

Time-warps

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Abstract (180 words)

Time and space are conflated in time-warps when asleep we dream. Our wakeful cognitive ability to keep them separate indicates different ways of envisaging self-hood. Awareness that dream-time and life-time are separate is itself a propensity of human minds that has evolved by natural selection with adaptive developments in cerebral neuronal circuitry that underpin human behavioural complexity. The contrast is highlighted between how memory of our spatio-temporal experiences appears to be treated by our brain one way when we are wide-awake and thus well aware of our unfolding personal life-history, and in a different way when, fast asleep, our dreaming dissolves any such awareness and our

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oneiric self-hood seems different from our real-life self-hood. The difference is considered from two angles. Regarding simultaneous memory-storage for an experience recalled from alternative perspectives, a theoretical possibility is proposed of a pro-system comparable to Cantor Ternary Set topology. Regarding the brain, the difference is considered in relation to the temporo-spatial theory of consciousness and the compatible concept of active inference forestalling surprisal by minimising variational free energy according to the free energy principle.

Word-length

The following text has 4,150 words. (These do *not* include the abstract, legends to figures, bibliographical references, or the 40 words of foot-note 1.)

1 Introduction

The familiar stages of my everyday 120-minute walk are recoverable from my episodic memory in orderly sequence that I can imagine easily during my waking hours. Yet when I awoke from sleep, I recalled having dreamt of experiencing *not* the sequential stages of the 120-minute walk, but of having been stuck within the stage that customarily I walk between minutes 75 and 80, as if in my vivid dream I was stranded in a “time-warp” walking tirelessly but never advancing. (I think it unlikely, but cannot know, if also I had been dreaming about the other 115 minutes of my daily walk which on waking up were not recalled.) It as if space and time were transformed reversibly into one another in disconcertingly surprising ways.

Psychiatry, psychophysics, and neuroscience are increasing our knowledge rapidly about how the brain constructs dreams, though some fundamental matters remain unclear, including the neuronal basis of “time-warp” and bizarre dreaming about ourself (“oneiric consciousness”), or the evolution of some aspects of dreaming that are peculiar to humans (e.g., “lucid dreaming” and “sleep-talking”) and plausibly associated with our awareness of “self-hood” and ability to talk about it.

Evidently, although it is unclear just how it does so, our brain keeps separate, thankfully, its episodic memory of the (often bizarre) experiences of our oneiric unreal self from its episodic memory about our real-life personal narrative that is known to our awake-self. Daily life would be mayhem if we fought with someone because our awake-self felt aggrieved by something inflicted on our oneiric self! Natural selection must be thanked for our avoidance of such disasters, but it is unclear at what level it has acted, because, if, as is likely, other mammals dream when asleep, dreaming ought to be an archetypical, genetically-determined, ingrained aspect of mammalian cerebral networks and circadian biorhythms characterised by regular coupling and uncoupling between awake and oneiric consciousness.

2 Dreaming and consciousness in our brain

Compatible, coherent, neuroscientific explanations of the cerebral neuronal underpinning of dreaming are those of psychiatrists Karl Friston and Georg Northoff (e.g., Hobson and Friston, 2012; Northoff et al., 2023). Neurobiologist Joseph LeDoux has published an eloquent, authoritative background to the modern neuroscientific study of consciousness (LeDoux, 2020; LeDoux et al., 2022), and the burgeoning scientific literature about dreaming during normal sleep and induced and pathological states has been reviewed by Scarpelli et al. (2022). The divisibility of an awake person’s consciousness was shown fifty years ago in “split-brain” patients whose cerebral hemispheres had been separated surgically (LeDoux et al., 1977). Therefore, the concept of a particular oneiric consciousness is not unreasonable, and it is reinforced by psychophysical investigation of “lucid dreaming” (Konkoly et al., 2021).

It has been proposed

that sleep is a natural optimization process that is disclosed by the nightly removal of precise sensory information; in other words, the brain can take itself off-line with impunity, so that synaptic plasticity and homoeostasis... can reduce the complexity it has accrued during wakefulness” (Hobson and Friston, 2012).

Dreaming while asleep shows us that cerebral consciousness is not abolished despite our greatly diminished awareness of external impingements and ability to respond to them, hence “unawareness” is a more accurate word to describe the state of the sleeping brain than is “unconsciousness”. Dreaming is reported to happen more often during lighter, “rapid eye movement” (REM) sleep than during deeper, non-REM sleep about which nonetheless there exist reports of dreaming. If dreaming when asleep is characteristic of Mammalia, whose primordial fossils are over two-hundred million years old, circadian rhythms that included sleeping likely had characterized their ancestral Vertebrata for at least twice as long ago. Presumably natural selection was involved in the evolution of both sleep and dreaming while asleep.

Plausibly, physiological homeostatic thermodynamic considerations supported circadian biorhythms that ensured reproductive success (“adaptive”, “Darwinian” fitness). Hobson and Friston (2012) considered

sleep in terms of model optimization, under the free energy principle. The basic idea here is that the brain uses sleep to optimize its generative model of the world during wakefulness. This is akin to post hoc model selection, in which redundant parameters are removed to minimize model complexity and provide a more parsimonious internal model. Whether model optimization of this sort is sufficient to explain why sleep is subject to evolutionary pressure is not an

easy question to answer; however, this perspective provides a principled explanation for the utility of sleep that underwrites the homeostatic and autopoietic imperatives for biological organisms.

They address the matter of,

What is the basic explanation of sleep on offer here? As put nicely by one of our reviewers: “On the one hand, there is a requirement that the senses are shut down so complexity can be minimized. On the other hand, there is the idea that the senses are shut down because there is a learnt regularity about precision – namely that it drops at night – and that the brain simply continues free energy minimization under these conditions. Perhaps these ideas can be combined but, at least initially, they are different: the former says there is evolutionary pressure to sleep, the latter that sleep is a contingent upshot of the fact our free energy minimization happens on a planet that spins.” The reviewer favored the latter explanation and we tend to agree: free energy minimization is all about making the brain a good model of its environment. This means that the brain – and indeed the phenotype – cannot be divorced from its environment. In this setting, natural selection can be regarded as selecting phenotypes (models) with the lowest free energy—or maximizing free fitness in evolutionary theory. . . This means that there is no necessary requirement to suppress sensory input to minimize free energy (or complexity); however, certain species have found a local optimum in a free fitness landscape. . . that exploits night-time to focus on minimizing complexity. In this view, sleep is an opportunistic – and highly effective – process that allows the brain to concentrate on statistical housekeeping and can be regarded as an example of meta-selection — the selection of selective processes. In short, evolution has selected brains that sleep and sleep selects the synaptic connections that constitute brains, where both evolution and sleep minimize free energy or maximize free fitness (Hobson and Friston, 2012).

Their explanation does not account fully for the part played by dreaming when we sleep. For instance, how (and why) can dreaming while asleep involve an unnerving, vivid experience of being stranded in bizarre “time-warps”, sometimes dreamt in colour, sometimes in black-and-white, that are only in part referable to a real-life spatio-temporal narrative that we can recall from episodic memory when wide awake? Moreover, because our dream can include music or conversation (even bilingual conversation), oneiric consciousness may permit cross-modal cerebral processing or recognition of sequences of symbolic referents independent of ostensive spatial reference that are drawn from long-term memory.

Consideration must be given also to the part played by the frontal pole that is far bigger in our prefrontal cerebral cortex than in any great ape and unrecognisable in other Anthropoid Primates (Semendeferi et al., 2011). It is active particularly in controlling

our decisions and plans regarding their alternative advantages or disadvantages, possible rewards *vis-à-vis* likely unwelcome outcomes. LeDoux (2020) proposes that

non-conscious prefrontal representations that fuse sensation and schema-based memory to make visual stimuli meaningful might also constitute penultimate states that are antecedent to conscious experiences. . . . And in some situations, especially involving top-down mental modeling, sensory states may not be needed at all. Better understanding of the penultimate non-conscious state (or states) in various kinds of conscious experiences is crucial, as conscious experience is always preceded by such non-conscious events. . . . the explanatory scope of consciousness research might be enriched by recognizing the complex nature of the connectivity between sensory and memory circuits, and between these and higher cognitive circuits that mostly involve prefrontal cortex. A virtue of the model is that it is potentially applicable to any and all kinds of experiences, whether they involve external stimuli, body states, inner thoughts, or emotions. All are viewed as being rendered conscious through memory-informed conceptualizations that create nonconscious working memory states, which, in turn, are antecedent to conscious experiences.

Sometimes we wake up feeling comforted by a “rewarding” dream. At other times we wake up feeling that our peace of mind has been upset by dreamt details that assail it. Here, the unnerving illusion of a dream is not unlike the vertiginous disconcertment that is evoked by the dizzyingly dream-like artistic use of technical intricacy by Maurits Cornelis Escher (Escher, 2016[1989] or Giovanni Battista Piranesi’s intimidating, exquisitely-detailed drawings of lugubrious imaginary prisons (Angelini and Celli, 2007). Spookiness is an unsettling experience suffused with an eery intuition that time and space seem somewhat to be out of joint with regard to our usual everyday experiences.

Bizarre dreams could be frequent outcomes of the cerebral default-mode network that enhances the predominance of slow, low frequency, brain waves, recorded by electroencephalography (EEG) during REM sleep. These could foster the fusion of various temporal lobe elements that, plausibly, during wakefulness are held separately when EEG records indicate higher-frequency, faster waves (Northoff et al., 2023). From the standpoint of the TTC or “temporo-spatial theory of consciousness” (Northoff and Huang, 2017), Northoff and Zilio (2022) regard the wakeful brain’s register of time and space as involving the transformation of scale-free “temporal duration” by an “intrinsic” autocorrelation mechanism and associating it with “spatial extension” of “intrinsically” scale-free space, so as to attain temporo-spatial “alignment” of stochastically-grounded sequences of stimuli which may result in long-term spatio-temporal alignment. Attainment of alignment could be disrupted by the default-mode network that fosters increasingly slow, low frequency, brain waves during increasingly deeper sleep, leading to a consequent outcome of dreaming with bizarre scale-free temporo-spatial experiences.

The reasoning is compatible with empirical data, though the authors accept that their sufficient explanation does not imply a necessary one (let alone a unique one). It is also compatible (Northoff et al., 2023, Box 1) with the proposal (Hobson and Friston, 2012) that REM sleep is a plausible way of optimising the brain’s generative model of the world by means of neuronal predictive processing, involving a Bayesian model-evidence approach to error prediction, “active inference” (AI) and minimisation of free energy under the FEP or “free energy principle” (for AI-FEP, see Parr et al., 2022 and references therein). Furthermore, a fundamental part played by the environment characterises both the TTC and AI-FEP approaches to understanding the interaction between brain, mind, and behaviour.

An intriguing matter concerns the relation between space and time which seem to be treated by our brain as aspects of our entanglement with the environment. They are far from always being treated by the brain as if they are confined exclusively within separate “silos” bereft of intercommunication. Not only are closed-off “silos” regarded by neuroscientists as unlikely entities in the human brain, but also, moreover, our memories of space and time are interwoven and transposed in the performing and visual arts, dance, song, music, poetry, religion, writing, language and everyday speech. From AI-FEP considerations, and using the information theoretic “surprisal” as coterminous with “surprise” (cf., Parr et al., 2022), it is a plausible proposal (Manrique and Walker, 2023a,b) that in the genus *Homo* there has evolved a broader and more permeable “zone of bounded surprisal” (ZBS) than those “zones” (ZBS) that are the phylogenetical baggage of other Primates (and all other Vertebrata). If that be so, then our human ZBS enables many complex behavioural abilities - including several that involve interpersonal activities unique to our species - by virtue of the cerebral evolution in Hominoid Primates of the underlying Mammalian “hierarchically mechanistic mind” (HMM) (Badcock, 2012; cf., Badcock et al., 2019a,b). Working memory in humans outstrips that of great apes (Read et al., 2022) and undergoes development during a relatively longer proportion of an individual’s life-span (Manrique et al., 2024a).

3 Life-time, linearity, self-hood

A significantly differential aspect is human communicative fluency that likely enhanced technological abilities after perhaps one million years ago or less (Manrique et al., 2024b). The spread of spoken language may have enabled our ancestors’ HMM to interweave space and time in ways that are unavailable to great apes and which allow us to envisage our life-time narrative sequentially, as if in a virtual world, rather than disjointedly as disconnected happenings in disparate spatial contexts. Our spoken utterances have evolved to form sequences of phonemes and symbolic morphemes that enhance mutual comprehension of reciprocal interpersonal discourse. We can arrange and re-arrange real-life experiences in alternative ways that rarely may be envisaged by great apes. Hobson and Friston (2012) mused that

finding order in the real world may not be the same as finding order in the virtual world ... This suggests an optimal balance between rehearsing what has already been learned about the world and exploring new hypotheses and possibilities that could be experienced. This may, in part, explain the curious nature of dream content and be related to the creative and synthetic capacity of the brain that can be harnessed in wakefulness.

Interestingly, speech often occurs when we are dreaming, not only dreamt speech but also speaking aloud while we dream asleep according to bystanders who hear our words. Undoubtedly, this must be a uniquely human aspect of dreaming during sleep.

A remarkable aspect of dreams, nevertheless, is that while dreaming our sense of selfhood is preserved. Even when we lose the ability to recognise that what we are experiencing is indeed happening, we nonetheless perceive this “unreality” as happening to us. In short, we are aware of ourself being the subject of experience. More remarkable yet is that so often we recall what we had dreamt. Why do we store information of things that never happened?

We experience dreams as episodes that involve us personally. That we recall them later shows that we had formed episodic memories of them. We regard it appropriate to refer to consciousness during dreams as oneiric consciousness, to differentiate it from our consciousness when awake. It is no less appropriate to talk of oneiric episodic memories about the personal events experienced and recorded while dreaming. Yet the puzzle still remains: Why does our brain store false episodic events? More to the point: How is the brain able to keep separate the threads of what our awake-self experiences and stores, from what our oneiric-self experiences and stores?

This ability to *not* conflate actual and oneiric personal stored events is extraordinary. The likely explanation is that the human self is a construction that is linked directly to episodic recollection. Thus, we accumulate personal experiences that give us a narrative of who we are. We place things that happened to us personally in an imaginary time-line in which events are arranged in succession and in a unique, irreversible direction where things that happened before are placed first in the line. As long as this temporal line is uninterrupted, our sense of selfhood or of existence across time will remain whole. The temporal narrative corresponds to the ability of our brain to keep separate the different threads of what the awake-self and the oneiric one experience, whilst at the same time keeping actual episodic memories separate from – and impervious to the effect of – fictitious oneiric episodic events. The awake-self stores information in a time-line that is continuous and flowing in a “forward” direction, whereas the oneiric-self’s recollections are not stored in relation to any time-frame.

Therefore, no life-history of our oneiric-self ever develops or evolves in parallel to that of our awake-self. We have actual memories of our infancy and all the different stages of our lives, and we mark special events to create a narrative of our awake-self’s life. In contrast, we fail to produce a narrative for our oneiric-self. Things that happen to the

oneiric-self are disconnected to each other, no doubt because they fail to be registered in a narrative time-line that flows in one direction - they happen off-line, as it were. For instance, I am unable to recall what my oneiric-self did, felt, or experienced when it was eighteen-years-old, or how it felt when he reached thirty. Did my oneiric-self experience a mid-life crisis in his forties? These are meaningless questions because we have failed to create a narrative for our oneiric-self which is disconnected from the time-line of our life. Figuratively speaking, the awake-self is recording events in a tape which is running continuously forward, whereas the tape is stopped when the episodic memories are created and stored for the oneiric-self.

We change as we mature and grow old whilst retaining the sense of our continuity, that is to say, the awareness of being ourselves at different ages. This could be on account of the temporal storage of episodic events in our memory. In other words, we create a first-person narrative of our lives by storing events that happened to us in a unidirectional time line, from the earliest event to the latest. The mental ability to re-visit those past events is what gives us a sense of continuity and integral selfhood as opposed to one of multiple discontinuous “selves” that emerged independently at different times. Our self-aware consciousness reflects a narrative account of our life.

In contrast, there is no sense of ageing connected to our oneiric-self. There is no life-history for our oneiric-self. We cannot create a time-line to recount the life of our oneiric-self because it is not autobiographical. That is so, because the things that happened to our oneiric-self are not stored relative to time. Our oneiric-self shows no awareness of our unfolding life-history’s time-line. While our awake-self has a history, there is no equivalent history for our oneiric-self. Plausibly, it is the subjective perception of time by our awake-self that enables the separation between threads of stored episodic memories about what in fact happened, and those created by our oneiric-self. In our awake-self, episodic memories are stored and recalled relative to a particular time in our past, whereas the oneiric episodes are time-less. This perception of time could also help to explain that in our dreams while asleep we often experience temporal loops where we feel unable to complete an action, or are condemned to repeat the same behaviour as though we are trapped in a time-warp. In dreams, the subjective perception of time is distorted or inexistent. This is because the cerebral default mode during sleep blocks even that persistent part of the consciousness of the drowsy awake-self that is not wholly blocked when we day-dream or take hallucinogenic substances.

4 The oneiric-self as a pro-system

It is not unreasonable to regard narrative time as constructed from the recall of stored episodic memories about waking-state events, which are connected in a topological space, carrying the standard topology and admitting a linear ordering. Thus, the topology of our awake-self’s episodic memory of waking-state events is interpretable in formal terms of

the topology of storage. In stark contrast, oneiric-state “events” occur disconnected from one another in a profinite space that is a compact, Hausdorff topological space, namely, a profinite Stone Space where “events” dreamt by our oneiric-self are *not* stored relative to linear time; hence, a plausible model for dream-time is a ring of p -adic integers, \mathbb{Z}_p , considered as a topological space.

The p -adic number system is a non-Archimedean system that is the completion of the rational numbers \mathbb{Q} under the p -adic metric $|\cdot|_p$ where p denotes a prime number. Interestingly, p -adic numbers are close if their difference is a high power of p . The p -adic numbers have no notion of linear ordering and their shape resembles a self-similar set (Koblitz 1984). Informally, the (ring of) p -adic integers, \mathbb{Z}_p , are written in base p , and admit infinite expansions to the left of the decimal point. In particular, p -adic integers take the form:

$$\sum_{n \geq 0} a_n p^n$$

for $0 \leq a_n < p$. Addition and multiplication are performed on the p -adic integers with the *carrying* method used to add and multiply ordinary integers.

The set of “events” experienced by our oneiric-self can be modelled as a profinite set, where an “event” is a “complete, instantaneous, experienced event” (Dobson and Fields, 2023). From a formal standpoint, a profinite set (Stone space) is a compact, Hausdorff, totally disconnected, topological space. The Hausdorff criterion ensures there is no simultaneous experience occurring in the oneiric dream state. That is, every point in the profinite space uniquely represents an “event” that has occurred oneirically ¹.

In order to understand the criterion of total disconnectedness, account must be taken of another difference between awake-time and dream-time.

A key aspect of the construction of [waking]-time, however, is the assumption that between any two distinct events experienced by some agent A, other things happened (i.e. non-agent-specific “events” occurred) that A did not experience, but some other agent B may have experienced” (Dobson and Fields, 2023).

This is not the case for any “event” that is experienced in an oneiric state, because its “events” are totally disconnected. To understand this, consider the canonical example of a Stone Space, which is the Cantor Ternary Set that is nowhere dense, has no interior points, is negligible, and has measure zero (Lapidus and van Frankenhuijsen, 2013). Thus, if the oneiric-self is composed of “events” that occur in a Stone Space such as the Cantor Ternary Set, then that oneiric-self would be like *Cantor dust* and hence negligible. The *self-similarity* of the Cantor Ternary Set, i.e. its *fractal-like behavior* (Lapidus and van

¹A set of events can be rendered countable by using light profinite sets (Clausen and Scholze 2024). It is possible also to extend the representation to an extremely disconnected profinite set, where the closure of disjoint open subsets is disjoint.

Frankenhuijsen, 2013), could represent the “time-loops” experienced by our oneiric-self. (Figure 1, Figure 2).

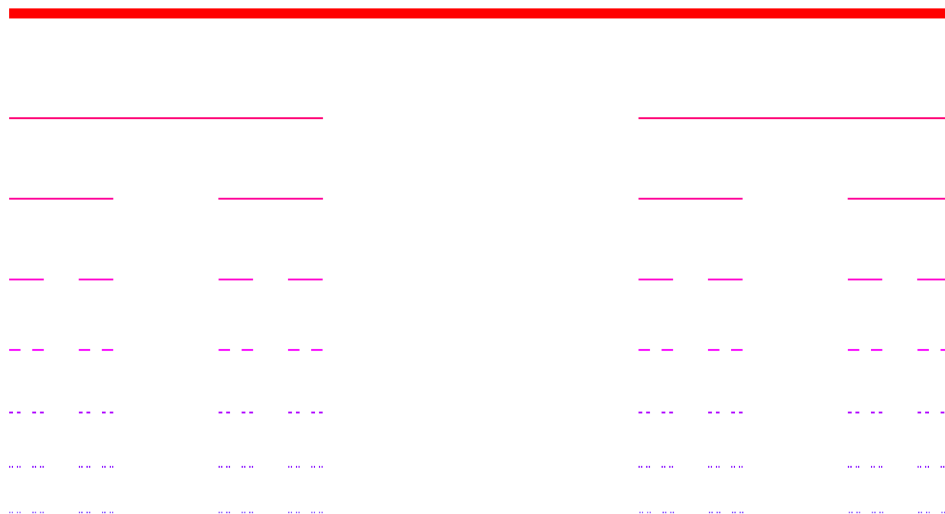


Figure 1: *Cantor Ternary Set*. The Cantor Ternary set is constructed iteratively from an infinite process as follows: beginning with the closed interval $[0,1]$, remove the open middle third $(\frac{1}{3}, \frac{2}{3})$ from it; now remove the open middle third from each of the remaining line segments $[0, \frac{1}{3}] \cup [\frac{2}{3}, 1]$. This process is continued infinitely. The Cantor Ternary Set consists of all points that remain, that are never deleted from the original interval $[0,1]$. The Cantor Ternary Set is uncountably infinite, having the same number of points as the interval $[0,1]$ (Lapidus and van Frankenhuijsen, 2013). The image depicts the first seven steps of the infinite construction. [Image@ Shanna Dobson]

For an alternative explanation, it should be remembered that the p -adic metric induces a non-Archimedean norm on the rational numbers, \mathbb{Q} . A non-Archimedean norm on \mathbb{Q} has the curious property that every point in an open ball is a centre of the ball (Koblitz, 1984). Thus, absence of a familiar everyday experience could lead to an oneiric experience of a “loop” phenomenon. Plausibly, therefore, the set of all “events” occurring in an oneiric state is a pro-system, i.e., a system of projective limits, which is a particular inverse system. (Figure 3).

In brief, waking-state events occur in a connected space admitting a linear ordering; i.e. a well-defined notion of before and after; a notion which is not admitted in the profinite setting. Thus, events that occur during our waking state could be stored with linear ordering. The intuition that the waking-state stores episodic memories in linear time can be understood topologically as “consistency of typing across a collection of memories is

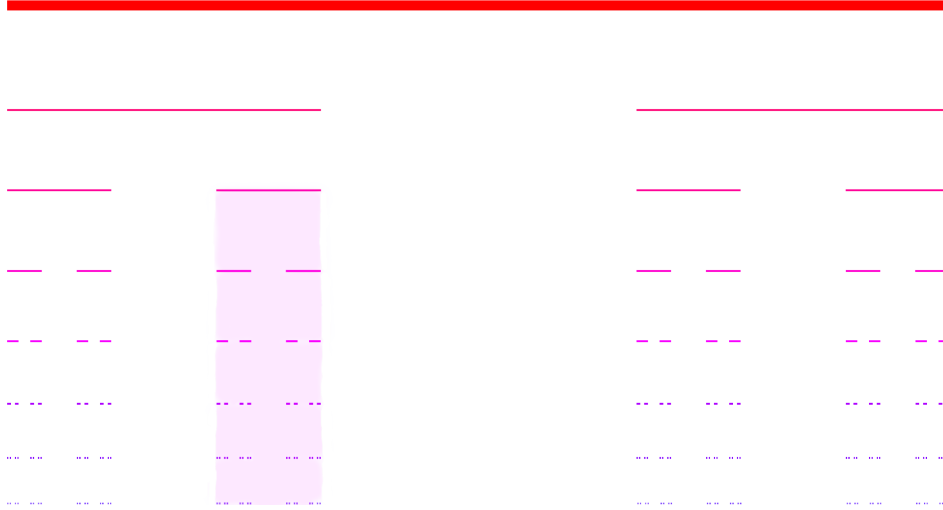


Figure 2: *Self-Similarity of the Cantor Ternary Set.* The Cantor Ternary Set exhibits self-similar behavior; i.e. *fractal behavior*. That is, it is similar to itself upon magnification and translation of its pieces. In particular, two copies of the Cantor Ternary set, each diminished by a factor of $\frac{1}{3}$ and translated, produces the original Cantor Ternary Set (Lapidus and van Frankenhuijsen, 2013). The pink region is a visual depiction of one of these pieces. [Image@ Shanna Dobson]

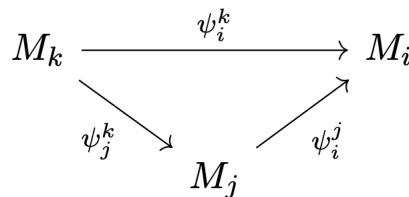


Figure 3: *Inverse System.* (Rotman, 2000) Given a partially ordered set I and a category \mathbf{C} , an inverse system in \mathbf{C} is an ordered pair $((M_i)_{i \in I}, (\psi_i^j)_{j \geq i})$ abbreviated $\{M_i, \psi_i^j\}$, where $(M_i)_{i \in I}$ is an indexed family of objects in \mathbf{C} and $(\psi_i^j : M_j \rightarrow M_i)_{j \geq i}$ is an indexed family of morphisms for which $\psi_i^i = 1_{M_i}$ for all i , and such that the following diagram commutes whenever $k \geq j \geq i$.

effectively a gluing condition” (Dobson and Fields, 2023). Nevertheless, cerebral neuronal microcircuitry has yet to be demonstrated which maintains alternative storage systems and causes switching between them.

5 Dream and surprisal

Several published studies relate dreamt elements of memory to possible future behaviours. One such study leads Wamsley (2024) to suggest that “prospective dreams may be an emergent phenomenon occurring when various fragments of future-relevant episodic and semantic memory are co-activated and combined in novel ways” such that “offline reactivation of future-relevant past memory could potentially function to help prepare us for the future”. Because many remembered dreams lack clear-cut prospective relevance, an over-arching explanation could be that dreaming asleep enables elements held in memory to be juxtaposed by neuronally-mediated, cerebral, combinations and permutations which would challenge our common sense when wide-awake, but which have evolved so as to adapt hierarchically mechanistic minds (HMM) by honing active inference, and the underlying predictive processing, such that there has developed an exceptionally broad band-width of the phylogenetical human Zone of Bounded Surprisal. From the standpoint of adaptive fitness, it could be advantageous to have access to and awareness of two simultaneous storage systems of memory about self-hood: one for memory focused on combinations of successive events in our life-history and another that envisages permutations of temporo-spatially likely and unlikely experiences or “events”. Plausibly, experiencing bizarre combinations of real and surreal experiences helps to reduce surprisal. Thus, dreaming could facilitate the minimisation of free energy and help to broaden the band-width of our ZBS when wide-awake. Indeed, when we are wide-awake, our retention and awareness of oneiric episodic memories may help towards the minimisation of free energy during our wakeful hours. Whenever we experience something so very outlandish, unexpected, and extraordinarily surprising that it fails to register in our HMM, - because the surprisal is ignored on account of differing so greatly from anything that had been predicted by our HMM - , then, to all intents and purposes, it is as if the “something” had not been sampled by the brain at all. However, if outlandish things have been experienced in dreams, then they cease to be surprising, whence it should follow that were something extraordinary to happen during our wakeful hours, the level of surprise evoked might no longer be beyond our phylogenetical Zone of Bounded Surprisal. Awareness in lucid dreaming and wide-awake awareness of the difference between our oneiric self and real-life self implies the presence of a metacognitive mechanism, plausibly involving the frontal pole of the human prefrontal cortex.

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