introduces the notion of a completion. Of course, there are important differences between completions of these three different types. Presumably, for example, if the fact that Goethe loves Buff exists then Goethe loves Buff. This is usually thought not to be so in the case of the state of affairs of Goethe's loving Buff, or in that of the proposition that Goethe loves Buff. For simplicity, I restrict my attention primarily to states of affairs in what follows.

[4] Like Donnelly, in her development of relative positionalism, I consider only relations of fixed *finite* arity.

possesses the same advantage Donnelly's, Fine's, and MacBride's accounts of relations have over absolute positionalism, which, it is well known, cannot handle all such relations.

In section 4, I turn to the task of evaluating directionalism, with my previous results in mind, in relation to other accounts of relations that avoid the problem of symmetric relations, viz., Fine's, MacBride's, and Donnelly's. I argue that directionalism has advantages over each of these views, including relative positionalism, in that it is not, like relative positionalism, committed to the involvement of relative properties in every irreducibly relational claim (i.e., in every relational claim which cannot be construed as a claim involving the instantiation of only ordinary non-relative properties). I close by conceding, in section 5, that, as Donnelly notes, directionalism is committed to the primitive relation of ordered relational application. But I don't find this notion as mysterious as Donnelly does. I conclude that, even if one construes this feature of directionalism as a drawback, the two views are at worst at a draw, other things being equal, since this drawback is mitigated by the advantage directionalism has over relative positionalism. Unfortunately, I won't have the space to properly address all of the objections that have been leveled against directionalism over the years, including Williamson's and Fine's, and instead leave replies to these objections for another occasion.

Directionalism is usually formulated in terms of binary relations only.[5] It is typically taken to consist of three central theses.

[5]See Russell's own (1903: §§94–95 and §§218–19) formulations of directionalism, as well as those of Fine (2000: §1), MacBride (2007: 25 and 2014: 1–2), Gaskin and Hill (2012: §1), Leo (2014: 263), Liebesman (2014: 409), Donnelly (2016: §5.2), and Ostertag (2019: §2.1). Fine (2000: 3) and Donnelly (2016: 83–85) discuss some elements of a generalization of the view, though, as I discuss below, Donnelly suggests that directionalism *can't* be generalized. Others, including Gaskin and Hill (2012: 167) and MacBride (2014: 4), appear to acknowledge that directionalism can be generalized to cover relations of any arity, though they provide few details about how they think such a generalization could be carried out. Russell himself (1913: 123) appears to recognize the relevance of algebra to the question of individuating completions of relations, but he did not himself give a general statement of directionalism. Thanks to Gregory Landini (personal communication) for bringing this passage to my attention. As I suggest in the first sentence of the introduction, directionalism is not particularly popular, at least in the literature on the metaphysics of relations. But it appears to be standardly assumed, or at least major components of it are, in the tradition of higher-order metaphysics, at least implicitly. Many working in this tradition employ a higher-order language, often simple type theory with lambda abstraction (as in Dorr 2016 and Bacon 2020), that allows one to attribute to higher-order entities even higher-order properties and relations. To express the idea that a binary relation R applies to objects a and b in that order, one would say in such a language that $(\lambda X^{\langle e,e\rangle}.Xa^e b^e)R^{\langle e,e\rangle}$, which says of the binary relation R whose domain encompasses first-order objects (type e entities) that it applies to a and b. But the fact that 'a' and 'b' must appear in a specific order in such an expression forces an interpretation of relational application in such a language as being order-sensitive. There is a semantic difference between the expression above and '$(\lambda X^{\langle e,e\rangle}.Xb^e a^e)R^{\langle e,e\rangle}$'. In addition to this, many working in higher-order metaphysics distinguish between each (non-symmetric) relation and its converse, as does the directionalist (see (D3) below), since a necessary condition on the identity of second-order entities is that they are coextensive. And the extensions of a (non-symmetric) relation and its converse *are* distinct; the ordered pairs which populate them consist of pairs of the same objects but those objects oppositely ordered in those pairs in the two extensions. See Trueman 2021: 141–42 and Skiba 2021a: 3.

(D1) Every relation has a *direction* (what Russell calls a 'sense'). It applies to its relata in an order, proceeding from one to another.

The relation *loving*, for example, is understood by the directionalist as applying first to Goethe then to Buff when Goethe loves Buff, or, alternatively, proceeding from Goethe to Buff.[6]

(D2) Every relation R has a *converse*, which applies to x and y in the opposite order to that in which R applies whenever R applies to x and y.

The converse of *loving*, for example, is *being loved by*. It applies first to Buff and second to Goethe when Goethe loves Buff — in the opposite order or direction to that in which *loving* applies to them under the same condition.

(D3) Every necessarily symmetric relation is identical to its converse, while every other relation is distinct from its converse,

where a (binary) relation R is necessarily symmetric if and only if, necessarily, Rxy if and only if Ryx, and is non-symmetric otherwise. So while *loving* is distinct from its converse *being loved by*, a symmetric binary relation, like *being next to*, is its own converse.

Donnelly's criticism of directionalism emerges clearly even in the case of binary symmetric relations. She says,

> If the different ways R can hold among $x_1, \ldots, x_n$ amount to just different orders of application of R to $x_1 \ldots, x_n$, then *any* difference in the order of $x_1, \ldots, x_n$ should correspond to a different way for R to hold among $x_1, \ldots, x_n$. (Donnelly 2021: 6, ital. orig.)[7]

Donnelly is concerned that, because the directionalist imparts a direction to *every* (binary) relation, not just non-symmetric ones, she will be forced to say that, just as a non-symmetric binary relation like *loving* can apply to two objects in two ways, a symmetric binary relation like *being next to* will have to too. Note, however, that (D2) saves the directionalist from this consequence. Since *being next to* is necessarily symmetric, by (D3), it is its own converse, and so (D2) demands that, when it applies to two objects like Goethe and Buff in that order, it must also apply to them in the opposite order. So there is only one way for it to apply to Goethe and Buff: the way in which it applies to Goethe and Buff both in that order *and* the opposite order. Contrast that with how directionalism treats *loving*. Since it is non-symmetric, by (D3), it is distinct from its converse

[6] While relations are characterized as having *directions* or *senses*, or applying in an *order*, according to directionalism, this needn't be understood as involving the reification of any of these things. What is important is that, according to (D1), a relation applies *first* to one relatum *then* to the other, or, alternatively, it proceeds *from* one *to* the other.

[7] See Donnelly 2016: 83 for an earlier statement of the objection. Gaskin and Hill (2012: 175) also take directionalism to be incapable of properly treating relations with partial symmetries.

being loved by. (D2) demands that, when *loving* applies to Goethe and Buff in that order, *being loved by* must apply to them in the opposite order (and vice versa). But this yields *two* ways for *loving* (and *being loved by*) to apply to Goethe and Buff: the way in which *loving* applies to Goethe and Buff in that order and *being loved by* does so in the opposite order, and the way in which *loving* applies to Buff and Goethe in *that* order and *being loved by* does so in the opposite order. Of course many countenance relations of arity greater than two, and such relations exhibit a variety of different symmetry structures, and as I will discuss later, Donnelly takes her concern to generalize to many of these structures. So the directionalist's response can't be as simple as this. To understand Donnelly's criticism in full, and the directionalist's response to it, we first need to see the full picture of the possible symmetry structures relations can have. This task I undertake in the next section.

2. Relations of Arbitrary Symmetry

Following Donnelly (2016), I represent a relation's symmetry (structure) by its *symmetry group*. A *group* is a set of objects that is closed under an associative operation $\cdot$, the *group operation*, which has a unique *identity element* e such that x, $e \cdot x = x \cdot e = x$ and, for each element x, a unique *inverse element* x^{-1} such that $x \cdot x^{-1} = x^{-1} \cdot x = e$. A symmetry group of an n-ary relation is a group of permutations of $\{1, 2, \ldots, n\}$ (i) whose group operation is function composition, $\circ$, (ii) whose identity element is the identity permutation (i.e., the permutation that maps 1 to 1, 2 to 2, ..., and n to n), and (iii) for which the inverse of each element is that element's inverse permutation. In particular,

> **Definition of Symmetry Groups.** The *symmetry group* of an n-ary relation R, where $n \in \{2, 3, \ldots\}$, is the set Sym_R of permutations of $\{1, \ldots n\}$ such that, for each member p of Sym_R, necessarily, for all $x_1, \ldots, x_n$, $Rx_1 \ldots x_n$ iff $Rx_{p(1)} \ldots x_{p(n)}$.[8]

As Donnelly notes (2016: 83, incl. fn 10), the symmetry group of any n-ary relation will be a subgroup of the group of *all* possible permutations of $\{1, \ldots, n\}$, i.e, of the *symmetric group of degree n*, or S_n.[9]

A question arises at this point, for each n-ary relation R, whether the fact that, necessarily, for all $x_1, \ldots, x_n$, $Rx_1 \ldots x_n$ iff $Rx_{p(1)} \ldots x_{p(n)}$ really is sufficient for p to be in R's symmetry group, as

[8] Henceforth, when I introduce an arbitrary n-ary relation, I leave it implicit that $n \in \{2, 3, \ldots\}$ unless specified otherwise.

[9] That is, the set is a subset of that group and itself forms a group under the group operation of permutation composition.

the above definition of symmetry groups stipulates, or whether instead $[Rx_1 \ldots x_n]$ must be *identical to* $[Rx_{p(1)} \ldots x_{p(n)}]$ to guarantee this to be the case, where $[Rx_1 \ldots x_n]$ and $[Rx_{p(1)} \ldots x_{p(n)}]$ are completions of the same type (viz., facts, states of affairs, or propositions). But since 'R' appears on both sides of the biconditional in the definition, there will presumably be no cases in which $[Rx_1 \ldots x_n]$ is distinct from $[Rx_{p(1)} \ldots x_{p(n)}]$. It is plausible that, for any relations R and R', when $R = R'$, if necessarily, $Rx_1 \ldots x_n$ iff $R'x_{p(1)} \ldots x_{p(n)}$, then $[Rx_1 \ldots x_n] = [R'x_{p(1)} \ldots x_{p(n)}]$, even if this is implausible when $R \neq R'$. So an intensional definition of symmetry groups should be adequate. For this reason, I'll allow myself to move back and forth between talk of (non-)identity of completions and (non-)equivalence of relational claims in what follows.

The discussion of relations' symmetry groups has been pretty abstract so far, so I'll consider some examples. I'll begin with the symmetry groups of the binary relations *being next to* and *loving.* Since, necessarily, for any x_1 and x_2, x_1 is next to x_2

- iff x_1 is next to x_2 (equivalently: $x_{[1\,2](1)}$ is next to $x_{[1\,2](2)}$),
  and
- iff x_2 is next to x_1 (equivalently: $x_{[2\,1](1)}$ is next to $x_{[2\,1](2)}$),

where $\ulcorner[x_1\, x_2 \ldots x_n]\urcorner$ denotes the permutation of $\{1, 2, \ldots, n\}$ that maps 1 to x_1, 2 to x_2, ..., and n to x_n, the symmetry group of *being next to*,

$$Sym_{being\ next\ to} = \{[1\,2], [2\,1]\}.$$

In other words, every permutation of x_1 and x_2 results in an equivalent claim. But since (i) necessarily, for any x_1 and x_2, x_1 loves x_2 iff

- x_1 loves x_2 (equivalently: $x_{[1\,2](1)}$ loves $x_{[1\,2](2)}$)

but (ii) it is not the case that, necessarily, for any x_1 and x_2, x_1 loves x_2 iff

- x_2 loves x_1 (equivalently: $x_{[2\,1](1)}$ loves $x_{[2\,1](2)}$),

the symmetry group of *loving*,

$$Sym_{loving} = \{[1\,2]\}.$$

In other words, the only permutation of x_1 and x_2 that results in an equivalent claim is the identity permutation, i.e., the permutation that leaves the two terms where they are.

An n-ary relation such that, necessarily, for all $x_1, \ldots, x_n$, $Rx_1 \ldots x_n$ iff $Rx_{p(1)} \ldots x_{p(n)}$ for *every* permutation $p \in S_n$, is *completely symmetric*, while one that is such that this is true *only* when p is the identity permutation of S_n, $[1\,2\,\ldots\,n]$, is *completely non-symmetric*. *Being next to* is an example of the former, and *loving* the latter. Indeed, any binary relation can only be either completely symmetric or completely non-symmetric, since there are only two subgroups of the group of S_2, viz., S_2 itself, and the group that consists of just the identity permutation of S_2, i.e., $\{[1\,2]\}$. There are, of course, also completely symmetric and completely non-symmetric n-ary relations for $n > 2$ as well, though I will not consider any here.

Fine (2000: 17–18, including. fn. 10) argues that absolute positionalism is unable to properly treat fixed arity relations with certain symmetries.[10] According to absolute positionalism, relations are neutral (directionless), but feature positions, which have been interpreted as worldly correlates of thematic roles in linguistics that their relata fill (as in Orilia 2011 and 2014), or as entities akin to holes which their relata occupy (as in Gilmore 2013 and 2014 and Dixon 2018). Such views properly treat relations with some symmetries just fine. But there are relations with other symmetries that they cannot properly treat. They can properly treat any completely symmetric or completely non-symmetric relation one might throw at them.

For a theory of relations to properly treat a given n-ary relation, I mean that the theory has the resources to ensure that that relation can apply in the ways that we think it should be able to apply. But what is a way for an n-ary relation to apply? And, for a given such relation, what are the ways that it *should* be able to apply? The ways such relation can apply can be identified with the *left cosets* of that relation's symmetry group. For a given ordering of n objects, yielding a certain completion of an n-ary relation R, Sym_R includes exactly those permutations of that ordering that yield the same completion of R, which of course include the identity permutation. This amounts to one way the relation can apply. For some relations (any relation that is not completely symmetric), there will be non-identity permutations of that initial ordering (in S_n but not in Sym_R) that yield distinct completions of a given sort (facts, states of affairs, or propositions). Consider such a relation R and such a non-identity permutation q. Then $[Rx_1 \ldots x_n] = [Rx_{[1\,\ldots\,n](1)} \ldots x_{[1\,\ldots\,n](n)}] \neq [Rx_{q(1)} \ldots x_{q(n)}]$. And $[Rx_{q(1)} \ldots x_{q(n)}]$ will be identical to every other completion (of the same sort) that results from permuting the arguments of R in $[Rx_{q(1)} \ldots x_{q(n)}]$ by some permutation in Sym_R. The sets of permutations identified by considering every $q \in S_n$ form the *left cosets* of Sym_R in S_n, and represent the ways R can apply to n fixed

[10] See also Donnelly 2016: §5.3.

objects. More formally,

> **Definition of Left Cosets of the Symmetry Group of a Relation.** For any n-ary relation R, the *left cosets* of Sym_R in S_n are the sets $\{q \circ p : p \in Sym_R\}$ for each $q \in S_n$.

The left cosets of the symmetry group of an n-ary relation R partition S_n into between 1 and $n!$ equally-sized sets of permutations, depending on R's symmetry group. And by Lagrange's theorem, which implies that the number of left cosets of a subgroup H of a group G equals $|G| \div |H|$, the number of left cosets of $Sym_R = |S_n| \div |Sym_R|$. So there are $|S_n| \div |Sym_R|$ ways for R to apply to n objects.[11,12]

A completely symmetric n-ary relation R will therefore be able to apply to n objects in only $|S_n| \div |Sym_R| = |S_n| \div |S_n| = 1$ way, corresponding to the single coset of Sym_R in S_n. The single way *being next to* can apply to two objects, for example, corresponds to the single coset of $Sym_{\textit{being next to}} = \{[1\,2], [2\,1]\}$ in $S_2 = \{[1\,2], [2\,1]\}$, viz., $\{[1\,2], [2\,1]\}$ itself.[13] ($|\mathrm{S}_2| \div |Sym_{\textit{being next to}}| = 2 \div 2 = 1$.) A completely non-symmetric n-ary relation, on the other hand, will be able to apply in $|S_n| \div |Sym_R| = |S_n| \div |\{[1\ 2\ \ldots\ n]\}| = |S_n| \div 1 = n!$ ways to n objects, corresponding to the $n!$ cosets of Sym_R in S_n. The two ways *loving* can apply to two objects, for example, correspond to the two cosets of $Sym_{loving} = \{[1\,2]\}$ in $S_2 = \{[1\,2], [2\,1]\}$, viz., $\{[1\,2]\}$ and $\{[2\,1]\}$. ($|\mathrm{S}_2| \div |Sym_{loving}| = 2 \div 1 = 2$.)

The absolute positionalist can say that a completely symmetric relation has just one position which can take up to n arguments. This results in there being just one way for such a relation to apply to n objects: that constituted by each of those objects being assigned to that single position. So, for example, the absolute positionalist would say that *being next to* has one position, p_1, which can take up to two arguments, and so there is only one way for it to apply to two objects like Goethe and Buff. Goethe and Buff can only both be assigned to p_1. And, as mentioned, there is indeed only one way for *being next to* to apply to two objects like Goethe and Buff. Goethe's being

[11] See Gallian 2013: 147–48 for a statement and proof of Lagrange's theorem.

[12] R can apply to m objects in fewer ways when $m < n$. Certain combinatorial possibilities collapse in such cases because a relation's/predicate's argument cannot be permuted with itself and yield a new completion/non-equivalent claim. See Donnelly 2016: 83–84, fn. 11.

[13] The left coset $[1\,2] \circ Sym_{\textit{being next to}} = \{[1\,2] \circ p : p \in Sym_{\textit{being next to}}\} = \{[1\,2] \circ [1\,2], [1\,2] \circ [2\,1]\} = \{[1\,2], [2\,1]\}$. The left coset $[2\,1] \circ Sym_{\textit{being next to}} = \{[2\,1] \circ p : p \in Sym_{\textit{being next to}}\} = \{[2\,1] \circ [1\,2], [2\,1] \circ [2\,1]\} = \{[2\,1], [1\,2]\}$. These cosets are identical and exhaustive of the permutations in S_2, and so $Sym_{\textit{being next to}}$ has only a single coset in S_n. Remember that $\circ$ is function composition. For permutations p and q of $\{1, \ldots, n\}$, $p \circ q$ is the permutation that maps each $i \in \{1, \ldots, n\}$ to $p(q(i))$. In other words, it is the result of first applying q to i, getting the result, and then applying p to that result. So $[1\,2] \circ [2\,1] = [2\,1]$, for example, because (i) $([1\,2] \circ [2\,1])(1) = [1\,2]([2\,1](1)) = [1\,2](2) = 2$ and (ii) $([1\,2] \circ [2\,1])(2) = [1\,2]([2\,1](2)) = [1\,2](1) = 1$.

next to Buff is the same state of affairs as Buff's being next to Goethe. The absolute positionalist can say that a complete non-symmetric n-ary relation has n positions, each of which can take just a single argument. This results in there being $n!$ ways for such a relation to apply to n objects, each corresponding to a different assignment of those n objects to those n positions. For example, the absolute positionalist would say that *loving* has two positions, p_2 and p_3, each of which can take just a single argument, and so there are two ways for it to apply to two objects, such as Goethe and Buff. Goethe can be assigned to p_2 and Buff to p_3, or Buff can be assigned to p_2 and Goethe to p_3. And, as mentioned, there are indeed two ways for *loving* to apply to two objects like Goethe and Buff: one in which Goethe is doing the loving, and Buff is being loved, and one in which Buff is doing the loving, and Goethe is being loved.

In addition to *completely* symmetric and non-symmetric n-ary relations for $n > 2$, however, there are also *partially* (*non-*)*symmetric* such relations. The symmetry group of a partially symmetric n-ary relation is a proper non-trivial subgroup of S_n. That is, it will contain some, though not all, non-identity permutations of $\{1, \ldots, n\}$. The ternary relation *being between* is an example of such a relation. Since (i) necessarily, for any x_1, x_2, and x_3, x_1 is between x_2 and x_3

- iff x_1 is between x_2 and x_3 (equivalently: $x_{[1\,2\,3](1)}$ is between $x_{[1\,2\,3](2)}$ and $x_{[1\,2\,3](3)}$), and
- iff x_1 is between x_3 and x_2 (equivalently: $x_{[1\,3\,2](1)}$ is between $x_{[1\,3\,2](2)}$ and $x_{[1\,3\,2](3)}$),

but (ii) this is false of every other permutation of $\{1, 2, 3\}$, the symmetry group of *being between*,

$$Sym_{\textit{being between}} = \{[1\,2\,3], [1\,3\,2]\}.$$

Absolute positionalist views can properly treat *some* partially symmetric relations, like this one. The absolute positionalist can say that such a relation, while ternary, has only two positions, p_4 and p_5, the first of which can take only a single argument, while the other can take up to two (see Dixon 2018: 208). This results in there being three ways for such a relation to apply to three objects, such as Larry, Curly, and Moe. Larry can be assigned to p_4 and the other two to p_5, or Curly can be assigned to p_4, and the other two to p_5, or Moe can be assigned to p_4, and the other two to p_5. And there are, indeed, three ways for such a relation to apply to Larry, Curly, and Moe. Larry could be between the other two, or Curly could be, or Moe could be. These three ways correspond to the three left cosets of $Sym_{\textit{being between}} = \{[1\,2\,3], [1\,3\,2]\}$ in $\mathrm{S}_3 = \{[1\,2\,3], [1\,3\,2], [2\,1\,3], [2\,3\,1], [3\,1\,2], [3\,2\,1]\}$, viz., $\{[1\,2\,3], [1\,3\,2]\}$, $\{[2\,1\,3], [2\,3\,1]\}$, and $\{[3\,1\,2], [3\,2\,1]\}$. ($|\mathrm{S}_3| \div |Sym_{\textit{being between}}| = 6 \div 2 = 3$.)

But absolute positionalist views cannot handle *all* partially symmetric relations. The ternary relation *being arranged clockwise in that order* is such a relation.[14] Since (i) necessarily, for any x_1, x_2, and x_3, x_1, x_2, and x_3 are arranged clockwise in that order

- iff x_1, x_2, and x_3 are arranged clockwise in that order
(equivalently: $x_{[1\,2\,3](1)}$, $x_{[1\,2\,3](2)}$, and $x_{[1\,2\,3](3)}$ are arranged clockwise in that order),
- iff x_2, x_3, and x_1 are arranged clockwise in that order
(equivalently: $x_{[2\,3\,1](1)}$, $x_{[2\,3\,1](2)}$, and $x_{[2\,3\,1](3)}$ are arranged clockwise in that order),
and
- iff x_3, x_1, and x_2 are arranged clockwise in that order
(equivalently: $x_{[3\,1\,2](1)}$, $x_{[3\,1\,2](2)}$, and $x_{[3\,1\,2](3)}$ are arranged clockwise in that order),

but (ii) this is false of every other permutation of $\{1, 2, 3\}$, the symmetry group of *being arranged clockwise in that order*,

$$Sym_{being\ arranged\ clockwise\ in\ that\ order} = \{[1\,2\,3], [2\,3\,1], [3\,1\,2]\}.$$

The absolute positionalist appears to have only four options for treating *being arranged clockwise*. But none of these options yields the correct number of possible ways for it to apply to three objects. If it has one position, then it can apply in only one way. If it has two positions, one which can take only a single argument while the other can take up to two, it can apply in three ways (as was the case with in the previous example). If it has two positions, either of which can take up to two arguments, then it can apply in six ways. And if it has three positions, it can apply in six ways. But there are *two* ways for such a relation to apply to three objects, like Larry, Curly, and Moe. Larry, Curly, and Moe could be arranged clockwise in that order. Or Larry, Moe, and Curly could be arranged in *that* order instead. These two ways correspond to the two left cosets of $Sym_{being\ arranged\ clockwise\ in\ that\ order}$ in $\mathrm{S}_3 = \{[1\,2\,3], [1\,3\,2], [2\,1\,3], [2\,3\,1], [3\,1\,2], [3\,2\,1]\}$, viz., $\{[1\,2\,3], [2\,3\,1], [3\,1\,2]\}$ and $\{[1\,3\,2], [3\,2\,1], [2\,1\,3]\}$. ($|\mathrm{S}_3| \div |Sym_{being\ arranged\ clockwise\ in\ that\ order}| = 6 \div 3 = 2$.)

3. Generalizing Directionalism

It is the shortcoming of absolute positionalism just related which has motivated others to develop alternative accounts of relations. This includes Donnelly, who develops relative positionalism,

[14] This nominalization and the corresponding predicate '..., ..., and ... are arranged clockwise in that order' presuppose a particular vantage point on one side of the plane in which the objects are arranged. The nominalization also makes essential reference to the order of terms with respect to the argument places of the predicate . A name for the relation that avoids the latter issue (though not the former) is '*being clockwise in front of from the perspective of*'. See Donnelly (2016: 92 ff).

which provably yields the correct possible completions of any fixed arity relation. She recognizes that the problem of symmetric relations is at its heart an algebra problem, and uses this to draw insights about what relations would have to be like to avoid the problem. But she thinks that directionalism is unable to do the same. Donnelly (2016: 83–85 and 2021: 6) takes her concern about directionalism's ability to deal with symmetric relations, which I explicated at the end of section 1, to generalize to any relation that is anything but completely non-symmetric. Stated generally, her concern is that, because each n-ary relation R applies to n relata in a total order, there will always be $n!$ ways for R to apply to n relata, clashing with our intuitive judgement about n-ary relations that are anything but completely non-symmetric that they apply in m ways where $m < n!$. In this section, I show how directionalism can properly treat relations of *any* fixed arity relation.

It is clear that directionalism, as formulated in section 1, like absolute positionalism, cannot properly treat the relation *being arranged clockwise in that order*. This is for the simple reason that directionalism was formulated there in terms of binary relations only, and a relation expressed by the predicate '. . . , . . . , and . . . are arranged clockwise in that order' is presumably ternary. (For the same reason, directionalism, as formulated above, can't even handle *being between*—something that I noted absolute positionalism *can* do.) But all the directionalist needs to do is construe (D1) (see section 1) as allowing for some relations to take more than two relata. The direction of such a relation can be understood as the ordering of those relata, proceeding from the first relatum to the second, to the third, . . . , to the nth.

Then, once a couple more adjustments are made to the original formulation of directionalism, it becomes clear that directionalism can treat these relations, and indeed relations of *any* fixed arity, and that it can do so properly, no matter these relations' symmetries. First, for any n-ary relation R and each possible ordering of n relata, $x_1, \ldots, x_n$, the directionalist posits a unique converse for R which applies to $x_1, \ldots, x_n$ in that ordering of them exactly when R applies to $x_1, \ldots, x_n$ in *that* order. More precisely,

> **p-Converse Existence.** For any n-ary relation R and any permutation p of $\{1, \ldots, n\}$, R has exactly one *p-converse*,

where

> **Definition of p-Converses.** For any n-ary relations R and R' and any permutation p

of $\{1, \ldots, n\}$, R' is the *p-converse* of R, i.e., $R' = R_p =_{df}$ (i) R' is a converse of R, and (ii) necessarily, for all $x_1, \ldots, x_n$, $Rx_1 \ldots x_n$ iff $R'x_{p(1)} \ldots x_{p(n)}$.

p-Converse Existence effectively replaces (D2) (see section 1).

I will not define the notion of a converse of a relation, as it appears in clause (i) of the above definition. A straightforward way to do so — in terms of a p-converse of that relation (given a definition of p-converses which omits clause (i) of the above definition) — is as follows.

For any n-ary relations R and R' and any permutation p of $\{1, \ldots, n\}$, R' is a *converse* of $R =_{df} R'$ is a p-converse of R for some permutation p of $\{1, \ldots, n\}$.

But if one thinks that there are distinct though intensionally equivalent relations, this definition would be too permissive. For example, it would seem that, necessarily, for any x_1 and x_2, x_1 is triangular and taller than x_2 iff x_2 is shorter than x_1 and x_1 is trilateral. But presumably *being a y and z such that y is shorter than z and z is triangular* — and not *being a y and z such that y is shorter than z and z is trilateral* — is the single distinct converse of the completely symmetry binary relation *being triangular and larger than*. Such cases would also prevent one from supposing that the p-converse of a relation is unique, as I stipulate in the above definition of p-converses.

I will prevent such cases from causing problems by instead taking the notion of a converse as primitive, regarding facts about which relations are which relations' converses as brute, and adopting the following principle.

Converse-p-Converse Link. For any n-ary relations R and R' and any permutation p of $\{1, \ldots, n\}$, if R' is a converse of R, then R' is a p-converse of R for some permutation p of $\{1, \ldots, n\}$.

I assume that every relation is (one of) its own converse(s), so that $R = R_{[1 \ldots n]}$ for every n-ary relation R. Thus the notion of a converse I have in mind, and which I will employ in what follows (mainly to simplify the discussion), is different than that given by (D3) (see section 1) — the claim that every necessarily symmetric binary relation is identical to its converse, while every other binary relation is distinct from its converse. But even if revised according to this new terminology, (D3) will still entail a difference between necessarily symmetric relations and all other fixed arity relations; each of the former is its own *only* converse, while each of the latter has at least one converse distinct from itself.

To be able to properly treat any n-ary relation R, no matter its symmetry, all the directionalist needs to do is identify those p-converses of R whose orderings of relata $x_1, \ldots, x_n$, when $Rx_1 \ldots x_n$, "can be transformed into one another by a permutation in the symmetry group" of R (Donnelly 2016: 94). More precisely,

> p-**Converse Identity.** For any n-ary relation R, R's q-converse $=$ R's q^*-converse ($R_q = R_{q^*}$) iff there is some $p \in Sym_R$ such that $q^* = p \circ q$.

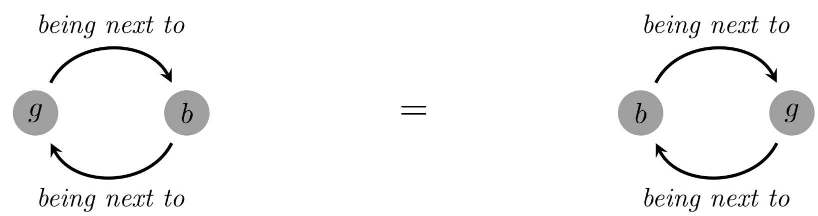
p-Converse Identity effectively replaces (D3) (see section 1). I assume that the symmetry structure of any n-ary relation R is represented by some subgroup of S_n.[15] Whatever subgroup of S_n Sym_R turns out to be, p-Converse Identity guarantees that $R = R_p$ iff $p \in Sym_R$. ($R = R_{[1\,\ldots\,n]}$, and so p-Converse Identity implies that $R = R_{q^*}$ iff there is some $p \in Sym_R$ such that $q^* = p \circ [1\ \ldots\ n] = p$.) This ensures that the ways R can apply to n objects correspond to the left cosets of Sym_R, as they should. This is because, by the definition of p-converses, $R = R_p$ iff, necessarily, for any $x_1, \ldots, x_n$, $Rx_1 \ldots x_n$ iff $Rx_{p(1)} \ldots x_{p(n)}$, and so the directionalist has ensured that $p \in Sym_R$ iff, necessarily, for any $x_1, \ldots, x_n$, $Rx_1 \ldots x_n$ iff $Rx_{p(1)} \ldots x_{p(n)}$, which is in agreement with the definition of symmetry groups. And I explained in the previous section why the ways an n-ary relation R can apply correspond to the left cosets of Sym_R in S_n. This in turn ensures, of course, that the number of ways R can apply to n objects equals $|S_n| \div |Sym_R|$, as it should.

But directionalism must also imply that the symmetry group of any converse of an n-ary relation R is isomorphic to that of R. What I've just said establishes that any n-ary relation will, according to directionalism, be able to apply to n objects in the ways we think it should. But we also expect R's (non-identical) converses (if it has any) to apply to n objects in the same ways as R (or, at least, in ways that are *structurally* the same). To show that this isomorphism holds, consider any n-ary relation R and any permutation p of $\{1, 2, \ldots\}$. There is bijective function f_p from Sym_R to Sym_{R_p} such that, for any $i, j \in Sym_R$, $f_p(i \circ j) = f_p(i) \circ f_p(j)$. $f_p(q) = p \circ q \circ p^{-1}$ fits this bill. (Recall that p^{-1} is the inverse of p. See section 2 above.) In other words, $f_p(q)$ is the permutation in S_n that maps a to b iff q maps $p^{-1}(a)$ to $p^{-1}(b)$, where $a, b \in \{1, \ldots, n\}$, i.e., $f_p(q)(a) = b$ iff

[15] This follows assuming that every relation can be expressed by a predicate which is *order-determined*, i.e., by a predicate that is such that implications of a relational claim that involve the predicate "concerning the order of relational application are completely determined in some fixed way by the order of the terms denoting the relata" relative to the predicate (Donnelly 2016: 84, fn. 13). Donnelly makes this assumption in her development of relative positionalism as well. It means that, according to directionalism, every relation must be expressible by a relational predicate that has a fixed number of singular argument places, and relates to directionalism's (and relative positionalism's) inability to accommodate variable arity relations. See fn. 24 in section 4 below.

$q(p^{-1}(a)) = p^{-1}(b)$.[16] In general, $Sym_{R_p} = \{p \circ q \circ p^{-1} : q \in Sym_R\}$.[17]

This discussion has been very abstract, so to provide the reader with a better idea of how directionalism handles relations of different arities and symmetries, and to highlight some interesting differences between directionalism, understood as applying to relations of any fixed arity, as compared to the binary formulation of it that I gave in section 1, I'll show how directionalism, as formulated above, treats the examples I discussed in section 2. I've already noted that, according to directionalism, a (completely) symmetric binary relation is its own only converse, while a (completely) non-symmetric binary relation has a single converse distinct from it. (Though now even a non-symmetric binary relation is a converse of itself.) But it will be instructive to see how p-Converse Existence and p-Converse Identity result in these treatments. Consider first the binary (completely) symmetric relation *being next to*. p-Converse Existence implies that *being next to* has p-converses *being next to*$_{[1\,2]}$ and *being next to*$_{[2\,1]}$. Since $[1\,2], [2\,1] \in Sym_R = \{[1\,2], [2\,1]\}$, by p-Converse Identity, *being next to* $=$ *being next to*$_{[1\,2]}$ $=$ *being next to*$_{[2\,1]}$. So by the definition of p-converses, *being next to* has a single converse, viz., itself. This means that, according to directionalism, *being next to* can apply to two things, such as Goethe and Buff, in only one way. If *being next to* applies to Goethe and Buff in that order, then *being next to*'s converse must apply to them in the opposite order. And if *being next to*'s converse applies to Goethe and Buff in that order,



Goethe's being next to Buff   Buff's being next to Goethe

Figure 1. The single possible application of *being next to* to Goethe and Buff

In this diagram and the one to follow, a relation applying to x_1 and x_2 in that order is represented by an arrow going from x_1 to x_2.

[16]Because every permutation is a bijection and the composite of bijections is a bijection, f_p is a bijection. To show it is an isomorphism, consider arbitrary $i, j \in Sym_R$. $f_p(i \circ j) = p(q(p^{-1}(i \circ j)))$. Then

$$\begin{aligned} f_p(i \circ j) &= p \circ i \circ j \circ p^{-1} && \text{by the definition of } f_p \\ &= p \circ i \circ e_n \circ j \circ p^{-1} && \text{recall that } e_n \text{ is the identity element of } \mathrm{S}_n \\ &= p \circ i \circ p^{-1} \circ p \circ j \circ p^{-1} && e_n = p^{-1} \circ p \\ &= f_p(i) \circ f_p(j) && \text{by the definition of } f_p. \end{aligned}$$

[17]Sym_R and Sym_{R_p} are *conjugate subgroups*. Many thanks to Maureen Donnelly and Jan Plate (personal communications) for helpful suggestions about the reasoning in this section.

then *being next to* must apply to them in the opposite order. But since *being next to* is its own converse, there is no difference between these two possibilities, which are depicted in figure 1. By assigning Goethe to 1 and Buff to 2, it is clear that this single manner of application corresponds to the single left coset $\{[1\,2], [2\,1]\}$ of $\mathrm{SYM}_{\textit{being next to}}$ in S_2.

Things go the same in the case of a non-symmetric relation like *loving*, except that, because $[2\,1] \notin \mathit{Sym}_{\textit{loving}} = \{[1\,2]\}$, it follows by p-Converse Identity that $\textit{loving} = \textit{loving}_{[1\,2]} \neq \textit{loving}_{[2\,1]}$. So by the definition of p-converses, *loving* has two converses, one of which is itself, the other, presumably, being *being loved by*. This means *loving* can apply to two things, such as Goethe and Buff, in two ways. If *loving* applies to Goethe and Buff in that order, then *loving*'s distinct converse, *being loved by*, must apply to them in the opposite order. And if *being loved by* applies to Goethe and Buff in that order, then *loving* must apply to them in the opposite order. But since $\textit{loving} \neq \textit{being loved by}$, these are two different possibilities, which are depicted in figure 2.

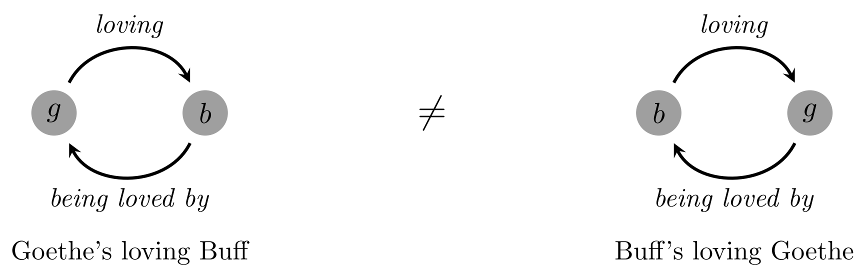


Figure 2. The two possible applications of *loving* and its single (distinct) converse to Goethe and Buff

By assigning Goethe to 1 and Buff to 2, it is clear that these two manners of application correspond to the two left cosets $\{[1\,2]\}$ and $\{[2\,1]\}$ of $\mathrm{SYM}_{\textit{loving}}$ in S_2.

Things become more complicated for ternary relations. Consider *being between*. Recall that

$$\mathit{Sym}_{\textit{being between}} = \{[1\,2\,3], [1\,3\,2]\}.$$

By p-Converse Existence, the directionalist would say that *being between* (R for now) has p-converses $R_{[1\,2\,3]}$ $(= R)$, $R_{[1\,3\,2]}$, $R_{[2\,1\,3]}$, $R_{[2\,3\,1]}$, $R_{[3\,1\,2]}$, and $R_{[3\,2\,1]}$. And because

(i) $[1\,3\,2] \in \mathit{Sym}_{\textit{being between}}$ and $[1\,3\,2] \circ [1\,2\,3] = [1\,3\,2]$,
(ii) $[1\,3\,2] \in \mathit{Sym}_{\textit{being between}}$ and $[1\,3\,2] \circ [2\,1\,3] = [3\,1\,2]$,
and
(iii) $[1\,3\,2] \in \mathit{Sym}_{\textit{being between}}$ and $[1\,3\,2] \circ [2\,3\,1] = [3\,2\,1]$,

the directionalist would say, by p-Converse Identity, that (i) $R_{[1\,2\,3]} = R_{[1\,3\,2]}$, (ii) $R_{[2\,1\,3]} = R_{[3\,1\,2]}$, and (iii) $R_{[2\,3\,1]} = R_{[3\,2\,1]}$. But because

(iv) there is no permutation $p \in Sym_{being\ between}$ such that, e.g., $p \circ [2\,1\,3] = [1\,2\,3]$,

(v) there is no permutation $p \in Sym_{being\ between}$ such that, e.g., $p \circ [2\,3\,1] = [1\,2\,3]$, and

(vi) there is no permutation $p \in Sym_{being\ between}$ such that, e.g., $p \circ [2\,3\,1] = [2\,1\,3]$,

the directionalist would say, by p-Converse Identity, that $R_{[1\,2\,3]} \neq R_{[2\,1\,3]}$, $R_{[1\,2\,3]} \neq R_{[2\,3\,1]}$, and $R_{[2\,1\,3]} \neq R_{[2\,3\,1]}$ (and so $R_{[1\,3\,2]} \neq R_{[2\,3\,1]}$ and $R_{[3\,1\,2]} \neq R_{[3\,2\,1]}$).

By the definition of p-converses, this means that *being between*, according to the directionalist, has three converses, one of which is itself. To identify plausible interpretations of the two converses distinct from *being between*, suppose Larry is between Curly and Moe, and consider the following diagram.

Figure 3. Larry's being between Curly and Moe

Being between applies to Larry, Curly, and Moe in that order, and also to Larry, Moe, and Curly in *that* order (see (i) above). But what relation applies to Curly, Larry, and Moe in that order and to Moe, Larry, and Curly in *that* order (as (ii) above demands)? A plausible interpretation of this relation is *being on the far side of from the perspective of.* Curly is on the far side of Larry from the perspective of Moe, and Moe is on the far side of Larry from the perspective of Curly. And what relation applies to Curly, Moe, and Larry in that order and to Moe, Curly, and Larry in *that* order (as (iii) above demands)? A plausible interpretation of this relation is *being on the opposite side as from the perspective of.* Curly is on the opposite side as Moe from the perspective of Larry, and Moe is on the opposite side as Curly from the perspective of Larry.[18]

Given the way the application conditions of these three relations are connected to one another, there are, according to directionalism, three possible ways for each of them to apply to three objects, like Larry, Curly, and Moe. These three manners of application are depicted in figure 4. The reader can check, by assigning Larry to 1, Curly to 2, and Moe to 3, that these three manners of application correspond to the three left cosets $\{[1\,2\,3], [1\,3\,2]\}$, $\{[2\,1\,3], [2\,3\,1]\}$, and $\{[3\,1\,2], [3\,2\,1]\}$ of $Sym_{being\ between}$ in S_3.

[18] Donnelly's (2021: 16) interpretations of the three relative properties associated with the predicate '... is between ... and ...' are similar.

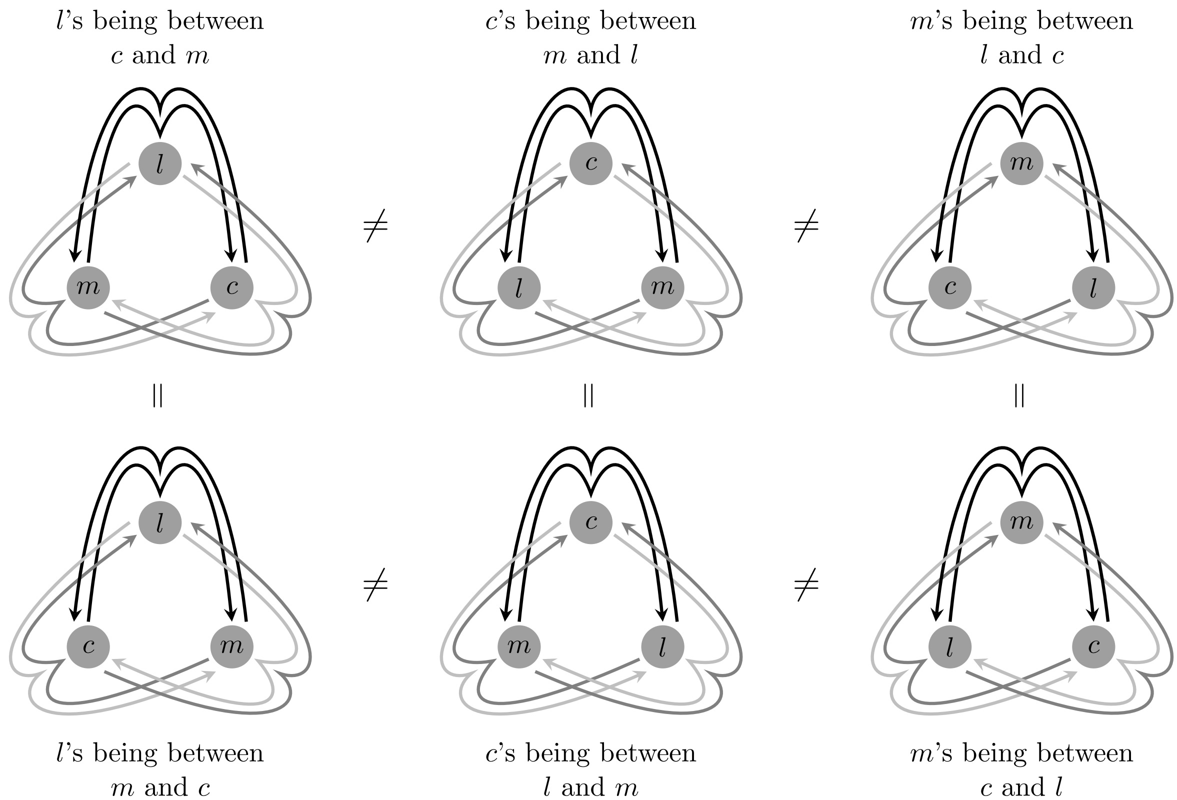


Figure 4. The three possible applications of *being between* and its two distinct converses to Larry, Curly, and Moe

In this diagram, a relation applying to x_1, x_2, and x_3 in that order is represented by an arrow going from x_1 to x_2, then to x_3. Light grey arrows depict applications of *being between*, black arrows depict applications of *being on the far side of from the perspective of*, and dark grey arrows depict applications of *being on the side opposite as from the perspective of*.

Being between is noteworthy because it has more than one converse distinct from it, which undermines the idea, expressed in (D2), that the order in which a relation R's converse applies to its relata is *opposite* that in which R does; they are merely *different*. When Larry is between Curly and Moe, *being between* applies in the orders $[l\,c\,m]$ and $[l\,m\,c]$, *being on the far side of from the perspective of* applies in the orders $[c\,l\,m]$ and $[m\,l\,c]$, and *being on the opposite side as from the perspective of* applies in the orders $[c\,m\,l]$ and $[m\,c\,l]$. But neither of these two pairs of orders seem to be *opposite* the two orders in which *being between* applies; they appear only to be *different*. It is somewhat plausible that the first and third of these relations apply in opposite orders. It is, after all, Larry who is the one who is privileged in the scenario under consideration (i.e., when Larry is between Curly and Moe). And Larry is at opposite ends of the light grey and dark grey arrows in the leftmost column of figure 4, which depicts this scenario. But neither of the first and third relations could plausibly be understood to apply in orders opposite to those in which the *second* relation (depicted by the black arrow) applies, and the second relation is nonetheless a converse of each of the other two. This, in conjunction with my choice to count every relation — even every completely non-symmetric relation — as its own converse means that the most we can hang onto as far as (D2) goes is that, except in cases of completely non-symmetric relations, a relation R's converse (even in the case when it is its own converse) applies to its relata in an order that is *different*, not *opposite*, from the order in which R applies to them.

Our discussion of *being between* also helps to illustrate why (D3), which covers only (completely) symmetric and (completely) non-symmetric binary relations, needs to be replaced with something, like p-Converse Identity, that can accommodate complete non-symmetries and partial symmetries which arise in relations of higher arities. According to (D3), a symmetric binary relation is identical to its converse, while a non-symmetric one is distinct from its converse. But a completely non-symmetric n-ary relation, where $n \in \{3, 4, \ldots\}$, will have more than one converse distinct from it, $n! - 1$, to be exact. And a partially symmetric relation will have more than one converse (some factor of $n!$ between 1 and $n!$), though one of those converses will be identical to it. Of completely symmetric relations of any arity, the directionalist can say that it has a single converse, viz., itself.

Consider last the ternary relation *being arranged clockwise in that order* — the relation with a symmetry structure that causes problems for the absolute positionalist. Recall that

$$Sym_{\textit{being arranged clockwise in that order}} = \{[1\,2\,3], [2\,3\,1], [3\,1\,2]\}.$$

By p-Converse Existence, the directionalist would say that *being arranged clockwise in that order* (R for now) has p-converses $R_{[1\,2\,3]}$ $(= R)$, $R_{[1\,3\,2]}$, $R_{[2\,1\,3]}$, $R_{[2\,3\,1]}$, $R_{[3\,1\,2]}$, and $R_{[3\,2\,1]}$. And because

(i) $[2\,3\,1] \in Sym_{\textit{being arranged clockwise in that order}}$ and $[2\,3\,1] \circ [1\,2\,3] = [2\,3\,1]$
and
(ii) $[3\,1\,2] \in Sym_{\textit{being arranged clockwise in that order}}$ and $[3\,1\,2] \circ [1\,2\,3] = [3\,1\,2]$,

the directionalist would say, by p-Converse Identity, that (i) $R_{[1\,2\,3]} = R_{[2\,3\,1]}$ and (ii) $R_{[1\,2\,3]} = R_{[3\,1\,2]}$ (and so $R_{[2\,3\,1]} = R_{[3\,1\,2]}$). Similarly, because

(i) $[2\,3\,1] \in Sym_{\textit{being arranged clockwise in that order}}$ and $[2\,3\,1] \circ [1\,3\,2] = [2\,1\,3]$
and
(ii) $[3\,1\,2] \in Sym_{\textit{being are arranged clockwise in that order}}$ and $[3\,1\,2] \circ [1\,3\,2] = [3\,2\,1]$,

the directionalist would say, by p-Converse Identity, that (i) $R_{[1\,3\,2]} = R_{[2\,1\,3]}$ and (ii) $R_{[1\,3\,2]} = R_{[3\,2\,1]}$ (and so $R_{[2\,1\,3]} = R_{[3\,2\,1]}$). And finally, because there is no permutation $p \in Sym_{being\ arranged\ clockwise\ in\ that\ order}$ such that, e.g., $p \circ [1\,3\,2] = [1\,2\,3]$,

we know by p-Converse Identity that $R_{[1\,2\,3]} \neq R_{[1\,3\,2]}$ (and so $R_{[1\,2\,3]} \neq R_{[2\,1\,3]}$, $R_{[1\,2\,3]} \neq R_{[3\,2\,1]}$, $R_{[1\,3\,2]} \neq R_{[2\,3\,1]}$, and $R_{[1\,3\,2]} \neq R_{[3\,1\,2]}$).

By the definition of p-converses, this means that *being arranged clockwise in that order* has two converses, one of which is itself. A plausible interpretation of the converse of *being arranged*

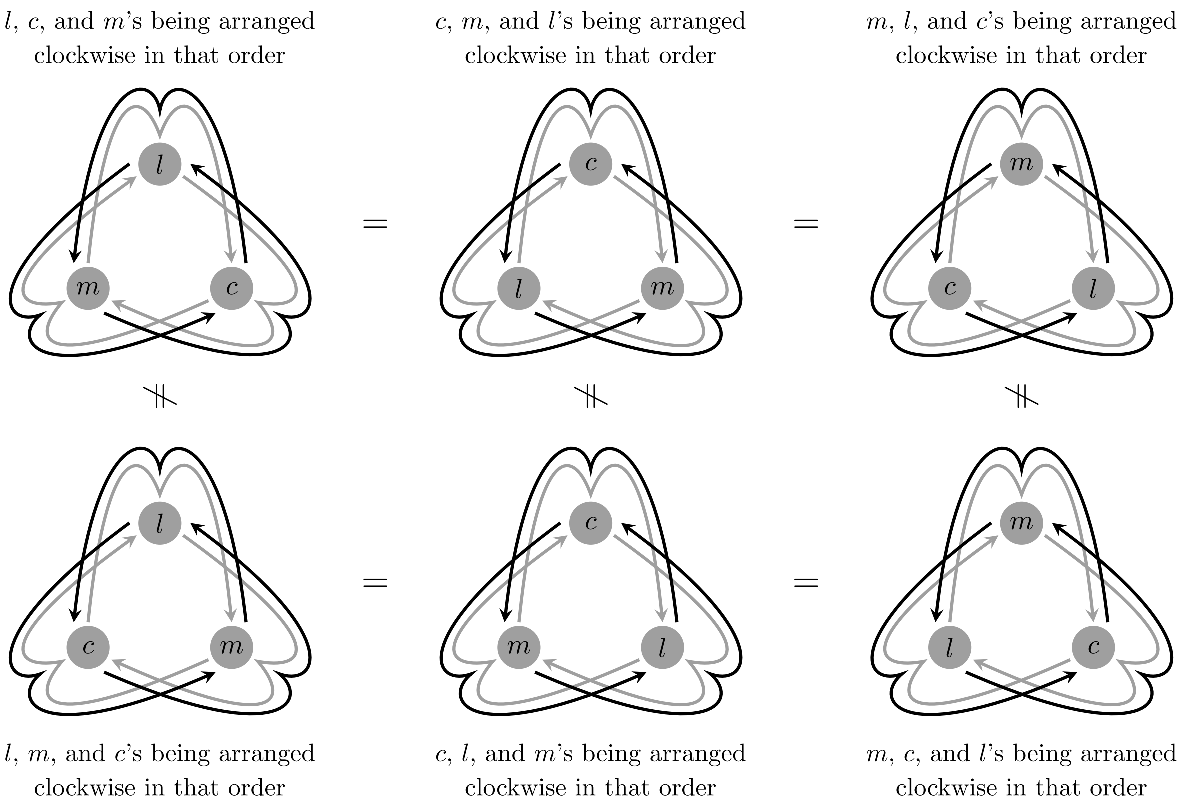


Figure 5. The two possible applications of *being arranged clockwise in that order* and its single distinct converse to Larry, Curly, and Moe

In this diagram, a relation applying to x_1, x_2, and x_3 in that order is represented by an arrow going from x_1 to x_2, then to x_3. Grey arrows depict applications of *being arranged clockwise in that order*, while black arrows depict applications of *being arranged counterclockwise in that order*.

clockwise in that order distinct from it is *being arranged counterclockwise in that order*.[19] Given the way the application conditions of these two relations are coordinated, there are, according to

[19] The name for this relation and the associated predicate are subject to the same issues I mentioned in connection with '*being arranged clockwise in that order*' in fn. 14 above. It presupposes a vantage point on one side of the plane in which the objects are arranged, and it makes essential reference to the order of terms with respect to the argument places of the corresponding predicate, in this case '…, …, and … are arranged clockwise in that order'. It could be analogously replaced with '*being clockwise behind from the perspective of*' to avoid the latter issue (though not the former).

directionalism, two possible ways for each of them to apply to three objects, like Larry, Curly, and Moe. These three manners of application are depicted in figure 5. The reader can check, by assigning Larry to 1, Curly to 2, and Moe to 3, that these two manners of application correspond to the two left cosets $\{[1\,2\,3], [2\,3\,1], [3\,1\,2]\}$ and $\{[1\,3\,2], [3\,2\,1], [2\,1\,3]\}$ of $Sym_{being\ arranged\ clockwise\ in\ that\ order}$ in S_3.

4. Directionalism's Advantages Over Its Closest Competitors

I've shown how directionalism avoids Donnelly's charge, in that it is able to properly treat any fixed arity relation with any symmetry such a relation can have. As such, it possesses the same advantage over absolute positionalist theories that is enjoyed by Donnelly's relative positionalism, Fine's (2000) antipositionalism, and MacBride's (2014) relational primitivism. In this section, I describe some advantages that directionalism has over each of these three accounts of relations. First, directionalism, unlike primitivism, supplies an explanation of why a given relation can apply in the ways it can. Second, directionalism, unlike antipositionalism and primitivism, supplies an explanation of why two relations can apply in the same or different ways (as the case may be). And third, directionalism, unlike relative positionalism, isn't committed to the involvement of relative properties in every irreducibly relational claim (i.e., in every relational claim which cannot be captured by a claim involving the instantiation of only ordinary non-relative properties). I'll describe each of these advantages in that order, explaining the views along the way as necessary.

Relations can apply in a variety of ways. But *why* is a given relation able to apply in the ways in can? Not all accounts of relations answer this question. Directionalism does. For example, directionalism explains why the binary relation *being next to* can apply to two objects in the single way it can. This is because it can apply to up to two objects (i.e., it is a binary relation), it is its own unique converse, and necessarily, for any x_1 and x_2, if it applies to x_1 and x_2 in that order (i.e., if x_1 is next to x_2), then its converse applies to x_2 and x_1 in *that* order (i.e., x_2 is next to x_1). The binary relation *loving*, on the other hand, can apply to two objects in the two ways it can, according to the directionalist, because (i) it can apply to up to two objects, (ii) it has a single converse distinct from it, viz., *being loved by*, and (iii) necessarily, for any x_1 and x_2, if *loving* applies to x_1 and x_2 in that order (i.e., if x_1 loves x_2), then its distinct converse applies to x_2 and x_1 in *that* order (i.e., x_2 is loved by x_1). This ensures that it is possible for *loving* to apply to x_2

and x_1 in that order *whether or not* it applies to x_1 and x_2 in *that* order, and vice versa, yielding two ways in which it can apply to two objects. In general, for any n-ary relation R, R can apply to n objects in the ways it can because R can take up to the number of relata it can, it has the number of converses it does, and the application conditions of it and its p-converses are necessarily connected in the ways that they are.

Contrast this with MacBride's relational primitivism, which is the view that, in general, there is no explanation for why any given relation can apply in the ways it can. It is, according to the primitivist, a matter of brute fact, for example, that *being next to* can apply in the single way it can, and that *loving* can apply in the two ways it can. By refusing to explain such facts, the primitivist avoids postulating any machinery that might treat a relation improperly (as does the absolute positionalist's machinery). Thus primitivism can properly treat any relation directionalism can properly treat. But primitivism has a pro tanto disadvantage compared to directionalism, in that it does not supply an explanation of the behavior of each relation it can properly treat, whereas directionalism does.

Now to directionalism's second advantage. Some relations seem able to apply in the same ways as one another, while others seem able to apply in different ways from one another. Consider *loving* and *hating*. Each of these relations can apply to two objects in two ways. Moreover, they seem to be applicable in the *same* two ways. $Sym_{loving} = Sym_{hating} = \{[1\,2]\}$, and so the two left cosets of each these relations' symmetry groups are the same; they are the two left cosets of $\{[1\,2]\}$ in S_2, viz., $\{[1\,2]\}$ and $\{[2\,1]\}$. The single way in which the binary relation *being next to* can apply to two objects is distinct from each of the two ways in which *loving* or *hating* can do so. That way is represented by the single left coset of $Sym_{being\ next\ to} = \{[1\,2], [2\,1]\}$ in S_2, viz., $\{[1\,2], [2\,1]\}$ itself. Directionalism supplies explanations of the identities and distinctions between the ways any two fixed arity relations can apply to appropriate numbers of objects in terms of the relation's arity, the number of converses it has, and how the application conditions of it and its p-converses are necessarily connected.

In the case of *loving* and *hating*, the directionalist says that these relations can apply in the same two ways because each has arity two, each has two converse, one of which is itself and the other distinct from it, and each is such that, necessarily, for any x_1 and x_2, if it applies to x_1 and x_2 in that order, then its distinct converse applies to x_2 and x_1 in *that* order. The way in which *being next to* can apply to two objects is different from the two ways in which *loving* (or *hating*) can do so, according to directionalism, because, while these relations have the same arity, the former relation is its own only converse, while the latter relation is distinct from one of its converses. As a result, while the latter can apply to two objects in two ways, the former, whenever it applies to x_1 and x_2 in that order, it must, as its own converse, apply to x_2 and x_1 in *that* order as well, yielding only a single way in which it can apply.

Some relations, while able to apply in the same number of ways, can nonetheless apply in different ways from one another. The ternary relation *being arranged clockwise in that order*, for example, can apply to three objects in two ways. But these two ways are different than the two in which *loving* or *hating* can apply. An intuitive explanation for this is that the first two ways can involve up to three objects, while the latter two can't. In terms of cosets, this can be explained by the fact that the two cosets of $Sym_{\textit{being arranged clockwise in that order}}$ in S_3, viz., $\{[1\,2\,3], [2\,3\,1], [3\,1\,2]\}$ and $\{[1\,3\,2], [3\,2\,1], [2\,1\,3]\}$ are pairwise distinct from the two left cosets that represent the ways *loving* or *hating* can apply. The directionalist can explain these differences by appealing to the fact that *being arranged clockwise in that order* has a different arity than each of *loving* and *hating*.

Some relations have the same arity, apply in the same number of ways, but nonetheless apply in different ways. Such relations, though of the same arity, still have non-isomorphic symmetry groups, and thus the way such relations can apply are still represented by different left cosets. For example, the six ways in which the quaternary *being arranged clockwise in that order*^{4} (as in Alice, Bob, Carol, and Diane are arranged clockwise in that order) can apply to four objects are pairwise distinct from the six ways in which *being closer together than* (as in Alice and Bob are closer together than Carol and Diane) can apply to them.[20] I will not go to the trouble of listing these cosets, but instead just briefly explain why the symmetry groups of these two relations, viz.,

$$Sym_{\textit{being arranged clockwise in that order}^4} = \{[1\,2\,3\,4], [2\,3\,4\,1], [3\,4\,1\,2], [4\,1\,2\,3]\}$$

$$Sym_{\textit{being closer together than}} = \{[1\,2\,3\,4], [1\,2\,4\,3], [2\,1\,3\,4], [2\,1\,4\,3]\}.$$

are not isomorphic. This is illustrated by the fact that the latter relation yields the same completion if certain pairs of relata are transposed in its application to them, while the former does not. For example,

[20] As with the ternary version of this clockwise arrangement relation, this name and the associated predicate presuppose a particular vantage point on one side of the plane in which the objects are arranged. See fn. 14 above.

Alice and Bob's being closer together than Carol and Diane = Bob and Alice's being closer together than Carol and Diane,

but

Alice, Bob, Carol and Diane's being arranged clockwise in that order ≠ Bob, Alice, Carol, and Diane's being arranged clockwise in that order

(see Dixon 2019: 68–69 for discussion of this point). These relations have the same number of converses (six). The directionalist will explain these differences in possible applications by appealing to differences in the ways the application conditions of these relations are necessarily connected.[21]

In contrast, neither Fine's antipositionalism nor MacBride's relational primitivism supplies explanations of the identities or differences in the ways distinct relations can apply. The primitivist supplies no explanation for why any given relation can apply in the ways it can, and so, ipso facto, can supply no explanation for why two relations can apply in the same or different ways as the case may be.[22] According to antipositionalism, relations do not have positions. What determines

[21]It is worth emphasizing the fact that the explanandum and explanans involved in each of these explanations are distinct. As was hopefully clear in the discussion above concerning the directionalist explanation for why any given n-ary relation R can apply to n objects in the ways it can, the directionalist explains why R can apply in these ways by appealing to R's arity, to the number of converses R has, and to the ways the application conditions of R and its p-converses are necessarily connected. The former fact is distinct from each of these latter facts. The same is going on when explaining why two relations can apply in the same ways (or different ways, as the case may be), except that it involves a comparison between the former and latter sorts of facts for two relations instead of one. The distinctness of the ways in which an n-ary relation R can apply to n objects and the ways in which the application conditions of R and its p-converses are necessarily connected can be further illustrated. The former correspond to the left cosets of Sym_R, as described in section 2, while the latter correspond to the *right* cosets of Sym_R, where

Definition of Right Cosets of the Symmetry Group of a Relation. For any n-ary relation R, the *right cosets* of Sym_R in S_n are the sets $\{p \circ q : p \in Sym_R\}$ for each $q \in S_n$.

But the left and right cosets of some n-ary relations differ, depending on their symmetry structures. For example, while the ways in which *being between* can apply to three objects are represented by $\{[1\,2\,3],[1\,3\,2]\}$, $\{[2\,1\,3],[2\,3\,1]\}$, and $\{[3\,1\,2],[3\,2\,1]\}$, the ways in which its application conditions are necessarily connected are best represented by $\{[1\,2\,3],[1\,3\,2]\}$, $\{[2\,1\,3],[3\,1\,2]\}$, $\{[2\,3\,1],[3\,2\,1]\}$. The reader can check that these latter three ways are the orders in which *being between* and its two distinct converses apply to Larry, Curly, and Moe when Larry is between Curly and Moe by consulting the left column of figure 4 above. The converses of *being between* must apply to x_1, x_2, and x_3 in certain orders, represented by the right costs of $Sym_{being\ between}$, exactly when *being between* applies to them in that order. This in turn determines, and, according to the directionalist, explains the ways, represented by the left cosets of $Sym_{being\ between}$, in which *being between* can apply to three objects. The general claim that the ways the application conditions of a relation and its p-converses are necessarily connected correspond to the right cosets of its symmetry group can be shown by considering any n-ary relation R and its q-converse R_q for any permutation q of $\{1,\ldots,n\}$. Whether $q \in Sym_R$ (and so $R_q = R$), or $q \notin Sym_R$ (and so $R_q \neq R$), it follows by the definition of p-Converses that, necessarily, $Rx_1 \ldots x_n$ iff $R_q x_{q(1)} \ldots x_{q(n)}$. By p-Converse Identity, for every $p \in Sym_{R_q}$, necessarily, $R_q x_{q(1)} \ldots x_{q(n)}$ iff $R_{p\circ q} x_{p\circ q(1)} \ldots x_{p\circ q(n)}$. Since Sym_{R_q} and Sym_R are isomorphic (see proof above), for every $p \in Sym_R$, necessarily, $R_q x_{q(1)} \ldots x_{q(n)}$ iff $R_{p\circ q} x_{p\circ q(1)} \ldots x_{p\circ q(n)}$. So the orders in which every converse of R (potentially including itself) applies to $x_1,\ldots,x_n$ exactly when R applies to them in that order constitute one right coset of Sym_R. And because we must consider each of R's q-converse for every permutation q of $\{1,\ldots,n\}$, every right coset of Sym_R contains exactly those orders in which some converse of R (potentially including itself) applies to $x_1,\ldots,x_n$ exactly when R applies to them in that order.

[22]The primitivist might recognize identities and differences between distinct relations' arities, and thus be able to

the ways in which a given relation can apply to some things — its manners of completion — are not facts about the internal structure of the completions that result from its application. Instead, the ways a relation can apply are determined by identity and distinctness relationships that hold between completions of that relation by different sets of objects. In previous work, I say,

> the manner in which Goethe and Buff complete *loving*... in Goethe's loving Buff is the same, on Fine's view, as exactly one of the two manners in which W. B. Yeats and Maud Gonne complete that relation in Yeats's loving Gonne and Gonne's loving Yeats, and it is distinct from the other. Which identity and distinctness relationships hold of these two possible but mutually exclusive sets of possibilities is, according to the antipositionalist, a matter of brute fact. (Dixon 2019: 65)

Antipositionalism can properly treat any relation that directionalism (and relative positionalism) can treat, as long as any such relation is instantiated by enough distinct sets of objects. But, as I note (see Dixon 2019: 70, fn. 17), because Fine defines the identity of manners of completions of relations R and R' only when $R = R'$, the antipositionalist is left without a way to compare manners of completions of distinct relations. Here is Fine's statement of the definition:

> to say that s is a completion of a relation R by $a_1, a_2 \ldots, a_{\mathrm{m}}$, in the same manner as t is a completion of R by $b_1, b_2, \ldots, b_{\mathrm{m}}$ is simply to say that s is a completion of R by $a_1, a_2 \ldots, a_{\mathrm{m}}$ that results from simultaneously substituting $a_1, a_2 \ldots, a_{\mathrm{m}}$ for $b_1, b_2, \ldots, b_{\mathrm{m}}$ in t (and vice versa). (Fine 2000: 25–26)

Moreover, I also note, it is not clear that Fine's definition could be modified in such a way that it could apply when $R \neq R'$. There will be no principled way to identify the manner in which a non-symmetric relation R applies to some things with any one of the manners in which a distinct non-symmetric relation applies to some other things rather than any of the other ways R' applies to those other things. Why, for example, should Goethe's loving Buff result from simultaneously substituting Goethe and Buff for Yeats and Gonne (and *loving* for *hating*) in Yeats's hating Gonne rather than in Gonne's hating Yeats? Only if this question has an answer will the antipositionalist have a way to explain why the way in which *loving* applies to Goethe and Buff in Goethe's loving Buff is identical to the way in which, say, *hating* applies to Yeats and Gonne in Yeats's hating Gonne and distinct from the way in which *hating* applies to them in Gonne's hating Yeats rather than vice versa. There does not seem to be a non-*ad hoc* way to answer questions like this, and

supply the same explanation that the directionalist does of why relations with different arities can apply in different ways. But she will be unable to explain why relations with the same arity that can nonetheless apply in different ways, like *being arranged clockwise in that order*[4] and *being closer together than*, can do so.

so the antipositionalist seems to be left unable to compare the manners of completions of distinct relations.[23,24]

Directionalism has an advantage over relative positionalism too. Relative positionalism is the view that, when a relation applies to some things, its doing so consists in those things occupying positions of the relation relative to one another. But the positions of a relation are not understood, on relative positionalism, as roles that objects fill, or holes that they occupy, as they are understood on absolute positionalist views. Instead, they are construed as unary *relative properties*, which relata instantiate relative to one another. A relative property is a property that can be instantiated by a thing only relative to a thing or some things, while a non-relative property is a property that can be instantiated by a thing *full stop*. If *being north* is a property, rather than a binary relation, it is presumably a relative property, since something can be north, it would seem, only relative to something or some things. It makes no sense, for example, to say that Washington, D.C. is north. Washington, D.C. is north *relative to something*, such as Kingston, Jamaica. In contrast, many would take a property like *being spherical* to be non-relative. Exceptions to even the latter sort of case are certain endurantists, who regard putative non-relative properties as relative properties that can be instantiated only relative to a time. More on this below.

Structurally, relative positionalism and directionalism are quite similar. The directionalist sees the application of each relation as being order-sensitive, and involving attendant order-sensitive applications of its converse(s). And while the relative positionalist regards each relation as neutral (directionless), she also regards each as having one or more relative properties — equal in number to

[23] MacBride (2007: 45 ff.) raises the issue even for single relations; there is no reason the way in which *loving* applies to Goethe and Buff in Goethe's loving Buff should be identical to the way in which it applies to Yeats and Gonne in Yeats's loving Gonne and distinct from the way in which it applies to them in Gonne's loving Yeats and not vice versa. Admittedly, the antipositionalist may be able to employ the same algebraic analysis of manners of completion as I provide, instead of the substitution-based analysis. And she could accept the idea that the ways in which two distinct n-ary relations, such as *loving* and *hating*, can apply to n objects are the same, without identifying any pair of ways one of which is a way in which one of the relations can apply while the other is one in which the other can apply. See Dixon 2019: 68, fn. 15. But the view faces other problems, e.g., MacBride's (2007: 48 and 2014: 14) objection that the antipositionalist cannot say anything about the ways a relation can apply unless it is instantiated at least twice. See MacBride 2007: §8 and Gaskin and Hill 2012: §§3–4 for other objections to antipositionalism.

[24] I argue in Dixon 2019 that relative positionalism has these same explanatory advantages over antipositionalism and primitivism, and that they are at least enough to offset the fact that the latter two accounts can accommodate variable arity relations, while relative positionalism cannot. Directionalism, as I have formulated it above, is also unable to handle variable arity relations, and for a perfectly analogous reason that relative positionalism cannot. According to directionalism, some relations with different arities have different numbers of converses, and thus must be distinct. For example, the ternary *being arranged clockwise in that order* has two converses, but the quaternary *being arranged clockwise in that order*^{4} has four. But directionalism's explanatory advantages over antipositionalism and primitivism similarly offset this disadvantage.

the number of converses a relation has according to directionalism — which are instantiated by $x_{p(1)}$ relative to $x_{p(2)}$, ..., relative to $x_{p(n)}$ in exactly those orders that the directionalist would have her relation and its distinct converse apply to $x_1, \ldots, x_n$. So according to the relative positionalist, *being next to* has one relative property, which one might interpret as *being adjacent*, that is instantiated by x_1 relative to x_2 and by x_2 relative to x_1 whenever x_1 is next to x_2, yielding only a single way for *being next to* to apply two objects. *Loving*, on the other hand, has two relative properties, *being a lover* and *being beloved*, the first of which is instantiated by x_1 relative to x_2 when x_1 loves x_2 and the second of which is instantiated by x_2 relative to x_1, and vice versa when x_2 loves x_1, yielding two ways for *loving* to apply to two objects. The ternary relation *being between* has three relative properties, resulting in it being able to apply in the three ways discussed in section 2, while the ternary relation *being arranged clockwise in that order* has two, resulting in it being able to apply in the two ways discussed in section 2.[25] Like directionalism, relative positionalism can provably properly treat any fixed arity relation with any symmetry such a relation can have (see Donnelly 2016: 94–96).

Directionalism possesses an advantage over relative positionalism in that it is not, while relative positionalism is, committed to the involvement of relative properties in every irreducibly relational claim. An irreducibly relational claim is a claim which cannot be captured by a claim involving the instantiation of only ordinary non-relative properties. For example, the claim that Goethe and Buff are mortal can be captured by the claim that Goethe is mortal and Buff is mortal, which, if it involves the instantiation of properties at all, is most plausibly understood as involving the instantiation of ordinary non-relative properties, viz., the property *being mortal*. I'm putting aside some endurantists' view, mentioned above, that anything that we might have thought is a non-relative property is actually a relative property which can be instantiated only relative to a time. But even the claims I've been discussing at length, like 'Goethe is next to Buff' and 'Goethe loves

[25] For n-ary relations where $n > 2$, the relative properties Donnelly must invoke are, like the two just mentioned in the main text, not instantiated by something relative to just one thing. Instead, they are instantiated by something relative to a thing, relative to a thing, ..., relative to a thing, with the exact number of relativizations equal to $n-1$. The existence of such *multiply relativized* properties is not wholly implausible. A candidate is that of *closeness*; San Francisco is close relative to (i.e., as compared to) Seattle relative to (i.e., from the perspective of) Los Angeles. In newer work, Donnelly (2021: 13) explicates the instantiation of multiply relativized properties in terms of *embedded standpoints*. According to Donnelly, to embed one object's standpoint within another's "is to supply external structure in terms of which other objects may be, e.g., *front* or *behind*, *closer* or *farther*, *more beloved* or *less beloved*" (2021: 15). From the standpoint of L.A., San Francisco is closer than Seattle. In this example, the standpoint of Seattle is embedded in that of L.A.

Buff', could as easily be regarded as irreducibly relational by such endurantists as by others, since people across that divide think that such claims cannot be adequately paraphrased as claims that involve the instantiation of only non-relative properties.

Relative positionalism's commitment to both relations and relative properties is problematic for the simple reason that it makes that view ontologically less parsimonious than directionalism, as the latter view is committed to only one type of entity, viz., relations. In answer to a different objection, Donnelly (2016: 98–99) considers a version of relative positionalism according to which there *are* no relations, just relative properties; relational predicates are associated immediately with a certain number of relative properties.[26] Adopting this *relationless* relative positionalism would enable the relative positionalist to do away with relations altogether, and be committed to the same number of types of entities as the directionalist. But directionalism possesses an advantage over relationless relative positionalism as well. Directionalism is a theory of non-relative relations only, and makes claims only about *their* application. It explains why a given non-relative relation R can apply in the ways it can in terms of the fact that it has a certain number of converses, all of whose application conditions are necessarily connected in a certain way. It says nothing about relative properties. It does not explain the application of non-relative relations in terms of relative properties, and it does not posit relative properties anywhere else. But it is *compatible* with their existence. Directionalism is perfectly compatible with the existence of relational claims that involve the instantiation of relative properties rather than the application of relations; it just won't say anything about *why* these relative properties can be instantiated in the ways they can. That is

[26]Donnelly (2016: §5.5) considers the objection that relative positionalism is committed to the primitive relation of relative instantiation, the relation that relative properties stand in to those objects which instantiate them. This relation is to be contrasted with the more familiar non-relative instantiation, the relation that non-relative properties and relations stand in to those objects which instantiate them, to which certain theories of relations are committed. Donnelly concedes that this is a cost of her view, and introduces relationless relative positionalism (see coming discussion in main text) in an effort to answer it. But I think she concedes too much. The matter would be particularly serious if neither of these relations could be defined in terms of the other, thus saddling her view with two *primitive* instantiation relations, in contrast to many other theories of relations which require only one primitive instantiation relation (see Donnelly 2016: 98). But non-relative instantiation can be defined in terms of relative instantiation as follows.

> **Non-Relative Instantiation.** $x_1, \ldots, x_n$ instantiate $R =_{df} R$ has between 1 and $n!$ relative properties and (i) each of those relative properties is instantiated by one of $x_1, \ldots, x_n$, relative to another, ..., relative to the remaining one, and (ii) every ordering of $x_1, \ldots, x_n$ is such that at least one of those relative properties is instantiated by the first, relative to the second, ..., relative to the nth.

Thus the relative positionalist who countenances both non-relative relations and relative properties need only be committed to *one* primitive notion of instantiation — no more than to which many a competing theory of relations is committed.

the job of a theory of relative properties — something which directionalism does not purport to be. Relationless relative positionalism, on the other hand, is committed to the claim that any irreducibly relational claim involves the instantiation of relative properties and not the application of relations.

Thus relationless relative positionalism is compatible with a narrower range of epistemic possibilities than directionalism, and is therefore methodologically inferior. It is incompatible with the existence of relations, while directionalism is not similarly incompatible with the existence of relative properties. In addition to this, however, there is reason to think that, while some irreducibly relational claims are best understood in terms of the instantiation of relative properties, others are best understood in terms of the application of relations. Jack Spencer (2016) argues that this is the case. Spencer is interested in *relativity*, the phenomenon of something's being a certain way relative to a thing or some things.[27] One of Donnelly's examples of a relative property, which I mentioned above, is that something is *north* only relative to a location (or an object in a location). Each example that Spencer has in mind is something that, at least on its face, seems like it can be appropriately construed as the instantiation of a relative property, like *being north*, as Donnelly conceives of it, or of a relative relation like *being closer than*, as in San Francisco's being closer than (i.e., as compared to) Seattle relative to (i.e., from the perspective of) Los Angeles.

Spencer argues that there are at least two ways to cash out talk about relativity, only one of which invokes genuine relative properties (or relative relations). According to the first, *relationalism*, a putative relative property or relation is actually a non-relative relation of greater arity. Instead of there being a genuine relative n-ary property or relation that is instantiated relative to a thing, there is in fact a non-relative $n+1$-ary relation. *Being north*, on this view, is not a unary property, instantiated relative to a location, but is instead a binary relation, which takes a location as its second argument. According to the second way to cash out talk of relativity, *variabilism*, a relative n-ary property or relation is understood as being genuinely n-ary, and its relativity is captured by the fact that the extension of that property or relation can change when the value of a parameter associated with that property or relation (an *index*) changes. *Being north*, on this view,

[27] This is a more general sense of 'relativity' than the sort involved in the instantiation of relative properties. As I will discuss, the latter is one way to cash out the former notion. But, as I'll also discuss, there is another way, which invokes only relations and not relative properties. Spencer's notion of relativity is more akin to the irreducible relationality associated with what I've been calling 'irreducibly relational claims'.

is a genuine unary property. But its extension function has a location parameter, and can yield different extensions when that parameter takes different values. So, for example, when the location parameter is Lima, Peru, the extension of *being north* includes Kingston, Jamaica, whereas when the location parameter is Washington, D.C., it does not.

On Spencer's account, the difference between relationalism and variabilism, and thus between relative and non-relative properties and relations, is substantive. Relative properties' and relations' extensions vary across parameters, which can take different values, while non-relative properties' and relations' extensions do not, since they don't have such parameters. This means that a relative property or relation is always instantiated relative to at least one thing whenever it is instantiated at all, while a non-relative property or relation is never so instantiated. If instantiation is itself non-relative, then the instantiation relation that n objects stand in to a non-relative n-ary property or relation R can be at most $n+1$-ary (due to the fact that it will take R as an argument in addition to up to n other arguments). But the instantiation relation that n objects stand in to an n-ary *relative* property or relation R' can be up to $n + k + 1$-ary, where k is the number of parameters relative to which R' may be instantiated. If, on the other hand, the instantiation relation is relative for relative relations and non-relative for non-relative relations, then R and R' will stand in different instantiation relations altogether.

Spencer notes that there are certain tests to which we can subject a putative relative property or relation that can tell us whether it is a genuine example of such an entity, or whether it is in fact a non-relative relation with a higher-than-expected arity. Moreover, these tests deliver examples of both sorts of entity — both genuine relative properties and relations and non-relative relations.[28] The first test Spencer discusses is the *switch-the-index* test. (For simplicity, I'll explain Spencer's tests in terms of relative properties and non-relative binary relations only.) Suppose that x instantiates a property F relative to some putative index i. Now pick a property G that is incompatible with F (i.e., x can't instantiate both F and G relative to the same putative index), and let x instantiate G relative to a different parameter j. If, intuitively, a change in x has taken place, then F and G are genuine relative properties. If, on the other hand, intuitively, no change in x has taken place, then each is a non-relative binary relation.

[28]The interested reader can look to Spencer's paper, which includes treatments of other cases of relativity which I will not discuss. These result in more examples of both relative properties and non-relative relations in addition to the ones I discuss, further substantiating my claim that we have reason to believe that both sorts of entity exist.

Consider the examples Spencer uses to illustrate how this test works. David Lewis (1986: 202–04) argues that the endurantist faces a challenge because they are apparently committed to the idea that the very same object is both bent and straight, since they are committed to the view that objects persist by being wholly present at each moment at which they exist. One way of responding to this challenge is to claim that properties like shape are relative, instantiated relative to times (as in Haslanger 1989: 123). That they are in fact relative properties and not disguised binary relations between objects and times can be shown by applying the switch-the-index test. Suppose Lewis instantiates *being bent* at t_1 and *being straight* at t_2. (These two properties are incompatible.) Has Lewis undergone a change in properties between these times? Intuitively, yes. So *being bent* and *being straight* are indeed relative properties. Contrast the case of shape with that of size. A big mouse, Remy, is big compared to other mice. But when the average-sized mice surrounding him are replaced with larger animals, say dogs, Remy is no longer big. Has Remy undergone a change through this replacement? Intuitively, no. So *being large* is not instantiated relative to anything, but instead '. . . is large' expresses a binary relation — presumably something like *being large compared to* — which holds between an object and the objects in certain groups. A difference in properties over time implies genuine change; a thing's simply being related by different relations to different things at different times doesn't. This is what the switch-the-index test is supposed to capture. And it delivers results that imply that both genuine relative properties and non-relative relations exist, assuming that these claims involving shape and size are true and irreducibly relational.

Now to Spencer's second test, the *real similarity* test. Suppose that x instantiates a property F relative to some putative index i. Now switch x out with a different object y, and switch the value of i to a new acceptable value j. If, intuitively, x is exactly similar to y with respect to F, then F is a genuine relative property. If, on the other hand, intuitively, x is *not* exactly similar to y with respect to F, then F is a non-relative binary relation. The rationale for these conclusions is, roughly, that similarity is a matter of sharing properties, not of instantiating relations to different objects (see Spencer 2016: 443). Consider what this test says about the two examples discussed above. Begin by supposing that Lewis instantiates *being bent* at t_1, and then switch Lewis with Haslanger and t_1 with t_2 to yield the result that Haslanger instantiates *being bent* at t_2. Intuitively, Haslanger is exactly similar to Lewis with respect to *being bent*, and therefore *being bent* is a genuine relative property. *Being large*, on the other hand, is a non-relative relation according to the real similarity test. Remy instantiates *being large* relative to mice. Now replace Remy with Jupiter and replace mice with planets of the solar system. Intuitively, Remy is not exactly similar to Jupiter with respect to *being large*.

According to Spencer's account of relativity, there is a real difference between relative properties and relations on the one hand and non-relative properties and relations on the other. And in light of the deliverances of Spencer's tests, I'm happy to grant that relative properties exist. But the relationless relative positionalist is committed to an analysis of *every* instance of relativity (i.e., every irreducibly relational claim) in terms of relative properties. Indeed, even the relative positionalist who countenances relations is so committed. Directionalism, on the other hand, can provide an account of relativity in exactly those cases which we have reason to believe involve relations only, and is simply silent in those cases which we have reason to believe involve relative properties only, if any such cases exist.[29]

Consider one of the examples of relativity I have been discussing all along — that connected with someone loving someone. Suppose that Buff is beloved relative to Goethe. Is there a genuine relative property *being beloved*? Or does this case of relativity involve a non-relative relation, *loving*, instead? According to the switch-the-index test, it is the latter that is the case. First, consider the fact that Buff is beloved relative to Goethe. Next, consider the fact that she is not beloved relative to Joseph II. (*Being beloved* and *not being beloved* are incompatible.) However, intuitively, Buff has not undergone a change. To briefly summarize the advantage I have argued directionalism has over relative positionalism: if the relative positionalist countenances relations as well as relative properties, her ontology is more profligate than that of the directionalist. But if she dispenses with relations, then she is forced to posit the involvement of relative properties in relational claims which we have reason to believe involve relations only, while the directionalist is not forced to do the reverse.[30]

[29] Spencer's view (2016: 441–42, incl. n. 20) is that variabilists should accept the existence of the corresponding non-relative $n + k$-ary relation along with the relative n-ary property or relation (where k is the number of parameters of the relative property or relation). But whichever way the variabilist decides to go, the relative positionalist, relationless or not, will be in trouble. Even if the variabilist decides to reject the corresponding non-relative relation in cases for which Spencer's tests prescribe a relative relation, this variabilist will still countenance only non-relative relations in cases of relativity for which Spencer's tests prescribe only non-relative relations. And this is incompatible with both varieties of relative positionalism.

[30] A believer in relative properties could certainly adopt the view that some apparently irreducibly relational claims actually involve the instantiation of relative properties and not the application of relations, but leave open whether

5. Where Directionalism Stands Now

I've shown how directionalism can rise to Donnelly's challenge and properly treat fixed arity relations with any symmetry such a relation can have. Consequently, directionalism has a distinct advantage over absolute positionalist views of relations. Granted, other views, like Fine's antipositionalism, MacBride's relational primitivism, and Donnelly's own relative positionalism can solve this problem as well. But unlike primitivism, directionalism supplies an explanation of why each relation can apply in the ways it can. And unlike both primitivism and antipositionalism, it supplies explanations of why distinct relations can apply in the same or only different ways (as the case may be). Directionalism has an advantage over relative positionalism as well, in that it is not, like relative positionalism, committed to the involvement of relative properties in every irreducibly relational claim.

Still, more remains to be said before we can conclude that directionalism wins the day. I've dealt with only one objection — the problem concerning symmetric relations that Donnelly poses for relative positionalism. But Donnelly gives another objection to directionalism; she charges the directionalist's primitive notion of order-sensitive relational application with being obscure (Donnelly 2016: 82 and 97–98 and 2021: 5–6), since the ordering of a relations' relata by it can't be understood to be "a process which unfolds over time or across space" (Donnelly 2016: 82). She adds,

> [I]t is hard to see how the idea of an order of relational application could be filled out. It is not as though relata are somehow fed into a relation as paper is fed into a printer or wood into a chipper. Relations are not the kinds of things that can "pick up" their relata in a temporal or spatial succession. Perhaps there is some other way for relations to apply to their relata in an order, but no one has tried to explain what this is supposed to be. (Donnelly 2021: 6)

I won't try to explain what order-sensitive relational application is supposed to be, but I'm not as concerned about this as Donnelly is. It's not clear to me that the directionalist is on the hook to provide a general account of this notion, given that relational predicates are themselves order-sensitive. Of course, I doubt Donnelly would be satisfied by this. But this problem strikes me as being no worse than the problem I identified for relative positionalism in the previous section. So,

some such claims involve the application of relations and not the instantiation of properties. But this is a different view than relationless relative positionalism, which is committed to the claim that *every* irreducibly relational claim involves the instantiation of relative properties and not the application of relations. The former view is actually the view I prefer, with relations understood as being directed. Spencer's tests will tell us which irreducibly relations claims should be understood to involve the application of (on my view, directed) relations, and which should be understood instead to involve the instantiation of relative properties.

other things being equal, the two views are at worst on a par. Of course, everything depends on whether other things really are equal between the two views. As I've mentioned, there are other objections to directionalism that still warrant replies, notably Fine's and Williamson's, mentioned in the introduction. There are also important concerns raised by MacBride (e.g., 2014: 5–6) and others. I must leave replies to these objections for another occasion.[31,32]

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[31] See Liebesman 2014 and Trueman 2021: ch. 10, §4 for replies to Fine's objection. See Liebesman 2013 for a reply to Williamson's.

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