Plural Slot Theory[1]

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T. Scott Dixon
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ABSTRACT: Kit Fine (2000) breaks with tradition, arguing that, *pace* Russell (e.g., 1903: 228), relations have neither directions nor converses. He considers two ways to conceive of these new "neutral" relations, positionalism and anti-positionalism, and argues that the latter should be preferred to the former. Cody Gilmore (2013) argues for a generalization of positionalism, slot theory, the view that a property or relation is n-adic if and only if there are exactly n slots in it, and (very roughly) that each slot may be occupied by at most one entity. Slot theory (and with it, positionalism) bears the full brunt of Fine's (2000) symmetric completions and conflicting adicities problems. I fully develop an alternative, *plural slot theory* (or *pocket theory*), which avoids these problems, key elements of which are first considered by Yi (1999: 168 ff.). Like the slot theorist, the pocket theorist posits entities (pockets) in properties and relations that can be occupied. But unlike the slot theorist, the pocket theorist denies that at most one entity can occupy any one of them. As a result, she must also deny that the adicity of a property or relation is equal to the number of occupiable entities in it. By abandoning these theses, however, the pocket theorist is able to avoid Fine's problems, resulting in a stronger theory about the internal structure of properties and relations. Pocket theory also avoids a serious drawback of anti-positionalism.

1. Introduction

Bertrand Russell (1903: 228) introduces the distinction between a relation and its converse, stating that every non-symmetric relation has a converse, where

> **Complete Symmetry**
> Where $n \in \{1, 2, \ldots\}$, a relation $\Lambda y_1 \ldots y_n \, \varphi(y_1, \ldots, y_n)$ is *completely symmetric* $=_{df}$ necessarily, for any pairwise distinct $x_1, \ldots, x_n$ and any x_i and x_j among them, where $i, j \in \{1, 2, \ldots\}$ and $i, j \leq n$, $\varphi(x_1, \ldots, x_i, \ldots, x_j, \ldots, x_n)$ if and only if $\varphi(x_1, \ldots, x_j, \ldots, x_i, \ldots, x_n)$,

and a relation is non-symmetric just in case it is not completely symmetric.[2,3] A relation holds of

[1] Many thanks to Cody Gilmore, Daniel Nolan, and Ted Sider for extremely helpful comments and suggestions.

[2] 'Λ', when concatenated with one or more variables, yields a term-forming operator which takes a sentence term denoting a property. In general, $\ulcorner \Lambda y_1 \ldots y_n \, \varphi \urcorner$ abbreviates $\ulcorner$*being* $y_1 \ldots y_n$ *such that* $y_1 \ldots y_n \, \varphi \urcorner$. See Fine 2012: 67–68.

[3] A relation may not be completely symmetric, but still allow for arbitrary changes among some of its arguments that do not result in non-equivalent claims. It is this more general notion of symmetry that is ultimately what is of interest in the discussion that follows, so I characterize it here.

> **Argument-Relative Symmetry**
> Where $n \in \{1, 2, \ldots\}$, a relation $\Lambda y_1 \ldots y_n \, \varphi(y_1, \ldots, y_i, \ldots, y_j, \ldots, y_n)$ is *symmetric with respect to* i *and* j, where $i, j \in \{1, 2, \ldots\}$ and $i, j \leq n =_{df}$ necessarily, for any pairwise distinct $x_1, \ldots, x_n$, $\varphi(x_1, \ldots, x_i, \ldots, x_j, \ldots, x_n)$ if and only if $\varphi(x_1, \ldots, x_j, \ldots, x_i, \ldots, x_n)$.

A relation is *partially* symmetric if it satisfies Argument-Relative Symmetry with respect to at least two of its argument places, while it is completely symmetric only if it satisfies Complete Symmetry as well.

some objects in an order if and only if its converse holds of them in some other order. The converse of *being taller than*, for example, is *being shorter than*. Russell conceived of non-symmetric relations as having a direction — what he calls a 'sense'. This view of relations became very influential among philosophers and has come to be known as 'the standard view of relations'.[4] Kit Fine (2000) breaks with tradition, arguing that, *pace* Russell, non-symmetric relations do not have directions, nor do they have converses.[5] He considers two ways to conceive of these new "neutral" relations, positionalism and anti-positionalism, and argues that the latter should be preferred to the former. There are not, for example, two relations, *being taller than* and *being shorter than*, but only a single neutral relation according to these two views. These views are also distinctive insofar as they are consistent with the following two principles, on which Fine (2000) rests his critique of the standard view.[6]

> **Identity**
> Any completion of a relation with a converse is identical to a completion of (each of) its converse(s).
>
> **Uniqueness**
> Nothing is the completion of two distinct relations.

Put informally, a completion of a property or relation is any entity that amounts to some objects fully saturating it. According to Fine (2000: 4–5), who first introduced the notion of a completion, propositions, facts, and states of affairs might all count as completions. Identity guarantees that, for example, the proposition that a is taller than b is identical to some completion of *being shorter than* (presumably, the proposition that b is shorter than a). Uniqueness guarantees that, for example, the proposition that a is taller than b has only a single *predicative* element (i.e., an element of a completion that is the relation, the complete saturation of which by some objects results in that completion).

These two principles, which Fine takes to be plausible, are at odds with the standard view. According to the standard view, a non-symmetric relation like *being taller than* has a converse, *being shorter than*, which is distinct from it. But then the proposition that a is taller than b cannot

[4] See Fine 2000 and MacBride 2007: 25.

[5] See Williamson 1985 for another argument against the idea that relations have converses. See Liebesman 2013 and 2014, respectively, for replies to Williamson's and Fine's arguments.

[6] There are challenges to the plausibility of these two principles. See, for example, MacBride 2007 and Liebesman 2014.

be the same completion as the proposition that b is shorter than a (as it seems it should be), since, if it were, that completion would have to contain *two* predicative elements. And there is no reason it should be the completion of one of them rather than the other. Positionalism and anti-positionalism do not face this problem. Identity is satisfied vacuously on these views, since they eschew converses altogether. Uniqueness is also satisfied, since there are not two relations, *being taller than* and *being shorter than*, which are distinct from and converses of one another, but only a single neutral relation, *being taller than/being shorter than*. The proposition that a is taller than b is identical to the proposition that b is shorter than a; they are each the completion of that single neutral relation by a and b.

What distinguishes positionalism and anti-positionalism from one another is that, according to the former, properties and relations have *positions* or *argument places* in them. When some things R, it is because each of them occupies a certain position in the relation *being R*. The state of affairs of a's being taller than b, for example, is the result of a occupying one position in the neutral relation *being taller than/being shorter than* and b occupying the other. Certain completions of some relations are distinguished from one another by those objects occupying different argument places in them. The state of affairs just mentioned, for example, is distinguished from the state of affairs of b's being taller than a by the fact that the former is the result of a's occupying one position in *being taller than/being shorter than* and b's occupying the other, while the latter is the result of b's occupying the other position and a's occupying the one. Anti-positionalism, as its name suggests, eschews positions. Instead, things complete relations in certain manners. For example, the manner in which a and b complete *being taller than/being shorter than* in the former state of affairs is distinct from the manner in which they do so in the latter. The anti-positionalist takes these manners to be basic and unanalyzable. She gives no account of the difference between manners, nor of why one relation can only by completed in some manners by some objects while other relations can only be completed in others by those same objects.[7]

Cody Gilmore (2013) argues for a generalization of positionalism, which he calls 'slot theory'. It takes on board the positionalist thesis that a property or relation is n-adic if and only if there are exactly n slots (positions) in it, and another thesis which is only implicit in both positionalism, as Fine presents it, and slot theory, as Gilmore presents it: very roughly, that each slot may be

[7] See Gaskin and Hill 2012: 178–82.

occupied by at most one entity.[8] Others appear to endorse slot theory, including Armstrong (1997: 121–22) and King (2007: 123).[9] But slot theory bears the full brunt of Fine's (2000) symmetric completions and conflicting adicities problems, which Fine originally raises against positionalism. I fully develop an alternative to these views, *plural slot theory* (or *pocket theory*), which avoids these problems. Key elements of pocket theory have been considered by Yi (1999: 168 ff.), Fine (Fine 2000: 17, fn. 10), McKay (2006: 13), MacBride (2007: 39–40), and Gilmore himself (2013: 229–30). Like the slot theorist, the pocket theorist posits entities (pockets) in properties and relations that can be occupied. But unlike the slot theorist, the pocket theorist denies that at most one entity can occupy any one of them. As a result, she must also deny that the adicity of a property or relation is equal to the number of occupiable entities in it. By abandoning these theses, however, the pocket theorist is able to avoid Fine's problems, resulting in a stronger theory about the internal structure of properties and relations.

In what follows, I provide reasons for thinking that pocket theory should be preferred to slot theory. My case will largely rest on the fact that the latter faces Fine's symmetric completions and conflicting adicities problems, while the former does not. But it is worth mentioning that Yi (1999: 168 ff.) provides independent reasons for holding that at least some argument places may be occupied by more than one entity. Rather than rehashing these arguments, I merely direct the reader to the relevant section of that paper.

2. Slot Theory and Its Problems

As mentioned, slot theory is the view that a property or relation is n-adic if and only if there are exactly n slots in it, and that each slot may be occupied by at most one entity. A slot is akin to a hole in an ordinary concrete object, except that, unlike a hole, it may not be spatiotemporal, and

[8] As I will show in section 3, this formulation is inadequate. But since (an adequate version of) the claim is implicit in Fine 2000, I take Fine to mean the same thing by 'position' and 'argument position' as Gilmore does by 'slot'—at most one entity may occupy any one of them. But I reserve 'position' and 'argument position', along with 'argument place', to refer to *any* occupiable entity in a property or relation, whether or not more than one entity may occupy it. Also, as far as I can tell, Yi's (1999: 167) *principle of singular instantiation* amounts to an adequate version of the claim.

[9] Fine (2000: 16) and Dorr (2004: 175) also recognize the naturalness of positionalism, though they reject the view in the end. In addition to explicit endorsements to slot theory, the literature is replete with implicit endorsements of the view, wherein authors refer to specific argument places (or positions) in properties and relations (using phrases like those of the form ⌜the nth argument place of the relation *being* R⌝) or quantify over them (using phrases like those of the form ⌜the relation *being* R has n argument places⌝). For examples of these phenomena, see Zalta 1983: 21, 23–24, 32, 174 ch. 1 en. 6 and 1988: 28, 49, 52, 57–58, 79, 163–64, 218, Williamson 1985: 251 ff., Menzel 1993: 68 ff., Swoyer 1998: 303, Newman 2002: 68 ff., McKay 2006: 8 ff., and King 2007: 20 ff. For references to other endorsements of slot theory, explicit and implicit, see Gilmore 2013: fn. 3 and 2014: fn. 43.

may not be in a concrete object. It may, for example, be in a property or relation, understood as an abstract Platonic universal. The slot(s) in a property or relation is (are) its argument place(s). So, for example, according to slot theory, each of the properties *being 5 kg in mass* and *moving pianos* has exactly one slot in it, while each of the dyadic relations *being taller than* and *being exactly as tall as* has two.

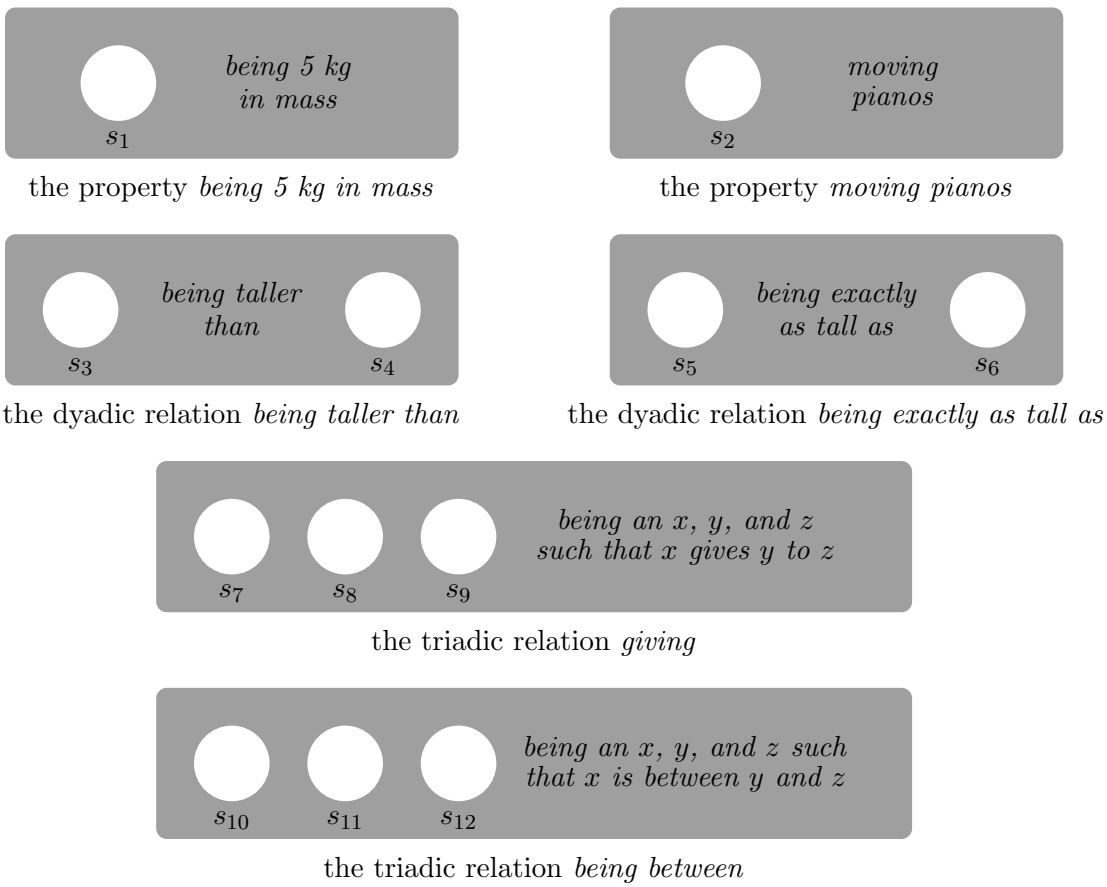


Figure 1. two properties and four relations

Similarly, each of the triadic relations *being an x, y, and z such that x gives y to z* and *being an x, y, and z such that x is between y and z* has three slots in it.

Note that slot theory is technically compatible with the standard view of relations. The slot theorist is free to say that a relation like *being taller than* has a converse that is distinct from it, though, by doing so, the slot theorist would inherit the difficulties Fine outlines concerning Identity and Uniqueness. It will be impossible to count the proposition that a is taller than b and the proposition that b is taller than a as the same completion, for reasons outlined above. If

paired with a neutral view of relations, slot theory results in a view much closer to positionalism, as characterized by Fine. There is no need to make a decision about these matters in what follows, however, since the problems for slot theory with which I will be concerned arise for both versions of the view. They are problems for slot theory *per se*, and not for particular developments of that basic view.

It will be helpful to pair slot theory from the outset (and, later, pocket theory) with the following view about the nature of properties and relations.

> **PRP-ism**
> Properties, relations, and propositions are all species of a single ontological category: PRPs.[10]

Usually, PRP-ism is understood as the stronger claim that properties and relations are certain types of relations. Either conception will do for my purposes. Moreover, it will be helpful in what follows to understand PRP-ism as also possibly including states of affairs and/or facts instead of or in addition to propositions as members of this single ontological category.[11] I assume PRP-ism in what follows for a couple of related reasons. First, as I will discuss next, pairing the PRP-ism with slot theory results in a powerful theory of properties, relations, and propositions. This provides reason to think that slot theory *should* be paired with PRP-ism. Eventually, I'll show this is no less true of pocket theory. Second, because of this, PRP-ism serves as a helpful backdrop in front of which to compare and contrast slot theory and pocket theory.

As I note (in Dixon 2017), slot theory, together with PRP-ism, results in a very powerful theory of properties, relations, and propositions. Slot theory (and, as I will show, also pocket theory) provides a natural explanation for why properties, relations, and propositions are of the same ontological category: they are all saturations of the same sort of entity to some degree. Peter van Inwagen calls them '*assertibles*'. He characterizes propositions as "things that can be said" (2004: 131–32, 2006b: 27), properties as "things that can be said *of* something" (2004: 131–32, 2006a:

[10] My terminology is inspired by Bealer 1983: 1. For other endorsements of PRP-ism, see Zalta 1983: 6, 59, 61 and 1988: 7, 57, Menzel 1993: 66–67, and van Inwagen 2004: 131, 2006a: 454 ff., and 2006b: 27. Swoyer (1998: 303 and 322 en. 4) identifies properties with monadic relations, and, while he does not himself identify propositions with 0-adic relations, his reasons for this seem to be entirely programmatic (see 1998: 297). He is always careful to mention how his system can be modified in order to accommodate this further thesis (see 1998: 303 and 322 en. 4).

[11] The view could be known as 'PRC-ism', for 'properties, relations, and *completions*'. If one does this, one will need to answer the question of how propositions, states of affairs, and facts are distinguished from one another. Due to limitations on space, I ignore this issue in what follows.

472, and 2006b: 27, italics added), and relations as "things that can be said of [some] *things*" (2006a: 472, italics added). Propositions are, for van Inwagen, fully saturated assertibles (2004: 136, 2006a: 472, and 2006b: 30–31), properties are singly unsaturated assertibles (2004: 132, 2006a: 472, and 2006b: 27), while relations are multiply unsaturated assertibles (2006a: 472).

An example will serve to illustrate the role that slot theory can play in filling out van Inwagen's characterizations. According to slot theory, *being taller than*, for example (see Figure 1 above), has more than one slot in it that may be occupied by something. It is multiply unsaturated, in van Inwagen's sense, and thus is a relation. *Being 5 kg in mass* has a single slot in it that may be occupied. It is singly unsaturated, in van Inwagen's sense, and thus is a property. *Being taller than b* is too. While it has two slots, one is occupied by b, and the other is unoccupied. It therefore also has a single slot in it that may be occupied.

the property *being taller than b*

Figure 2. another property

Finally, the proposition that a is 5 kg in mass has one slot, which is occupied by a. It is fully saturated, in van Inwagen's sense, and is thus a proposition. Similarly, the proposition that a is taller than b has two slots, one of which is occupied by a and the other by b. It is also fully saturated, in van Inwagen's sense, and is thus a proposition.

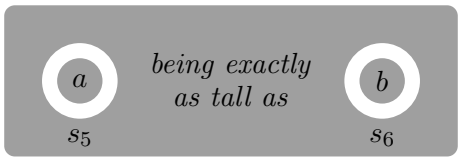
the proposition that a is 5 kg in mass

the proposition that a is taller than b

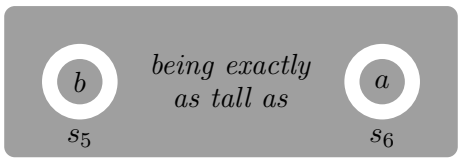
Figure 3. two propositions

Something analogous is true for every other property, relation, and proposition. Slot theory, then, explains why properties, relations, and propositions belong to the same category. They are all the sort of things that have slots in them. Some properties are partially saturated relations. Propositions are fully saturated properties and relations.

As mentioned in the introduction, slot theory bears the full brunt of Fine's symmetric completions and conflicting adicity problems. I'll now present each of these in detail. Consider first the symmetric completions problem. Fine (2000: 17–18) argues that slot theory results in too many completions of symmetric relations. (Recall that a completion of a property or relation is anything that results from fully saturating it, and that propositions, facts, and states of affairs might all count as completions.[12]) Consider the dyadic symmetric relation *being exactly as tall as*. According to slot theory, this relation has two slots in it: s_5 and s_6. Given two entities a and b, the slot theorist must say there are two ways to complete the relation, resulting in two propositions, two facts, and/or two states of affairs: one that is the result of a occupying s_5 and b occupying s_6, and another that is the result of b occupying s_5 and a occupying s_6.



the proposition that a is exactly as tall as b



the proposition that b is exactly as tall as a

Figure 4. the symmetric completions problem

Intuitively, however, there is only one proposition, one fact, and/or one state of affairs that results from the completion of *being exactly as tall as* with two objects such as a and b—one that happens to go by two names. (A third name for it which makes its uniqueness more obvious is: the proposition that a and b are exactly as tall as one another.)

[12] For the time being, I must ask the reader to rely on her intuitive understanding of the notion of saturation, as does van Inwagen in 2004, 2006a, and 2006b. I'll define the notion below in terms of occupation.

Fine (2000: 22) also argues that slot theory cannot accommodate variably polyadic relations, i.e., relations that can relate more than one number of entities. According to slot theory, the number of slots in a relation determines the adicity of that relation, and given that the number of slots must be unique, its adicity must be unique as well. Fine seems to think there is little the slot theorist can do to avoid this problem. Presumably, this is because he thinks that, in order to accommodate variably polyadic relations, the slot theorist would need to posit the existence of a relation (actually, many such relations) that has n and m slots in it, where $n \neq m$. This move, however, seems distasteful to say the least. Consider an example: the putatively variably polyadic relation *moving pianos*, and suppose that a (a bodybuilder) moves pianos on one occasion, while b and c (not bodybuilders) move pianos on another occasion. *Moving pianos*, therefore, must both have an adicity of 1 and have an adicity of 2.

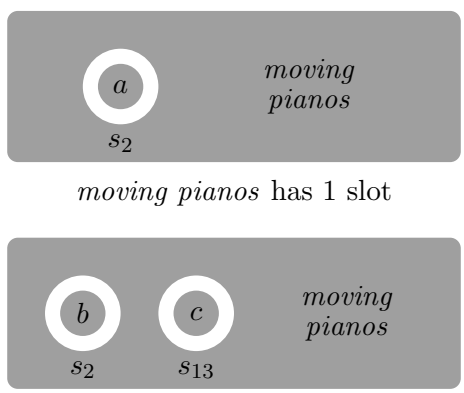


Figure 5. the conflicting adicities problem

But if the slot theorist is right, then *moving pianos* has exactly 1 slot in it *and* it has exactly 2 slots in it. Slot theory, it seems, results in a contradiction.

3. Stating Slot Theory More Precisely

So far, I have not provided a complete characterization of either slot theory or pocket theory. I have just been trying to get across enough of the former to illustrate Fine's two problems. In this section, I aim to characterize slot theory adequately. In the following two sections, I will fully develop pocket theory, and show how it solves the Fine's problems. One of the core components of slot theory is:

The Slot-Adicity Relationship

S1. A property or relation is n-adic if and only if there are exactly n slots in it.

There is, of course, much more to the view than this, such as the theses that there are such things as slots in properties and relations, and that when some things stand in a relation, this amounts to their occupying those slots. But the plural slot theorist does not deny these other theses, and so (S1) is worth singling out for the discussion that follows.

In the introduction, I also included in my characterization of slot theory the claim that each slot may be occupied by at most one entity. But slots theorists do not say exactly this, and for good reason, as it is an oversimplification. The thesis is worth properly stating, as it is also denied by the pocket theorist. The problem with the initial formulation is that more than one thing *can* occupy a single slot. Consider two entities, b and c. Then, presumably, the proposition that a is taller than b exists, but so does the proposition that a is taller than c. Now each of these completions is the result of b and c each occupying the same slot in *being taller than*.[13]

the proposition that a is taller than b

the proposition that a is taller than c

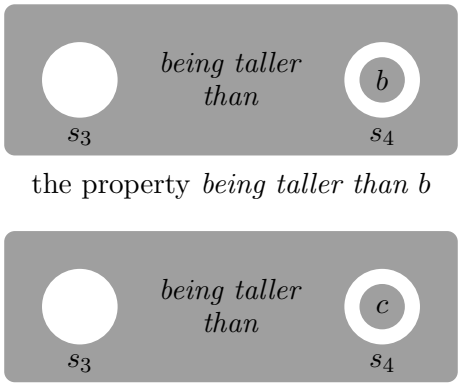
Figure 6. occupation of a single slot by more than one thing

A straightforward way to solve this problem is to say that something occupies a slot only *relative* to a completion.[14] This means that what slot theory requires is that each slot may be occupied by at most one entity *in a given completion*.

[13] As suggested in fn. 11, I intend my discussion to apply to whatever sort of completions one might want, be it propositions, states of affairs, or facts, though I take propositions as my exemplars throughout. For simplicity, I will speak as if the relationship between a property or a relation and the things that saturate it in a proposition is occupation, though this sort of relationship is usually restricted to facts and obtaining states of affairs.

[14] Gilmore can deal with this issue with the resources of his 2014 paper. There he introduces a four-place parthood relation, and replaces claims like "Mars occupies slot s_{14} in *being a planet* in the proposition that Mars is a planet" as "Mars at s_{14} is a part of *being a planet* at slot s_{15} in some property, relation, or connectant x", where a connectant is the semantic value of a sentential operator such as the conjunction operator. On his view (but in my preferred

But, actually, this isn't quite right either. Consider again the two entities b and c. Presumably, the property *being taller than b* exists, but so does the property *being taller than c*. Each of these properties is the result of b and c each occupying the same slot in *being taller than*.

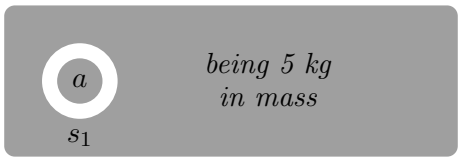


the property *being taller than b*

the property *being taller than c*

Figure 7. occupation of a single slot by more than one thing

To accommodate these cases, I must introduce the notion of a *partial completion*, a generalization of the notion of a completion. Recall that, according to the earlier characterization, a completion of a property or relation by some objects is anything that results from fully saturating that property or relation with those objects. I will formally define the notion of a completion and of saturation later. For now, I just need to provide the reader with enough of an idea of a completion to allow her to generalize to the notion of a partial completion (which I will also formally define later). On this informal characterization, both entities depicted in Figure 6 will count as completions as will the following (assuming there is a property *being 5 kg in mass*).



the proposition that a is 5 kg in mass

Figure 8. another completion

language), any completion itself occupies many slots. The proposition that Mars is a planet, for example, occupies the first slot (s_{15}) in the conjunction connectant in the proposition that Mars is a planet and Pluto is not a planet. (There will be infinitely many such slots in infinitely many such propositions that this proposition will occupy.) In Gilmore's preferred language, Mars is a part at s_{14} of *being a planet* in the proposition that Mars is a planet at s_{15} of the conjunction connectant. I do not follow Gilmore in my characterizations of slot theory and pocket theory, as I think there is a way to formulate these views that does not commit one to a specific view about parthood that is worth exploring.

A partial completion is like a completion, except that it is not the case that each slot in a property or a relation must be occupied (though at least one must be). Thus each of the entities depicted in Figures 6, 7, and 8 will count as partial completions. A properly partial completion (also to be formally defined later) is a partial completion that is not complete. So the partial completions depicted in Figure 7 are properly partial, though those depicted in Figures 6 and 8 are not.

Since a similar problem to that depicted in Figure 6 appears in Figure 7, slot-occupation should be taken to hold relative to a *partial completion*. For this reason, I will officially express occupation claims with a three-place predicate.

x occupies w in z

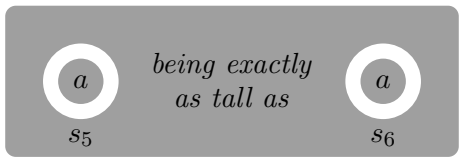
According to the slot theorist, the occupation relation is governed by the following axioms.

Occupation Axioms for Slot Theory

S2. If x occupies w in z, then, for all v, if v occupies w in z, then $v = x$.

S3. It is not the case that if (i) x occupies w in z, (ii) x' occupies w' in z, and (iii) $w \neq w'$, then $x \neq x'$.

The first axiom is a precise statement of what sets slot theory apart from pocket theory: the claim that each slot may be occupied by at most one entity *in a given partial completion*. The second ensures that a single object may occupy more than one slot in a property or relation in a given partial completion. This is to accommodate the fact that a single object can occupy more than one slot in a property or relation in a given partial completion. Consider, for example, the following two completions.



the proposition that a is exactly as tall as a

the proposition that a is taller than a

Figure 9. occupation of more than one slot by a single thing

a occupies more than one slot of the relevant relation in each of these completions.

I will define the notion of a partial completion, along with the more specific notions of a completion and a properly partial completion, in terms of slot-occupation. But it will be helpful first to provide necessary and sufficient conditions for the existence of any entity whatsoever with respect to which something may occupy a slot in a property or relation. It will of course turn out that any such entity will be a partial completion, and perhaps also a completion of a more specific type.

Existence Axiom for Partial Completions

S4. Necessarily, for any property or relation y with pairwise distinct slots $w_1, \ldots, w_n$, and any pairwise distinct $x_1, \ldots, x_m$, where $n, m \in \{1, 2, \ldots\}$, there exists a z such that, for every $i \leq m$, x_i occupies w_j in z for some $j \leq n$, where $i, j \in \{1, 2, \ldots\}$, if and only if $m \leq n$.

This axiom states every single thing among some things occupies some pocket or other of an n-adic property or relation in some entity z just in case there are at most n of those things.[15]

The notions of a completion and saturation can, for my purposes, be used nearly interchangeably. I define them and their cognates in a single pass. ('Γ' is a plural variable.)

Completions and Saturation

- z is a *partial completion* of y by Γ (Γ *partially saturate* y in z) $=_{df}$ (i) y is a property or relation, (ii) for every x among Γ, there is a slot in y that x occupies in z, and (iii) for every x, if there is a slot in y that x occupies in z, then x is among Γ.
- z is a *completion* of y by Γ (Γ *fully saturate* or *complete* y in z) $=_{df}$ (i) z is a partial completion of y by Γ (Γ partially saturate y in z) and (ii) for every slot w in y, there is an x among Γ that occupies w in z.
- z is a *properly partial completion* of y by Γ (Γ *properly partially saturate* y in z) $=_{df}$ (i) z is a partial completion of y by Γ (Γ partially saturate y in z) and (ii) z is not a completion of y by Γ (Γ do not fully saturate y in z).

[15] This axiom is best understood as stating conditions for the existence of propositions or states of affairs, rather than facts, since, for example, the proposition that $\varphi(x_1, \ldots, x_m)$ and x_1's, ..., x_m's φ-ing would presumably exist whether or not $\varphi(x_1, \ldots, x_m)$. For axioms governing the existence of facts, one would seem to need the extra condition that $\varphi(x_1, \ldots, x_m)$. Moreover, one should note that his axiom is rather permissive about what counts as a completion in another respect, since it allows for propositions and properties that conflict with the essential natures of things, e.g., the proposition that the number 2 is in my refrigerator, and the property *being the color of set theory*. It also might allow one to build contradictory partial completions like the property *being a non-self-instantiator*, if non-instantiation is a relation. Thanks to Daniel Nolan for these points. There will no doubt be further restrictions that will have to be imposed on what counts as a partial completion to prevent the existence of ones such as these. One can take (S4) as defining a boundary among entities inside of which must fall all the partial completions of even the most permissive theories of them.

The first of these definitions ensures that (S4) acts as a kind of comprehension axiom for partial completions. A full statement of slot theory, then, as I am understanding it, consists of the core thesis (S1) that a property or relation is n-adic if and only if there are exactly n slots in it, conjoined with the two occupation axioms (S2) and (S3), the existence axiom for partial completions (S4), and the definitions of completions and saturation.

4. An Initial Formulation of Pocket Theory

In this section, I provide an initial formulation of pocket theory. This formulation is the most natural first step to take away from slot theory in light of the symmetric completions and conflicting adicity problems. While this formulation does effectively solve these problems, I show that it faces a problem of its own. I discuss this problem in the following section, and provide a revised formulation of pocket theory which avoids this new problem as well as Fine's two.

Like the slot theorist, the pocket theorist posits occupiable entities (*plural slots* or *pockets*) in each property or relation.

Postulation of Pockets
P1. Every property or relation has at least one plural slot (pocket) in it.

Perhaps the most fundamental difference between slot theory and pocket theory is that, according to the latter, an occupiable entity in a property or relation may be occupied by more than one thing in a given partial completion. As a result, the pocket theorist drops the occupation axiom (S2). But she retains (S3).

Singular Occupation Axiom
P2. It is not the case that if (i) x occupies w in z, (ii) x' occupies w' in z, and (iii) $w \neq w'$, then $x \neq x'$.

Recall that this axiom states that a single object may occupy more than one slot in a property or relation in a given partial completion.

It is this allowance on the pocket theorist's part that more than one object may occupy a single occupiable entity in a property or relation in a given partial completion that gives these entities their name: *plural* slots or *pockets*. For each, there will be a minimum and maximum number of entities which may occupy it — its *minimum* and *maximum occupancy limits*.

Occupancy Limits

P3. Each pocket w_i in every property or relation has

a. a minimum occupancy limit $m_i \in \{1, 2, \ldots\}$, which is the minimum number of entities which may occupy that pocket in a partial completion,
and

b. a maximum occupancy limit $k_i \in \{m_i, m_{i+1}, \ldots\}$ (which may also take the value ∞, to be explained later), which is the maximum number of entities which may occupy the pocket in a partial completion.

As a result, according to pocket theory, a complete specification of the internal structure of a property or relation requires several parameters. (Slot theory, recall, requires only one: the adicity n of the property or relation, which is the number of slots in it.) First, pocket theory requires a parameter which states the number n of pockets in the property or relation. Second, it requires $2n$ more parameters, which specify the minimum and maximum occupancy limits of those pockets. I'll refer to these parameters of a property or relation collectively as its '*p-adicity*'. Officially, I will specify the p-adicity of a property or relation with a string of numerals denoting positive integers of length $2n$, where the $2i-1$th and $2i$th elements, m_i and k_i, specify the minimum and maximum number of entities that may occupy the property or relation's ith pocket, respectively. Officially, then, the p-adicity of any property or relation will be specified with an expression of the form

$$m_1, k_1, m_2, k_2, \ldots, m_n, k_n.$$

The p-adicity of a PRP with no unoccupied pockets, such as a proposition, fact, or state of affairs, is indicated with '0'.[16]

For properties and relations, a more reader-friendly notation results from understanding the $2n$-length string as an n-length string of 2-length substrings, delimited by occurrences of '(' and ')'. Understood in this way, the number of members in the n-length string specifies the number of pockets in the property or relation, while the first and second members of the ith substring, m_i and k_i, specify the minimum and maximum occupancy limits of the property or relation's ith pocket, respectively. Reader-friendliness may be improved further by stating the length of the string (i.e.,

[16] Oliver and Smiley (2004: 615–18) make a similar move regarding predicates rather than relations. They distinguish between the *places* and *positions* of a predicate. The number of pockets n in a given property or relation is akin to the number of places in a multigrade predicate, while the minimum and maximum occupancy limits of a pocket in a property or relation are akin to constraints on the number of positions at each place of a multigrade predicate. But there are differences, e.g., a predicate may not be symmetric with respect to two arguments in positions of a single place (618–19), while, according to pocket theory, they are when they occupy a single pocket. See also Oliver and Smiley 2013: 162–64.

the number of pockets in the property or relation) first, and separating it from the string itself by an occurrence of ':'.

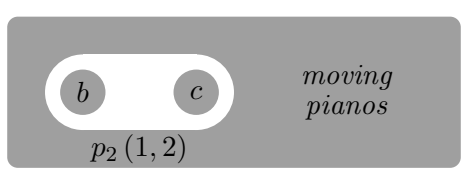
$$n : (m_1, k_1), (m_2, k_2), \ldots, (m_n, k_n).$$

While every pocket will have a minimum occupancy limit of at least 1, the maximum may be infinite. I indicate this with the symbol '∞'.[17] The easiest way to begin to get a feel for p-adicity is by way of several examples.

PRP	**p-adicity**	**alternate p-adicity**
the proposition that grass is green	0	
being a single individual	$1 : (1, 1)$	
being a single intrinsic duplicate of	$1 : (1, 2)$	
being exactly as tall as	$1 : (1, 2)$	$1 : (1, \infty)$
being triplets	$1 : (1, 3)$	
moving pianos	$1 : (1, \infty)$	
cooperating	$1 : (2, \infty)$	
being taller than	$2 : (1, 1), (1, 1)$	$2 : (1, \infty), (1, \infty)$
being between	$2 : (1, 1), (1, 2)$	
giving	$3 : (1, 1), (1, 1), (1, 1)$	

Note that it is not always obvious what the p-adicity of a property or relation is. In such cases, I indicate the two most plausible specifications of it.

According to pocket theory, each pocket in a property or relation is symmetric, in the sense that when more than one entity occupies it, they do so in no particular order. So, for example, the one and only pocket in *moving pianos* is symmetric in this sense, since, if b and c move pianos (together), they occupy the single pocket in it in no particular order.



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[17] I ignore the question of whether the maximum occupancy limit of a pocket can take specific infinite cardinal numbers as its value, though I suspect it can.

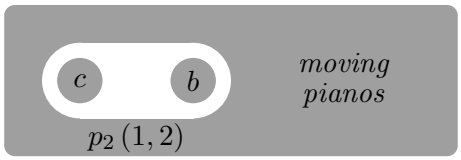


Figure 10. the proposition that a and b move pianos (together)

Similarly, the second pocket in *being between* is symmetric in this sense, since, if a is between b and c, b and c occupy the second pocket in that relation in no particular order. Contrast this relation with *giving*, which has three pockets in it.

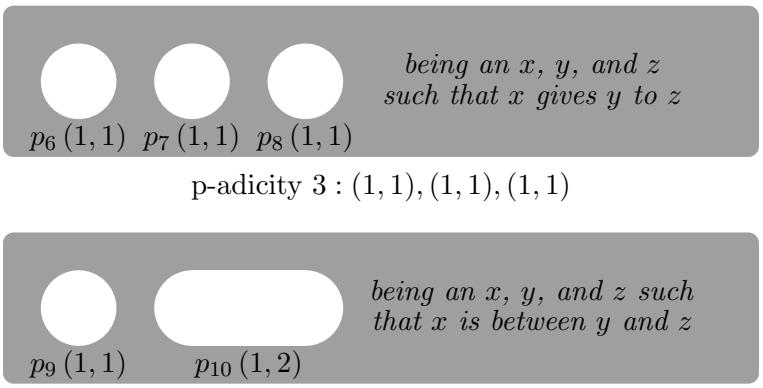


Figure 11. two triadic relations according to pocket theory

Indeed, it is this feature of pocket theory that enables the pocket theorist to avoid the symmetric completions problem. Recall that the proposition that a is exactly as tall as b is presumably identical to the proposition that b is exactly as tall as a, even when a and b are distinct. As we have seen, the slot theorist encounters a problem here, since she will say that the relation *being exactly as tall as*, just like *being taller than*, is dyadic, and so it has two slots, s_5 and s_6. The slot theorist will, therefore, be forced to say that there are two ways for a and b to saturate this relation: (1) for a to occupy s_5 and b to occupy s_6, and (2) for b to occupy s_5 and a to occupy s_6, resulting in two propositions where there should only be one. The pocket theorist, however, will avoid this problem by saying that the p-adicity of *being exactly as tall as* is $1:(1,2)$, and so it has a single pocket, p_5, which may take either one or two entities in no particular order.

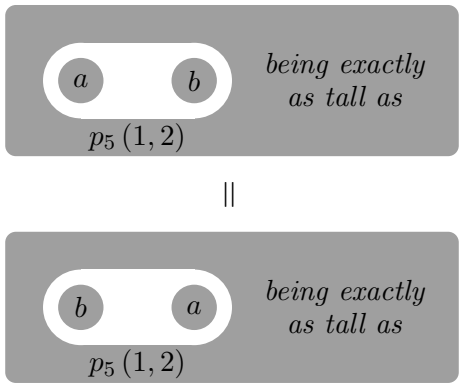


Figure 12. the single completion of a dyadic symmetric relation

As a result, pocket theory says there is only *one* way for two entities such as a and b to saturate *being exactly as tall as*: for a and b to occupy p_5 in no particular order, resulting in only a single proposition, fact, and/or state of affairs.

Yet, while accommodating symmetric completions, the pocket theorist does not lose the ability to properly treat *non*-symmetric completions. Like the slot theorist, the pocket theorist is able to accommodate the fact that order *does* matter in some cases. So, for example, the proposition that a is taller than b is distinct from the proposition that b is taller than a when a and b are distinct. In slot theory, this is borne out by the fact that the relation *being taller than* is dyadic, and so it has two empty slots, s_3 and s_4. According to slot theory, then, there are two ways for a and b to saturate it: (1) for a to occupy s_3 and b to occupy s_4, and (2) for b to occupy s_3 and a to occupy s_4, resulting in two propositions, facts, and/or states of affairs. The pocket theorist will accommodate this in a similar way, by saying that the p-adicity of *being taller than* is $2:(1,1),(1,1)$, and so it has two empty pockets, p_3 and p_4, each of which must take exactly one entity.

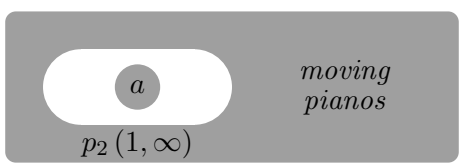
the proposition that a is taller than b

the proposition that b is taller than a

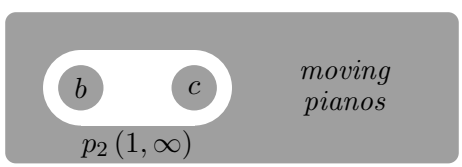
Figure 13. the two completions of a dyadic non-symmetric relation

As a result, pocket theory also says there are two ways for two entities such as a and b to saturate the relation: (1) for a to occupy p_3 and b to occupy p_4, and (2) for b to occupy p_3 and a to occupy p_4, also resulting in two propositions, facts, and/or states of affairs.

Remember that the slot theorist was also unable to provide an account of variably polyadic relations. Since the number of slots in a property or relation must, presumably, be unique, its adicity must be unique as well. The pocket theorist is also able to solve this problem due to the fact that more than one number of entities may occupy a pocket in a property or relation, as long as that number falls between the minimum and maximum occupancy limits for that pocket. Thus the pocket theorist could say that a variably polyadic relation, such as *moving pianos*, has a p-adicity of $1 : (1, \infty)$, and therefore has only a single pocket, p_2.



moving pianos has 1 pocket



moving pianos has 1 pocket

Figure 14. variable polyadicity according to pocket theory

So a may occupy p_2 to result in a completion of *moving pianos*, and so may b and c.

It should be clear from the pocket theorist's solution to the symmetric completions problem that one factor which helps us determine the p-adicity of a property or relation is whether it is symmetric. It is because necessarily, for all distinct x and y, if x is exactly as tall as y then y is exactly as tall as x, that we know that *being exactly as tall as* has a single pocket that may be

occupied by two objects. And it is because it is not the case that, necessarily, for all distinct x and y, if x is taller than y then y is taller than x, that we know that *being taller than* has two pockets in it, each of which may be occupied by at most one object. This idea can be captured more generally as follows.

The Symmetry-Pocket Relationship

P4. Every instance of the following schema is true. For any property or relation $\Lambda y_1 \ldots y_n \, \varphi(y_1, \ldots, y_n)$,

a. if necessarily, for all pairwise distinct $x_1, \ldots, x_i, \ldots, x_j, \ldots, x_n$, if $\varphi(x_1, \ldots, x_i, \ldots, x_j, \ldots, x_n)$ then $\varphi(x_1, \ldots, x_j, \ldots, x_i, \ldots, x_n)$, then there is at least one pocket w in $\Lambda y_1 \ldots y_n \, \varphi(y_1, \ldots, y_n)$ such that, for all $x_1, \ldots, x_i, \ldots, x_j, \ldots, x_n$, if $\varphi(x_1, \ldots, x_i, \ldots, x_j, \ldots, x_n)$, then there is a completion z of $\Lambda y_1 \ldots y_n \, \varphi(y_1, \ldots, y_n)$ such that x_i occupies w in z and x_j occupies w in z,
and

b. if it is not the case that necessarily, for all pairwise distinct $x_1, \ldots, x_i, \ldots, x_j, \ldots, x_n$, if $\varphi(x_1, \ldots, x_i, \ldots, x_j, \ldots, x_n)$ then $\varphi(x_1, \ldots, x_j, \ldots, x_i, \ldots, x_n)$, then there are at least two pockets w and w' in $\Lambda y_1 \ldots y_n \, \varphi(y_1, \ldots, y_n)$ such that, for all $x_1, \ldots, x_i, \ldots, x_j, \ldots, x_n$, if $\varphi(x_1, \ldots, x_i, \ldots, x_j, \ldots, x_n)$, then there is a completion z of $\Lambda y_1 \ldots y_n \, \varphi(y_1, \ldots, y_n)$ such that x_i occupies w in z and x_j occupies w' in z.[18]

Roughly, (P4) enables the pocket theorist to ground the symmetry of a relation in the fact that more than one entity can occupy certain of its pockets, and to ground the non-symmetry of a relation in the fact that at least two of its pockets can be occupied at most once.

We also saw that apparently variably polyadic relations can be accommodated as long as they have the appropriate p-adicities, i.e., as long as the minimum and maximum occupancy limits of at least one of their pockets differ in value. Admittedly, the pocket theorist has no straightforward way of systematically determining the p-adicity of a variably polyadic relation, but must instead determine them on a case-by-case basis. But, at least, they are not forced into as uncomfortable a position as the slot theorist with respect to variably polyadic relations.

The plural slot theorist adopts an axiom governing the existence of partial completions that is analogous to the one adopted by the slot theorist.

[18] Clauses (a) and (b) are required, rather than a single biconditional, if one wishes to allow for properties, relations, propositions, or states of affairs that are formed from uninstantiable properties or relations. (P4) does not, however say anything about the number of pockets uninstantiable properties and relations have. The pocket theorist will have little choice, I think, than to simply stipulate that they have the number of pockets they seem to have, based on similarities they have to instantiable properties and relations. Of course, the slot theorist will have to do something similar, stipulating the adicities of such properties and relations.

Existence Axiom for Partial Completions

P5. Necessarily, for any property or relation y with pairwise distinct pockets $w_1, \ldots, w_n$ with maximum occupancy limits $k_1, \ldots, k_n$, respectively, and any pairwise distinct $x_1, \ldots, x_m$, where $n, m, k_i \in \{1, 2, \ldots\}$ for each $i \leq n$ where $i \in \{1, 2, \ldots, n\}$, there exists a z such that, for every $i \leq m$ where $i \in \{1, 2, \ldots, n\}$, x_i occupies w_j in z for some $j \leq n$, where $i, j \in \{1, 2, \ldots\}$, if and only if $m \leq k_1 + \ldots + k_n$.

This axiom states that every single thing among some things occupies some pocket or other of an n-adic property or relation in some entity (a partial completion) z just in case the number of those things does not exceed the sum of the maximum occupancy limits of the pockets in that property or relation.

The pocket theorist may also define the notions of a completion and of saturation in a way analogous to that of the slot theorist.

Completions and Saturation

- z is a *partial completion* of y by Γ (Γ *partially saturate* y in z) $=_{df}$ (i) y is a property or relation, (ii) for every x among Γ, there is a pocket in y that x occupies in z, and (iii) for every x, if there is a slot in y that x occupies in z, then x is among Γ.
- z is a *completion* of y by Γ (Γ *fully saturate* or *complete* y in z) $=_{df}$ (i) z is a partial completion of y by Γ and (ii) for every pocket w_i in y, there are pairwise distinct $x_1, \ldots, x_n$ among Γ such that $m_i \leq n \leq k_i$, where m_i and k_i are the minimum and maximum occupancy limits of w_i, respectively, and each of $x_1, \ldots, x_n$ occupies w_i in z.

As is the case with slot theory, the first of these definitions ensures that (P5) acts as a comprehension axiom for partial completions. Also, since properly partial completions have been defined in terms of partial completions and completions, it can be carried forward without any changes. So I do not bother repeating it here. A full statement of pocket theory, then, as currently formulated, comprises (P1)–(P5), along with the definitions of the notions of a completion and saturation.

5. A Problem and a Revised Formulation

As I warned at the beginning of the last section, pocket theory, as formulated so far, is inadequate. Consider any relation that is symmetric with respect to at least two of its arguments. A simple such relation is *being exactly as tall as*. The most plausible p-adicities of this relation are (i) $1 : (2, 2)$ and (ii) $1 : (1, 2)$.[19] (i) is unappealing, since it would not allow the pocket theorist to fully saturate the

[19] For simplicity, I ignore the possibility that its pocket has no finite maximum occupancy limit.

relation with only a single object, making it impossible to build the proposition that, for example, a is exactly as tall as a.

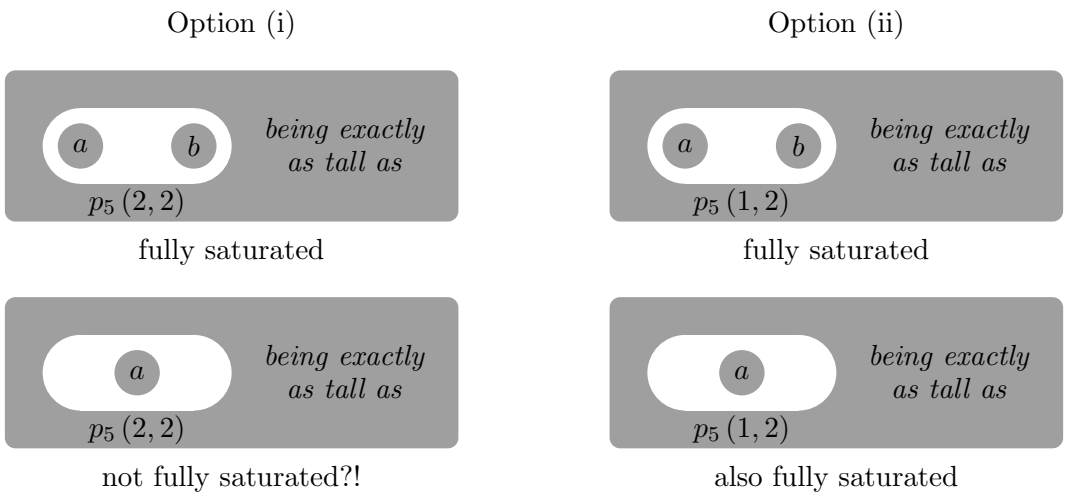
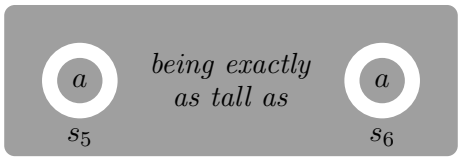


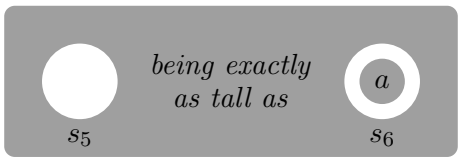
Figure 15. the p-adicity of *being exactly as tall as*

But (ii) is unappealing as well, since the pocket theorist will be unable to differentiate among, for example, the proposition that a is exactly as tall as a and the properties *being* (*an x such that x is*) *exactly as tall as a* and *being an x such that a is exactly as tall as x*. (Or, if one assumes that those italicized expressions pick out a single property, she will be unable to distinguish the aforementioned proposition from that single property.) Each is the result of a occupying the single pocket in *being exactly as tall as* (depicted on the bottom right of Figure 15). I'll call this problem 'the differentiation problem'.

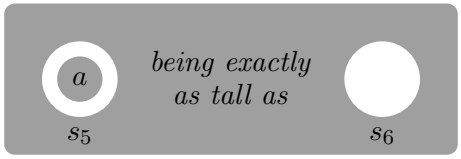
The slot theorist can differentiate among these entities quite easily. According to her, *being exactly as tall as* has adicity 2, and so has exactly two slots in it, s_5 and s_6. The proposition that a is exactly as tall as a is the result of a occupying both s_5 and s_6. On the other hand, *being exactly as tall as a* is the result of a occupying one of s_5 an s_6 and nothing occupying the other, while *being an x such that a is exactly as tall as x* is the result of a occupying the other and nothing occupying the one.



the proposition that a is exactly as tall as b



the property *being exactly as tall as a*



the property *being an x such that a is exactly as tall as x*

Figure 16. slot theory's solution to the differentiation problem

Now one might be concerned that, while the slot theorist is able to avoid the differentiation problem, she faces one of her own — one that is reminiscent of the symmetric completions problem. Anyone who is sympathetic to the idea that 'the proposition that a is exactly as tall as b' and 'the proposition that b is exactly as tall as a' refer to the same proposition will likely also be sympathetic to the idea that '*being exactly as tall as a*' and '*being an x such that a is exactly as tall as x*' refer to the same property. So slot theory results in two properties where it should only result in one. Moreover, the slot theorist will face the further difficulty that each of these phrases will indeterminately refer, or be ambiguous between, the two properties supplied by slot theory.[20] I'll call this problem 'the symmetric partial completions problem', and add it to the case against slot theory. But my immediate concern is how the pocket theorist can deal with the differentiation problem.

One way the pocket theorist might try to solve the problem is to outfit some pockets with subpockets, and allow a single thing to occupy more than one subpocket in a single pocket. While such an approach would allow her to distinguish between the proposition that a is exactly as tall as a, on the one hand, and the properties *being exactly as tall as a* and *being an x such that a is exactly as tall as x* on the other, it should be obvious that it results in her facing both versions

[20] Thanks to Ted Sider for bringing to my attention this additional complication involving indeterminate reference or ambiguity.

of the symmetric completions problem. It would also get the pocket theorist into trouble with variably polyadic relations, at least if the pocket theorist says that every subpocket in a property or relation must be occupied in order for that property or relation to be fully saturated. This would fix the number of subpockets in that property or relation in each completion, and the number of subpockets that would be fixed in different completions might have to differ, resulting in contradiction. Moreoever, these problems arise with more force against the pocket theorist than the slot theorist, since those very same problems, as they afflicted slot theory, have been used to motivate the pocket theorist's project over slot theory.

So can the pocket theorist solve the differentiation problem while not running afoul of the symmetric (partial) completions and conflicting adicity problems? I believe it can. The solution is to reimagine the occupation relation. Rather than being understood as a triadic relation that holds among an entity, a pocket, and a partial completion, it should be understood as a 4-adic relation that holds among an entity, a pocket, a natural number, and a partial completion.[21]

x occupies y zero times (x does not occupy y) in z
x occupies y once in z
x occupies y twice in z
x occupies y three times in z
x occupies y four times in z
$\vdots$

The singular occupation axiom for plural slot theory will need a minor adjustment to incorporate this new 4-adic occupation relation.

Revised Singular Occupation Axiom
P2′. It is not the case that if (i) x occupies w *at least once* in a partial completion z, (ii) x' occupies a w' *at least once* in z, and (iii) $w \neq w'$, then $x \neq x'$.

The minimum and maximum occupancy limits associated with each pocket must also be reimagined. The minimum occupancy limit of a pocket should not be understood as the minimum number of *entities* that may occupy it in a given partial completion, but rather as the minimum number of *times* one or more entities may do so. Similarly, the maximum occupancy limit of a pocket

[21] This move is akin to Bennett's (2013) move with respect to the parthood relation, allowing a thing to have something "multiple times over".

should not be understood as the maximum number of *entities* that may occupy it in a given partial completion, but rather as the maximum number of *times* one or more entities may do so.

Before this can be done, however, the notion of plural occupation must be introduced. After all, I have not yet indicated what it means to say that n things occupy a pocket t times, where $n > 1$. (I take the case where $n = 1$ as primitive, governed by the axiom (P2′).) The best way to introduce the idea is with an example. Suppose that p is a pocket whose minimum occupancy limit is 3, as is its maximum. The basic notion I want yields the following possibilities. (i) p can be occupied by a single object, as long as it occupies p three times. (ii) p can be occupied by two objects, as long as one of them occupies p twice. And (iii) p can be occupied by three objects, as long as each of them occupies p once.

	Singular Specification	**Plural Specification**
i.	a occupies p three times	a occupies p three times
ii.	a occupies p once and b occupies p twice	a and b occupy p three times
	a occupies p twice and b occupies p once	a and b occupy p three times
iii.	each of a, b, and c occupies p once	a, b, and c occupy p three times

In each of these three cases, p is occupied (by an object or objects) three times, and so each time its minimum occupancy limit is reached, and its maximum is not breached. Plural occupation can be defined in terms of singular occupation as follows.

Plural Occupation

Where $x_1, \ldots, x_n$ are pairwise distinct, $x_1, \ldots, x_n$ (*plurally*) *occupy* w t times in $z =_{df}$ (i) for each $i \leq n$, x_i (singularly) occupies w t_i times in z, where $n, i, t_i \in \{1, 2, \ldots\}$, (ii) $t = t_1 + t_2 + \ldots + t_n$, and (iii) for any x, if x (singularly) occupies w at least once in z, then x is among $x_1, \ldots, x_n$.

With this in mind, (P3) may be restated as follows.

Revised Occupancy Limits

P3′. Each pocket w_i in every property or relation has

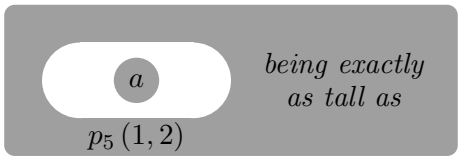
a. a minimum occupancy limit m_i, where $m_i \in \{1, 2, \ldots\}$, which is the minimum number of *times* one or more entities may occupy the pocket in a partial completion,
and

b. a maximum occupancy limit $k_i \in \{m_i, m_{i+1}, \ldots\}$ (which may also take the value ∞), which is the maximum number of *times* one or more entities may occupy the pocket in a partial completion.

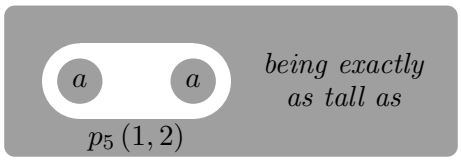
As before, a pocket with no finite maximum occupancy limit has a maximum occupancy limit of ∞.

In effect, the occupation (complete or partial) of a plural pocket by an entity or some entities can be modeled as a *multiset*, which, unlike a set, may contain some of its members "multiple times over". Multisets are not individuated by their members, as are sets, but rather by the number of *instances* of each of their members. I will adopt this language in what follows, and may speak of one or more instances of an object in a (some) pocket(s) from time to time. Moreover, as with sets, order doesn't matter in multisets, reflecting the fact that when some things occupy a pocket, they do so in no particular order.

How does all this help with the differentiation problem? Well, the pocket theorist may say that the adicity of *being exactly as tall as* is $1 : (1, 2)$, and yet have a way to distinguish between the proposition that a is exactly as tall as a and the properties *being exactly as tall as* a and *being an* x *such that* a *is exactly as tall as* x. Each of the properties is the result of a occupying the single pocket in the relation *once*, while the proposition is the result of a occupying it *twice*, just as two entities, such as a and b, might occupy it twice.



being exactly as tall as a/*being an* x *such that* a *is exactly as tall as* x



the proposition that a is exactly as tall as a

Figure 17. revised pocket theory on the differentiation problem

Note also that pocket theory, so conceived, does not face the partial symmetric completions problem. Since pockets are not order sensitive, there is no difference between the way a occupies the relation once in the case of *being taller than* a and the way it does so in the case of *being an* x *such that* a *is taller than* a. They are, on this revised formulation of pocket theory, the same property. Moreoever, for reasons already outlined in the last section, this revised formulation of pocket theory avoids the

original symmetric completions problem and the conflicting adicity problem for the same reasons as did the initial formulation.

To properly formulate pocket theory, those theses and definitions which were used to characterize pocket theory in the last section that are formulated in terms of 3-adic occupation must be reformulated in terms of 4-adic occupation. (P1) remains the same, and new versions of (P2) and (P3) have already been provided. A new version of (P4) follows. Changes are italicized.

The Revised Symmetry-Pocket Relationship

P4′. For any property or relation $\Lambda y_1 \ldots y_n \, \varphi\,(y_1, \ldots, y_n)$,

a. if necessarily, for all pairwise distinct $x_1, \ldots, x_i, \ldots, x_j, \ldots, x_n$, if $\varphi\,(x_1, \ldots, x_i, \ldots, x_j, \ldots, x_n)$ then $\varphi\,(x_1, \ldots, x_j, \ldots, x_i, \ldots, x_n)$, then there is a single pocket w in $\Lambda y_1 \ldots y_n \, \varphi\,(y_1, \ldots, y_n)$ and a completion z of $\Lambda y_1 \ldots y_n \, \varphi\,(y_1, \ldots, y_n)$ such that x_i occupies w *at least once* in z and x_j occupies w *at least once* in z, and

b. if it is not the case that necessarily, for all pairwise distinct $x_1, \ldots, x_i, \ldots, x_j, \ldots, x_n$, if $\varphi\,(x_1, \ldots, x_i, \ldots, x_j, \ldots, x_n)$ then $\varphi\,(x_1, \ldots, x_j, \ldots, x_i, \ldots, x_n)$, then there are two pockets w and w' in $\Lambda y_1 \ldots y_n \, \varphi\,(y_1, \ldots, y_n)$ and a completion z of $\Lambda y_1 \ldots y_n \, \varphi\,(y_1, \ldots, y_n)$ such that x_i occupies at least one of those pockets *at least once* in z and x_j occupies the other *at least once* in z.

As does (P4), (P4′) enables the pocket theorist to ground the symmetry of a relation in the fact that an entity or entities can occupy certain of its pockets more than once, and to ground the non-symmetry of a relation in the fact that at least two of its pockets can be occupied at most once.[22]

The existence axiom for completions and the definitions of completions and saturation must also be adjusted.

The Revised Existence Axiom for Partial Completions

P5′. Necessarily, for any property or relation y with pairwise distinct pockets $w_1, \ldots, w_n$ with maximum occupancy limits $k_1, \ldots, k_n$, respectively, and any pairwise distinct $x_1, \ldots, x_m$, where $n, m, k_i \in \{1, 2, \ldots, n\}$ for each $i \leq n$ where $i \in \{1, 2, \ldots\}$, there exists a z such that, for every $i \leq m$ where $i \in \{1, 2, \ldots\}$, x_i occupies* $w_1, \ldots, w_n$ t_i times in z, where $t_i \in \{1, 2, \ldots\}$, if and only if $t_1 + \ldots + t_m \leq k_1 + \ldots + k_n$,

[22] There are putative relations with certain symmetries, such as being arranged clockwise (see Fine 2000: 17–18, fn. 10 and 2007: 58–59, MacBride 2007: 40–44, Leo 2008: 356, Orilia 2011: 9, fn. 11, and Donnelly 2016: 88–90) and playing tug-of-war with (see MacBride 2007: 42–44), that admittedly cannot be handled in a straightforward manner by even this revised version of plural slot theory. Some of these sorts of relations can be easily accommodated by Maureen Donnelly?s new view, relative positionalism. I do not have the space to get into this issue here, but I consider the adequacy of Donnelly?s view with respect to this issue, and weigh the relative merits of plural slot theory and relative positionalism, in a work-in-progress.

where

Occupation*

x occupies* $w_1, \ldots, w_n$ t times in z, where $n \in \{1, 2, \ldots\} =_{df}$ (i) for each w_i among $w_1, \ldots, w_n$, where $i \in \{1, 2, \ldots\}$, x (singularly) occupies w_i t_i times in z, where $t_i \in \{0, 1, 2, \ldots\}$, and (ii) $t = t_1 + \ldots + t_n$.

Completions and Saturation

- z is a *partial completion* of y by Γ (Γ *partially saturate* y in z) $=_{df}$ (i) y is a property or relation, (ii) for every x among Γ, there is a pocket in y that x occupies *at least once* in z, and (iii) for every x if there is a slot in y that x occupies *at least once* in z, then x is among Γ.
- z is a *completion* of y by Γ (Γ *fully saturate* or *complete* y in z) $=_{df}$ (i) z is a partial completion of y by Γ and (iii) for every pocket w_i in y, there are pairwise distinct $x_1, \ldots, x_n$ among Γ such that $x_1, \ldots, x_n$ occupy w_i at least t times in z, where $m_i \leq n \leq t \leq k_i$.
- z is a *properly partial completion* of y by Γ (Γ *properly partially saturate* y in z) $=_{df}$ (i) z is a partial completion of y by Γ (Γ partially saturate y in z) and (ii) z is not a completion of y by Γ (Γ do not completely saturate y in z).

As before, the first of these definitions ensures that (P5') acts as a comprehension axiom for partial completions. Also, the definition of properly partial completions can once again be carried forward without any changes. A full statement of pocket theory, then, consists of (P1), (P2′)–(P5′), and the new definitions of plural occupation, occupation* completions, and saturation.

6. Concluding Remarks

I conclude with two points. First, it is worth mentioning that, like slot theory, pocket theory is technically compatible with the standard view of relations. The pocket theorist is free to say that a relation like *being taller than* has a converse that is distinct from it. But, as with slot theory, the pocket theorist who endorses this further view will run into difficulties with Identity and Uniqueness. It will be impossible to count the proposition that a is taller than b and the proposition that b is taller than a as the same completion. Thus pocket theory, like slot theory, is most naturally paired with a neutral view of relations.

Second, I must explain how pocket theory can fill in the details of PRP-ism. For the slot theorist, a proposition is a PRP that does not have any unoccupied slots, a property has exactly one, and a relation has more than one. This is perfectly consonant with van Inwagen's characterizations or properties, relations, and propositions. Things aren't quite as simple in the case of pocket theory,

though it provides an interesting answer. A proponent of it could take a proposition to be a completion of a PRP by some objects, a property to be a partial completion that can be occupied exactly one (more) time, and a relation to have either exactly one pocket that can be occupied more than one more time, or more than one pocket, each of which can be occupied at least one more time.

Some might be worried that this can't be right, since there are entities that meet more than one of these conditions. Consider the partial completion of *moving pianos* by a and b. This is a completion of that relation, since the number of times it is occupied by a and b is greater than or equal to the minimum occupancy limit (1) of its single pocket, p_2. So it would count as a proposition. But p_2 can be occupied more than two times, including three times or four (since, for example, a, b, and c can move pianos together, as can a, b, c, and d). Thus it would count as a property and a relation as well.

While some might think this is an undesirable result, I think it is palatable, and, further, intriguing, since it serves to reinforce the idea that the notion of a PRP is the fundamental one. Properties, relations, and propositions do not constitute a partition of PRPs according to pocket theory. There is substantial overlap among these categories. Really, it is not that important for the pocket theorist to give *any* independent characterizations of properties, relations, and propositions. This vocabulary can be replaced entirely with the term 'PRP'. Any of my uses of the phrase 'property or relation' above can be replaced with those of a phrase which picks out PRPs that meet the condition that at least one of their pockets may be occupied by at least one more object in some partial completion. And the notion of a PRP can be easily characterized in a way that is independent of the notions of property, relation, and proposition, as anything which has at least one pocket in it (whether or not it is occupied by something in a completion).[23]

Moreover, the pocket theorist can retain the common platitude that a fully saturated entity is

[23] One shouldn't be concerned with the fact that part of my motivation for pocket theory relied on the claim that the proposition that a is exactly as tall as a and the property *being exactly as tall as a* (or *being an x such that x is exactly as tall as a*) are distinct. I did not intend to rely on the claim that no propositions are properties. And, while the pocket theorist is committed to there being overlap among properties, relations, and propositions, she is by no means committed to the view that every member of each type of completion is identical to a member of one of the other types. I take the particular distinctness claim on which I have relied as intuitive. If there is a general principle that lies behind it, it is one that speaks to the distinctness of entities that have different numbers of instances of things as constituents. And I do not think that taking facts, general or particular, about the number of instances or occurrences of a thing as data presupposes the framework of plural slot theory. These data are out there for the taking. Wetzel (2009) and Bennett (2013), for example, appear to take such facts as data as well.

an entity that can be true or false, while an entity that is not fully saturated cannot, as long as full saturation is understood in the sense defined above. Granted, there will be another sense in which a fully saturated entity is not fully saturated if the minimum occupancy limits of its pockets have been reached but not the maximums. But this just means than an entity which can be true or false can be further saturated and result in another entity which can be true or false.[24] So I think that, despite its differences from slot theory, pocket theory, paired with PRP-ism, results in as powerful a theory of properties, relations, and propositions as does slot theory. It is just that those categories are not disjoint, as the slot theorist takes them to be. And it provides as natural an explanation as slot theory of why those entities that have traditionally been conceived as properties, relations, and propositions are all species of a single ontological category: PRPs.

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[24] Another concern in this general area is that, according to pocket theory, it is not the "shape" of a property or relation that guarantees that it is not fully saturated, as is the case according to slot theory. Rather, the minimum and maximum occupancy limits are "put in by hand". A related worry is that, according to pocket theory, expressions of the form $\ulcorner x$ occupies w t times in $z \urcorner$ are primitive. This means that fundamental facts about occupation make ineluctable reference to natural numbers. Thanks to Cody Gilmore and Ted Sider for these points. Gilmore notes further that this will make pocket theory vulnerable to Armstrong-style objections. As Armstrong points out (1978: 49), the resemblance nominalist must take the formal properties of resemblance as brute, while the realist may explain them by analyzing resemblance in terms of the sharing of properties, allowing the symmetry of resemblance to be explained by the symmetry of numerical identity. Similarly, the pocket theorist must take the fact that occupation holds of, among other things, a natural number, as brute, and cannot analyze this in terms of the "shape" of the relation of which it holds. I acknowledge that pocket theory does have these consequences. But they can be perceived as disadvantages only if one thinks it best that fundamental facts do not make reference to natural numbers. While there are doubtless plenty of exceptions to this rule, I suspect it is often nominalistic predilections that would make one wary of such fundamental facts. And those same predilections are also likely to make one wary of non-extensional accounts of properties and relations like pocket theory and slot theory. Thus I would not expect there to be many to whom these consequences of pocket theory would register as serious disadvantages. But even for those there are, I do not take these problems to be any more serious than those facing slot theory, including the symmetric (partial) completions and conflicting adicities problems.

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