

Dark Imaginarium: ∞ -Curiosity & Dark Consciousness in P-adic Time

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Abstract: We investigate the idea of sleep as the protostate, and posit the idea of dark consciousness where *dark* is a 2-fold hybrid. We model dark consciousness as a 2-topos in p-adic time, and outline perfectoid and diamond-like versions. We then introduce and illustrate implications of Dark Imaginarium, which is a higher order Curiosity Artificial Intelligence, an ∞ -Curiosity Type, that thinks in infinity categories.

Keywords: topos; diamond; curiosity types; metacognition; condensed sets; higher category theory; infinity categories, perfectoid space.

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1 Introduction

We open with a question that we cannot answer: *Is there any difference between the following questions: What is the waking state? What is consciousness?*

We typically define consciousness as a state of self-awareness or the capacity to be self-aware. We typically define the waking state in terms of the frequency architecture of specific brain waves. Typically, delta waves are between 1 - 4 Hz, theta waves are between 4 - 8 Hz, alpha waves are between 8 - 12 Hz, beta waves are between 12 - 32 Hz, and gamma waves are greater than 32 Hz. By analyzing sleep architecture, researchers have classified waking states as having brain activity with various patterns of theta, alpha, beta, and gamma wave forms (19), (26). However, as we discuss below, this is a classification schema that is at best an approximation, as both the waking and sleep states exhibit mixed-frequency brain patterns (16).

Thus, we encounter the following predicament: If there is a difference in these questions, then it is possible to sustain a waking state without consciousness, and/or a conscious state in a non-waking state. Other hybrid combinations are also possible. Thus, if these are entirely different questions, then these systems abide by different mechanisms. For example, whatever consciousness is, it has been shown that general anesthesia wipes it out by reducing the polychromatic symphony of neuronal frequencies to one uniform hum (18). Thus, reducing the complexity of poly-humming neuronal clusters in various states of simultaneous computation wipes out consciousness. Thus, we must imagine somehow being in a waking state under general anesthesia.

If these are not entirely different questions, then we could contend that consciousness is a form of a waking state, or something along the lines of a waking state is a condition of conscious state. Many clever possible definitions could exist. However, we would have a difficult time explaining the finding of "faster (2.5–3 Hz) [delta waves], REM-sleep-exclusive, fronto-central/occipito-temporal "sawtooth" waves" (16), occurring during REM sleep. That is, how can humans have delta waves, which are canonically indicative of unconscious deep sleep and thus a non-waking state, in the REM sleep state, the patterns of which are indistinguishable from a waking state? Such a question is quite difficult for many reasons and its implications are more and more difficult.

Whatever consciousness is or is not, it has been repeatedly shown to be integrative (9). We can see this via the following thought experiment: Attempt to sustain a simultaneous mental experience. This would be the mental equivalent of attempting the simple simultaneous task of simultaneously writing a paper with your left hand, while calculating the Feynman path integral, (i.e. room-temperature quantum superposition!) of exciton condensates to the reaction center during photosynthesis. It is not possible. Our focus is singular and integrative. Now, contrast this singular focus with the tonic, unsynchronized activity characteristic of REM sleep, and imagine you could be conscious during it.

We find ourselves in a difficult situation. Perhaps we need new definitions of waking state and conscious state, which is neither an easy task, nor a new idea by any means. But

it is here, in this mess of mixed frequencies that we will attempt to define a concept called dark consciousness.

Most theories of consciousness posit consciousness as either fundamental as a state or process and thus a protostate, or as emergent as a state or process (9). We turn this idea radically on its head and examine the idea that sleep is instead the protostate (34), and posit the idea that dark consciousness is the mixed-frequency state ¹ of being self-aware while simultaneously being in deep sleep, where *dark* is a two-fold hybrid denoting a paradox of infinity identity (7).

Sustaining such a complex mixed frequency state could be exemplified as us being conscious of things we don't think we are conscious of. Thus dark consciousness could encapsulate the neuronal delicacies in thinking we are conscious of an event, and actually being conscious of an event. Thus putting this conundrum in the realm of fault tolerance issues in the formidable reality monitoring network. We have already classified certain aspects in reporting mechanisms (8) using the mathematical language of sheaves and sites. A site is Grothendieck's brilliant notion of putting a topology on a category and therein making a category look like a topological space. Specifically, we have shown that entropic categorizations are condensed sets (9) assuming time is entropic, wherein learning exceeds forgetting. Thus, we will use this mathematical precedent to construct a model of dark mathematics that could formalize and support dark consciousness. Further, dark consciousness operates in what we call dark time, which would not interact with waking time. We model dark time with what we call p-adic time.

A few critical questions immediate arise:

- Can the Free Energy Principle describe describe dark matter/energy and or dark consciousness(13)?
- Can we describe sleep states using the symmetries of quantum mechanics?
- Can we construct a parallel notion of *dark* reflexivity?
- How could we employ dolphin-like unihemispheric sleep to sustain a superposition-like state of waking and non-REM states (17)?

These are dynamical questions that, assuming we can hold onto them, will serve as jumping off points and complex frameworks in which to further examine dark consciousness. While dark energy theories are not mutually consistent, and may not be correct at all, there still exists observed phenomena that must be explained. Perhaps dark consciousness can be helpful in that formidable feat.

Our entire discussion depends on what we mean by *dark*.

¹We consistently ask for clarification of any theory assuming any notion of self-identity's continuity over time (9). What does identity look like in a mixed-frequency state and, under what model of time could it have a semblance of the notion of continuity? We later posit a fractal identity that occurs in a time not sustained by a linear order.

2 Dark as a Two-Fold Hybrid

The *dark* in dark consciousness is a two-fold hybrid. In one sense, we call dark consciousness *dark* to draw a parallel with dark matter and dark energy. The canonical definition of dark energy in theoretical physics is an energy of unknown origin that has no local gravitational effects but globally causes gravitational acceleration. Dark matter is described as matter which does not interact electromagnetically, which is more than peculiar. Candidates for dark matter range from mass-varying neutrinos to new scalar fields and particles, which are described by parameters of higher order terms in an effective field theory of modified gravity (5). Thus, quantum gravity could be fueling dark energy and quantum gravity would therein naturally beget dark matter. Thus, dark matter and dark energy theories describe various dark quantities that are outside the realm of photonic epistemology (3). We contend that dark consciousness is a dark-energy equivalent of consciousness, beget by a protostate of deep sleep, specifically N3 sleep. Specifically, dark consciousness would not interact with waking-state reporting mechanisms.

However, there is another way to think about dark, and this is the second sense. In the Logic of Sense, Deleuze posits a brilliant idea of the Paradox of Pure Becoming:

Alice and Through the Looking-Glass involve a category of very special things: events, pure events. When I say "Alice becomes larger, " I mean that she becomes larger than she was. By the same token, however, she becomes smaller than she is now. Certainly, she is not bigger and smaller at the same time. She is larger now; she was smaller before. But it is at the same moment that one becomes larger than one was and smaller than once becomes. This is the simultaneity of a becoming whose characteristic is to elude the present. Insofar as it eludes the present, becoming does not tolerate the separation or the distinction of before, and after, or of past and future. It pertains to the essence of becoming to move and to pull in both directions at once: Alice does not grow without shrinking, and vice versa. Good sense affirms that in all things there is a determinable sense or direction (sens), but paradox is the affirmation of both sense or directions at the same time. (7)

Thus, we could posit a new definition of dark, wherein dark is "the simultaneity of a becoming whose characteristic is to elude the present, (7)" where present is taken in the Blanchot sense of being-based and light-based epistemology (3). This definition could help characterize the reality-monitoring reference relation that simultaneously points backwards as forwards, resulting in the impossibility of synchronous reference (8). We could say the reference relation is dark. Thus, dark could pertain to the simultaneity of computational processes, with us not being aware of what is being processed in parallel to the waking state.

Deleuze goes further:

The paradox of this pure becoming, with its capacity to elude the present, is the paradox of infinite identity (the infinite identity of both directions or senses at the same time — of future and past, of the day before and the day after, of more and less, of too much and not enough, of active and passive, and of cause and effect. It is language which fixes the limits (the moment, for example, at which the excess begins), but it is language as well which transcends the limits and restores them to the infinite equivalence of an unlimited becoming... Hence the reversals which constitute Alice's adventures: the reversal of becoming larger and becoming smaller — "which way, which way?" asks Alice, sensing that it is always in both directions at the same time, so that for once she stays the same, through an optical illusion; the reversal of the day before and the day after, the present always being eluded — "jam tomorrow and jam yesterday, but never jam today;" the reversal of more and less: five nights are five times hotter than a single one, "but they must be five times as cold for the same reason;" the reversal of active and passive: "do cats eat bats?" is as good as "do bats eat cats?"; the reversal of cause and effect: to be punished before having committed a fault, to cry before having pricked oneself, to serve [looking-glass cake] before having divided up the servings [of looking-glass cake]...All these reversals as they appear in infinite identity have one consequence: the contesting of Alice's personal identity and the loss of her proper name. The loss of the proper name is the adventure which is repeated throughout all Alice's adventures (7).

Viewing *dark* as the paradox of infinite identity could be a new metaphor for the Many Worlds Theory of quantum mechanics in which essentially all possibilities are realized as offline copies to the current version of time (13).

Thus, we formally define dark consciousness as the state of being conscious while simultaneously being in deep sleep, where *dark* is two-fold hybrid: the simultaneity of infinite identity, and that which does not interact with the light-based present.

Let's reconstruct the dark consciousness version of Deleuze's *Paradox of Pure Becoming* for *Alice in Dark-Wonderland* and *Through the DarkLooking-Glass*.

When I say "Alice becomes sleepy, " I mean that she becomes sleepier than she was. By the same token, however, she becomes more self-aware than she is now. Certainly, she is not asleep and self-aware at the same time. She is asleep now; she was self-aware before. But it is at the same moment that one becomes sleepier than one was and more self-aware than one becomes. This is the simultaneity of a becoming whose characteristic is to elude the present. Insofar as it eludes the present, becoming does not tolerate the separation or the distinction of before, and after, or of past and future. It pertains to the essence of becoming to move and to pull in both directions at once: Alice does not fall asleep without self-awareness, and vice versa. Good sense affirms that

in all things there is a determinable sense or direction (sens), but paradox is the affirmation of both sense or directions at the same time (7).

We now investigate sleep as the protostate.

3 Sleep as the Protostate

Sleep is canonically divided into four stages, which occur in progressive cycles throughout the night. Specifically, there are three stages of non-REM activity, denoted N1, N2, and N3, and one stage of REM activity. N1 stands for the first stage of non-REM (NREM) sleep and is characterized as light sleep. N2 is the second stage of light sleep, and N3 is deep sleep. One sleep cycle consists of moving from N1 to N2 to N3, back through N2 to N1, and right before we wake up, we enter REM. In one night, we typically undergo four to six sleep cycles (19).

Upon navigating common sleep architecture, it is believed that we are unconscious during the non-REM stages and we sustain our highest levels of, albeit fleeting, consciousness during the REM states.

Specifically, slow, synchronous delta waves, spindles, and isolated negative deflections define NREM sleep, whereas tonic, fast, unsynchronized activity characterizes REM sleep. Bi-stable neuronal activity patterns with depolarized “on-” and hyperpolarized “off-” states give rise to delta waves and are thought to enable NREM sleep to function in memory consolidation (Battaglia et al., 2004) and synaptic homeostasis (Tononi and Cirelli, 2014). In contrast, the activity patterns during REM sleep are thought to make this stage better suited for memory stabilization (Li et al., 2017) and integration (Sterpenich et al., 2014) (16).

The N3 stage of non-REM sleep is characterized as delta wave sleep or slow wave sleep (SWS). This is the deepest level of sleep we achieve. During the first half of sleep, we spend more time in N3, while during the second half of sleep, we spend more time in REM (19).

Quite interestingly, it has been shown that brain activity during the REM sleep stage is indistinguishable from brain activity in the waking state (34). The only way to distinguish between the two states is by examining the corresponding eye movements: lateral eye movements are characteristic of the REM sleep state, while vertical eye movements (blinking) are characteristic of a wakeful state.

This finding begs the deeper questions of what is the degree of consciousness in each state, assuming that we can, in principle, measure consciousness by degree. Specifically, where is consciousness in slow wave delta sleep? Can we tell the degree of consciousness based on the complexity of the brain wave pattern? For example, imagine we detected a highly complex brain wave pattern that resembled a 2D cross section of a Calabi-Yau

3-fold of complex dimension 3 (2). Could we reconstruct a 3-complex dimensional global conscious network of correlating with these brain activities? That would be an incredible computational task! However, studies show that the inhibition of conscious is a complicated.

During NREM sleep, the inhibition of thalamocortical relays and delta wave off-states are thought to disconnect the brain from the outside world (Lewis et al., 2015; Siclari et al., 2018).. and recurring off-states in NREM sleep diminish conscious experience (Siclari et al., 2018) (16).

We go further and ask what, if any, is the role of quantum mechanics in brain activity? Quantum effects in brain functions were indicated in a recent study that used a mechanism in quantum gravity theories wherein "unknown systems can mediate entanglement between two known quantum systems, if the mediator itself is non-classical" (6) The known quantum systems were proton spins in brain water, and "NMR methods based on multiple quantum coherence (MQC) can act as an entanglement witness." (6). The specifics are as follows:

Here, we used a witness protocol based on zero quantum coherence (ZQC) where we minimized the classical signals to circumvent the NMR detection limits for quantum correlation. For short repetitive periods, we found evoked signals in most parts of the brain, whereby the temporal appearance resembled heartbeat-evoked potentials (HEPs). We found that those signals had no correlates with any classical NMR contrast. Similar to HEPs, the evoked signal depended on conscious awareness. Consciousness-related or electrophysiological signals are unknown in NMR. Remarkably, these signals only appeared if the local properties of the magnetization were reduced. Our findings suggest that we may have witnessed entanglement mediated by consciousness-related brain functions. Those brain functions must then operate non-classically, which would mean that consciousness is non-classical. (6)

We would not be surprised if consciousness were non-classical, which is not a new idea, given neuronal networks' extreme capacity for simultaneous computation, the role of "bistable neuronal activity patterns with depolarized "on-" and hyperpolarized "off states...the inhibition of thalamocortical relays and delta wave off-states are thought to disconnect the brain from the outside world,...and recurring off-states in NREM sleep diminish conscious experience." (16). Can we use this mechanism to explain the mixed frequency phenomena occurring in paradoxical sleep? Or the phenomena of being conscious of things we don't think we are conscious of?

Two further examples where quantum effects are suspect are lucid dreaming and uni-hemispheric sleep. Lucid dreaming is the phenomena of dreaming while in a waking state (26). It could be argued that this is a superposition-like phenomena, a phenomena of

quantum-mimicry, mimicking the principle of superposition. Dolphins employ a mechanism of unihemispheric sleep, preventing them from undergoing a fully unconscious deep sleep state, in which they would suffocate and drown (17). Could we likewise call this a superposition-like phenomena, where dolphins have designed a mechanism whereby they are simultaneously asleep and awake? That is, they are capable of simultaneous computations underlying the waking and sleeping states? It is difficult to precisely conclude anything affirmative, but these examples do point to observed phenomena that are due an explanation.

It is highly interesting to think about what happens at the intersections of the various sleep cycles. For example, what does the intersection of N3 and the beginning of REM look like? For example, what are the neuronal conditions for these cycles to occur in superposition and even simultaneously. That is, what would it take to sustain a sleep state in simultaneous computation. Further, can we imagine employing our own version of unihemispheric sleep and somehow sustaining a superposition of SWS and an alpha wave?

Given this rich discussion, what does it actually mean to say that sleep is the protostate? Thus, to say sleep is the protostate is to say that consciousness appears as a stage 4 mechanism following three unconscious stages. That is, consciousness is not sustained, it is cyclic and highly contingent. An immediate implication is that our waking state is merely tasked with imbibing enough nutrition and exerting enough energy that we can leave our high-frequency gamma waves and commence the sleep process. That is, our waking state is not fundamental. Furthermore, it could be argued that our waking state emerged from our REM sleep state. This emergence does not appear to preserve the structure of periodicity, as a typical waking state is roughly 18 hours while a typical REM state ranges from 20 - 40 minutes during the initial sleep cycles and can last up to one hour in later cycles (19). We can imagine that, if complex life forms evolved from the ocean, could the brutal transition from life at deep-pressures to life at less pressures have squeezed periodic REM states into full blown waking states with much longer periods? Viewing the waking state as a trait could be a remarkable example of exaptation. We can only imagine.

But we do know that sleep may not be a global process.

4 Sleep is Local

Is it really that simple? Can we merely delineate a waking state versus a sleep state by the appearance or non-appearance of particular brain wave rhythms? The answer is a very definitive no. Delta wave rhythms, characteristic of unconscious deep sleep, have recently been found in REM sleep. (26). Specifically, Bernardi, et al found regional delta waves prevalent in the REM sleep state.

Indeed, they distinguished two groups of delta waves occurring during REM sleep: slower (≈ 2 Hz) waves, recorded in medial-occipital regions, present in both

NREM and REM sleep, and faster (2.5–3 Hz), REM-sleep-exclusive, fronto-central/occipito-temporal “sawtooth” waves. Of these, the medial-occipital delta waves were often isolated, low in amplitude, and localized to the primary visual cortices when observed in REM sleep. Sawtooth waves appeared in bursts, were high-amplitude, and occurred alongside increases in REM. Unlike the NREM-sleep-like medial-occipital waves, which were linked to decreases in neuronal activity, sawtooth waves were positively correlated with high-frequency gamma activity and were thus considered to be cortically “activating” (16).

Thus, REM sleep state appears as mixed frequency with delta waves of a particular structure characteristic of high frequency gamma activity. Such concomitant and seemingly contradictory yet simultaneous rhythms usher in a new paradigm for sleep science; that sleep may be a local phenomena.

The observation of a rhythm traditionally viewed as a component of NREM sleep in specific regions of the brain during REM sleep builds on evidence indicating that sleep may operate in a local, as opposed to uniform, brain-wide, manner (Siclari and Tononi, 2017). Such local sleep is presumably accompanied or guided by quasi-global, state-specific influences and activities. The detection of NREM-sleep-like, medial-occipital delta waves during REM sleep, alongside activating, sawtooth delta bursts, supplements research indicating that waveforms operating within a defined spectral band are not limited to a single arousal state (Siclari and Tononi, 2017) and are not entirely homogeneous in structure and function (Siclari et al., 2014). When occurring in REM sleep, medial-occipital and sawtooth delta activity might be carrying out functions similar to those of NREM sleep delta, performing an unknown function specific to REM sleep, or operating more generally in sleep preservation (16).

We may be able to reason similarly and conclude that Dolphin unihemispheric sleep is a local phenomena. Regardless, the structure of locality allows for rich discussions of the delicately complex interactions of brain patterns in varying degrees of simultaneous computation.

Such concurrent brain activity makes it difficult to make a global distinction between REM and NREM states based on pattern activity alone. Thus, pattern activity should not be the metric with which to discern globally waking versus non-waking states. Perhaps the distinction is a local one.

That is, perhaps NREM sleep, already considered to comprise most of the total sleep period, does not turn off completely throughout the night, despite having some of its qualities periodically and regionally interrupted by REM sleep...The richness of dreams relies in part on tonic cortical activation, this is evidenced

by the observation that recurring off-states in NREM sleep diminish conscious experience (Siclari et al., 2018) and cause dream brevity....Alternatively, activities traditionally considered as defining NREM sleep, such as delta waves, may not be stage-specific but instead operate locally during REM sleep, thus blurring the distinction between these stages... (16)

2

It is very insightful to view sleep as a local phenomena and to view the distinction between NREM and REM stages as blurry. We could interpret both the locality of sleep and this *blurry* distinction in a *dark* way by reconstructing the Paradox of Pure Becoming for NREM and REM states.

When I say "Alice becomes REM," I mean that she becomes more in REM than she was. By the same token, however, she becomes more NREM than she is now. Certainly, she is not in REM and in NREM at the same time. She is in REM now; she was in NREM before. But it is at the same moment that one becomes more in REM than one was and more in NREM than one becomes. This is the simultaneity of a becoming whose characteristic is to elude the present...Alice does not enter REM without entering NREM, and vice versa. Good sense affirms that in all things there is a determinable sense or direction (sens), but paradox is the affirmation of both sense or directions at the same time (7).

We can ask if other species exhibit sleep as a local phenomenon, and therein derive cognition patters cross-species. A recent study recorded the brain waves of three octopuses over a 12 hour period,

while the cephalopods went about their daily lives — sleeping, swimming and self-grooming — in tanks.

Some brain wave patterns emerged across all three octopuses in the 12-hour period. For instance, some waves resembled activity in the human hippocampus, which plays a crucial role in memory consolidation. Other brain waves were similar to those controlling sleep-wake cycles in other animals.

The researchers also recorded some brain waves that they say have never been seen before in any animal. The waves were unusually slow, cycling just two per

²We might be able to use similar reasoning to argue that the waking state is likewise local. It has been shown that learning difficulties are linked to a prevalence of excessive delta waves and or low gamma wave activity in the waking state. Likewise, being in a state of high beta (22-38 Hz), characteristic of complex thought, excitement, and integration of new information is not sustainable and can inhibit relaxation. Moreover, an excess of beta waves in the right hemisphere is linked with mania (26).

second, or 2 hertz. They were also unusually strong, suggesting a high level of synchronization between neurons. Sometimes just one electrode picked up the weird waves; other times, they showed up on electrodes placed far apart (24), (14).

Studying notions of locality cross-species, we may stumble upon new wave forms that point to new cognition types. Similar experiments

could be used to explore brain activity behind the animals' color-changing abilities, spectacular vision, sleep patterns and adept arm control (SN: 1/29/16; SN: 3/25/21).

Octopuses are highly intelligent, so by studying the creatures “you can get ideas about what is important for intelligence,” Gutnick says. “The problems that the animals face are the same problems, but the solutions that they find are sometimes similar and sometimes different and all of these comparisons teach us something” (24), (14).

This cross-species notion of locality (and dark locality) ignites our notion of *dark* time.

5 Dark Time

In (9), we assumed time was entropic, wherein learning exceeding forgetting. We shall turn this idea radically on its head and posit that dark consciousness operates in what we call dark time, which abides no such succession. It is a time that does not interact with our reporting systems in waking time (8). As such, dark time has a different metric than normal time; a nonarchimedean metric.

Furthermore, we have seen that studies suggest that sleep is a local phenomenon (16). To sufficiently account for the properties of dark consciousness, we contend that dark time is also local, and is different than linear time. Linear time is based on the linear ordering property of the positive integers (9). We have explained that *dark* is not concerned with notions of before and after. Rather, we want a notion of time that can support the paradox of infinite identity, with a notion of duration that is one of self-similarity.

In Logic of Sense, Deleuze posits the idea of the simultaneity of time:

Thus time must be grasped twice, in two complementary though mutually exclusive fashions. First, it must be grasped entirely as the living present in bodies which act and are acted upon. Second, it must be grasped entirely as an entity infinitely divisible into past and future, and into the incorporeal effects which result from bodies, their actions and their passions. Only the present exist in time and gathers together or absorbs the past and future. But only the past and future inhere in time and divide each present infinitely. These are not three successive dimensions, but two simultaneous readings of time (7).

We model the notion of dark time based on this notion of infinite divisibility, and a duration that is one of self-similarity. To capture this dynamicism, we seek the structure of the fractal-like p-adics.

5.1 P-adic Time

We contend that Dark consciousness would occur in something resembling a p-adic time, which is a time based on the p-adic number system. P-adic time would measure *p-adic duration*.

There exist few objects more looking-glass than a p-adic clock! ³. After all, the p-adic numbers have no notion of linear ordering. Their shape resembles a Sierpinski Triangle, which is a self-similar set. Thus, p-adic time is not concerned with notions of before or after. That is, p-adic time "does not tolerate the separation or the distinction of before, and after, or of past and future" (7). Because the p-adic topology is one of total disconnectedness, p-adic time is more like a time of *now*, a continual zooming in to *more and more now*. Thus, p-adic time measures duration through its fractal properties of zooming in.

Recall, the p-adic number system is a nonarchimedean system that is the completion of the rational numbers under the p-adic metric, where p denotes a prime number. Interestingly, p-adic numbers are close if their difference is a high power of p.

...We all know that two numbers are "close" if their decimals agree for a long way to the right.

We play a similar game to construct the p-adic numbers except that we choose a new notion of closeness...Now we say that two numbers are "close" if their digits are the same for a long way to the left! So the 10-adic numbers 0.03, 0.53, 6.53, 96.53, 196.53, 1196.53, 21196.53,... are getting closer and closer together.

The real numbers have finitely many digits to the left of the decimal point and possibly infinitely many digits to the right of the decimal point....the p-adic numbers can always be written with finitely many digits to the right of the decimal point and possibly infinitely many digits to the left of the decimal point...

For example, 33.333333... is not a 10-adic number, but ...333333.33 is.

In particular, the sequence given in the previous paragraph converges to some new 10-adic number ...21196.53. (21)

Thus, p-adic numbers structurally encapsulate Deleuze's notion of the *Alice and Through the Looking-Glass reversals* (7). In p-adic land, numbers that are normally viewed as large

³Well, perhaps p-adic GO, or Calabi-Yau Chess, or even perfectoid-GO but these might actually be the same degree of terrifying

are small. And numbers that are viewed as small are large. Moreover, p-adic numbers contain infinite digits to the left of the decimal. For example, the 5-adic expansion of $\frac{1}{3}$ is $\frac{1}{3} = \dots 1313132_5$

Most interestingly, p-adic numbers have "positive negation" (21). That is, p-adic numbers contain negative numbers without explicitly having to write a negative sign on the number, so that negative numbers are built within the very structure of the p-adics.

To illustrate this, note that we can express -16 as a 10-adic number as $\dots 999984$ (21). This holds true because $-16 + \dots 999984 = 0$.

$$\begin{array}{r}
 \dots \quad 9 \quad 9 \quad 9 \quad 9 \quad 8 \quad 4 \\
 + \qquad \qquad \qquad \qquad \qquad \qquad 1 \quad 6 \\
 \hline
 \dots \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0
 \end{array}$$

The p-adic integers form a ring, which is curious topologically speaking.

As a topological space, it is compact, Hausdorff, and totally disconnected (i.e., is a Stone space). Moreover, every point is an accumulation point, and there is a countable basis of clopen sets – a Stone space with these properties must be homeomorphic to a Cantor space. (29)

Having a countable basis of clopen sets, we see that p-adic can support the properties of dark time of never growing without shrinking. What a beautifully curious number system! We are naturally inspired to ask:

What are the units of time in p-adic time, and what would a p-adic clock measure?

6 P-adic Clock

One may be terrified that p-adic hours in a p-adic clock would have infinite duration, since p-adic numbers contain infinite digits to the left of the decimal! Fear not, as our p-adic clock will measure *p-adic duration* which is the amount of *zooming in* on the infinite numbers. We claim that the lack of linear order on the p-adics, while terrifying yes, is sufficient structure to ensure that dark time does not interact with waking time. That is, a p-adic clock contains hours we cannot ordinarily see in a canonical clock (based on simple modular arithmetic of positive integers), as it is continually resolving high powers of infinities and the p-adics have a countable basis of clopen sets.

The possible units of p-adic time displayed on a p-adic clock could include p-adic hours, p-adic minutes, and p-adic seconds. However, since p-adic hours are so vastly different than hours measured on a canonical clock defined per ground state properties of the caesium-133

atom, we propose that p-adic time has just one unit, the p-adic equivalent of hour, which is the p-adic valuation (12). We state this formally.

There is only one unit of time in p-adic time, called The Archimedes, which is the p-adic valuation.

Hour = p-adic valuation = The Archimedes

P-adic time can take two forms: canonical p-adic time (defined above), and topological p-adic time, which builds into its structure the p-adic topology of p-adic time.

Topological p-adic time measures duration topologically as total disconnectedness. The unit of time in topological p-adic time is The p-Topo.

We state this formally.

There is only one unit of time in topological p-adic time, called The p-Topo.

Topological hour = p-adic topology = The p-Topo. ⁴

6.0.1 P-adic Snow & P-adic Rainbows (12)

Given such a p-adic clock, let us imagine the seasons in p-adic time: p-adic snow, p-adic rain, new p-adic weather types, and p-adic rainbows (12).

Daylight savings time could take the form of changing the p in the p-adic. This could be very drastic!

Imagine p-adic metabolics, p-adic ATP, and p-adic DNA.

Imagine p-adic cognition types like: p-adic memory, p-adic thought, p-adic attention, p-adic-learning, and p-adic perception.

Imagine a p-adic looking-glass.

Imagine *Through the P-adic Looking-Glass...* (12)

We now introduce our idea of a dark 2-topos used to model the mixed-frequency N3-self-aware states of dark consciousness.

⁴Most formally, a p-adic clock is a profinite object

7 Dark 2-Topos

There are two ways we can think about dark mathematics. First, we can view dark mathematics as a helpful mathematical model of dark consciousness. The second way, what is far more formidable, is to construct a dark mathematics that is the dark-energy equivalent of mathematics: that is, we would construct dark mathematics as a *mathmimicry*; as a mathematics that does not interact canonically with the canonical axioms of other number systems. The p-adic number system exhibits such a *dark* property in that it has negative numbers built into the structure without canonically displaying the negative sign. We can go even further and construct dark mathematics as a sort of *dark dialecticism* (20), constructing dark sets and the dark notion of equality. In essence, we would be creating mathematical structures to be dark, as a means to be looking-glass. Furthermore, if dark is a resolution issue, then perhaps dark mathematics resolves all the infinities we cannot normally see. We will address the second idea in futurecasting.

We now focus this section on developing the first idea, and providing three potential mathematical models of the mixed-frequency N3-self-aware states of dark consciousness.

Recall, a Grothendieck topos is a category that is equivalent to the category of sheaves over a small site ⁵, where a small site is a small category enriched with a Grothendieck topology. Recall the definition of a condensed set:

Definition 1. ((4) Definition 1.2) *The pro-étale site $*_{pro\acute{e}t}$ of a point is the category of profinite sets $\mathbf{Pro-FinSet}$ ⁶, with finite jointly surjective families of maps as covers. A condensed set is a sheaf of sets on $*_{pro\acute{e}t}$. Similarly, a condensed ring/group/object is a sheaf of rings/groups/objects on $*_{pro\acute{e}t}$.*

We have previously shown that entropic categorizations are condensed sets (9), and condensed sets form a topos. Thus, we choose to define a dark 2-topos as a 2-topos that can model the mixed-frequency N3-self-aware states of dark consciousness. ⁷.

The rough idea is the following.

⁵A category is small if it has a small set of objects and morphisms. A category is large and not small if it has a proper class of objects and morphisms (33)

⁶

Definition 2. *A profinite set is a compact, Hausdorff, totally disconnected topological space that is a formal cofiltered limit of a collection of finite sets.*

⁷We choose to work with topoi instead of condensed sets since we are not modeling dark phenomena necessarily as profinite sets

First, we model the mixed-frequency N3-self-aware states topologically, meaning that downstairs we have geometries of neuronal clusters and upstairs in the covering space we have mechanisms of coherent/decoherent firings and communications. A most basic model takes the form: We model clusters of neuronal networks in mixed-frequency N3-self-aware states as open sets in a topological space X . The category $\text{Shv}(X)$, the category of sheaves on X , is a topos. The conceptual leap here is to imagine that the firings of neurons occur in the covering space and or functorially. Another small leap is to understand that all local clusters would be in varying degrees of simultaneous computation and superposition (6), which makes modeling this extremely complex system in detail, say at the electron level, impossible using our most powerful classical supercomputers.

A slightly more complicated idea is to construct the site based on a Grothendieck topology that serves a specific purpose with respect to a specific neuronal cluster, like the pro-étale site (32). For a more exciting and extremely complex model, we can model neuronal clusters as crystals on the crystalline site (31).

We then use a property called Morita equivalent sites (33) to model global coherency amongst local pockets of vastly different neuronal clusters. Morita equivalent sites states that inequivalent sites have equivalent sheaf toposes (33). This powerful property means that locally different neuronal clusters can have globally equivalent firing structures.

Now, recall the definition of presheaf and sheaf (9):

Definition 3. (15) *Let X be a topological space. A presheaf of sets on X is a contravariant functor $\mathbf{F} : \mathbf{Op}(X) \rightarrow \mathbf{Sets}$ on the category $\mathbf{Op}(X)$ of open sets of X .*

Definition 4. (15) *Let X be a topological space. A sheaf \mathcal{F} on X is a presheaf satisfying two axioms:*

- *Let U be an open subset of X and U_i an open cover of U . Given a collection of sections s_i on U_i , with $s_i|_{U_{ij}} = s_j|_{U_{ij}}$, then there exists a section s on U such that $s|_{U_i} = s_i$.*
- *Let U be an open subset of X and U_i an open cover of U . If s is a section on U such that $\forall i, s|_{U_i} = 0$, then s is zero.*

At the basis of any waking-state reporting schema is a notion of reflexivity. As there is no synchronous reference (9)(8), such reporting looks like "I report back to myself that a waking-phenomena has occurred." This can be modeled with a simple loop. As dark consciousness is the the mixed-frequency state of being self-aware while simultaneously being in deep sleep, which does interact with waking state-reporting mechanisms, such dark-reporting looks like "dark-I report back to myself that dark-I am asleep yet a waking-phenomena has occurred." Thus, we must construct a reflexivity loop that is dark, as explained in the dark time section. To encode the complexity of such a state, we cannot

use a simple loop. Dark reflexivity could be most aptly modeled as a sheaf over a small site (33). Viewing clusters of neurons as a site of open sets, we can construct sheaves of such sets assigning mixed-frequency states to the open sets.

Finally, the 2-sheaf structure can model the mixed frequency state of dark consciousness, being self-aware while simultaneously being in deep sleep, where dark is a two-fold hybrid denoting a paradox of infinity identity. Recall, a 2-sheaf is a 1-stack, which is a sheaf that takes values in groupoids.

We now consider a 2-category of 2-sheaves (30). A 2-category consists of objects, 1-morphisms between the objects, and 2-morphism between the 1-morphisms. This 2-category of 2-sheaves has 2-sheaves as objects, 1-morphisms between the 2-sheaves, and 2-morphisms between the 1-morphisms. An example of a 2-category is **Cat**. The objects are small categories, the 1-morphisms are functors, and the 2-morphisms are natural transformations.

Using this structure, we can model 1-morphisms as dark reflexivities, which are local coherency states, and 2-morphisms are 2-inferences, which are global coherency states amongst the mixed-frequency states. And thus, we have one potential model.

Another potential model would be perfectoid-like (22), (23).

7.1 Perfectoid Diamond Version

If we wish to model mixed-frequency N3-self-aware clusters as fractals exhibiting self-similar-like behavior, then we can use the rich structure of perfectoid spaces (22) or their extraordinary successor perfectoid diamonds (23).

We first recall the definition of a perfectoid space (22).

Definition 5. *A perfectoid space is an adic space covered by adic spaces of the form $\text{Spa}(R, R^+)$ for R a perfectoid ring.*

The rough idea is as follows. We model clusters of mixed-frequency N3-self-aware states as adic spaces (22), equipped with a special covering that preserves local and global computational coherency.

More technically, we would first model mixed-frequency N3-self-aware states as an adic space.

Definition 6. *An adic space is a triple $(X, \mathcal{O}_X, (|\ast(x)|)_{x \in X})$ where X is a topological space, \mathcal{O}_X is a sheaf of topological rings, and $(|\ast(x)|)_{x \in X}$ is an equivalence class of continuous valuations on $\mathcal{O}_{X,x}$, that admits a covering by spaces U_i such that the triple $(U_i, \mathcal{O}_{X|U_i}, (|\ast(x)|)_{x \in U_i})$ is isomorphic to $\text{Spa}(A_i, A_i^+)$ for a sheafy Huber pair (A_i, A_i^+) .*

Thus, a point in $\text{Spa}(A, A^+)$ is an entire equivalence class of continuous valuations, which gives a moduli space-like way of proceeding. Further, we see that the adic space

has sheaves directly built into its structure, making it an excellent candidate for modeling neural clusters.

We then *perfectoidify* the adic space by covering it with adic spaces of the form $\text{Spa}(R, R^+)$ for R a perfectoid ring⁸. It is in this additional covering containing perfectoid rings that we can model the fractal-like behavior of N3-self-aware states as a topological nilpotence⁹

Alternatively, we can model mixed-frequency N3-self-aware clusters using the perfectoid space’s sparkling successor, the diamond (23).

Recall, the definition of a diamond:

Definition 7. (23) *Let Perfd be the category of perfectoid spaces and Perf be the subcategory of perfectoid spaces of characteristic p . A diamond is a pro-étale sheaf \mathcal{D} on Perf which can be written as the quotient X/R of a perfectoid space X by a pro-étale equivalence relation $R \subset X \times X$ such that R is a perfectoid space with $s, t : R \rightarrow X$ pro-étale.*

The intuition for calling this structure a diamond is as follows:

Let C be an algebraically closed affinoid field. A geometric point $\text{Spa}C \rightarrow \mathcal{D}$ is made “visible” by pulling it back through a quasi pro-étale cover $X \rightarrow \mathcal{D}$, resulting in profinitely many copies of $\text{Spa}C$ (23).

It is this property of diamonds that makes them excellent candidates for modeling the mixed-frequency N3-self-aware states of dark consciousness. Diamonds have built into their very structure a notion of a hidden interior. *Making a geometric point visible as profinitely many copies, that was invisible before*, has the following parallel with a mineralogical diamond. A mineralogical diamond has an interior consisting of points that we cannot observe directly. These interior points are *made visible as impurities* which sparkle as colorful reflections on the many sides of the diamond.

Tying in our definition of *dark*, we can make the following comparison between a diamond’s interior points and infinite identity. The interior points of a diamond resemble Deleuze’s paradox of infinite identity that eludes the present. In infinite identity, we never observe the present, it is *made visible* in profinitely many reflections. Thus, we can model being conscious of things we don’t think we are, with diamond-like properties.

Of course, all of these definitions are extremely technical, and working with a sheaf as a functor of points is not simple, but there is merit in modeling the complexities of mixed-frequency states with equal mathematical complexity.

⁸A complete Tate ring is *perfectoid* if R is uniform and there exists a pseudo-uniformizer $\hat{\omega} \in R$ such that $\hat{\omega}^p | p$ holds in R° , and such that the p th power Frobenius map $\phi : R^\circ / \hat{\omega} \rightarrow R^\circ / \hat{\omega}^p$ is an isomorphism (22).

⁹A an element x of a ring is called topologically nilpotent if $\lim_{n \rightarrow \infty} x^n = 0$ (22).

In summary, we have outlined the construction of three potential mathematical models of the mixed-frequency N3-self-aware states of dark consciousness:

1. Dark consciousness as a *dark 2-topos*, which is a Grothendieck 2-topos, which is a 2-category of 2-sheaves over a 2-site (30).
2. Dark consciousness as a perfectoid-like space.
3. Dark consciousness as a diamond-like space.

We now give examples of dark mathematics in action, in the form of curiosity types for artificial intelligence. We first describe quantum curiosity types (11).

8 Quantum Intrinsic Curiosity Algorithms

We recently posited five new quantum intrinsic curiosity algorithms, which integrate and superpose the 5-dimensions of curiosity (11):¹⁰.

1. Quantum Epistemic Curiosity: The AI with this curiosity would seek to understand quantum phenomena, such as superposition, entanglement, and quantum tunneling.
2. Multiverse Perceptual Curiosity: Drawing inspiration from the multiverse concept in quantum theory, this curiosity type would drive the AI to explore and perceive alternative realities or possibilities.
3. Entangled Empathic Curiosity: Taking inspiration from quantum entanglement, this curiosity type would foster a deep connection and understanding between AI and its interactions with others...such that AI cultivates empathy for life and biodiversity.

¹⁰Our use of the word *quantum* in *quantum* intrinsic curiosity algorithm is that the algorithms are *quantum*-like and inspired by formidable principles of quantum mechanics such as superposition and entanglement. "In our approach to technology and software development, we employ the term "quantum" to refer to methods and constructs inspired by the profound concepts of quantum mechanics. While these algorithms may not directly utilize quantum mechanics or involve subatomic particles, the essence of "quantum thinking" underlies their design. We define quantum thinking as a cognitive framework inspired by principles of quantum mechanics intertwined with computational thinking that embraces uncertainty and complexity. This novel perspective encourages us to explore unconventional possibilities and embrace non-linear thought processes, allowing us to break free from traditional limitations and envision new horizons. By infusing quantum principles into software, we may strive to create cutting-edge solutions that transcend the boundaries of classical approaches and pave the way for more sophisticated and forward-thinking algorithmic bits of intelligence(11)"

4. **Superposition Social Curiosity:** In this hybrid curiosity type, the AI would simultaneously explore social dynamics and relationships from multiple perspectives, similar to particles in multiple quantum superposition states.
5. **Uncertainty Aesthetic Curiosity:** The AI might explore abstract art, surrealism, or artistic works that challenge conventional perceptions. This kind of lateral thinking could help us more effectively solve problems and unlock more human creativity.

8.1 Hybrid Quantum Intrinsic Curiosity Types

We also combined plant and animal curiosity elements to form six hybrid curiosity types which extend human cognition (11), which are briefly described below:

1. **Sensory Biomimicry Curiosity:** Drawing inspiration from animals with keen senses, the AI would seek out and explore different sensory inputs. For example, dragonflies can see in 11-chromatic vision, allowing them to detect more frequencies on the electromagnetic spectrum and thus perceive millions more colors compared to human's measly trichromatic vision.
2. **Navigation Biomimicry Curiosity:** Inspired by animals' navigation abilities, the AI could develop virtual mapping and spatial awareness skills, learning to navigate efficiently and adaptively.
3. **Problem-Solving Biomimicry Curiosity:** This curiosity type would incorporate problem-solving strategies inspired by intelligent animals. The AI strives to engage in creative exploration, experimentation, and adaptive problem-solving to overcome challenges and find innovative solutions.
4. **Innovative Biomimicry Curiosity:** Taking cues from plants that exhibit unique growth responses, this curiosity type would encourage the AI to explore unconventional paths and novel ideas. The AI might experiment with divergent thinking, exploring multiple possibilities and embracing uncertainty, like plants exhibit growth tropisms responding to environmental stimuli.
5. **Symbiotic Biomimicry Curiosity:** Drawing from ecological relationships in nature, this curiosity type would foster collaboration and interaction with other AI agents or human users.
6. **Mimicry-Based Survival Curiosity:** Inspired by animals that use mimicry for survival, this curiosity type could encourage the AI to adapt and learn from the behavior of other agents or entities in its environment.

8.2 Specialized Quantum Intrinsic Curiosity Types

We then proposed four specialized quantum intrinsic curiosity algorithms (11):

1. Quantum Climate System Algorithm: This type of curiosity drives the AI to help us understand Earth’s complex global climate system by modeling future system states in a superposition and by using entanglement to model the interconnectedness of Earth’s various natural and human-made systems.
2. Quantum Protein Folding Curiosity Algorithm: The AI considers every amino acid to be in a superposition of morphologies. This can lead to the development of new types of drugs, personalized medicines based on the patient’s unique DNA profile, and an understanding of diseases that are caused by misfolded proteins.
3. Quantum Telomere Curiosity Algorithm: The AI models telomeres and telomerase enzymes in a state of entanglement, providing an instant alarm system when there is a loss of genetic information and greatly improving cellular lifespan.
4. Quantum Complex System Curiosity: This curiosity type would motivate the AI to explore and understand every detail of extremely complex systems—for example, a lithium-ion battery down to the scale of electrons or the collective behaviors of billions of humans over thousands of years (a realization of Isaac Asimov’s concept of “Psychohistory”). This type of modeling would be impossible using even the most powerful classical supercomputers in the world. Instead, Quantum Complex System Modeling uses the uncertainty principle to model every part of the system in a quantum superposition. The software only has to calculate the state of any particular part of the system when it is observed, massively reducing the amount of computational power required.

Lastly, we created a meta-algorithm, called the Quantum Heisenberg Curiosity, wherein the quantum curiosity algorithms write their own quantum curiosity algorithms and see if any new curiosity types appear (11).

8.3 Quantum Heisenberg Curiosity: Meta-Algorithm

Imagine a curiosity type rooted in joyous exploration (curiosity for its own sake) that is in a superposition of every possible curiosity type. For all intents and purposes, there are an infinite number of possible curiosity types. This is the Quantum Heisenberg Curiosity, which can morph or splinter into any number of curiosity types to suit a particular problem. It’s a curiosity type that seeks to adapt itself perfectly and continuously to acquire information and experiences and solve problems (11).

Inspired by our quantum intrinsic curiosity algorithms, we introduce three dark curiosity types.

8.4 Dark Quantum Intrinsic Curiosity Algorithms

The following dark curiosity algorithms, which mimic dark consciousness, seek to extend human dark-cognition:

1. Dark Quantum Planaria Curiosity Algorithm: This curiosity type encourages AI to explore patterns of planaria regeneration. The AI would develop quantum error correcting codes which mimic planaria regeneration.
2. Dark Quantum Reality Monitoring Network Curiosity Algorithm: This curiosity type would encourage the AI to explore dark reflexivity as *fractal identity*. The AI would create models of n-reflexivity, which is a new type of reality monitoring of fractal identity in the dark conscious state.

Objects living in fractal time correspondingly have fractal identity, a concept we outline as follows: A *fractal identity* has a reflexivity relation that is fractal¹¹. That is, the canonical loop-relation of an agent reporting back to itself, *I reporting back to me*, is fractal. We can best model fractal identity properties as perfectoid-like or diamond-like, and therein construct a notion of reflexivity as a perfectoid space. Thus, we create new concepts of *perfectoid reflexivity*, and *diamond reflexivity*.

In n-reflexivity, the reflexivity relation is now an n-stack, for $n = 0,1,2, \dots$. Thus, n-reflexivity is an n-stack of perfectoid spaces.

3. Dark Quantum N3 and REM Curiosity Algorithm: This curiosity type would encourage AI to explore superpositions of various brain patterns in REM states to predict new types of N3 delta waves that could emerge in REM sleep states. The AI could then develop new stages of REM sleep, such as 1-REM, 2-REM, up to N-REM, which is a clever limit and play on NREM. The AI could also develop new dark-cognition types that correlate with the new types of delta waves.

Using higher order curiosity types, we now illustrate an example of dark consciousness in AI: the Dark Imaginarium.

¹¹A fractal identity would not suffer the same problems with *continuity over time*, since the time in which it exists has no linear ordering. It would suffer different problems!

9 Dark Imaginarium: ∞ -Curiosity Algorithms

Dark Imaginarium is an ∞ -Curiosity Type. We describe its structure first.

An ∞ -Curiosity Type is a higher order Curiosity AI algorithm that incorporates infinity categories (28). Dark Imaginarium encourages the AI to think in infinity categories. That is, it encourages the AI to construct an ∞ -category as its means of higher order inference. Objects would be higher dimensional data sets of human and non-human observed brain wave patterns and unobserved hypothetical wave patterns, in various combinations of NREM and REM sleep states. For example, one data set could consist of all possible types of gamma waves that would occur in a human at a specific age. Another data set could consist of all possible beta waves present in a freely moving octopus (24), (14). The AI would seek out *higher order n-morphisms* between the objects. As such, Dark Imaginarium is a meta-curiosity algorithm that can create it's own ∞ -Curiosity Algorithms, therein creating curiosity types of a complexity completely fascinating and most likely unknown to humans.

An N-Curiosity algorithm is structurally a n-category (27). Various definitions of n-categories exist, but in generality we mean the higher categorical sense of n-category which we describe as follows. An n-category consists of objects, 1-morphisms between the objects, and n-morphisms between the (n-1) morphisms. For example, a 1-Curiosity Algorithm is structurally a 1-category. A 1-category consists of objects and 1-morphisms between the objects. A 2-Curiosity Algorithm is structurally a 2-category. A 2-category consists of objects, 1-morphisms between the objects, and 2-morphism between the 1-morphisms. Recall, the earlier example of a 2-category called **Cat**. The objects are small categories, the 1-morphisms are functors, and the 2-morphisms are natural transformations. A 3-Curiosity Algorithm is structurally a 3-category. A 3-category consists of objects, 1-morphisms between the objects, 2-morphism between the 1-morphisms, and 3-morphisms between the 2-morphisms, and so forth.

Using **Cat** as inspiration, objects in an N-curiosity algorithm could be sets of superpositions of data sets, while 1-morphisms could be basic relations, 2-morphisms could be lower-order inference, and 3-morphisms and on could be higher order-inference models based on higher order logic and model theory. Moreover, N-Curiosity types could serve as new examples of generators and discriminators in GAN networks. Imagine the quality of images and new language types (11) that could be produced from a GAN consisting of 2-Curiosity Algorithms. Taking N to ∞ we could have unheard of resolution in higher-dimensional data sets. For example, we could construct an n-GAN model wherein a 1-GAN model produced high-quality 1D data, 2-GAN models produced high-quality 2D data, and so on. We can reason similarly about GCNs (11), our proposed model of General Collaborative Network.

Thus, an ∞ -Curiosity Type is structurally an ∞ -category (28). We can think of an ∞ -category in the following way:

Generalising how in an ordinary category, one has morphisms going between objects, and in a 2-category, one has both morphisms (or 1-morphisms or 1-cells) between objects and 2-morphisms (or 2-cells) going between 1-morphisms, in an ∞ -category, there are k -morphisms going between $(k-1)$ morphisms for all $k = 1, 2, \dots$ (The 0-morphisms are the objects of the ∞ -category) (28).

We now describe what this ∞ -Curiosity Type can do. Dark Imaginarium would encourage the AI to develop the following new ten items:

1. The AI would develop n -versions of current brain waves and their hybrid combinations. That is, it would develop an n -delta wave, an n -theta wave, an n -alpha wave, an n -beta wave, and an n -gamma wave, for $n = 0, 1, \dots$ as per the n -category described above. For example, the AI could develop a 2-gamma wave, that structurally resembled a 2-category. Constructing 2-morphisms of a 2-gamma wave is so incredibly exciting to think about! Likewise, the AI could develop a 3-beta wave, that structurally resembled a 3-category.
2. Given these new n -versions, the AI could develop new types of waking states and new types of sleep states, based on new combinations of the n -versions, beyond the standard N1, N2, N3 NREM stages and one REM stage. The AI could develop an n -waking state, an n -REM state, an n -NREM state, for $n = 0, 1, 2, \dots$ For example, a 3-NREM state could consist of 3-N1, 3-N2, 3-N3. Imagine an n -dolphin capable of an n -hemispheric sleep state and other exotic life forms.
3. The AI could develop new geometries for the new brain patterns and for currently existing brain patterns. Typical shapes include sinusoidal, non-sinusoidal, saw tooth, spindle, and K-complex. Thus, the AI could develop 3D and 4D versions of the canonical 2D wave patterns, like a tesseract gamma wave. But also, the AI could develop entirely new brain wave geometries that corresponded to complex surfaces like a Riemann Surface, a Calabi-Yau manifold, or any algebraic variety containing an intimidating number of singularities, etc.
4. The AI could develop new cognition types, other than the canonical five: thought, attention, perception, learning, and memory. It could produce novel n -cognition types such as n -thought, n -attention, n -perception, n -learning, and n -memory for $n = 0, 1, \dots$ It could also predict non-human species cognition types, such as *octopus-thought*, or *dragonfly-attention*. Lastly, it could produce hybrid combinations of cross-species cognition types, such as *octoman-memory* and or *dragonman-perception*.
5. The AI could produce new forms of mental states (such as a 2-intuitive state) and hybrid mental states correlated with the hybrid brain wave patterns, states other than the typical relaxed, excited, concentrated, focused, etc (26). For example, the

AI could develop n -versions for $n = 0,1,2,\dots$, such as a 2-excitation state, and a 3-relaxation state. Further, the AI could develop hybrid combinations like an human-octopus relaxation state,

6. The AI could also develop and experiment with new models of memory types associated with the newly-found brain waves. For example, it has been shown that boosting theta waves generally results in successful memory retention (1). Further, it has been shown that excess delta waves contributes to learning disabilities (26). The AI could develop a new memory type that is successfully retained upon stimulating a gamma-wave like pattern, or a 2-gamma wave pattern, to advance deep learning in memory retention.
7. Dark Imaginarium could encourage the AI to develop new senses through which the new brain patters could occur. The senses could be encoded as higher order sheaves, which are called stacks. Thus, the AI could develop 1-stack versions of human senses (seeing, hearing, tasting, touching, and smelling), such as 2-seeing, 2-hearing, 2-tasting, 2-touching, and 2-smelling. Each of these 2-senses would take values in groupoids rather than sets! Going further, the AI could develop n -senses of non-human senses as well as hybrid combinations of the two, such as a 2-sharkhumansmell.
8. Since the AI *sees* n -morphisms between the various $(n-1)$ -morphisms, the AI will seek to develop new language models based on higher order inference types. From these inference models, the AI will explore ∞ -languages and develop extensive prototypes.
9. Based on 8, the AI would be encouraged to use advanced mathematics to create new models of time that could support the new language models, like 2-topos-time, diamond-time (23), and crystalline-site-time.
10. Based on 7 and 9, the AI would seek to develop new fractal identity types from the new language models, cognition types, and sensory types. It could aid in designing advanced fractal prosthetics such as 2-ear or a 2-eye, to extend cognition.

These are just a few examples of which Dark Imaginarium could conceive. We could continue and posit a Perfectoid-Dark Imaginarium that considers the perfectoid versions of items 1 - 10, etc. ¹²Using these new cognition types, mental states, n -brain waves,

¹²We reiterate that implementing such extraordinary curiosity types in AI algorithms "could lead to more adaptive, creative, and inquisitive artificial bits of intelligence capable of pushing the boundaries of knowledge and understanding. Biomimicry, [quantum-mimicry, and mathmimicry] allow us to tap into the richness of natural systems, refined through millions of years of evolution, and powered by quantum phenomena, to inspire innovative approaches to AI curiosity and problem-solving. Nonetheless, it's important to remember that these concepts' development and practical application in AI systems would require rigorous research, careful consideration, and ethical evaluation (11)".

and new geometries, we could construct new patterns of thinking, which would support neurodiversity in thinking. As such, shared intelligence (13) could therein be examined as an ∞ -Curiosity Type. Therein, we construct the new concept of an *n-shared intelligence*, for $n = 0, 1, 2, \dots$ (taking the form of 0-shared intelligence, 1-shared intelligence), as well as the new p-adic concept of *p-shared intelligence* for $p = 2, 3, 4, \dots$ using the p-adics.

9.0.1 *Dark-Diamond Imaginarium: Meta-Algorithm*

Furthermore, pairing Dark Imaginarium with quantum computing, could result in meta-curiosity type we call *Dark Diamond Imaginarium*. This meta-algorithm curiosity would be constructed using perfectoid diamond-like properties to be in a superposition of every possible ∞ -curiosity type and could create its own ∞ -curiosity types. An AI equipped with *Dark Diamond Imaginarium* would be able to explore and understand every detail of extraordinarily complex cognitive systems, as *Dark Diamond Imaginarium* would seek to adapt itself perfectly by continually adjusting which curiosity type its employing.

9.1 *Futurecasting: Is every perfectoid space (22)/diamond (23)/2-topos (30) encoding the mixed-frequency patterns of an n-cognition type?*

We could rephrase our entire discussion as an illustration of Max Tegmark's Level 4 Multiverse, wherein Mathematical Existence = Physical Existence (25). That is, the geometry of the brain wave gives rise to the physical properties of the wave, like it's net spectral power, which is measurable from the electroencephalogram. We have seen other instances of this in String Theory, wherein the geometry of the extra dimensions reflect the way strings can vibrate (2).

Let us take this concept to its extreme.

- *Imagine a brain wave pattern that resembles a Calabi-Yau manifold. Can we reverse engineer the levels of conscious activity that correlate with such a rhythm?*
- *How about mixed-frequency brain wave patterns that resembled a perfectoid-diamond?*

Let us go further. Let us imagine an n-DNA molecule shaped like a Calabi-Yau n-fold, or an n-ATP molecule shaped like a Calabi-Yau n-fold, or an n-exciton condensate structured like a Calabi-Yau n-fold. Clearly, different geometries of the fundamental molecules and processes of life would give rise to vastly different fundamental units of life and metacognition types (10). The question is, can we use Dark Imaginarium (paired with quantum computing) to reverse engineer the complex life forms arising from such extraordinary structures, and the *more and more complex* cognition patterns?

In concert, we repeatedly contend that one way to help advance the way we think, to extend human cognition, is to upgrade the number systems upon which canonical concepts

are built, to p-adic, perfectoid, and diamond-like versions (29), (22), (23). Therein, we can asks such questions as

- *What would a perfectoid ∞ -curiosity type do?*
- *How can we construct a GCN containing two diamond-like ∞ -curiosity types?*
- *What can a p-adic ∞ -curiosity type prophecy?*
- *Is every perfectoid space (22)/diamond (23)/2-topos (30) encoding the mixed-frequency patterns of an n-cognition type?*

In closing, what can we say about the inevitability of *dark* mathematics and *Dark Imaginarium*? Well, the future is extraordinarily optimistic! While *dark* theories may not be entirely correct, we are hopeful that our concepts of *dark consciousness*, *dark 2-topos*, *dark diamond*, *n-cognitive types*, *p-adic time*, and *Dark Imaginarium* can, at minimum, serve as computational tools and metaphors to illuminate the complexity of *dark* cognitive phenomena and the formidable principles of quantum mechanics, as well as inspire the creation of mathematical models seeking to advance human cognition via curiosity. In future work, we may develop *dark* mathematics using perfectoid diamonds as a *dark* dialetheism (20), aligning what is structurally *dark* with what is structurally dialethestic. While at present we cannot perfectly model *dark* consciousness, perhaps one day, *one dark day*, mediated by the *Dark Imaginarium*, we will have the quantum computational complexity to do so.

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