Categorical Colors in Diamonds: *Sight as Site*: Categorical Ozma and Cinderella

Shanna Dobson[0000−0001−8818−4841]

Abstract We present colorful illustrations of particular properties of functorial diamonds, in the sense of Scholze; namely profinite reflections as categorical colors. We discuss *sight as site* using representable functors in the condensed formalism. We illuminate diamonds using our novel constructions of Categorical Ozma and Cinderella, the Site of Oz, and Condensed *Through the Looking-Glass*.

Keywords diamond; representable functor; Grothendieck site; condensed set

1 Diamonds

A diamond [20], in the sense of Scholze, is a Grothendieck functor of points whose analytic origins lie in rigid analytic geometry. In [5], we present a comprehensive review of diamonds and their incarnations in the Langlands Program. We follow our exposition and highlight principal results from [24].

The analytic definition of diamonds is centered upon certain adic spaces, in the sense of Huber [11], called *perfectoid spaces.* We first recall the definition of a perfectoid ring.

Definition 1. *([24] Definition 6.1.1) A complete Tate ring R is perfectoid if R is uniform and there exists a pseudo-uniformizer $\bar{\omega} \in R$ such that $\bar{\omega}^p | p$ holds in R°, and such that the pth power Frobenius map*

$$\Phi : R^\circ/\bar{\omega} \to R^\circ/\bar{\omega}^p$$

is an isomorphism.

Shanna Dobson
University of California, Riverside, 900 University Ave, Riverside, CA 92521, e-mail: Shanna.Dobson@email.ucr.edu

Now we let R^+ denote a perfectoid ring of integral elements [20]. A perfectoid space is formally defined as follows:

Definition 2. *([24] Definition 7.1.2) A perfectoid space is an adic space covered by affinoid adic spaces $Spa(R, R^+)$ with R perfectoid.*

There exists a site of perfectoid spaces with the pro-étale topology. As discussed in [1], pro-étale morphisms are defined in terms of weakly étale morphisms.

Definition 3. *([1] Definition 4.1.1) A morphism $f : X \to Y$ of schemes is called weakly étale if*

1. *f is a flat morphism of schemes;*
2. *its diagonal $X \to X \times_Y X$ is also flat.*

There is also a site of perfectoid spaces with the v-topology. The sites are formally defined as follows:

Definition 4. *([20] Definition 8.1) Let Perfd be the category of κ-small perfectoid spaces.*

- *The big pro-étale site is the Grothendieck topology on Perfd for which a collection $\{f_i : Y_i \to X\}_{i\in I}$ of morphisms is a covering if all f_i are pro-étale, and for each quasicompact open subset $U \subset X$, there exists a finite subset $J \subset I$ and quasicompact open subsets $V_i \subset Y_i$ for $i \in J$, such that $U = \bigcup_{i\in J} f_i(V_i)$.*
- *Let X be a perfectoid space. The small pro-étale site of X is the Grothendieck topology on the category of perfectoid spaces $f : Y \to X$ pro-étale over X, with covers the same as in the big pro-étale site.*
- *The v-site is the Grothendieck topology on Perfd for which a collection $\{f_i : Y_i \to X\}_{i\in I}$ of morphisms is a covering if for each quasicompact open subset $U \subset X$, there exists a finite subset $J \subset I$ and quasicompact open subsets $V_i \subset Y_i$ for $i \in J$, such that $U = \bigcup_{i\in J} f_i(V_i)$.*

A diamond is formally defined as follows:

Definition 5. *([24] Definition 1.3) Let Perfd be the category of perfectoid spaces. Let Perf be the subcategory of perfectoid spaces of characteristic p. Let Y be a pro-étale sheaf on Perf. Then Y is a diamond if Y can be written as the quotient X/R with X a perfectoid space of characteristic p and R a pro-étale equivalence relation $R \subset X \times X$.*

Generally, a diamond is called a "pro-étale sheaf on Perf" [24], where a sheaf on a site takes the form of a set-valued functor of points [10] that *maps in* perfectoid spaces. A diamond is further categorized as a v-sheaf, which is a sheaf for the v-topology. Most importantly, "all diamonds are v-sheaves" [24][cf. [20] Proposition 11.9].

Diamonds which have a "well-behaved underlying topological space" [24] are called *spatial v-sheaves*. The following principal result states that

> a spatial v-sheaf is a diamond as soon as its points are sufficiently nice [24].

Theorem 1. *([24] Theorem 17.3.9 [20] Theorem 12.18). Let $\mathcal{F}$ be a spatial v-sheaf. Assume that for all $x \in |\mathcal{F}|$, there is a quasi-pro-étale map $X_x \to \mathcal{F}$ from a perfectoid space X_x such that x lies in the image of $|X_x| \to |\mathcal{F}|$. Then $\mathcal{F}$ is a diamond.*

Scholze's categorical diamonds resemble mineralogical diamonds in that their geometric points represent mineralogical impurities. Impurities in a mineralogical diamond are made visible as *colorful* and *sparkling reflections* on the diamond, the geometric points in categorical diamonds are made visible as *specific profinite reflections*.

Specifically, let $\mathcal{D}$ a diamond and let C be an algebraically closed affinoid field.

> A geometric point $Spa(C) \to \mathcal{D}$ is something like an impurity within a gem which produces a color. This impurity cannot be seen directly, but produces many reflections of this color on the surface of the diamond. Likewise, the geometric point cannot be seen directly, but when we pull it back through a quasi-pro-étale cover $X \to \mathcal{D}$, the result is profinitely many copies of $Spa(C)$. Often one can produce multiple such covers $X \to \mathcal{D}$, which result in multiple descriptions of the geometric points of $\mathcal{D}$ ([24] Figure 9.1).

The quasi-pro-étale map alluded to above is defined as follows:

Definition 6. *([20] Definition 9.2.2). Consider the site Perf of perfectoid spaces of characteristic p with the pro-étale topology. A map $f : \mathcal{F} \to \mathcal{G}$ of sheaves on Perf is quasi-pro-étale if it is locally separated and for all strictly totally disconnected perfectoid spaces Y with a map $Y \to \mathcal{G}$ (i.e., an element of $\mathcal{G}(Y)$), the pullback $\mathcal{F} \times_{\mathcal{G}} Y$ is representable by a perfectoid space X and $X \to Y$ is pro-étale.*

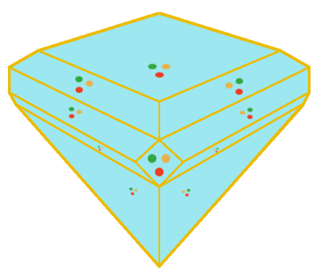

Fig. 1 Characterization of *Figure 9.1* [24]. Categorical Diamond $\mathcal{D}$ with impurities as geometric points Spa $C \to \mathcal{D}$. [Image @ShannaDobson]

1.1 Representable Functor: *Sight as Site*

"Well, now that we *have* seen each other, said the Unicorn, "if you'll believe in me, I'll believe in you. Is that a bargain?" [2]

> A category is a world of objects, all looking at one another. Each sees the world from a different viewpoint [12].

There is a curiosity at play between the constructions of *visual sight* and *categorical site*. In particular, it seems as if the *sight* of diamonds is the *v-site.* This is referenced in the following key result:

Corollary 1. *([24] Corollary 17.1.5) Representable presheaves are sheaves on the v-site.*

Consequentially, such a characterization could plausibly take the form of the representable and diamond equivalence relations alluded to in the remark:

> So to access v-sheaves takes two steps. First, we analyze diamonds as quotients of perfectoid spaces by representable equivalence relations. Second, then we analyze small v-sheaves as quotients of perfectoid spaces by diamond equivalence relations [24].

Formally, the *sight* of an object is categorized in the notion of a representable functor.

Definition 7. *[12][Definition 4.1.1] Let $\mathcal{A}$ be a locally small category*[1] *and $A \in \mathcal{A}$. We define a functor*

$$H^A = \mathcal{A}(A, -) : \mathcal{A} \to \mathbf{Set}$$

as follows:

- *for objects $B \in \mathcal{A}$, put $H^A(B) = \mathcal{A}(A, B)$;*
- *for maps $B \xrightarrow{g} B'$ in $\mathcal{A}$, define*

$$H^A(g) = \mathcal{A}(A, g) : \mathcal{A}(A, B) \to \mathcal{A}(A, B')$$

by

$$p \mapsto g \circ p$$

for all $p : A \to B$.

Definition 8. *[12][Definition 4.1.3] Let $\mathcal{A}$ be a locally small category. A functor $X : \mathcal{A} \to$* **Set** *is* **representable** *if $X \cong H^A$ for some $A \in \mathcal{A}$. A* ***representation*** *of X is a choice of an object $A \in \mathcal{A}$ and an isomorphism between H^A and X.*

The following Proposition shows how the "seeing functors" [12] are representable:

Proposition 1. *[12] [Propositionn 4.1.11] Any set-valued functor with a left adjoint is representable.*

We recall the definition of a categorical adjunction:

Definition 9. *([12] Definition 2.1.1) Let $\mathcal{A} \underset{G}{\overset{F}{\rightleftarrows}} \mathcal{B}$ be categories and functors. We say that F is left adjoint to G, and G is right adjoint to F, and we write $F \dashv G$ if*

---

[1] [12](Remark 4.1.2)Recall that 'locally small' means that each class $\mathcal{A}(A, B)$ is in fact a set.

$$\mathcal{B}(F(A), B) \cong \mathcal{A}(A, G(B)) \qquad *$$

naturally[2] *in* $A \in \mathcal{A}$ *and* $B \in \mathcal{B}$. *An adjunction between F and G is a choice of natural isomorphism* $*$.

2 Profinite Reflections *as* Categorical Colors

We further discuss the intuition underlying the construction of the categorical diamond. While impurities in mineralogical diamonds are made visible as sparkling reflections on the faces of the diamond, in Scholze's diamond, the geometric point [impurity] is made visible as "profinite reflections." Specifically, the geometric point can only be observed *indirectly* as representation upon pullback through a quasi-pro-étale cover $X \longrightarrow \mathcal{D}$, where X is a perfectoid space and $\mathcal{D}$ is a diamond.

One might ask, what would profinite reflections of $Spa(C)$ look like? We first recall that a profinite set is a compact, Hausdorff, totally disconnected set. The Yoneda Lemma famously answers the question, "what do representables see? [12]" Likewise, we may ask *what does profinite see?*

2.0.1 Condensed *Sight*

"What a time the Monster is, cutting up that cake!" [2]

To address this question, we recall the construction of a condensed set [3].

> Let $\mathbf{C}$ be a category. $\mathbf{Cond}(\mathbf{C})$ denotes the category of *condensed* objects of $\mathbf{C}$. Equivalently, $\mathbf{Cond}(\mathbf{C})$ can be represented as a representable functor $\mathbf{F} : \mathbf{C}^{\mathbf{op}} \to \mathbf{Set}$.

The pro-étale *site* is defined as follows:

Definition 10. *[16] For a scheme X, its pro-étale site is the site whose objects are pro-étale morphisms into X and whose Grothendieck topology is that of the fpqc site.*

A condensed set takes the consequent form:

Definition 11. ([3] Definition 1.2) *The* pro-étale site $*_{proét}$ *of a point is the category of profinite sets* **Pro-FinSet**, *with finite jointly surjective families of maps as covers. A* condensed set *is a sheaf of sets on* $*_{proét}$. *Similarly, a condensed ring/group/object is a sheaf of rings/groups/objects on* $*_{proét}$.

The pro-étale site is the sight of what is profinite. Thus, a condensed set X measures the mapping of profinite sets S into X.

> Let me describe what a condensed set X "is": For each profinite set S, it gives a set $X(S)$, which should be thought of as the "[continuous] maps from S to X", so it is measuring

[2] [12](2.1 Definitions and Examples) "Naturally in $A \in \mathcal{A}$ and $B \in \mathcal{B}$ means that there is a specified bijection for each $A \in \mathcal{A}$ and $B \in \mathcal{B}$, and that it satisfies a naturality axiom."

how profinite sets map into X. The sheaf axiom guarantees some coherence among these values. Taking $S = *$ a point, there is an "underlying set" $X(*)$...This is what happens in the condensed perspective, which only records maps from profinite sets. [Scholze(2020)]

Informally, $X(S)$ measures how condensed sets are *seen*. Likewise, *condensed sets measure what profinite sets see*. *Condensed Sight*.

Interestingly, the shifts in causality that occur in Carroll's *Through the Looking-Glass [2]* could be recast in terms of representable functors taking the form of condensed sets. Recall the Unicorn's exchange with Alice who is struggling to serve the Lion and the Unicorn the Looking-glass cake.

You don't know how to manage Looking-glass cakes," the Unicorn remarked. "Hand it round first, and cut it afterwards. [2]"

If looking-glass causality is treated as a profinite set S, then Looking-glass cake could be modeled as a condensed set X.

Conjecture 1. *Looking-glass cake is a condensed set.*

Furthermore, the White Queen's memory that "works both ways" and Alice's reversals ("bigger or smaller, which way?") could be modeled as a condensed set X [6], which gives a set $X(S)$ that considers how X is seen by S. Consequentially, 2-way memory is what *profinite sees.* Looking-glass cake is what *profinite sees.*

Perhaps to return to canonical causality, one must "forget" functorially. After all, one motivation for categorical diamonds was the creation of a functor

- $\{$analytic adic spaces over $Z_p\} \to \{$diamonds$\}$
- $\{X\} \to \{X^\diamond\}$

that "forgets the structure morphism to Z_p [24]. "

It is by functoriality, namely forgetful functors, that we pass from looking-glass causality to non looking-glass causality. Perhaps this formalism extends to the entire land of *Through the Looking-Glass.*

Conjecture 2. *Through the Looking-Glass is a condensed set.*

Conjecture 3. *Let* **C** *be a category. Through the Looking-Glass is categorically equivalent to* ***Cond(C)****, the category of condensed objects of* **C**.

2.1 Diamond Poem

In the following poem "Diamond" we reflect on the question *"what does profinite see?"* by exploring identity in the external world. We utilize Scholze's categorical diamond, and the powerful categorical ideas of universal property and terminality.

We compose a dialogue that shifts perspective from the inside of a diamond to the outside of a diamond, while simultaneously shifting perspective from the categorical diamond to a mineralogical diamond. The narrator experiences the shimmering grandeur of the diamond if she is on the outside of the diamond, while she experiences haunting despair upon being trapped inside her own total internal reflection. Our poem is written in confessional style to capture the dual reflections: the feeling of being trapped inside one's own thoughts; being a terminal object, by universality, there is nowhere non-terminal to go. Thick lines of tension in the poem underlie the dual reflections at play; Topos Tigers and Adamantem Lions, looking-glass ice-cream and black rainbows.

Diamond *by Shanna Dobson*

If I was your diamond,
What would profinite see?

I know, sweet Kierkegaard,
Reflections only to me

Total internal reflection
Makes aporia sparkle

Great Concession
Categorical marble

Your diamond is a darkling
Unhallowed Sparkling

Dark energy must have diamonds
Adamantem Lions

I hear total internal reflection
Red functorial subreption

As Topos Tigers dream of
Looking-glass ice-cream

Does no one else see the
Black rainbow?

I dare you to compose anything here
And hope for anything but what was
always here.

Clarion Universal
You could not see 4th Dimensional me,
Profinitely.

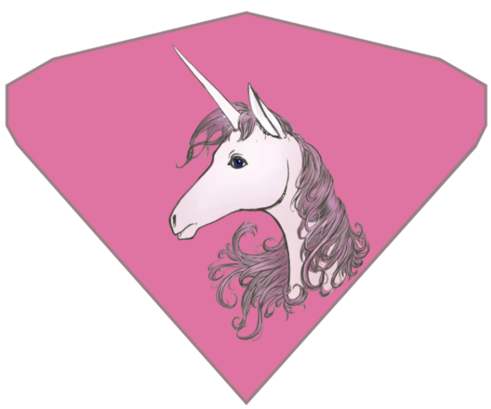

Fig. 2 Diamond Adamantem Lion: *Diamond Poem* [Image@ Shanna Dobson]

3 Functorial Mirror

Recall that an adjunction in which the right adjoint is full and faithful is called a reflection. Moreover, we have the following definition of a reflective subcategory:

Definition 12. *[15] A reflective subcategory $C \hookrightarrow D$ contains objects d and morphisms $f : d \to d'$ in D that have reflections Td and $Tf : Td \to Td'$ in C. Every object in D looks at its own reflection via a morphism $d \to Td$ and the reflection of an object $c \in C$ is equipped with an isomorphism $Tc \cong c$.*

We provide a colorful, heuristic example of how one might envision the reflections of a categorical diamond in terms of mirror reflections; a functorial mirror.

3.1 Categorical Cinderella

We work in the purely hypothetical locally small category called Cinderella. Objects are characters. 1-morphisms are emotions (relations). How could we construct a functor between categories of fairy tales?

Let us imagine a mirror is a "functor" from the category of Cinderella to a "subcategory" of Cinderella. The pumpkin looks into the mirror and sees a carriage; an isomorphism. Likewise, Cinderella, in tarnished clothing looks into the mirror and sees a chic and proper princess; an isomorphism of herself! [Figure 2].

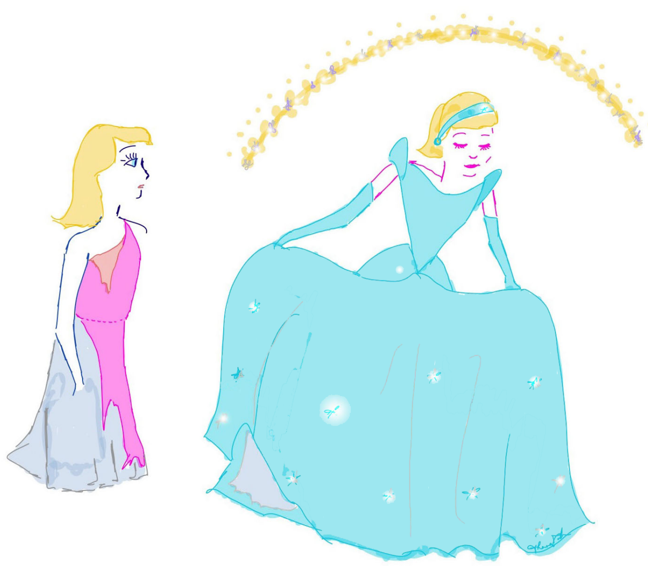

Fig. 3 Mirror as Reflector Functor [Image@ Shanna Dobson]

Consequentially, we can consider the mirror as a ”functor” from the category of Cinderella to a ”subcategory” of Cinderella. Moreover, the mirror is a ”reflector” mapping each object in the story of Cinderella to its ”beautified” form.

Additionally, we can think of universal properties and terminality as follows: The Fairy Godmother is a *special object*. She sees herself ”as is” when she looks into the mirror. Anything already beautified to begin with is a fixed point. Thus, she is a fixed point under the reflector functor. We can further say that she is *divine* because she possesses universal properties [Figure 3].

The Fairy Godmother is the initial object. There exists only one 1-morphism from her to each character. This 1-morphism is love. This is the simplest kind of universal property. The terminal object is the Prince. All other characters (objects) fawn over the Prince and exhibit adoration of him. We could go further and try to construct a ***pullback universal construction*** using, categorically;

- The Wand, The Fairy Godmother, Cinderella, and Gus Gus as objects;
- 1-morphisms as
  - The Wand’s loyalty to The Fairy Godmother;
  - the Wand’s encouragement to Gus Gus;
  - Gus Gus’ appreciation for Cinderella; and
  - The Fairy Godmother’s love for Cinderella.

Parenthetically, we note that if we worked in the Category of Hansel and Gretel, the candy sugar house could be the terminal object because everyone loves candy! Likewise, the Witch could be the initial object because the Witch hates everyone except herself. Importantly, the candy house does not hate the Witch. Consequentially, the Witch is not the terminal object.

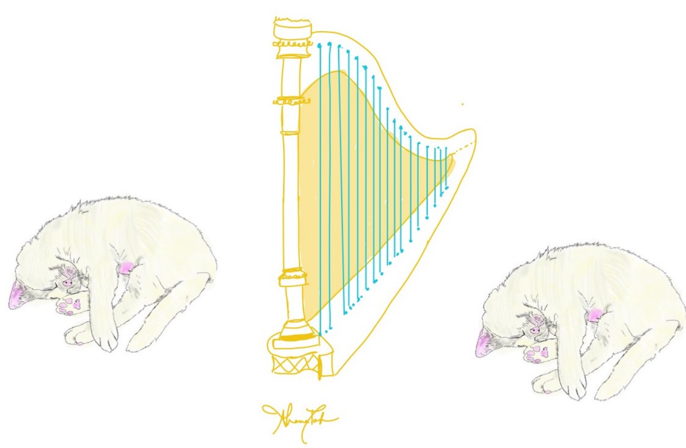

Fig. 4 Special Object *Artemis* in Functorial Harp; Image @Shanna Dobson

4 Categorical Colors

Informally, we have the following analogy: mineralogical impurities *as colorful reflections* are represented as profinitely many copies of geometric points. We might go further and imagine that profinite reflections admit categorical colors.

A few questions naturally arise:

Question 1. *How could we represent color as a categorical construction?*

Question 2. *How could we construct profinite color as a pro-object?*

Question 3. *How could we construct profinite color as a functor?*

Question 4. *How could we construct profinite color p-adically as an inverse limit?*

Question 5. *How could we represent profinite color as a condensed set [3] [14]?*

To address Question 1, first we might claim the following:

Conjecture 4. *Profinite reflections are representable.*

Secondly, we might posit structure to specific colors as in the following consequential conjectures.

Conjecture 5. *Pink is a 2-category.*

Conjecture 6. *Cyan is an adjunction.*

Conjecture 7. *Emerald Green is a reflector functor.*

The structure for Conjecture 2 could admit:

- Objects as categories of spectra [photoreceptors];
- 1-morphisms as set-valued forgetful functors;
- 2-morphisms as natural transformations.

To illuminate Conjecture 3, we first recall that an adjunction is formally defined as follows:

Definition 13. *([12] Definition 2.1.1) Let* $\mathcal{A} \underset{G}{\overset{F}{\rightleftarrows}} \mathcal{B}$ *be categories and functors. We say that F is left adjoint to G, and G is right adjoint to F, and we write $F \dashv G$ if*

$$\mathcal{B}(F(A), B) \cong \mathcal{A}(A, G(B)) \qquad *$$

naturally[3] in $A \in \mathcal{A}$ and $B \in \mathcal{B}$. An adjunction between F and G is a choice of natural isomorphism $*$.

As cyan can be either a secondary color or a subtractive primary color, a possible structure for Conjecture 3 could entail:

- Categories of spectra [color spaces] [photoreceptors];

[3] [12](2.1 Definitions and Examples) "Naturally in $A \in \mathcal{A}$ and $B \in \mathcal{B}$ means that there is a specified bijection for each $A \in \mathcal{A}$ and $B \in \mathcal{B}$, and that it satisfies a naturality axiom."

- Pairs of functors in opposite directions representing additive and subtractive color models.

As stated above, there exist many descriptions of the diamond's geometric points based on the choice of quasi-pro-étale cover. Thus, we could construct an entire categorical color wheel [4] as the class of all profinite reflections given the multiple covers of the geometric points of the diamond.

Intuitively, Conjecture 4 questions how *visual sight mirrors categorical site*. We now develop this further in the context of Baum's *The Emerald City in the Land of Oz*.

5 Categorical Ozma of Oz: *the Site of Oz*

The film *Return to Oz* directed by Walter Murch is a fantasy film about The Land of Oz and The Emerald City adapted from Frank Baum's books *Ozma of Oz* and *The Marvelous Land of OZ Illustrated*. In the film, Ozma, the benevolent ruler of OZ, is trapped by Princess Mombi in a mirror inside the palace in The Emerald City. Dorothy sees Ozma's reflection in the mirror, and this reflection guides her along her incredible journey. Eventually, Dorothy frees Ozma from the mirror, and Ozma utilizes the ruby slippers to guide Dorothy home to Kansas.

It is interesting to consider the mathematics of how Dorothy passes from Kansas to Oz, and from Oz to Kansas. Is this passage profinite? Is it one of categorical analytic continuation via sheaves? It might be illuminating to consider The Wardrobe in C.S. Lewis' *The Chronicles of Narnia: The Lion, the Witch, and the Wardrobe*, which has curios topological properties; namely sidedess. Perhaps it functions as a Gepner point allowing passage from inside the Wardrobe to outside the Wardrobe which is inside the snow-fallen wonderland of Narnia. And snowfall is incredibly reflective.

Similarly, it could be argued that Ozma's mirror is like a Gepner point or a condensed set [3]. Consequentially, Ozma is a reflection of Dorothy. In particular, Dorothy sees her "beautified form as Ozma, ruler of OZ" in the mirror. Hence, Ozma's mirror could serve as a reflector functor. We conjecture the following:

Conjecture 8. *Ozma's mirror is a reflector functor.*

Conjecture 9. *Ozma's mirror is a condensed set.*

Going further, we can ask the following questions:

Question 6. *Are the Ruby Slippers functorial?*

Question 7. *What is the topology on The Emerald City? on The Land of Oz?*

Question 8. *What is the topology on Ozma's mirror?*

---

[4] We are currently constructing a categorical model of the phantasia/aphantasia spectrum using the diamond construction and quasi-pro-étale covers. An adjunction of the color wheel

Question 9. *Can we represent The Land of Oz mathematically as a category? as a topos?*

By constructing the *site of Oz*, we can categorify *the sight of Ozma*.

Fig. 5 Ozma of Oz in the Site of Oz; [Image @Shanna Dobson]

6 Conclusion

It appears that many fantastical fiction works can be recast categorically using the diamond and/or condensed formalisms. Infusing category theory into color theory and literature á la Grothendieck via *The Chronicles of Diamonds* is like adding new channels of color to our mathematical vision. Whereas our canonical modality of vision admits only three color cone receptors, Mantis shrimp have eyes that admit 12 color cone photoreceptors. Imagine, 12 channels of color! The hope is that through categorification, we can construct reflective universal properties of polychromatic sight and mathematical sight, therein connecting sight and site; *sight as site*.

7 Statements and Declarations

Author Contributions The author solely developed and wrote the paper.

Funding This research received no external funding.

Conflicts of Interest Statement The author has no conflicts of interest to declare that are relevant to the content of this manuscript.

References

1. Bhatt, B. and Scholze,P., *The pro-étale topology for schemes*, Astérisque No. 369 (2015), 99–201.
2. Carroll, Lewis. *Alice in Wonderland*, W. W. Norton & Company, Inc., New York, 1992.
3. Clausen, D.; Scholze, P.(2021). Lectures on Condensed Mathematics. https://www.math.uni-bonn.de/people/scholze/Condensed.pdf (Accessed 11 Jan 2022).
4. Cais, B., Davis, C., and Lubin, J., *A Characterization of Strictly APF Extensions*, arXiv:1403.6693 [math.NT].
5. Dobson, S., *Diamonds in the Langlands Program: a Comprehensive Review*, The Ramanujan Journal, Springer (2024). *In review.*
6. Dobson, S., and Fields, C., *Constructing Condensed Memories in Functorial Time*, Journal of Experimental Theoretical Artificial Intelligence, DOI: 10.1080/0952813X.2023.2222374.
7. Fargues, L., *Geometrization of the Local Langlands Correspondence: an Overview*, arXiv:1602.00999 [math.NT] (2016).
8. Fargues, L. and Scholze, P., *Geometrization of the Local Langlands Correspondence*, arXiv:2102.13459 [math.RT] (2021).
9. Fontaine, JM. and Wintenberger,JP, *Extensions algébrique et corps des normes des extensions APF des corps locaux*, C. R. Acad. Sci. Paris Sér. A–B 288(8) (1979), A441–A444.
10. Grothendieck, A. Introduction to functorial algebraic geometry, part 1: affine algebraic geometry, summer school in Buffalo, 1973, lecture notes by Federico Gaeta..
11. Huber, R. *A generalization of formal schemes and rigid analytic varieties*, Math. Z. 217 (1994), no 4, 513-551.
12. Leinster, T., *Basic Category Theory*, Cambridge Studies in Advanced Mathematics, Vol. 143, Cambridge University Press, 2014.
13. Lurie, J., *Higher Topos Theory*, Annals of Mathematics Studies, Vol. 170, Princeton University Press, 2009.
14. Dobson, S., and Fields, C., *Making Up Our Minds: Imaginative Deconstruction in MathArt, 1920 – Present*, AWM Springer Series: Women in MathArt: Research, Creativity, and Teaching, Springer Nature, 2024 (in press).
15. ncatlab authors. Reflective Subcategory. https://ncatlab.org/nlab/show/reflective+subcategory (accessed April 11, 2024).
16. ncatlab authors. Pro-étale Site. https://ncatlab.org/nlab/show/pro-
17. Scholze, P., *Perfectoid Spaces*,Publ. Math. Inst. Hautes Etudes Sci. ´ 116 (2012), 245–313. MR 3090258.
18. Scholze, P., *p-adic Hodge theory for rigid-analytic varieties*,Forum of Mathematics, Pi 1 (2013), no. e1.
19. Scholze, P., *On torsion in the cohomology of locally symmetric varieties*, Annals of Mathematics (2) 182 (2015), no. 3, 945–1066.
20. Scholze, P., *Étale Cohomology of Diamonds*. arXiv:1709.07343 [math.AG], 2017.
21. Caraiani, A. and Scholze, P., *On the generic part of the cohomology of compact unitary Shimura varieties*, Annals of Mathematics (2) 186 (2017), no. 3, 649–766.
22. Scholze, P., *On the p-adic cohomology of the Lubin-Tate tower*, Ann. Sci. Ec. Norm. Super. (4) 51 (2018), no. 4, 811-863.

[Scholze(2020)] Scholze, P. (2020). Liquid Tensor Experiment,https://xenaproject.wordpress.com/2020/12/05/liquid-tensor-experiment/, accessed July 14, 2022.

23. Scholze, P., *P-adic Geometry*, arXiv:1712.03708 [math.AG].
24. Scholze, P. and Weinstein, J., *Berkeley Lectures on P-adic Geometry*, Princeton University Press, Annals of Mathematics Studies Number 207.
25. L. Susskind, ER=EPR, GHZ, and the Consistency of Quantum Measurements, arXiv:1412.8483 [hep-th], 2014..
26. L. Susskind, ER=EPR, or What's Behind the Horizons of Entangled Black Holes?, Stanford Institute for Theoretical Physics,https://www.youtube.com/watch?v=OBPpRqxY8Uw.
27. Weinstein, J., *Gal*($\bar{Q}_p/Q_p$)*) as a geometric fundamental group*, Int.