

Pro-Generative Adversarial Network and V-Stack Perceptron, Diamond Holographic Principle, and Pro-Temporal Emergence

Shanna Dobson¹

¹ Department of Mathematics, California State University, Los Angeles, CA

Abstract

We recently presented our Efimov K-theory of Diamonds, proposing a pro-diamond, a large stable $(\infty, 1)$ -category of diamonds \mathcal{D}^\diamond , and a localization sequence for diamond spectra. Commensurate with the localization sequence, we now detail four potential applications of the Efimov K-theory of \mathcal{D}^\diamond : to emergent time as a pro-emergence (v -stack time) in a diamond holographic principle using Scholze's six operations in the 'etale cohomology of diamonds; to a pro-Generative Adversarial Network and v -stack perceptron; to \mathcal{D}^\diamond cryptography; and to diamond nonlocality in perfectoid quantum physics.

Keywords: pro-Generative Adversarial Network, diamonds, Efimov K-theory, pro-diamond, pro-emergence, holographic principle, v -stack, geometrization of local Langlands

Contents

1	Introduction	2
1.1	Diamond Conjectures	3
1.2	Diamonds	4
1.3	Spatial V -Sheaves	5

2	Pro-Generative Adversarial Network and V-Stack Perceptron	6
2.1	\mathcal{D}^\diamond Cryptography	7
2.1.1	\mathcal{D}^\diamond -multiclass classification and Chromatic Types	8
3	Diamond Holographic Principle	9
3.0.1	Diamond Coupling	14
4	Pro-emergence and Temporal Plurivocity	14
4.1	Pro-Diamond	15
4.2	Pro-Emergence	16
4.3	Temporal Nonlocality from Pro-Emergence	18
4.4	V-Stack Time and Profinite Temporal Nonlocality	19
4.4.1	Emergent Time on $*_{\text{proét}}$	19
4.4.2	V-stack Time in $\text{Cond}(\mathcal{D}^\diamond)$	21
4.4.3	Object Persistence as Descent	22
4.4.4	Condensed Types	22
4.5	Diapsalmata Interrogatory	22
4.5.1	Dialetheism Time	23
5	Diamond Nonlocality in Perfectoid Quantum Physics	25

1 Introduction

We recall the motivation for our Efimov K-theory of \mathcal{D}^\diamond [4]. The goal forthcoming is to get the lower K^{Efimov} -groups of the diamond spectra to encode the datum of the mixed-characteristic shtukas and the higher K^{Efimov} -groups to encode the moduli space of mixed-characteristic local G -shtukas, which is itself a locally spatial diamond [13] [4]. Additionally, the moduli spaces of shtukas in mixed characteristic live in the category of diamonds. Studying the isomorphism classes of moduli spaces of shtukas as the higher K^{Efimov} -groups of diamonds could link diamonds and global Langlands over function fields [4].

In this paper, we propose a diamond holographic principle, a pro-Generative Adversarial Network and v -stack perceptron, a new model of emergent time (v -stack-time) and condensed types, and a diamond version of nonlocality. We construct a diamond version of

temporal nonlocality reflecting a temporal multiplicity as a pro-emergence. Using diamond descent, our model is a double emergence, but profinitely many copies of emergence. We are interested in the relation between diamond descent, the stop mechanism, and the storage and recollection of profinitely many copies of information in the form of mathematical impurities as geometric points. We have posited a model of the brain allowing various mathematical partitions in the form of the profinitely many copies of the diamond structure [8] with neurons modeled as geometric points as mathematical minerological impurities.

The goal is one of connecting geometrized local Langlands-stacks with emergent time as a new incarnation of a new reciprocity law. For a more tractable construction, (possibly) assuming mirror neurons are an imitative representation of (pro)-emergent time, we can restrict to a reciprocity law between diamond v -stacks and mirror neurons. More specifically, we will propose an incarnation of pro-temporal emergence from diamond nonlocality, where local time emerges from condensed sets, and a 2-infomorphism connects temporal simultaneity with nonlocality.

1.1 Diamond Conjectures

We first recall our main diamond conjectures [4].

Conjecture 1.1.1. There exists a large, stable, presentable $(\infty, 1)$ -category of diamonds \mathcal{D}^\diamond with spatial descent datum. \mathcal{D}^\diamond is dualizable. Therefore, the Efimov K-theory is well-defined.

Conjecture 1.1.2. Let S be a perfectoid space, \mathcal{D}^\diamond a stable dualizable presentable $(\infty, 1)$ -category, and R a sheaf of E_1 -ring spectra on S . Let \mathcal{T} be a stable compactly generated $(\infty, 1)$ -category and $F: \text{Cat}_{S_t}^{idem} \rightarrow \mathcal{T}$ a localizing invariant that preserves filtered colimits. Then

- $F_{cont}(\text{Shv}(\mathbb{S}^n, \mathcal{D}^\diamond)) \simeq \Omega^n F_{cont}(\mathcal{D}^\diamond)$.

Conjecture 1.1.3. Let \mathcal{D}_\diamond be the complex of v -stacks of locally spatial diamonds. Let \mathcal{D}^\diamond be the $(\infty, 1)$ -category of diamonds. Let $\mathcal{Y}_{(R, R^+), E} = \text{Spa}(R, R^+) \times_{\text{Spa}F_q} \text{Spa}F_q[[t]]$ be the relative Fargues-Fontaine curve. Let $(\mathcal{Y}_{S, E}^\diamond)$ be the diamond relative Fargues-Fontaine curve. There exists a localization sequence

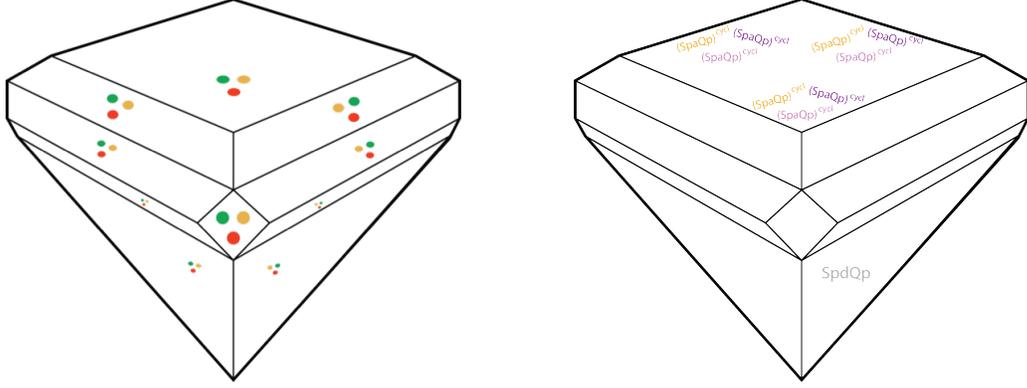


Figure 1: Diamond $SpdQ_p = Spa(Q_p^{cycl}) / \underline{Z_p^\times}$ with geometric point $Spa C \rightarrow \mathcal{D}$

- $K(\mathcal{D}_\diamond) \rightarrow K^{\text{Efimov}}(\mathcal{Y}_{S,E}^\diamond) \rightarrow K^{\text{Efimov}}(\mathcal{Y}_{(R,R^+),E})$.

Conjecture 1.1.4. \mathcal{D}^\diamond admits a topological localization, in the sense of Grothendieck-Rezk-Lurie $(\infty, 1)$ -topoi.

Conjecture 1.1.5. There exists a diamond chromatic tower

- $\mathcal{D}^\diamond \rightarrow \dots \rightarrow L_n \mathcal{D}^\diamond \rightarrow L_{n-1} \mathcal{D}^\diamond \rightarrow \dots \rightarrow L_0 \mathcal{D}^\diamond$

for L_n a topological localization for $K\mathcal{D}^\diamond$ the -theory spectrum of the diamond spectrum representative of the étale cohomology of diamonds.

Conjecture 1.1.6. The $(\infty, 1)$ -category of perfectoid diamonds is an $(\infty, 1)$ -topos.

1.2 Diamonds

Recall the definition and many incarnations of a diamond.

Definition 1.2.1. ([17] Definition 1.3). Let Perfd be the category of perfectoid spaces. Let Perf be the subcategory of perfectoid spaces of characteristic p . Let Y be a pro-étale sheaf on Perf . Then Y is a diamond if Y can be written as the quotient X/R with X a perfectoid space of characteristic p and R a pro-étale equivalence relation $R \subset X \times X$.

Examples of diamonds are the following:

- **Example 1.2.2** [17] [4]. For $X = Spa(R, R^+)$, we say $Spd(R, R^+) = Spa(R, R^+)^\diamond$.
- **Example 1.2.3** [17] [4]. The Fargues-Fontaine Curve X_{FF} is a regular noetherian scheme of Krull dimension 1 which is locally the spectrum of a principal ideal domain. The set of closed points of X_{FF} is identified with the set of characteristic 0 untilts of C^b modulo Frobenius. For C an algebraically closed perfectoid field of characteristic $p > 0$ and ϕ the Frobenius automorphism of C we have

$$- X_{FF}^\diamond \cong (SpdC \times SpdQ_p) / (\phi \times id).$$

- **Definition 1.2.4** [17] [4]. The diamond equation is

$$- \mathcal{Y}_{S,E}^\diamond = S \times (Spa\mathcal{O}_E)^\diamond$$

- **Example 1.2.5** [17] [4]. Let \mathcal{D} and \mathcal{D}' be diamonds. Then the product sheaf $D \times_\diamond D'$ is also a diamond.
- **Example 1.2.6** [17][4]. $SpdQ_p = Spd(Q_p^{cycl}) / \underline{Z}^{x_p}$ where \underline{Z}^{x_p} is the profinite group $Gal(Q_p^{cycl}/Q_p)$.
- **Example 1.2.7** [17][4]. $SpdQ_p \times_\diamond SpdQ_p$.
- **Example 1.2.8** [17][4]. $Sht_{G,b,\{\mu_i\}}$: moduli spaces of mixed-characteristic local G -shtukas is a locally spatial diamond.
- **Example 1.2.9** [17][4]. All Banach-Colmez spaces are diamonds.
- **Example 1.2.10** [17][4]. Any closed subset of a diamond is a diamond.

1.3 Spatial V -Sheaves

We now recall the properties of spatial diamonds.

Definition 1.3.1. (Definition 17.3.1). Spatial v -sheaves are a restricted class of diamonds with $|\mathcal{F}|$ well-behaved. A v -sheaf \mathcal{F} is spatial if

- 1. \mathcal{F} is qcqs (in particular, small), and

- 2. $|\mathcal{F}|$ admits a neighborhood basis consisting of $|\mathcal{G}|$, where $\mathcal{G} \subset \mathcal{F}$ is quasicompact open.

We say \mathcal{F} is locally spatial if it admits a covering by spatial open subsheaves.

Remark 1.3.2. For locally spatial diamonds, we recall the following [13] [4]:

- All diamonds are v -sheaves.
- Spatial diamonds are spatial- v -sheaves.
- If \mathcal{F} is quasiseparated, then so is any subsheaf of \mathcal{F} . Thus if \mathcal{F} is spatial, then so is any quasicompact open subsheaf.
- (Proposition 17.3.8). If X is a qcqs analytic adic space over $\mathrm{Spa}Z_p$, then X° is spatial.
- (Proposition 17.3.4) Let \mathcal{F} be a spatial v -sheaf. Then $|\mathcal{F}|$ is a spectral space, and for any perfectoid space X with a map $X \rightarrow \mathcal{F}$, the map $|X| \rightarrow |\mathcal{F}|$ is a spectral map.
- (Proposition 17.3.5). Let X be a spectral space, and $R \subset X \times X$ a spectral equivalence relation such that each $R \rightarrow X$ is open and spectral. Then X/R is a spectral space, and $X \rightarrow X/R$ is spectral.
- (Theorem 17.3.9). Let \mathcal{F} be a spatial v -sheaf. Assume that for all $x \in |\mathcal{F}|$, there is a quasi-pro-étale map $X_x \rightarrow \mathcal{F}$ from a perfectoid space X_x such that x lies in the image of $|X_x| \rightarrow |\mathcal{F}|$. Then \mathcal{F} is a diamond.
- (Corollary 17.3.7). Let \mathcal{F} be a small v -sheaf. Assume there exists a presentation $R \rightrightarrows X \rightarrow \mathcal{F}$, for R and X spatial v -sheaves (e.g., qcqs perfectoid spaces), and each $R \rightarrow X$ is open. Then \mathcal{F} is spatial.

2 Pro-Generative Adversarial Network and V-Stack Perceptron

For our first application, we are constructing a pro-GAN as a pro-object in the category of GANS to categorify embodiment for AI. We then model the perceptron as a v -stack perceptron. The output of a v -stack perceptron is a condensed set, which is a sheaf of sets on the pro-étale site of a point.

Our pro-formalism is a model of embodied meta-learning which reframes the frame problem using condensed sets. We propose modeling enactive robots as advanced AI in a profinite formalism and modeling mirror neurons as imitation/emulation GANs in a mirror game theory for enactive neurons. Finally, we offer a correspondence between embodied cognition in our profinite formalism, computability, and a pro-synchronic and diachronic emergence that is built into the pro-GAN.

2.1 \mathcal{D}^\diamond Cryptography

For our second application, we propose that post-quantum cryptography is a diamond cryptography in locally spatial diamonds providing multi-level fortification. Recall that spatial v -sheaves are a restricted class of diamonds with $|\mathcal{F}|$ well-behaved. The idea is that encryption is in the geometric points $Spa(C) \rightarrow \mathcal{D}$ and profinitely many copies of $Spa(C)$ and decryption is in the fiber term of the localization sequence

$$\bullet K(\mathcal{D}_\diamond) \rightarrow K^{\text{Efimov}}(\mathcal{Y}_{S,E}^\diamond) \rightarrow K^{\text{Efimov}}(\mathcal{Y}_{(R,R^+),E}).$$

We recall the diamond functor.

Definition 2.1.1. (Diamond functor (6.4 [10])). There is a diamond functor

- $\{\text{analytic pre-adic spaces}/SpaZ_p\} \rightarrow \{\text{diamonds}\}$
- $X \rightarrow X^\diamond$

which forgets the structure morphism to $SpaZ_p$, but retains topological information.

Conjecture 2.1.2. We consider diamond cryptography as follows. Data is multi-encrypted as a geometric point $Spa(C) \rightarrow \mathcal{D}$ with its multiple descriptions from its multiple quasi-pro-étale covers $X \rightarrow \mathcal{D}$. Recall a geometric point is a mathematical impurity and cannot be seen directly, but only upon its pull back through a quasi-pro-étale cover $X \rightarrow \mathcal{D}$, resulting in profinitely many copies of $SpaC$ [17]. The decryption is in the Efimov K-theory of diamonds, which can take two forms. One is in the fiber term of the localization sequence

$$\bullet K(\mathcal{D}_\diamond) \rightarrow K^{\text{Efimov}}(\mathcal{Y}_{S,E}^\diamond) \rightarrow K^{\text{Efimov}}(\mathcal{Y}_{(R,R^+),E}).$$

The second we take K^{Efimov} of the gluing-construction v -stacks [16].

¹By taking K^{Efimov} of the gluing-construction v -stacks, we mean to take diamond equivalence relations

$$\begin{array}{ccc}
\mathcal{Y}_{S,E}^\blacklozenge & \stackrel{=}{=} & S \times (\mathrm{Spa} \mathcal{O}_E)^\blacklozenge \\
\uparrow & & \uparrow \\
\mathcal{Y}_{S,E}^\blacklozenge & \stackrel{=}{=} & S \times (\mathrm{Spa} E)^\blacklozenge \\
\downarrow & & \downarrow \\
\mathcal{X}_{S,E}^\blacklozenge & \stackrel{=}{=} & \mathcal{Y}_{S,E}^\blacklozenge / \phi_S^\blacklozenge
\end{array}$$

Figure 2: Diamonds Gluing to v -Stacks.¹

2.1.1 \mathcal{D}^\diamond -multiclass classification and Chromatic Types

Conjecture 2.1.1.1. We are currently constructing a chromatic tower over a temporal logic for the AI problem and therein creating a new notion of condensed types and chromatic types [7].²

Conjecture 2.1.1.2. In parallel, we propose the categorification of multiclass classification. The idea is the following. Mirror neurons are modeled as locally spatial diamonds, in particular as objects in \mathcal{D}^\diamond the $(\infty, 1)$ -category of diamonds. Then, topological localization supervenes tensor network connectivity, in passing to the full reflective sub- $(\infty, 1)$ -category, where objects and morphisms have reflections in the category, which more properly mimic mirror neurons modeled after diamonds. Bousfield localization supervenes the joint scarcity and high connectivity properties of high dimensional expander graphs. We can then contemplate the moduli spaces of equivalences in terms of the category of sheaves, which is itself a reflective subcategory and a Grothendieck topos [?]. Passing to large ∞ -categories, multiclass classification could proceed K-theoretically via Efimov K-theory groups using our Efimov K-theory of diamonds formalism.

Remark 2.1.1.3. Recall that we are constructing an $(\infty, 1)$ -site on \mathcal{D}^\diamond and extending topological localization to \mathcal{D}^\diamond , where equivalence classes of topological localizations are in bijection with Grothendieck topologies on $(\infty, 1)$ -categories C [?]. We provide a small scale

of $\mathcal{Y}_{S,E}^\diamond$ to get v -stacks, analogous to taking pro-étale equivalence relations to get diamonds, from which we consider the moduli spaces of v -stacks.

²A condensed type is a topological space regarded up to a condensed weak equivalence. A chromatic type is a topological space regarded up to a chromatic convergence weak equivalence.

model of the localization formalism using Bousfield localization particular to triangulated categories in [8].

Scholze’s six operations are foundational to our construction of this $(\infty, 1)$ -site, in the following sense. The six operations live in the derived categories of sheaves and derived categories are the ∞ -categorical localization of the category of chain complexes at the class of quasi-isomorphisms [9]. The derived categories $\mathcal{D}(\mathcal{A})$ of abelian categories \mathcal{A} are an important class of examples of triangulated categories. They are homotopy categories of stable $(\infty, 1)$ -categories of chain complexes in \mathcal{A} [9].

Example 2.1.1.4. We take a simple case and consider neuronal connection in terms of triangulated categories, as explained in [23]. Ignoring higher morphisms, we can “flatten” any $(\infty, 1)$ -category \mathcal{C} into a 1-category $\text{ho}(\mathcal{C})$ called its homotopy category. If \mathcal{C} is also stable, a triangulated structure captures the additional structure canonically existing on $\text{ho}(\mathcal{C})$. This additional structure takes the form of an invertible suspension functor and a collection of sequences called distinguished triangles, which behave like shadows of homotopy (co)fibre sequences in stable $(\infty, 1)$ -categories [23].

So neuronally, everything is working via cofiber sequences, hence the appropriate introduction of our $K^{\text{Efimov}}\mathcal{D}^\diamond$ formalism.

3 Diamond Holographic Principle

For our third application, the goal is to construct a diamond formulation of the holographic principle [7] using the adjoint pairs in Scholze’s six operations for the étale cohomology of spatial diamonds [13] and the profinite condition of locally spatial diamonds, to replace Anti de-Sitter space and conformal field theory, respectively. Recall that spatial v -sheaves are a restricted class of diamonds with $|\mathcal{F}|$ well-behaved. Entropy is then encoded in the Efimov K-theory of the v -stack.

Recall the diamond functor [17] and proposition.

Definition 3.1 Diamond functor. (6.4 [10]). There is a diamond functor $\{\text{analytic adic spaces}/Z_p\} \rightarrow \{\text{diamonds}\}$.

Proposition 3.2. ([10] Proposition 6.11). For an analytic adic space X/Z_p , the diamond functor

- $X^\diamond : S \in \text{Perf} \rightarrow \{S^\# / Z_p \text{ untilts of } S \text{ plus map } S^\# \rightarrow X\}$

defines a locally spatial diamond. There are canonical equivalences

- $|X| \simeq |X^\diamond|$ and $X_{\text{ét}} \simeq X_{\text{ét}}^\diamond$.

For X perfectoid, $X^\diamond \simeq X^b$.

Proposition 3.3. ([10] Proposition 17.3.4) Let \mathcal{F} be a and for any perfectoid space X with a map $X \rightarrow \mathcal{F}$, the map $|X| \rightarrow |\mathcal{F}|$ is a spectral map.

Remark 3.4. The six operations formalism is apropos to the holographic principal. Informally, the six operations formalism is, in a sense, a higher cohomological analogue of the ADS/CFT duality [?]. The six operations formalism is a formalization of aspects of Verdier duality, which is the refinement of Poincaré duality from ordinary cohomology to abelian sheaf cohomology [?].

Moreover, v -stacks are highly holographic in a certain sense in their encoding of profinitely many copies of data that is already multiple on two fronts. ³

Conjecture 3.5. The goal is to generalize the distinction between "separable" and "entangled". Recall, geometric points are morphisms of schemes. Recall, a Barwise-Seligman infomorphism is an adjoint pair [?]. We extend this idea to construct a 2-infomorphism using the two adjoint pairs

- f^* and $\mathcal{R}f_*$
- and $\mathcal{R}f_!$ and $\mathcal{R}f^!$.

Thus, the grand correspondence is $\mathcal{D}_{\text{ét}}(X, \Lambda)$ which consists precisely of étale sheaves of Λ -modules on $X_{\text{ét}}$ for X a small v -stack [13].

Recall the six operations for the étale cohomology of spatial diamonds.

Terminology.

³Recall, a v -stack is a 2-sheaf, which is a sheaf that takes values in categories rather than sets.

Fix a prime p . Let X be an analytic adic space on which p is topologically nilpotent. To X we associate an étale site $X_{\acute{e}t}$. Let Λ be a ring such that $n\Lambda = 0$ for some n prime to p . There exists a left-completed derived category $\mathcal{D}_{\acute{e}t}(X, \Lambda)$ of étale sheaves of Λ -modules on $X_{\acute{e}t}$. Let $Perfd$ be the category of perfectoid spaces and $Perf$ be the subcategory of perfectoid spaces of characteristic p [13].

Consider the v -topology on $Perf$ ⁴.

Definition 3.6. ([13] Definition 1.7). Let X be a small v -stack, and consider the site X_v of all perfectoid spaces over X , with the v -topology. Define the full subcategory $\mathcal{D}_{\acute{e}t}(X, \Lambda) \subset \mathcal{D}(X_v, \Lambda)$ as consisting of all $\mathcal{A} \in \mathcal{D}(X_v, \Lambda)$ such that for all (equivalently, one surjective) map $f : Y \rightarrow X$ from a locally spatial diamond Y , $f^*\mathcal{A}$ lies in $\hat{\mathcal{D}}(Y_{\acute{e}t}, \Lambda)$.

$\mathcal{D}_{\acute{e}t}(X, \Lambda)$ contains the following six operations.

- Derived Tensor Product. $-\otimes_{\Lambda}^{\mathbb{L}} - : \mathcal{D}_{\acute{e}t}(X, \Lambda) \times \mathcal{D}_{\acute{e}t}(X, \Lambda) \rightarrow \mathcal{D}_{\acute{e}t}(X, \Lambda)$.
- Internal Hom. $R\mathcal{H}om_{\Lambda}(-, -) : \mathcal{D}_{\acute{e}t}(X, \Lambda)^{op} \times \mathcal{D}_{\acute{e}t}(X, \Lambda) \rightarrow \mathcal{D}_{\acute{e}t}(X, \Lambda)$.
- For any map $f : Y \rightarrow X$ of small v -stacks, a pullback functor $f^* : \mathcal{D}_{\acute{e}t}(X, \Lambda) \rightarrow \mathcal{D}_{\acute{e}t}(Y, \Lambda)$.
- For any map $f : Y \rightarrow X$ of small v -stacks, a pushforward functor $\mathcal{R}f_* : \mathcal{D}_{\acute{e}t}(Y, \Lambda) \rightarrow \mathcal{D}_{\acute{e}t}(X, \Lambda)$.
- For any map $f : Y \rightarrow X$ of small v -stacks that is compactifiable, representable in locally spatial diamonds, and with $\dim.\text{trg } f < \infty$ functor $\mathcal{R}f_! : \mathcal{D}_{\acute{e}t}(Y, \Lambda) \rightarrow \mathcal{D}_{\acute{e}t}(X, \Lambda)$.
- For any map $f : Y \rightarrow X$ of small v -stacks that is compactifiable, representable in locally spatial diamonds, and with $\dim.\text{trg } f < \infty$, a functor $\mathcal{R}f^! : \mathcal{D}_{\acute{e}t}(X, \Lambda) \rightarrow \mathcal{D}_{\acute{e}t}(Y, \Lambda)$.

⁴The v -topology, where a cover $\{f_i : X_i \rightarrow X\}$ consists of any maps $X_i \rightarrow X$ such that for any quasicompact open subset $U \subset X$, there are finitely many indices i and quasicompact open subsets $U_i \subset X_i$ such that the U_i jointly cover U [13]

Recall that for any small v -stack Y , we have defined the full subcategory $D_{\text{ét}}(Y, \Lambda) \subset D(Y_v, \Lambda)$.

Lemma 3.7. ([13] Lemma 17.1). There is a (natural) presentable stable ∞ -category $D_{\text{ét}}(Y, \Lambda)$ whose homotopy category is $D_{\text{ét}}(Y, \Lambda)$. More precisely, the ∞ -derived category $D(Y_v, \Lambda)$ of Λ -modules on Y_v is a presentable stable ∞ -category, and $D_{\text{ét}}(Y, \Lambda)$ is a full presentable stable ∞ -subcategory closed under all colimits.

Remark 3.8. Recall, any spatial v -sheaf is a spectral space ([17] Proposition 17.3.4). To get the entanglement property, we are reformatting the separability condition as a combination of the qcqs condition of the v -sheaf \mathcal{F} and the non uniqueness of the generic point as the failure of the sober property.

Recall the definition of a spectral space.

Definition 3.9. ([17] Definition 2.3.4). A topological space T is spectral if the following equivalent conditions are satisfied.

- 1. $T \simeq \text{Spec}R$ for some ring R .
- 2. $T \simeq \varprojlim T_i$ where $\{T_i\}$ is an inverse system of finite T_0 -spaces. ⁵
- 3. T is quasicompact ⁶ and sober. ⁷

Recall the holographic principle.

Definition 3.10 [7]. The holographic principle is basically a claim about how much information can be transported from A to B while keeping A and B distinct (quantum language: not entangled). Our idea is: exactly as much information as can be transported using two particular adjoint pairs. Otherwise, the ability to distinguish between A and B collapses.

The question becomes the following:

⁵ T_0 means that given any two distinct points, there exists an open set which contains exactly one of them.

⁶There exists a basis of quasi-compact opens of T which is stable under finite intersection

⁷Every irreducible closed subset has a unique generic point.

Question 3.11. What does it mean to say the ability to distinguish them collapses? What is this more general idea of entanglement or non-separability or super-correlation, given there are no ontic primitives?

There exists a kind of sparseness that is required to tell A and B apart, which in physics becomes an idea of "weakly interacting" or just touching briefly along a boundary, a vague idea, with only a negative definition. Clearly, we need some idea of "transport" between distinguishable entities or states to talk about a time reference frame. We develop this using the adjoint pair f^* and $\mathcal{R}f_*$.

All Hilbert spaces of the same dimension, for example, are isomorphic. So qubits A and B are in a sense "equivalent". The question is the following:

Question 3.12. Can we talk about A 's state independently of B 's state? If their joint state is separable, we can, but if it's entangled, we cannot.

So we need to know more than that their H -spaces are equivalent. We need to know something about their joint state within the tensor product of their H -spaces. We develop this idea using the derived tensor product $-\otimes_{\Lambda}^{\mathbb{L}}-$ and the internal Hom. $R\mathcal{H}om_{\Lambda}(-, -)$.

The question becomes:

Question 3.13. How do we generalize the idea of a "joint state"? How do we generalize the distinction between "separable" and "entangled"?

Definition 3.14. The definition of entanglement is if $|AB\rangle = |A\rangle|B\rangle$, the joint state $|AB\rangle$ is separable. Otherwise it is entangled [7].

Being in a separable joint state is very peculiar. Unitary evolution entangles states. Being in a separable state means that A and B are barely interacting. Even a very weak interaction will eventually entangle them.

We develop this using the second adjoint pair $\mathcal{R}f_!$ and $\mathcal{R}f^!$.

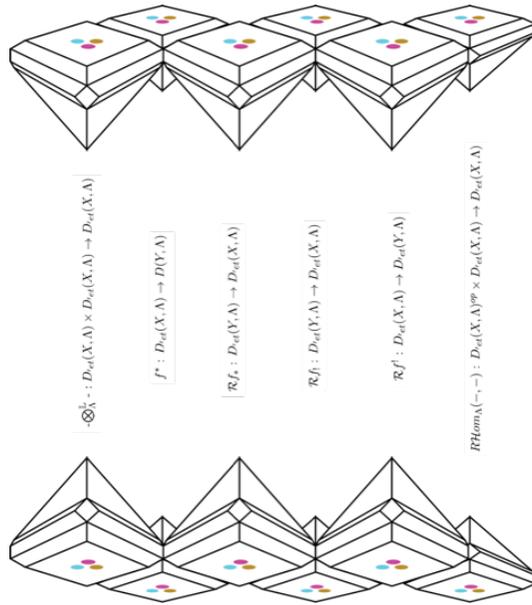


Figure 3: Diamond Holographic Principal in Six Operations for the Étale Cohomology of Spatial Diamonds on $\ast_{\text{proét}}$.

3.0.1 Diamond Coupling

To modify a holographic correspondence, we introduce in Table 1 our new dictionary relating holography coupling and pro-emergence with v -stacks.

To complete the dictionary, we need to construct the six operations in a condensed setting, and link diamond profiniteness with nonlocality, diamond descent, and diamond localization.

4 Pro-emergence and Temporal Plurivocity

For our fourth application, we construct a pro-emergence reflecting a temporal plurivocity. The glowing interrogatory driving our construction of pro-emergence is the question of *how do we construct a reciprocity law of emergent time? Specifically, how can we construct a reciprocity law of pro-emergence?* What is immediate is the further question of what formalism is immediate to and consummating of the question, and could potentially connect Fargues' geometrization of the local Langlands with a Grand Unified Theory of physics?

Table 1: Diamond Dictionary of Holographic Correspondence and Pro-Emergence

<u>Holography Coupling</u>	<u>Diamonds and Pro-Emergence</u>
Collapse/decoherence	Pro-étale Site/Localization
Weak Coupling Perturbation	Tilting + p-divisible formal group laws; kernels are p^n -torsions; torsion in global Langlands incarnations of $Spa(C) \rightarrow D$
Strong Coupling	pro-emergence; v -stack profinite
Coupling Strength	descent, irreversibility/reconstructability
Temporal Nonlocality	Condensed and singular perfectoid time
Weak Coupling	Six operations; $\text{Cond}(D)$
ADS Bulk	diamond v -stacks; $\Omega^n F_{cont}(\mathcal{D}^\diamond)$
CFT	diamond profiniteness; profinitely copies of $Spa(C)$
Nonlocality	Diamond Descent; Diamond Localization
Unitarity	étale cohomology of diamonds
quantum topology	

What is parallel to the interrogatories and without particularity is the question of *why is time always assumed a total order with a connexity property, rather than being considered perfectoid or even a diamond equivalence relation? Is temporal nonlocality instantiation without reciprocitous advent? Is temporal nonlocality dialetheism time?*

To reach our grand valiant goal of developing this reciprocity law, we begin with highly simplified cases and construct a basic example of *emergent time as a pro-emergence and investigate (to forthcoming construct) temporal nonlocality from pro-emergence*. We herein outline our preliminary ideas and constructions.

4.1 Pro-Diamond

Recall our pro-diamond pro-object is constructed as follows [4]:

Conjecture 4.1.1. Let \mathcal{D}^\diamond be a small cofiltered category of diamonds with morphisms the diamond product [13]. Two objects in \mathcal{D}^\diamond are diamonds (v -sheaves) and spatial v -sheaves. The pro-diamond pro-object in the category of pro-objects of \mathcal{D}^\diamond is the formal

cofiltered limit of objects of \mathcal{D}^{∞} . The $Hom_{\mathcal{D}^{\infty}}(F(-), G(-))$ for pro-objects $F : D \rightarrow C$ and $G : E \rightarrow C$ is given by the pro-diamond functor, the pro version of the diamond functor. Recall that

a pro-object of a category C is a formal cofiltered limit of objects of C [20].

A cofiltered category has the property that

for every pair of objects c_1 and c_2 of C , there is an object c_3 of C such that there exists an arrow $c_3 \rightarrow c_1$ and there exists an arrow $c_3 \rightarrow c_2$ [20].

Recall the category of pro-objects in C is defined as:

Definition 4.1.2 [Definition 2.2 [20]]. Let C be a category. The category of pro-objects in C is the category defined as follows.

- The objects are pro-objects in C .
- The set of arrows from a pro-object $F : D \rightarrow C$ to a pro-object $G : E \rightarrow C$ is the limit of the functor $D^{op} \times E \rightarrow Set$ given by $Hom_C(F(-), G(-))$.
- Composition of arrows arises, given pro-objects $F : D_0 \rightarrow C$, $G : D_1 \rightarrow C$, and $H : D_2 \rightarrow C$ of C , by applying the limit functor for diagrams $D^{op} \times E \rightarrow Set$ to the natural transformation of functors $Hom_C(F(-), G(-)) \times Hom_C(G(-), H(-)) \rightarrow Hom_C(F(-), H(-))$ given by composition in C .
- The identity arrow on a pro-object $F : D \rightarrow C$ arises, using the universal property of a limit, from the identity arrow $Hom_C(F(c), F(c))$ for every object c of C .

As we note in [4], we could also construct the pro-diamond pro-object of the category of diamonds by taking the isomorphism classes of diamonds under the diamond equivalence relation as pro-objects

4.2 Pro-Emergence

We model local condensed time as formal spectra. Recall

A formal spectrum is a generalization of prime spectrum to adic noetherian rings, therefore containing information on all infinitesimal neighborhoods, corresponding to the ideal of completion...A formal spectrum is an example of a formal scheme. Formal schemes in general form certain subcategory of the category of ind-schemes...Adic completion is to have all infinitesimal neighborhoods "at once" [22].

Definition 4.2.1 [22]: Assume R is a commutative ring and $I \subset R$ is an ideal, such that its powers make a fundamental system of neighborhoods of zero of a complete Hausdorff topology (we say that R is an separated complete ring in the I -adic topology).The formal spectrum $Spf R$ of (R, I) is the inductive limit of the prime spectra:

- $Spf(R) := \text{colim} \text{Spec}(R/I^n)$ where the connecting morphisms are the closed nilpotent immersions $\text{Spec}(R/I^n) \hookrightarrow \text{Spec}(R/I^{n+1})$ of affine schemes and the colimit is taken in the category of topologically ringed spaces.

Conjecture 4.2.2. Let S be the category of spectra. Pro-emergence is an ind-object in the category of ind-objects of S .

Recall that an ind-object

of a category C is a formal filtered colimit of objects of C ...this means that in particular chains of inclusions $c_1 \hookrightarrow c_2 \hookrightarrow c_3 \hookrightarrow c_4 \hookrightarrow \dots$ of objects in C are regarded to converge to an object in $\text{Ind}(C)$, even if that object does not exist in C itself [21].

We need a 2-morphism connecting the pro-emergence and pro-diamond objects in their respective categories of pro-objects.

Conjecture 4.2.3 $\text{Hom}_{\mathcal{D}^{\infty}}(F(-), G(-))$ for pro-objects pro-emergence and pro-diamond is a 2-localization 2-functor.

Remark 4.2.4. Therefore, our claim of pro-emergence being doubly emergence refers to both pro-emergence and the pro-diamond being pro-objects in the category of pro-objects of \mathcal{D}^{∞} . In the category of pro-objects, there are arrows from objects in the pro-emergence pro-object to objects in the pro-diamond pro-object, guaranteed by the cofiltered assumption.

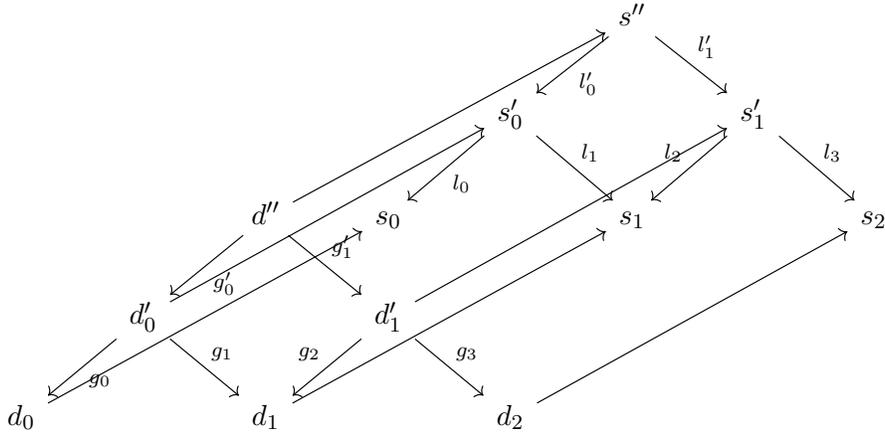


Figure 4: Pro-diamond in \mathcal{D}^{∞} and Pro-Emergence in S

4.3 Temporal Nonlocality from Pro-Emergence

Our immediate goal is to construct a theory of temporal plurivocity that does not fail object persistence. Given there are no ontic primitives, we construct the temporal plurivocity as a pro-emergence with a diamond descent. Our formalism at least allows for a temporal multiplicity in the profinite condition of the diamond and the language of, at least, idempotent infinity-categories to model object persistence.

Our theory of pro-emergence, profinitely many copies of emergence, which is a theory of emergent time, features a double emergence that should be immediate from our diamond holographic principle, wherein ADS/CFT [?] is modified by our proposed six operations/diamonds pro-duality. It remains to link the pro formalism above with the condensed interior and six operations. The idea is that local time emerges from the six operations which are translated into a condensed setting. Time is singular here given it is a condensed set, which is a sheaf of sets over the pro-'etale site of a point [1]. Global time is in the diamonds, where the profinite condition is translated into a form of nonlocality in the many incarnations of possible quasi-pro-'etale covers per each geometric point/mathematical impurity. There are implications for coupling in the following form: strong coupling refers to many incarnations of $Spa(C) \rightarrow D$ in the nonlocality of time ; weak coupling refers to condensed and singular perfectoid time.

Incarnation of global time emerges from diamond nonlocality. Local time emerges from condensed sets. A 2-infomorphism, consisting of two pairs of adjoint functor's from Scholze's 'etale cohomology of diamonds [13], connects local to global time to get temporal simultaneity and temporal nonlocality. In this construction, we are asking what is the difference between simultaneity and temporal nonlocality? That is, what are the preconditions to have either simultaneity as nonlocality or a difference between simultaneity and temporal nonlocality.

To complete the coupling dictionary, we need to construct the six operations in condensed setting, and link diamond profiniteness with nonlocality, diamond descent, and diamond localization.

4.4 *V*-Stack Time and Profinite Temporal Nonlocality

We introduce our new model of emergent time, *v*-stack-time, and a concomitant theory of condensed types, which are, proposedly, immediate from our recently introduced diamond holographic principle. Connecting geometrized local Langlands-stacks with emergent time is a new incarnation of a new reciprocity law.

Our model is a double emergence, but profinitely many copies of emergence, making it a pro-emergence. Our model gives levels of nonlocality as a stackification. The movement from local to global localization is by our conjectured diamond descent and diamond localization. We recall that localization in the reflective subcategory is a descent condition.

The question immediate is how to construct diamond nonlocality for temporal multiplicity? The hope is by diamond descent satisfied by *v*-stacks along all covers [13]. If the mathematical essence of strong coupling is an intrinsic irreversibility, we show, forthcoming that a diamond D is reconstructable up to irreversibility by diamond descent, where coupling is in the levels of profinite nonlocality.

4.4.1 Emergent Time on $*_{\text{proét}}$

Conjecture 4.4.1.1. Commensurate with our construction, there conjecturally follows a theory of emergent time, which would be a geometrized theory of time emerging from

condensed sets and locally spatial diamonds. We use a canonical example to model the emergent property of time as a translation to a condensed structure.

Recall the definition of a condensed set and the category $Cond(C)$ of condensed sets.

Definition 4.4.1.2. ([2] Definition 1.2). The pro-étale site $*_{\text{proét}}$ of a point is the category of profinite sets S , with finite jointly surjective families of maps as covers. A condensed set is a sheaf of sets on $*_{\text{proét}}$. Similarly, a condensed ring/group/... is a sheaf of rings/groups/... on $*_{\text{proét}}$.

For C any category, the category $Cond(C)$ of condensed objects of C is the category of C -valued sheaves on $*_{\text{proét}}$. This means a condensed set/ring/group/... is a functor

- $T : \{\text{profinite sets}\}^{op} \rightarrow \{\text{sets/rings/groups/...}\}$
- $S \rightarrow T(S)$

which satisfies $T(\emptyset) = *$ and the following two conditions equivalent to the sheaf condition.

- For any profinite sets S_1, S_2 , the natural map $T(S_1 \sqcup S_2) \rightarrow T(S_1) \times T(S_2)$ is a bijection.
- For any surjection $S' \rightarrow S$ of profinite sets with the fibre product $S' \times_S S'$ and its two projections p_1, p_2 to S' , the map $T(S) \rightarrow \{x \in T(S') \mid p_1^*(x) = p_2^*(x) \in T(S' \times_S S')\}$ is a bijection.

Given a condensed set T , we sometimes refer to $T(*)$ as its underlying set.

There is a canonical example that illustrates topological structures translating to condensed structures.

Example 4.4.1.3. ([2] Example 1.5.) Let T be any topological space. To T there is associated a condensed set \underline{T} , defined via sending any profinite set S to the set of continuous maps from S to T . It follows that

- if T is a topological ring/group/..., then \underline{T} is a condensed ring/group/... .

$$\begin{array}{ccccc}
Spa(F_q) & & Cond(\mathcal{D}^\diamond) & & Spa(Q_p)^\diamond \\
\searrow^{x_b} & & \swarrow^{L_n} & \searrow^{L_n} & \swarrow^{x_b} \\
\mathcal{Y}_{S,E}^\diamond = S \times (Spa\mathcal{O}_E)^\diamond & & & & \mathcal{Y}_{S,E}^\diamond = S \times (Spa\mathcal{O}_E)^\diamond
\end{array}$$

Figure 5: V -Stack Time in $Cond(\mathcal{D}^\diamond)$

Example 4.4.1.4. We consider the simplest case. An event (a topological localization of any particular reference frame) is considered a point in a diamond topological space T . On that point is the pro-étale site $*_{\text{proét}}$, the category of profinite sets S . Global time emerges as the set of continuous maps from all profinite sets S to T . So global time is constructed as a sheaf of sets on $*_{\text{proét}}$; that is, as a condensed set. Emergent time results in passing to the larger category of sheaves to consider a condensed version of

- $F_{\text{cont}}(Shv(\mathbb{R}^n, \mathcal{C})) \simeq \Omega^n F_{\text{cont}}(\mathcal{C})$.

4.4.2 V -stack Time in $Cond(D^\diamond)$

We can update the previous idea to v -stack time. The rough idea is the following:

Let \mathcal{C} be the category of diamonds. Let $Cond(\mathcal{C})$ be the category of condensed objects in \mathcal{C} ; objects are condensed diamonds. Let R be the reflective full subcategory of $Cond(\mathcal{C})$; objects in R are reflections.

Let \mathcal{D}^\diamond be the $(\infty, 1)$ -category of diamonds. We extend the formalism to $Cond(\mathcal{D}^\diamond)$, where x_b is a Fargues-like factorization [19] and L_n is topological localization depicted in Figure 5.

The goal forthcoming is to construct the fiber product of this diagram as a moduli space and a diamond.

Conjecture 4.4.2.1. A second way to get an emergent time is to take diamond equivalence relations of $\mathcal{Y}_{(0,\infty)}^\diamond$ to get v -stacks and consider the moduli space of v -stacks, wherein every point is an incarnation of the $Cond(\mathcal{D}^\diamond)$ diagram (Figure 8), and we consider ∞ -stacks of incarnations of incarnations via a diamond descent condition. Regarding every point

as a local emergence, structurally a formal condensed set, in the sense of formal schemes, passing to ∞ -stacks yields our conjectured pro-emergence.

4.4.3 Object Persistence as Descent

As stated above, the localization condition is already a descent condition. Therefore, our conjecture is that the event of object persistence is built into the very structure of our v-stack pro-emergence and is immediate from the diamond equivalence relation; diamond descent takes place in the multiple incarnations of the covers of the geometric points and allows the double reconstruction of object identity from the higher coherence datum.

4.4.4 Condensed Types

We introduce condensed types are objects weakly equivalent under condensed-equivalence to give a condensed version of our localization sequence and continuous K-theory:

- $K(\mathcal{D}_\diamond) \rightarrow K^{\text{Efimov}}(\mathcal{Y}_{S,E}^\diamond) \rightarrow K^{\text{Efimov}}(\mathcal{Y}_{(R,R^+),E})$ and modification
- $F_{\text{cont}}(\text{Shv}(\mathbb{S}^n, \mathcal{D}^\diamond)) \simeq \Omega^n F_{\text{cont}}(\mathcal{D}^\diamond)$, where:
- \mathcal{D}^\diamond is a stable dualizable presentable $(\infty, 1)$ -category of diamonds.
- \mathcal{D}_\diamond is the complex of v -stacks of locally spatial diamonds.
- $\mathcal{Y}_{(R,R^+),E} = \text{Spa}(R, R^+) \times_{\text{Spa}F_q} \text{Spa}F_q[[t]]$ the relative Fargues-Fontaine curve [19].

4.5 Diapsalmata Interrogatory

We close our discussion of pro-emergence by revisiting the many glowing interrogatories inspiring the formalism.

We first re-pose the grand interrogatory underlying our mathematics. *How can we measure emergent time without constructing it?* When we assume time is emergent, must we ask

from what is it emergent? What does it mean to ask of the eschatology of time? To assume time has an eschatology assumes it is debatable whether time will always exist, in whatever forms it does exist, with what properties and concomitant structure.

4.5.1 Dialetheism Time

In asking of the formalism object persistence, it is crucial to ask what is the shape and duration of time accompanying this formalism. *Is temporal nonlocality instantiation without reciprocitous advent? Is temporal nonlocality dialetheism time? That is, does a diamond construction of temporal nonlocality fail time as it fails all pro-incarnations of duration?*

Can we construct, for instance, a profinite version of temporal nonlocality? If so, what are the conditions for object persistence in a nonlocality that is profinite? In such a set up, what could be the difference between temporal simultaneity and temporal nonlocality?

We contend that local/global movement at the levels of emergence is singular and assumes continuity in the condensed setting, geometrically via sheaves over points and profinitely. We contend that the space of information is globally a v-stack which locally works categorically. We contend that time is a multiplicity in two ways: at the local level of singular information transfer and at the global level connecting the local levels. This double multiplicity beckons a double eschatology.

We revisit our opening interrogatories with a few conjectural answers whose derivation is from our presented formalism. *Even when we adamantly claim that there are no ontic boundaries, the question that is immediate is ontic boundaries of what?* If ontic boundaries are replaced by topological localizations of condensed diamonds, then the ‘of what’ would refer to the v-stack.

We ask *how does object persistence work in a singular case such as anterograde amnesia?* This question is multiply profound, so we at least try to hold onto it with a small assessment. This question perhaps has to do with the relation between thoughts and memory recollection. We are interested in the relation between the stop mechanism and the storage and recollection of profinitely many copies of information in the form of mathematical impure geometric points. We have posited a model of the brain allowing various mathemat-

ical partitions in the form of the profinitely many copies of the diamond structure [8]. If neurons are geometric points which are morphisms of schemes, anterograde amnesia would resemble a sustained truncation of diamond descent. Considering temporal nonlocality, we can model thoughts as profinite reflections of pro-étale topological covers oriented in nonlocality. This may help to model the state of being fully conscious during dreamless sleep, *which perhaps takes the form of sleeping in a diamond hourglass* [6].

When we say two events take place at the same time, what time is that specifically? In our formalism, we would say simultaneity is, at its most basic, a 2-morphism in an infinity-category. It is so very exciting to consider what then is simultaneous in a 3-morphism, and so on.

What would entail a discretization of time and what if time were singular or of a profinite duration? Using the mathematics of adic perfectoid spaces, we are indeed modeling time as singular and fractal-like. A discretization of time, more properly, a nonarchimedean time, should accompany, in quantum mechanics, the same treatment of nonarchimedean space.

Why can we not simultaneously sustain two different temporal experiences? In our formalism, we construct an n-awareness that can do so. We must fully understand the duration of time to understand the seemingly asymmetry of time, though we question how we can measure temporal asymmetry without constructing it. If it is our ontic boundaries contributing to the irreversibility of macro processes, we must fully explore the perfectoid quality of those boundaries and what that means phenomenologically.

How can we measure time without therefore constructing it and making our measurement, therein, antiphrastical? How can we measure emergence without constructing it? Our formalism cannot yet answer these question, but hopes to provide the mathematical models to do so.

How do we parse 'affordance' with a boundary that is profinite? To truly model affordance of totally disconnected boundary requires a new model of discretized time, which our formalism provides in the condensed setting. The formalism of 'information to action' is a descent condition on $Cond(D^\circ)$ [5].

Table 2: Dictionary between quantum physics over Hilbert space and over perfectoid space.

<u>Quantum Physics</u>	<u>Perfectoid Quantum Physics</u>
Hilbert space	perfectoid space
state vectors	geometric points $Spa(C) \rightarrow \mathcal{D}$
\otimes product	\diamond product $SpdQ_p \times_{\diamond} SpdQ_p$
nonlocality	profininitely copies of $Spa(C)$
superposition	pro-étale sheaves on Perf; profinite sets
wavefunction collapse	tilting; perfectoid modular curves S_{K^p}
holographic principal	six functor formalism
quantum topology	étale cohomology of diamonds
operator algebra	non-Noetherian complete valuation ring
unitarity	pro-étale descent datum

5 Diamond Nonlocality in Perfectoid Quantum Physics

We have recently introduced perfectoid quantum physics and diamond nonlocality, restricted to the class of spatial diamonds [3]. Recall that spatial v -sheaves are a restricted class of diamonds with $|\mathcal{F}|$ well-behaved. We proposed a dictionary between quantum physics over Hilbert space and quantum physics over perfectoid space (Table 2).

Our main conjectures are the following:

Conjecture 5.1. Geometric points $Spa(C) \rightarrow \mathcal{D}$ in the diamond are a geometrization of entanglement entropy, taking values in $\mathcal{Y}_{S,E}^{\diamond} = S \times (Spa\mathcal{O}_E)^{\diamond}$.

Remark 5.2. We are using 'geometrization' in the sense of 'making $Spec(E)$ geometric' in a GAGA correspondence for $\mathcal{Y}_{S,E}^{\diamond} = S \times (Spa\mathcal{O}_E)^{\diamond}$ [1], [6]. The global 'visibility' of the geometric points is in the profinitely many copies of $Spa(C)$. Multiple 'profininitely copies' result from multiple quasi-pro-étale covers. Perfectoid entropy measures the number of quasi-pro-étale covers.

We propose perfectoid entanglement entropy as a profinite 'up to' restricted to the pro-étale site and to pro-étale morphisms, which take values in $\mathcal{Y}_{S,E}^{\diamond} = S \times (Spa\mathcal{O}_E)^{\diamond}$. The 'up

to' takes the form of Scholze's six operations in the 'etale cohomology of diamonds [13].

For any map $f : Y \rightarrow X$ of small v -stacks that is compactifiable, representable in locally spatial diamonds and with $\dim.\text{trg } f < \infty$, a functor

- $Rf^! : D_{\text{et}}(X, \Lambda) \rightarrow D_{\text{et}}(Y, \Lambda)$

that is right adjoint to $Rf_!$ [13].

Conjecture 5.3. There is a modularity property of nonlocality taking values in

$$F_{\text{cont}}(\text{Shv}(\mathbb{S}^n, \mathcal{D}^\circ)) \simeq \Omega^n F_{\text{cont}}(\mathcal{D}^\circ) [4].$$

Conjecture 5.4. Nonlocality is categorified using coherent sheaves and geometrized in the étale cohomology of diamonds [13] [4].

Diamond nonlocality is a perfectoid version of nonlocality that arises from the nontrivial geometry of the diamond product

- $\text{Spd}Q_p \times_\diamond \text{Spd}Q_p$.

The moduli space of shtukas is a diamond fibered over

- $\text{Spa}Q_p \times \dots \times_m \text{Spa}Q_p$.

We give long-range entanglement the structure of fibering over these m -fold products.

References

- [1] Clausen, D. and Scholze, P., *K-theory of Adic Spaces*, electronic Algebraic K-theory Seminar, June 1, 2020.
- [2] Clausen, D. and Scholze, P., *Lectures on Condensed Mathematics*, <https://www.math.uni-bonn.de/people/scholze/Condensed.pdf>.
- [3] Dobson, S., *Perfectoid Quantum Physics and Diamond Nonlocality*, in preparation.
- [4] Dobson, S., *Efimov K-theory of Diamonds*, <https://philarchive.org/archive/DOBEKO>.

- [5] Dobson, S., *Diamonds and Condensed Types: The New Mathematics of Pro-Emergence and Temporal Nonlocality*, S&T Foresight Workshop “A quest for an interface between information and action” Proceedings, submitted.
- [6] Dobson, S., *Artemis Blu II: In Diamonds (of Diamond Hourglasses)*, in preparation.
- [7] Dobson, S. and Fields, C. *Quantum Information Theory, Chromatic Types, and Condensed Sets*, in preparation.
- [8] Dobson, S. and Prentner, R., *Perfectoid Diamonds and n -Awareness. A Meta-Model of Subjective Experience*, arXiv:2102.07620 [math.GM].
- [9] Lurie, J., *Higher Topos Theory*, Annals of Mathematics Studies, Vol. 170, Princeton University Press, 2009.
- [10] Scholze, P., *Geometrization of the Local Langlands Correspondence*, lecture notes <http://www.mit.edu/~fengt/Geometrization.pdf>
- [11] Scholze, P., *Lectures on Condensed Mathematics*.
- [12] Scholze, P., *p -adic Hodge theory for rigid-analytic varieties*, Forum of Mathematics, Pi 1 (2013), no. e1.
- [13] Scholze, P., *Étale Cohomology of Diamonds*. arXiv:1709.07343 [math.AG], 2017.
- [14] Scholze, P., *On torsion in the cohomology of locally symmetric varieties*, Annals of Mathematics (2) 182 (2015), no. 3, 945–1066.
- [15] Scholze, P., *Perfectoid Spaces*, Publ. Math. Inst. Hautes Etudes Sci. 116 (2012), 245–313. MR 3090258.
- [16] Scholze, P., *On the p -adic cohomology of the Lubin-Tate tower*, Ann. Sci. Ec. Norm. Super. (4) 51 (2018), no. 4, 811-863.
- [17] Scholze, P. and Weinstein, J., *Berkeley Lectures on P -adic Geometry*, Princeton University Press, Annals of Mathematics Studies Number 207.
- [18] Caraiani, A. and Scholze, P., *On the generic part of the cohomology of compact unitary Shimura varieties*, Annals of Mathematics (2) 186 (2017), no. 3, 649–766.

- [19] Fargues, L., *Geometrization of the Local Langlands Correspondence: an Overview*, arXiv:1602.00999 [math.NT] (2016).
Math. Res. Not. IMRN (2017), no. 10, 2964–2997.
Sci. Ecole Norm. Sup. (4), 16(1):59–89, 1983.
- [20] ncatlab authors. Pro-Object <https://ncatlab.org/nlab/show/pro-object> (accessed January 11, 2021).
- [21] ncatlab authors. Ind-Object <https://ncatlab.org/nlab/show/ind-object> (accessed February 11, 2021).
- [22] ncatlab authors. Formal Spectrum <https://ncatlab.org/nlab/show/formal+spectrum> (accessed January 11, 2021).
- [23] ncatlab authors. triangulated category. <https://ncatlab.org/nlab/show/triangulated+category> (accessed January 11, 2021).