Events and Memory in Functorial Time I: Localizing Temporal Logic to Condensed, Event-Dependent Memories

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Abstract

We develop an approach to temporal logic that replaces the traditional objective, agent- and event-independent notion of time with a constructive, event-dependent notion of time. We show how to make this event-dependent time entropic and hence well-defined. We use sheaf-theoretic techniques to render event-dependent time functorial and to construct memories as sequences of observed and constructed events with well-defined limits that maximize the consistency of categorizations assigned to objects appearing in memories. We then develop a condensed formalism that represents memories as pure constructs from single events. We formulate an empirical hypothesis that human episodic memory implements a particular constructive functor.

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1 Introduction

Temporal logics in the lineage of A. N. Prior's (1957) tense logic **TL** are well developed and have been broadly applied to formalize temporal reasoning in ordinary language and, particularly since Pnueli (1977), execution traces generated by linear or concurrent computions (for reviews see Fisher, 2008; Hodkinson & Reynolds, 2007). "Time" in such logics may be discrete or continuous, linear or branching; in either case it is unidirectional, i.e. representable as a directed acyclic graph (DAG). "Events" are effectively possible worlds

(PWs), sets of propositions that may, but do not necessarily, refer to past or future events, including past or future events on specific branches and at specific times. While events are labeled by and hence notionally dependent on time as a parameter, time is treated as independent of the events that it labels. This "objective" notion of time is familiar from ordinary language, classical physics, and nonrelativistic quantum mechanics (e.g. Landau & Lifshitz, 1958).

Here we consider models in which events are primitive and time is an inferential construct. This view of time as event-dependent is motivated by observer-dependent, entropic definitions of time (Rovelli, 2017; Tegmark, 2012) and by cosmological models in which spacetime is emergent from underlying informational processes (e.g. Arkani-Hamed & Trnka J. , 2014; D'Ariano & Perinotti, 2017; Pastawski *et al.*, 2015; Swingle, 2012). Similar conceptions of time as nonobjective can be found in constructivist approaches to cybernetics (Fields *et al.*, 2017; von Foerster, 2003) and to the psychology of perception (Fields *et al.*, 2018; Hoffman, Singh & Prakash, 2015), and in some postmodernist philosophical thinking (Baudrillard, 1983; Deleuze & Guattari, 1987). We show how this event-dependent conception of time provides the flexibility needed to model phenomena such as episodic memory and object-identity tracking (Fields, 2016; Scholl, 2007) in a realistic way.

We begin by showing that events specified by finite numbers of n-ary relations on finite numbers of objects can be redescribed by finite multi-hypergraphs (MHGs). Events here correspond to "event files" as defined by Hommel (2004) in the short-duration limit; objects correspond to time-stamped "object tokens" as defined by Zimmer & Ecker (2010). These structures naturally give rise to an entropic conception of time. Time in this representation becomes both discrete and functorial, consistent with its functorial nature in topological field theories (Atiyah, 1988). We then reformulate events in the more expressive language of sheaves, and show how memories, whether retrospective or prospective, can be viewed as intermediate steps between an experienced event and a sheaf-theoretic limit that captures the maximum information available in that event. As a final step, we employ the methods of Clausen & Scholze (2021) to condense memories onto a notional point interpretable as the present, in the process demonstrating the construction of extended "past" and "future" representations from this point. This enables us to make a specific, formal prediction about the implementation of episodic memory in humans. We close by reformulating the standard operators `Always`, `Sometimes`, and `Never` as localized operators on memories constructed from the present.

2 Events as MHGs

Barwise & Perry (1983) define an "event" as a finite set of individual objects $\{a_1 \dots a_n\}$, a set of unary properties $P(x)$, a set of binary relations $R(x, y)$, and a collection of metadata including "situational state" and a spacetime location. Such events are clearly amenable to organization with standard spatial and temporal logics that treat spacetime location as event-independent.

Somewhat broader concepts of "events" have been introduced by psychologists, beginning with the "event file" defined by Hommel (2004), a transient representation of objects, motions, and actions as well as affective states and motivations of agents including the self, information that would be attached as metadata to an event as defined above. Event files capture an "instantaneous" situation, including occurent actions, in a short-duration limit of approximately 350 ms (Zmigrod & Hommel, 2011), but can also represent temporally-extended "events" when time is suitably coarse-grained (see also Altmann & Ekves, 2019; Cohn-Sheehy & Ranganath, 2017, for more recent extended event models).

Here we define an (instantaneous) *event* as follows:

Definition 1. *An (instantaneous)* event $\mathbb{V} = (\mathbb{A}, \mathbb{R}, \mathbb{M})$ *comprises a finite set* $\mathbb{A}$ *of* n objects*, a finite set* $\mathbb{R}$ *of unary to* $(n-1)$*-ary relations together with a unique* n*-ary relation* V*, and a finite list* $\mathbb{M}$ *of metadata.*

The relation $V(a_1, \dots a_n)$ indicates that the objects $a_1, \dots a_n$ all occur in the single named event $\mathbb{V}$, which can be thought of informally as a component of an instantaneous state of a PW. We will also use the simplified notion $a, b, c, \dots$ for objects and $P, Q, R, \dots$ for relations within an event. The metadata may include labels such as 'occurent percept' or '(episodic) memory' as discussed in §7 below. Definition 1 generalizes that of Barwise & Perry (1983) by allowing $(n-1)$-ary relations, but restricts it by removing spatial and temporal labels from the metadata. Spatial relations and hence spatial labels do not concern us here; temporal relations are constructed as discussed in §4 below.

While the notation of 1^{st} order logic is traditional, here we will employ the alternative notation of MHGs for reasons that will be come clear.

Definition 2. *A* multi-hypergraph (MHG) *comprises a finite set* $\mathbb{N}$ *of* m *labeled* nodes *and a finite set* $\mathbb{H}$ *of unary to* m*-ary labeled* hyperedges.

Note that the number of hyperedges on any collection of $1 \leq m$ nodes is unrestricted, unlike in a standard hypergraph. If two MHGs G and H each contain a node with some particular label, we will say that they *share* the node with that label.

Definition 3. *An* MHG morphism *is a map $f : G \to H$, G and H MHGs that share at least one node, that 1) adds or deletes one or more nodes to/from G and/or 2) adds or deletes one or more hyperedges to/from G.*

Each MHG G has a unique associated MHG morphism $\mathrm{Id}_G : G \mapsto G$ that leaves the nodes and hyperedges of G fixed, and MHG morphisms obviously compose associatively; MHGs and their associated morphisms thus define a category **MHG**.

We will be particularly interested in MHGs in which the nodes are interpreted as objects, the hyperedges are interpreted as relations, and there is only a single n-ary hyperedge V, n the number of nodes/objects, interpreted as indicating that the objects $a_1, \ldots a_n$ co-occur in a single event. We will consider such MHGs to have attached metadata and will for simplicity also refer to them as "events". MHG Morphisms that preserve these interpretations clearly compose associatively; hence we can think of these metadata-labeled MHGs as composing a subcategory of **MHG** that is effectively a "category of events" as described more precisely below.

3 Categorization and object identity

Beginning in early infancy, humans segregate perceived objects from the "background" and assign them to cognitive categories, e.g. as being a person, chair, tree, etc. based on their properties and relations (Baillargeon, Spelke & Wasserman, 1985; Xu, 1999). It is commonplace to treat categorization as hierarchical, with a category name such as <thing> as the root. Formally, we assume a finite, rooted DAG Cat in which each node i is labeled with a set $\mathbb{C}_i$ of finite descriptors and each downward-directed edge $i \to j$ represents specialization to a less-inclusive category j.

Definition 4. *A* categorization *is an assignment of one or more additional hyperedges, each labeled with some sub-DAG of* Cat*, to each of the nodes (i.e. objects) in an MHG representing an event.*

We employ the term "categorization" to emphasize that assigning a category descriptor to an object is a cognitive process that attaches further semantic information, beyond that of observed inter-object relations, to an event. We can represent an event with its attached categorization as in Fig. 1. When drawn this way, it is clear that the categorized event is itself an MHG; we can therefore view a categorization C as an MHG morphism:

$$C : (\mathbb{A}, \mathbb{R}, \mathbb{M}) \mapsto (\mathbb{A}, (\mathbb{R} \sqcup \oplus_i \mathbb{C}_i), \mathbb{M}) \tag{1}$$

with $\sqcup$ disjoint union and the $\mathbb{C}_i$ sets of descriptors labeling nodes in the relevant sub-DAG of Cat.

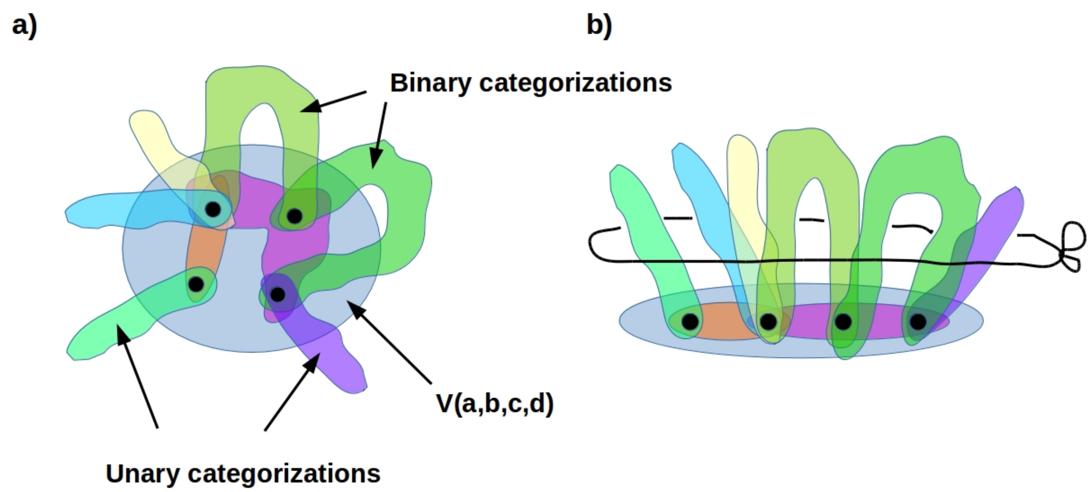


Figure 1: a) A 4-object event $\mathbb{V}(a, b, c, d)$ with 2-ary (orange) and 3-ary (magenta) relations between objects, "decorated" with unary and binary categorizations. b) The categorizations in a) can be regarded as "bundled together" (indicated by knotted string) over the event $\mathbb{V}$ to emphasize that they "add information" to $\mathbb{V}$.

An "object token" as defined Zimmer & Ecker (2010) represents a categorized object within a given event. *Identifying* an object as an individual assigns it to a *singular category*, a leaf node of Cat. The features, i.e. unary relations, of an object are often sufficient for individual identification, e.g. when recognizing a friend by her face. Beginning in childhood, humans also employ history information for individual identification (see e.g. discussion in Fields (2012)); we defer consideration of histories to §7.

4 Inter-event object identity induces temporal arrows

Now consider two events $\mathbb{V}_1(a, b, c)$ and $\mathbb{V}_2(a, b, c, d)$ as shown in Fig. 2. The objects a, b, and c are present in both events, but what constitutes the *evidence* that a, for example, reappears in $\mathbb{V}_2$? This is the question of individual identification, a question that remains unresolved despite decades of experimental work and millenia of philosophical speculation (Fields, 2016; Scholl, 2007). The only kind of evidence thus far defined is categorization: here the object a has been assigned the same (possibly singular) categorization in both $\mathbb{V}_1$ and $\mathbb{V}_2$, the object b has been assigned different categorizations in the two events, and the object c in $\mathbb{V}_1$ is joined by a new member, d of the same category in $\mathbb{V}_2$. We can consider these categorizations to indicate, effectively, "hypotheses" that a, b, and c are shared by $\mathbb{V}_1$ and $\mathbb{V}_2$, and that c and d are related but non-identical. With this interpretation, we can view the categorizations as inducing a map:

$$T_{12} : a \mapsto a; b \mapsto b; c \mapsto c; \mathbb{V}_1 \mapsto \mathbb{V}_2$$

This map is clearly an event morphism. We will call this map T_{12} a "time" map from $\mathbb{V}_1$ to $\mathbb{V}_2$ as it captures the intuition that time is what connects events that share at least one object. In general, we can write:

$$T_{ij} : \mathbb{V}_i \mapsto \mathbb{V}_j \tag{2}$$

for distinct events $\mathbb{V}_i$ and $\mathbb{V}_j$. Directionality is imposed by requiring that T be a partial order on the set of all events, with the maps T_{ij} becoming arrows or compositions of arrows in the Hasse diagram for T. Directionality in this sense forbids temporal loops, but allows any event to have arbitrarily many "pasts" and "futures"; hence it fully capures the branching time constructs common in concurrency applications. Note that past and future events may be isomorphic; no entropic or other means of distinguishing past from future from past events by content is assumed.

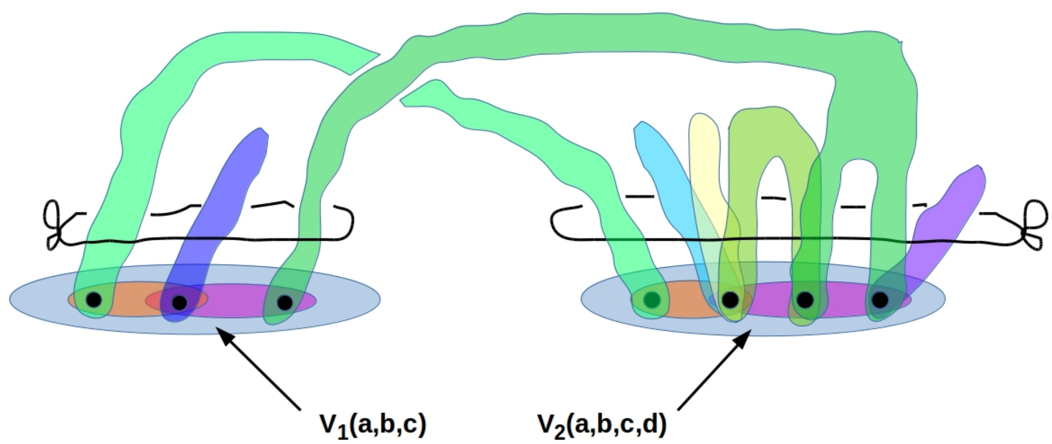


Figure 2: Two events joined by shared categorizations; T_{12} is the induced time morphism.

With this notation, we can characterize the **Evt** subcategory of **MHG** as the subcategory in which the objects are events with categorizations and the morphisms T_{ij} between distinct events are supplemented by identities that (abusing notation) can be written T_{ii}.

5 Entropic categorization as adjoint time

To further investigate the relationship between categorization and time, it is convenient to consider the functor:

$$\mathbf{f} : \mathbf{Evt} \rightarrow \mathbf{Chu}$$

where **Chu** is the category of Chu spaces $(O, A, \models)$, O a set of objects, A a set of attributes, and $\models \subseteq O \times A$ a satisfaction relation (Barr (1979); see Fields & Glazebrook (2019a) for discussion and examples). Morphisms within **Chu** are adjoint pairs $\overrightarrow{f}, \overleftarrow{f}$ such that for Chu spaces $(O, A, \models)$ and $(O', A', \models')$:

$$\begin{aligned}\overrightarrow{f} :& O \to O' \\ \overleftarrow{f} :& A' \to A\end{aligned}$$

such that $\forall o \in O$ and $\forall a' \in A'$, $\overrightarrow{f}(o) \models' a'$ iff $o \models \overleftarrow{f}(a')$. In the Channel Theory language of Barwise & Seligman (1997), the adjoint pair $\overrightarrow{f}, \overleftarrow{f}$ is an "infomorphism" between "classifiers" representable as Chu spaces (again see Fields & Glazebrook (2019a) for discussion and examples).

If we now consider events $\mathbb{V}_i$ with objects $\mathbb{A}_i$ and relations $\mathbb{R}_i$ and $\mathbb{V}_j$ with objects $\mathbb{A}_j$ and relations $\mathbb{R}_j$ such that:

$$\mathbf{f} : \mathbb{A}_i \mapsto O; \mathbb{A}_j \mapsto O'; \mathbb{R}_i \mapsto A; \mathbb{R}_j \mapsto A',$$

we can write $T_{ij} : \mathbb{V}_i \mapsto \mathbb{V}_j$ as the adjoint pair:

$$\overrightarrow{T_{ij}} : \mathbb{A}_i \to \mathbb{A}_j \tag{3}$$

$$\overleftarrow{T_{ij}} : \mathbb{R}_j \to \mathbb{R}_i \tag{4}$$

The forward component $\overrightarrow{T_{ij}}$ is the "time" through which objects evolve, while the backward, adjoint component $\overleftarrow{T_{ij}}$ is the "time" of (retrospective) memory, restricted to require that every relation – including every categorization – in $\mathbb{R}_j$ maps to a unique relation in $\mathbb{R}_i$. This restriction is effectively entropic: it requires $Card(\mathbb{R}_i) \leq Card(\mathbb{R}_j)$, which can be written as a "Second Law" $S(\mathbb{R}_i) \leq S(\mathbb{R}_j)$ by defining an "entropy" $S(\mathbb{X}) = \log(Card(\mathbb{X}))$ for any set $\mathbb{X}$. We can then define:

Definition 5. *An* entropic categorization *is a categorization of objects shared by events* $\mathbb{V}_i$ *and* $\mathbb{V}_j$ *for which the adjoint time operator* $\overleftarrow{T_{ij}}$ *in Eq.* (4) *is well-defined.*

In general, events involve varying numbers of objects and relations; hence a categorization may fail to be entropic if it joins a current event involving few relations to a past event involving many relations. Using Eq. (1) to write:

$$\mathbb{R} = \mathbb{R}^{core} \sqcup \oplus_k \mathbb{C}_k$$

We can see that the condition:

$$(S(\oplus_k \mathbb{C}_{jk}) - S(\oplus_k \mathbb{C}_{ik})) > (S(\mathbb{R}_j^{core}) - S(\mathbb{R}_j^{core})) \quad (5)$$

guarantees increasing entropy from $\mathbb{V}_i$ to $\mathbb{V}_j$ and hence well-defined adjoint time T_{ij}. We can interpret Eq. (5) as requiring that entropic categorizations "build in past relational information" about the objects they categorize. This lays a foundation for viewing entropic categorizations as "histories" of objects in §7. Hence while such categorizations carry no explicit temporal information, they can be viewed as encoding expectations about how an object will behave in future events. Such expectations enable prospective memory, i.e. prediction of and planning for future events.

6 Entropic categorizations as presheaves

We use the language of sheaves to investigate the construction of temporal relations between events, leading eventually to the construction of temporal histories of individual objects, and the binding together of such histories into episodic memories (Tulving, 2002) and generative models (Friston, 2010). We will see that such constructions can fail to be unique, producing a "plurivocity" of distinct times and "identities" for objects.

To begin, we note that by adding a null, i.e. 0-ary relation $\bar{\mathbb{V}}$ to each event, we can re-express events as discrete topological spaces in which every subset $\{a_k\} \in \mathbb{A}$ of objects related by some relation $R(a_k) \in \mathbb{R}^{core}$ is an open set. Recall the idea of a presheaf:

Definition 6. (Hartshorne, 1977) *Let X be a topological space. A* presheaf of sets *on X is a contravariant functor $\mathbf{F} : \mathbf{Op}(X) \rightarrow \mathbf{Sets}$ on the category $\mathbf{Op}(X)$ of open sets of X.*

Lemma 1. *Entropic categorizations are presheaves on events.*

Proof. Let $\{\mathbb{V}_i\}$ be a set of of events with an entropic categorization C. For any event $\mathbb{V}_i$, $\mathbf{F} : \mathbb{A}_i \rightarrow \{\mathbb{C}_{ik}\}$; i.e. $\mathbf{F}$ maps each object to its category labels as assigned by $C(\mathbb{V}_i)$. Hence $\mathbf{F}$ inherits compositionality and respect for identities from C. Contravariance of $\mathbf{F}$ is guaranteed by contravariance of $\overleftarrow{T_{ij}}$ whenever C is entropic. □

We can think of $\mathbf{F}$ as mapping each event $\mathbb{V}_i$ to its categorization $C(\mathbb{V}_i)$; hence we will abuse notation slightly and write:

$$\mathbf{F} : \mathbb{V} \mapsto C \tag{6}$$

to indicate that C is an entropic categorization of the set $\mathbb{V} = \{\mathbb{V}_i\}$ of events, viewed as a presheaf $\mathbf{F}$.

7 Constructed events and memories

Presheaves of events provide a natural representation for events as experienced. A key aspect of the construction of time, however, is the assumption that between any two distinct experienced events, other events occured that were not experienced. We all, for example, assume that various things happened between yesterday evening and this morning, and are eager to fill in knowledge of these happenings by checking the morning news. This "filling in" process is, moreover, essential to the maintenance of object identity through time, which requires the generation of fictive (i.e. unobserved) causal histories that explain what objects were doing between events of observation (Fields, 2012, 2013). Such fictive histories allow us to make immediate judgements about, for example, whether it is plausible that Jones was in Paris last weekend.

Consider now a set $\mathbb{V} = \{\mathbb{V}_i\}$ of observed events over which some entropic categorization C defines a presheaf $\mathbf{F}$ via Lemma 1 above. Now recall:[1]

Definition 7. *A* profinite set *is a compact, Hausdorff, totally disconnected topological space that is a formal cofiltered limit of a collection of finite sets.*

The elements of a profinite set constructed as a limit of $\mathbb{V}$ would "fill in" event-like elements "between" the observed events in $\mathbb{V}$ while maintaining the discrete topology. Call these filled-in elements *constructed* events $\widetilde{\mathbb{V}_j}$ and consider an entropic categorization $\widetilde{C}$ over a profinite set $\widetilde{\mathbb{V}} = \mathbb{V} \sqcup \{\widetilde{\mathbb{V}_j}\} = \{\mathbb{V}_i\} \sqcup \{\widetilde{\mathbb{V}_j}\}$ such that:

$$\begin{array}{ccc} \widetilde{C} & \dashrightarrow & C \\ \uparrow{\scriptstyle \widetilde{\mathbf{F}}} & & \uparrow{\scriptstyle \mathbf{F}} \\ \widetilde{\mathbb{V}} & \xrightarrow{Proj} & \mathbb{V} \end{array}$$

commutes, where $Proj$ projects the observed events $\mathbb{V}$ out of the profinite limit $\widetilde{\mathbb{V}}$. The

[1] All standard definitions not otherwise referenced are from `https://ncatlab.org/nlab/show/HomePage`.

induced arrow in this case renders $\widetilde{C}$ the limit, over $\widetilde{\mathbb{V}}$, of the observed categorization C. It "fills in" the appropriate category labels over the constructed extensions of the objects in the observed events $\mathbb{V}_i \in \mathbb{V}$, i.e. it creates their fictive causal histories.

As a limit, $\widetilde{\mathbf{F}}$ is unique. In practice, we will be interested in a sequence of events that are "between" $\widetilde{\mathbb{V}}$ and $\mathbb{V}$, and hence presheaves $\widetilde{\mathbf{F}_k}$ such that:

$$\begin{array}{ccccc} \widetilde{C} & \dashrightarrow & \widetilde{C_k} & \dashrightarrow & C \\ \uparrow{\scriptstyle \widetilde{\mathbf{F}}} & & \uparrow{\scriptstyle \widetilde{\mathbf{F}_k}} & & \uparrow{\scriptstyle \mathbf{F}} \\ \widetilde{\mathbb{V}} & \xrightarrow{Proj} & \widetilde{\mathbb{V}_k} & \xrightarrow{Proj} & \mathbb{V} \end{array}$$

commutes. Such a presheaf includes some, but not all, of the constructed events filled in to form $\widetilde{\mathbb{V}}$. The commutativity constraint can be expressed more succintly by requiring that for all k there are morphisms $\mathbf{G}_k, \mathbf{G}'_k$ such that:

$$\widetilde{\mathbf{F}} \xrightarrow{\mathbf{G}_k} \widetilde{\mathbf{F}_k} \xrightarrow{\mathbf{G}'_k} \mathbf{F} \tag{7}$$

These $\mathbf{G}_k, \mathbf{G}'_k$ are clearly associative and respect identity; hence they are functorial. We can, therefore, regard the nested presheaves $\widetilde{\mathbf{F}}, \dots \widetilde{\mathbf{F}_k}, \dots \mathbf{F}$ together with the functors $\mathbf{G}_k, \mathbf{G}'_k$ as forming a 2-category. The functors $\mathbf{G}_k, \mathbf{G}'_k$ pick out a particular intermediate presheaf $\widetilde{\mathbf{F}_k}$ that includes some, but not the maximal number, of constructed events with their associated entropic categorization.

The existence of $\mathbf{G}_k, \mathbf{G}'_k$ renders entropic time functorial: the local entropic time operators embedded in each presheaf $\widetilde{\mathbf{F}_k}$, and in the limits $\widetilde{\mathbf{F}}$ and $\mathbf{F}$, must associate and respect identities if the $\mathbf{G}_k, \mathbf{G}'_k$ do so. The sets $\widetilde{\mathbb{V}_k}$ of events, including the "observed" events $\mathbb{V}$ and the maximally "filled in" limit $\widetilde{\mathbb{V}}$ can, therefore, all be viewed as "small" categories. This categorical interpretation of the $\widetilde{\mathbb{V}_k}$ is natural given the neuroscience of "layers" of processing in which within-layer connections are interactions between representations of a given type, level of abstraction, and semantics, while "vertical" connections between layers are effectively maps between different types of representations at different levels of abstraction and with different semantics (see Fields & Glazebrook (2019b) for extensive discussion).

The entropic condition (5) can be viewed as "classicalizing" the functorial time evolution of events represented by coordinate-free (hence "topological") quantum states (Atiyah, 1988), allowing a functorial time in a setting containing bounded and hence classical objects (cf. the construction of "objects" from quantum interactions in Fields, Glazebrook & Marcianò (2021)). This classicalization can be viewed as a coarse-graining, confirming the dependence of entropic time on coarse-grained "macroscopic" degrees of freedom emphasized by Rovelli (2019).

Let us consider, as above, an entropic categorization $\mathbf{F}$ of "observed" events. We are now in a position to state:

Definition 8. *A* memory *associated with an "observed" entropic categorization* $\mathbf{F}$ *is a presheaf* $\widetilde{\mathbf{F}_k}$ *satisfying the commutativity constraint stated by Eq.* (7)*.*

A memory is *retrospective* if its events are arranged in positive temporal order from some past event to the present, and is *prospective* if its events are arranged in positive temporal order from the present to some future event. A prospective memory can also be considered a *plan* (see Schacter & Addis, 2007, for extensive discussion). We can also regard memories as either retrospective or prospective *histories* as this term is used in Fields (2012), bearing in mind that here "history" is as constructed by a remembering or planning agent, not "objective" in the sense of agent-independent.

We emphasize that between any two observed events $\mathbb{V}_1$ and $\mathbb{V}_\mathrm{N}$ there can exist many distinct memories that may include both additional observed events and different numbers of filled-in constructed events. Distinct memories may encode different fictive causal histories of the objects appearing in $\mathbb{V}_1$ and $\mathbb{V}_\mathrm{N}$ as illustrated in Fig. 3. These memories may impose inconsistent categorizations on the "boundary" events $\mathbb{V}_1$ and $\mathbb{V}_\mathrm{N}$.

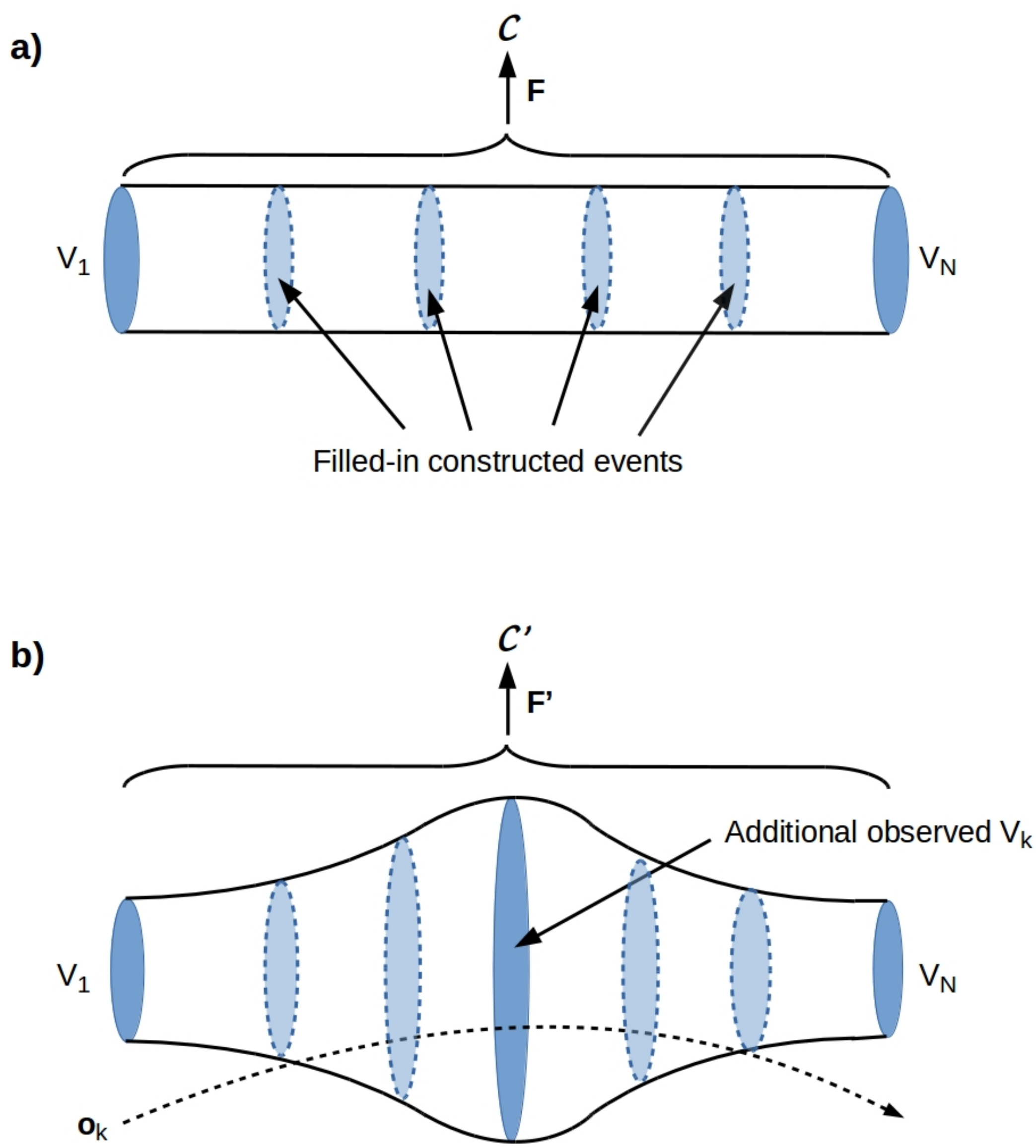


Figure 3: Two distinct memories connecting two observed events $\mathbb{V}_1$ and $\mathbb{V}_\mathrm{N}$. Panel a) has only observed events $\mathbb{V}_1$ and $\mathbb{V}_\mathrm{N}$, plus four constructeds events (dashed ovals). Panel b) adds an observed event $\mathbb{V}_k$ in which an object o_k (dashed trajectory) not included in either $\mathbb{V}_1$ or $\mathbb{V}_N$ appears. Because they contain different sets of observed events ($\{\mathbb{V}_1, \mathbb{V}_\mathrm{N}\}$ versus $\{\mathbb{V}_1, \mathbb{V}_\mathrm{k}, \mathbb{V}_\mathrm{N}\}$), they have different profinite limits that may, but may not, be consistent as categorizations.

Larger numbers of observed events further "classicalize" the functorial time evolution by imposing constraints on both the particular paths taken by the incorporated objects and, as illustrated in Fig. 3b, the co-occurence of new objects and hence new potential interactions in particular "internal" events. Memories with large numbers of observed events are analogous, in a quantum-theoretic setting, to trajectories with intervening "which-path" measurements; however, we will not pursue this analogy here.

8 Memories as sheaves

The event-dependent, constructed time of retrospective and prospective memories provides, in the current framework, an alternative to the objective time of **TL** and its descendants. Before defining modal operators on this constructed time, however, it is useful to complete the formal characterization of memories as sheaves. Recall that:

Definition 9. (Hartshorne, 1977) *Let X be a topological space. A sheaf $\mathcal{F}$ on X is a presheaf satisfying two axioms:*

- *Let U be an open subset of X and U_i an open cover of U. Given a collection of sections s_i on U_i, with $s_i|U_{ij} = s_j|U_{ij}$, then there exists a section s on U such that $s|U_i = s_i$.*
- *Let U be an open subset of X and U_i an open cover of U. If s is a section on U such that $\forall i,\ s|U_i = 0$, then s is zero.*

The collection $\{\mathbb{C}_k\}$ of category descriptors assigned by C to events $\mathbb{V}_i \in \mathbb{V}$ is clearly an open cover of the set $\mathbb{A} = \{\mathbb{A}_i\}$ containing any object that appears in any event contained in $\mathbb{V}$. Hence we immediately have:

Lemma 2. $\widetilde{\mathbf{F}}$ *is a sheaf on* $\widetilde{\mathbb{V}}$.

Proof. We consider the discrete topology on $\widetilde{\mathbb{V}}$. Both sheaf conditions are guaranteed by the functorial nature of the morphisms $\mathbf{G}_k, \mathbf{G}'_k$ between presheaves nested in $\widetilde{\mathbf{F}}$, i.e. by Eq. (7). □

Viewing the discrete topology on $\widetilde{\mathbb{V}}$ as a stratification, we will consider any entropic categorization as a constructible sheaf that is locally constant on the strata in our forthcoming Part II of this paper. Recall the following:

Theorem 1. Sheafification (Theorem 1.2.9 of Alper (2021)) *Let S be a site and $\mathbf{Sh}(S)$ and $\mathbf{Pre}(S)$ be the categories of sheaves and presheaves, respectively, on S. The forgetful functor $\mathbf{f} : \mathbf{Sh}(S) \rightarrow \mathbf{Pre}(S)$ admits a left adjoint $\mathbf{f} \rightarrow \mathbf{f^{sh}}$, called the sheafification.*

Recall from the discussion of Eq. (7) that the functors $\mathbf{G}_k, \mathbf{G}'_k$ render time functorial and hence the memories $\widetilde{\mathbb{V}_k}$ as well as their limits $\mathbb{V}$ and $\widetilde{\mathbb{V}}$ small categories. Hence $\widetilde{\mathbb{V}}$ can be considered a site. We have from Lemma 1 that $\widetilde{\mathbf{F}}$ is a sheaf on $\widetilde{\mathbb{V}}$. The functors $\mathbf{G}_k$ are clearly forgetful; hence Theorem 1 allows us to construct adjoints $\mathbf{G}_k^{sh}$.

The "upwards" construction in Fig. 1b has previously been shown (Fields & Glazebrook, 2019b) to have a "downwards" dual in which objects are viewed as labels ("instances") attached to the category descriptors, which now play the role of the "objects" being labeled. This duality has previously been explored in the setting of Barwise-Seligman classifiers and their associated infomorphisms. Here we consider this duality, for each memory k, as the specific left adjoint $\mathbf{G}_k^{sh}$ defined above as the sheafification. Whereas $\mathbf{G}_k$ expresses a consistency condition on objects that is imposed by an entropic categorization, the adjoint $\mathbf{G}_k^{sh}$ expresses a consistency condition on entropic categorizations that is imposed by (the assumption of) object identity. We can therefore dualize Eq. (7) as:

$$\widetilde{\mathbf{F}} \xleftarrow{\mathbf{G}_k^{sh}} \widetilde{\mathbf{F}_k} \xleftarrow{\mathbf{G}_k^{sh\prime}} \mathbf{F} \tag{8}$$

The $\mathbf{G}_k^{sh}, \mathbf{G}_k^{sh\prime}$ are effectively embeddings of memories within more-inclusive, but fully consistent, memories involving the same objects and categorizations, up to the limit specified by $\widetilde{\mathbf{F}}$.

Recalling the discussion of Fig. 3b above, it is clear that this sheafification-induced duality depends on consistent categorization at each step k of the embedding $\mathbf{G}_k^{sh}, \mathbf{G}_k^{sh\prime}$; Eq (4) can, therefore, be viewed as a consistency test. Failures of Eq (4) can be due to failures of object identity, e.g. an object losing some "essential" identifying property or relation in some incorporated event. "Mistakes" about object identity leading to unexpected consequences are common enough among humans to be a literary trope; see Scholl (2007) or Nichols & Bruno (2010) for examples and Fields (2012) for further discussion. They can also, however, be due to "intrinsic" (or "quantum") context changes, as discussed in this sheaf-theoretic context by Abramsky & Brandenburger (2011); see Fields & Glazebrook (2020) for further discussion. As a "context" in this sense is specified by a set of objects and relations, "enlarging" an event by embedding it in a larger events risks context change, and hence failure of Eq (4). Increasing the number of degrees of freedom of a joint system that are measured, for example, can introduce context shifts and hence violations of the Kolmogorov axioms by the joint distributions of observational outcomes in a quantum setting (Kochen & Specker, 1967). Events that introduce new objects and relations, as illustrated in Fig. 3b, must in principle be proved to introduce no significant context change. The question of how to construct such proofs is known in AI as the Frame Problem (McCarthy & Hayes, 1969); it is now known to be intractable (again see Fields & Glazebrook (2020) for discussion).

9 Sheaves over mutually-consistent memories

Consistency of categorization across a collection of memories is effectively a gluing condition; hence we can re-express the consistency condition implicit in Eq. (8) through a further sheaf construction. We follow a procedure one of us recently used to conjecture a pro-diamond (Dobson, 2021a) (Dobson, 2021b) towards a theory of pro-emergent time; here, we construct a pro-object of the category **Shv** of sheaves. Def. 7 can be generalized to:

Definition 10. *A pro-object of a category* **C** *is a formal cofiltered limit of objects of* **C**.

Note that a profinite set is a pro-object in **FinSet**. The category of pro-objects of an arbitrary category **C** is written **Pro-C**, and meets the following conditions:

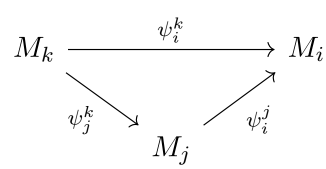
- The objects are pro-objects in **C**.
- The set of arrows from a pro-object $F : D \to \mathbf{C}$ to a pro-object $G : E \to \mathbf{C}$ is the limit of the functor $(\mathbf{D^{op}} \times E) \to \mathbf{Set}$ given by $\mathbf{Hom_C}(F(\cdot), G(\cdot))$.
- Composition of arrows arises, given pro-objects $F : D_0 \to C$, $G : D_1 \to C$, and $H : D_2 \to C$ of C, by applying the limit functor for diagrams $(\mathbf{D^{op}} \times E) \to \mathbf{Set}$ to the natural transformation of functors $(\mathbf{Hom_C}(F(\cdot), G(\cdot)) \times \mathbf{Hom_C}(G(\cdot),H(\cdot))) \to \mathbf{Hom_C}(F(-), H(-))$ given by composition in **C**.
- The identity arrow on a pro-object $F : D \to C$ arises, using the universal property of a limit, from the identity arrow $\mathbf{Hom_C}(F(c), F(c))$ for every object c of **C**.

We now state the following:

Theorem 2. *The sheaf over entropic categorizations of objects/events is a projective limit of a sheaf over objects/events. Therefore, the sheaf over categorizations is a pro-object in* **Shv***, the category of sheaves.*

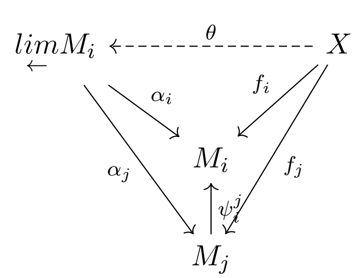
Proof. Let I be a partially ordered set. Recall the following:

Definition 11. *Rotman (2000) Given a partially ordered set I and a category $\mathbf{C}$, an inverse system in $\mathbf{C}$ is an ordered pair $((M_i)_{i\in I}, (\psi_i^j)_{j\geq i})$ abbreviated $\{M_i, \psi_i^j\}$, where $(M_i)_{i\in I}$ is an indexed family of objects in $\mathbf{C}$ and $(\psi_i^j : M_j \to M_i)_{j\geq i})$ is an indexed family of morphisms for which $\psi_i^j = 1_{M_i}$ for all i, and such that the following diagram commutes whenever $k \geq j \geq i$.*



Now let M_i be the graded sheaf over i-objects/events, **Shv** the category of sheaves, and $\{M_i, \psi_i^j\}$ an inverse system in **Shv** over I. Take the sheaf over categorizations of objects/events as an object $\underleftarrow{lim} M_i$. By definition of entropic categorization we have a family of projections $(\alpha_i : \underleftarrow{lim} M_i \to M_i)_{i\in I}$. For our inverse system to be a projective limit we need:

- i) $\psi_i^j \alpha_j = \alpha_i$ for $i \leq j$,
- ii) for every $X \in obj(\mathbf{Shv})$ and all morphisms $f_i : X \to M_i$ satisfying $\psi_i^j f_j = f_i$ for all $i \leq j$, there exists a unique morphism $\theta : X \to \underleftarrow{lim} M_i$ making the diagram commute.



Conditions i) and ii) are just the consistency conditions for sequentially embedding the entropic categorizations $\widetilde{C_k}$; they are met whenever (8) is satisfied as discussed above. $\square$

Theorem 2 provides a maximal consistent memory $\mathcal{M}$ between any two observed events. As noted above, there may be other memories, inconsistent with $\mathcal{M}$ and at least some of its components, that connect these events and include object identity changes or context shifts. We will focus on these in the forthcoming Part II of this paper; here we turn to our final constructive step before localizing the usual operators `Always`, `Sometimes`, and `Never` to individual memories.

10 Condensing memories onto the present

Any account of memory must eventually face questions of implementation, both for the memory itself as a data structure and for the recall mechanism that retrieves one or more memories in response to some cue. The "picture" of memories as sheaves, or as in Theorem 2, projective limits of sheaves, at least suggests the traditional view of (episodic) memories as explicit records stored in a "library" of sorts and recalled via some kind of indexing system. This explicit view of memory has largely been replaced by a constructive view of memory (Hassabis & McGuire, 2009; Nadel *et al.*, 2012; Schacter & Addis, 2007; Schwabe, Nader & Pruessner, 2014) in which even observed events are reconstructed "on the fly" and in a current-context dependent way. This constructive view suggests that what is remembered is not a set of explicitly-represented events, but rather a set of operators with which to construct such events. The previously-firm distinction between "observed" and "constructed" events thus drops away; all (episodic) memory becomes more or less constrained imaginative confabulation.

Here we employ methods developed by Scholze (2017) and Clausen & Scholze (2021) to re-express the previous sheaf-theoretic "picture" of memory in terms of a "condensed" object located at a single notional "point" that we interpret informally as "the present" without committing ourselves to any particular ontology. This condensed object can, in turn, be considered a representable functor, and so effectively a family of operators invokable at the present. These operators construct an extended, multi-event representation (i.e. a memory) of the (retrospective) past or the (prospective) future.

Let $\mathbf{C}$ be a category, and let $\mathbf{Cond}(\mathbf{C})$ denote the category of "condensed" objects of $\mathbf{C}$. Clausen & Scholze (2021) show that $\mathbf{Cond}(\mathbf{C})$ can be represented as the category of small sheaves on $\mathbf{C}$, or equivalently as a representable functor $\mathbf{F} : \mathbf{C^{op}} \to \mathbf{Set}$. More formally, we have:

Definition 12. (Clausen & Scholze (2021) Definition 1.2) *The* pro-étale site $*_{proét}$ *of a point is the category of profinite sets* **Pro-FinSet***, with finite jointly surjective families of maps as covers. A* condensed set *is a sheaf of sets on* $*_{proét}$*. Similarly, a condensed ring/group/object is a sheaf of rings/groups/objects on* $*_{proét}$*.*

Now consider the "current" event $\mathbb{V}_c$ located at "the present" regarded as a point. Recall from §3 that a categorization C_c of $\mathbb{V}_c$ is just an assigment of subgraphs of the category hierarchy Cat to the objects in $\mathbb{V}_c$. This assignment is a finite set of jointly surjective maps, i.e. $C_c : Cat \to \mathbb{A}_c$ can also be thought of as a set $\{\xi_i\}$ of maps $\xi_i : Cat \to a_i$ for each object $a_i \in \mathbb{A}_c$. We can, therefore, regard $\mathbb{V}_c$ as a pro-étale site on the present; these sites inherit the discrete (indeed Grothendieck[2]) topology of events described earlier.

With this identification of $\mathbb{V}_c$ with $*_{\text{proét}}$ on the present, we can state:

Theorem 3. *An entropic categorization is a condensed set.*

Proof. With the above identification, the local functor $\mathbf{F_c} : \mathbb{V}_c \to C_c$ is a presheaf on $*_{\text{proét}}$. Hence all that is required is to extend $\mathbf{F_c} \to \mathbf{F} \to \widetilde{\mathbf{F}}$, and we have a sheaf over $*_{\text{proét}}$ by Lemma 2. The required extension is, clearly, the adjoint pair $\mathbf{G}, \mathbf{G}^{sh}$ satisfying Eq. (7) and (8). These exist by construction for any entropic categorization. □

We can restate Theorem 3 in the language of representable functors.

Definition 13. *For a locally small category* $\mathbf{C}$*, a presheaf on* $\mathbf{C}$ *or equivalently, a functor* $\mathbf{f} : \mathbf{C^{op}} \to \mathbf{Set}$ *on the opposite category of* $\mathbf{C}$ *and with values in* $\mathbf{Set}$ *is* representable *if it is naturally isomorphic to a hom-functor:*

$$\mathbf{h_X} := \mathbf{hom_C}(\cdot, X) : \mathbf{C^{op}} \to \mathbf{Set}$$

that sends an object $U \in \mathbf{C}$ *to the hom-set* $Hom_{\mathbf{C}}(U, X)$ *in* $\mathbf{C}$ *and that sends a morphism* $\alpha : U' \to U$ *in* $\mathbf{C}$ *to the function which sends each morphism* $U \to X$ *to the composite* $(U' \stackrel{\alpha}{\to} U) \to X$*.*

[2]As we began in §2 with an informal notion of the relation between experienced events, we can simply stipulate that their topology is Grothendieck. Nothing we have done is inconsistent with this slightly stronger notion of discreteness.

As noted in the discussion of Eq. (8), the existence of $\mathbf{G}, \mathbf{G}^{sh}$ renders time functorial, and hence requires all diagrams with horizonal time arrows and vertical category-assignment arrows to commute. The above conditions are, therefore satisfied whenever Eq. (7) and (8) are satisfied, i.e. whenever a categorization is entropic.

This construction suggests a precise formal model of the implementation of human episodic memory, i.e. of the structure of the memory-encoding "engram" (Eichenbaum, 2016) as implemented by networks of neural connections:

Prediction: The (episodic) engram is a representation of $\mathbf{G}^{sh}$.

The observed context-dependence of episodic recall follows immediately from the definition of $\mathbf{G}^{sh}$ in this model. If this prediction is correct, humans reconstruct an experienced time, at each instant of recall, by reconstructing a memory. Hence time itself is condensed.

This condensed representation of memory, and therefore of event-dependent time, also provides a natural link to the traditional notion of an "objective" continuous time. Pro-étale sites simplify open sets, so condensed sets simplify topological spaces:

Proposition (Proposition 3.1 Clausen & Scholze (2021)): The forgetful functor from the category of topological spaces to condensed sets is a faithful functor. It becomes fully faithful when restricted to compactly generated spaces. This functor admits a left adjoint, which sends a condensed set $\mathcal{T}$ to the topological space given by the underlying set $\mathcal{T}*$ of $\mathcal{T}$ equipped with the quotient topology induced by the map $\bigsqcup_{S \to \mathcal{T}} S \to \mathcal{T}*$ where S runs over all (κ-small) profinite sets mapping into $\mathcal{T}$. The counit of this adjunction coincides with the counit $X^{cg} \to X$ of the adjunction between (κ-small) compactly generated spaces and topological spaces.

Hence we can see the discrete time constructed from the present by Theorem 3 as a local coarse-graining of a continuous time. Nothing in the construction, however, guarantees any straightforward relationship between distinct such coarse-grainings. The local "times" of different experiencing agents will, in general, not be commensurable.

11 Localizing `Always`, `Sometimes`, and `Never` to a memory

With this model of memory as a localized, condensed representation of $\mathbf{G}^{sh}$, we are in a position to define completely local analogs of the standard qualitative temporal-logic operators `Always`, `Sometimes`, and `Never`, as well as quantitative extensions such as `At least twice in the past`, etc. Let $R(a, b, \dots)$ be a relation on a finite set of objects

$\{a, b, \dots\}$. We can define, relative to an event $\mathbb{V}_c$:

- $\texttt{Always}(R(a, b, \dots) :=\ \forall$ sections s of $\widetilde{\mathbf{F}}$, $(R(a, b, \dots)$ on s.
- $\texttt{Sometimes}(R(a, b, \dots) :=\ \exists$ a section s of $\widetilde{\mathbf{F}}$, $(R(a, b, \dots)$ on s.
- $\texttt{Never}(R(a, b, \dots) :=\ \neg\exists$ a section s of $\widetilde{\mathbf{F}}$, $(R(a, b, \dots)$ on s.

Alternatively, $(R(a, b, \dots)$ on all, some, or no images of $\mathbf{G}^{sh}$ acting on $\mathbf{F}_c$.

Note how these definitions depend on time being entropic, i.e. on $\mathbb{V}_c$ encoding "past" relational information that applies equally to constrain the "future" of prospective memory. Such information can clearly be considered to include probability distributions, i.e. $\mathbb{V}_c$ can be taken to encode prior probabilities of predictable events. *Unpredictable* events fall outside this framework, i.e. represent failures of prospective memory.

12 Conclusions and extensions

In this Part I, we have developed the basic formalism needed to represent events and memory in a localized, event-dependent functorial time. This formalism allows us to free temporal logic from the ontological constraint of a continuous "objective" time equally shared by all experiencing agents. We are led quite naturally to a significant empirical prediction, that episodic memories are encoded as representations of a particular functor, $\mathbf{G}^{sh}$, that constructs retrospective pasts and prospective futures subject to the constraint of time being entropic, i.e. satisfying Eq. (5).

The constructions we report here leave open the question of how to most effectively represent categorization inconsistencies in either retrospective pasts or prospective futures. Planning, in particular, typically involves projecting multiple, mutually-inconsistent future environmental contingencies and courses of action. Retrospective memory can, however, also involve uncertainties about what actually happened, particularly during periods of non-observation.

One approach to these questions is suggested by Fig. 3 and the formalism of topological field theories: it is to reformulate memory in terms of cobordisms, and to allow topologically-complex evolutions that involve multiple intermediate boundaries. A second, more algebraic approach is suggested by Theorem 2: it is to generalize in the direction of derived categories and perverse sheaves. These approaches may, indeed, prove to be closely related. Working from the “diamond” construct of Scholze (2017), one of us recently conjectured a pro-diamond (Dobson, 2021a) towards a theory of pro-emergent time. This construction naturally suggests a holographic interpretation (Dobson, 2021b) and hence a formulation in terms of cobordisms.

To conclude, we suggest that viewing time as event-dependent, constructive, and indeed as condensed into a functorial operation on the present opens new opportunities for modeling memory, planning, and temporal reasoning. The formal methods enabling such a view have intruiging connections to field theories, particularly topological field theories. They enable novel predictions that are potentially testable as methods for analyzing neuronal networks in humans and other organisms are further developed. Finally, such methods suggest a deeper connection between models of time and constructive, experience and reference-frame dependent models of space.

Conflict of Interest

The authors declare that they have no conflicts of interest with the reported work.

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