A Metaphysical Puzzle for Neo-Fregean Abstractionists

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1. What is neo-Fregean Abstractionism?

As my title suggests, I will present a puzzle for neo-Fregean abstractionists (or "abstractionists", for short). Before explaining the puzzle, I would like briefly to describe some salient features of abstractionism. Øystein Linnebo's version of abstractionism will take centre stage, though much of what I will say is applicable, with modifications, to other versions.[1]

Abstractionism may be regarded as a theory about types and tokens (though it not usually described in *quite* these terms). We begin with a class of tokens, and an equivalence relation over that class. Tokens in the same equivalence class are *alike,* or *the same,* in some respect; they have something *in common*. Corresponding to each equivalence class is a type, which stands for this commonality. For example:

Tokens	Equivalence Relation	Types
Physical objects	x and y are similar (in the geometrical sense).[2]	Determinate shapes
Physical objects	x and y are exactly alike in colour.	Determinate colours
Pluralities[3]	x and y are equinumerous.	Cardinal numbers
Well-ordered sequences	x and y are isomorphic.	Ordinal numbers[4]

It will be helpful to focus on just one example; let's take the case of the shapes. Since it will be important to my argument later that things can and do change shape, I will focus on the shapes of *physical objects.* Abstracta (e.g. subsets of $\mathbb{R}^3$) can have shapes too, of course, but this is not my current concern.

[1] Linnebo's version of abstractionism is presented in Linnebo 2018. For other versions, see for example Wright 1983, or Hale and Wright 2001.

[2] According to one standard definition, two things are *similar* if they can be made to coincide by some combination of translation, rotation, reflection, and uniform scaling.

[3] In this paper, I will assume — following Linnebo — that cardinal numbers attach to *pluralities.* Frege of course talked about concepts rather than pluralities here. For now, this issue is of little importance, but it will come back into view in the closing section of the paper.

[4] The first two cases are much simpler than the last two, because they are "predicative" (see Linnebo 2018*,* section 6.2). The case of the ordinal numbers is particularly hairy, because of the Burali-Forti paradox (see Rumfitt 2018 and Hale 2020). One of the great strengths of Linnebo's new "dynamic" version of abstractionism, is that it can include ordinal numbers without inconsistency or adhocery. These issues are, however, orthogonal to my current concerns, and so I put them to one side. I will mostly confine my discussion to the simple predicative cases.

These relationships between types and tokens can be summarized with abstraction principles. In the case of the shapes of physical objects, the abstraction principle would normally be formulated like this:

(1) $\forall x\, \forall y\, (\text{Shape}(x) = \text{Shape}(y) \leftrightarrow x \text{ is similar to } y)$
(The variables range over physical objects.)

The abstractionist doesn't claim only that abstraction principles like this one are *true.* They also make two further claims, (a) an epistemological claim, and (b) a conceptual claim. Linnebo would add, (c), a metaphysical claim. The epistemological claim is that abstraction principles such as (1) are *a priori*, and do not require proof. I will have little to say about this epistemological claim in this paper, but I would like to discuss the conceptual and metaphysical components of abstractionism.

(b) The abstractionist's conceptual claim, as a slogan, is that *tokens are conceptually prior to types.* Consider the following two collections of capacities:

(i) The ability to refer to physical objects; the ability to understand claims about the similarity and non-similarity of physical objects.

(ii) The ability to refer to shape types; the ability to understand claims about the identity and distinctness of shape types.

The abstractionist claims that the capacities mentioned in (i) are *prior* to the capacities mentioned in (ii). That is, it is possible for a person to have the (i)-capacities without the (ii)-capacities, but not *vice versa,* and when someone *does* have the (ii)-capacities, they have them *because* they have the (i)-capacities. To be more specific, one acquires the (ii)-capacities by having an *independent* understanding of claims about the similarity and non-similarity of physical objects, and by accepting the relevant abstraction principle, and by learning to use the abstraction principle appropriately in particular cases.

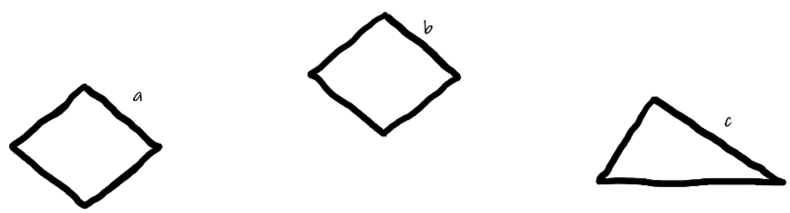
Linnebo develops this point by saying that, given any sentence about the identity and distinctness of shape types, one can use the abstraction principle (1) to generate an "assertibility condition" which can be stated just by reference to physical objects and the relations of similarity and non-similarity between them.[5]

[5] Linnebo 2018, ch. 8. Note that the procedure works only in the case of predicative abstraction principles (Linnebo 2018, section 6.4).

A sentence about shape types …	… and its assertibility condition
Shape(a) = Shape(b)	a is similar to b.
Shape(a) ≠ Shape(b)	a is not similar to b.
a, b, and c all have different shapes.	a is not similar to b; b is not similar to c; and c is not similar to a.
If Shape(a) = Shape(b), then Shape(a) = Shape (c).	If a is similar to b, then a is similar to c.
Every physical object either has the same shape as a or the same shape as b.	Every object is similar either to a or to b.

This means that if you understand the abstraction principle (1), and you are able to apply it in the right way in particular cases, then given any sentence about shape types, you can work out the conditions in which that statement is assertible. And so you know how to use sentences about shape types.

(c) Now let's consider Linnebo's take on the metaphysics of abstraction. For Linnebo, facts about the similarity and non-similarity of physical objects *metaphysically explain* the corresponding claims about the identity and distinctness of shape types (Linnebo 2018, 1.7). For example:



The shape type of a is identical to the shape type of b *because* a and b are similar. And the shape type of a is distinct from the shape type of c *because* a and c are not similar. Linnebo says that the similarity of a and b is "sufficient" for the identity of Shape(a) and Shape(b), adding that the relevant notion of sufficiency is a "species of metaphysical grounding" (2018, pg. 18).[6]

My task in the rest of the paper is to present and discuss a puzzle for abstractionists. The puzzle has both a conceptual and a metaphysical side, and it arises when we think about how to apply abstraction principles to sentences that contain tense operators and modal operators. I will begin with the conceptual side of abstractionism. In section 2, I consider tense; in section 3, modality. In section 4, I consider the metaphysical issues. In section 5, I consider the most discussed case of abstraction: the cardinal numbers. Section 6, which is tentative and exploratory, is a sketch of what may be a partial solution to the puzzle.

6 For discussion of ground, see Rosen 2010, Fine's 2012, or Schaffer 2009. Linnebo cites these three papers approvingly when he discusses the topic.

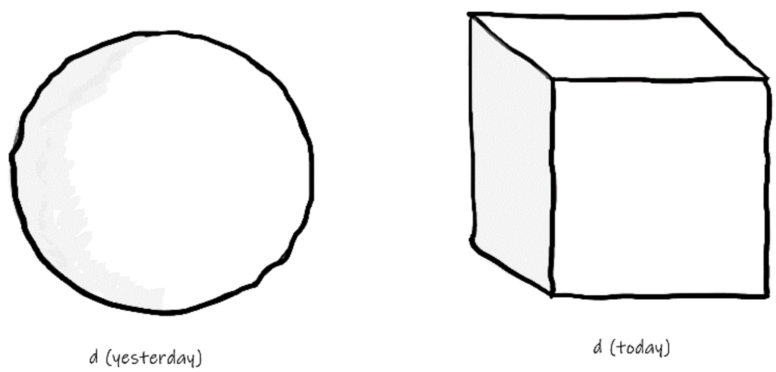
2. Abstraction and tense

For simplicity, I will continue to focus the example of the shape types of physical objects. I'll return to the more important case of cardinal numbers in section 5. The abstraction principle in our example would usually be formulated like this:

(1) $\forall x \, \forall y$ (Shape(x) = Shape(y) $\leftrightarrow$ x is similar to y)
(The variables range over physical objects.)

I assume for the moment that the predicate "is similar to" expresses a *relation* of similarity; I will revisit this contestable assumption later.

We begin with the simple observation that physical objects change shape over time. Suppose that d is a lump of plasticine which was spherical yesterday, but is cubic today. What is Shape(d)?



I suppose we could interpret (1) as saying only that given any *current* physical objects x and y, the shape type of x *now* is identical to the shape type of y *now* if and only if x is *currently* similar to y. But so understood, (1) is obviously far too weak — for it tells us nothing about the shapes of physical objects in the past and in the future.

It might seem at first that there is an easy solution to the problem:

(2) ALWAYS $\forall x \, \forall y$ (Shape(x) = Shape(y) $\leftrightarrow$ x is similar to y)
(The variables range over physical objects.)

"ALWAYS" here is a tense operator meaning "It is always the case that" or "It is at all times true that".

This formulation of the abstraction principle is true, but it is inadequate because it is too weak. To see this, consider the following story. Suppose that a house burnt down completely, so that it was reduced entirely to ash. The person who once owned the house decided to have a new house built in its place. The new house was built with entirely different materials, but it has the same shape type that the old house once had:[7]

As I say, f today has the same shape type that e had in 1950. But what does this mean? (2) doesn't tell us! (2) *does* tell us that, in 1950, something had the same shape type as e just in case it was similar to e. But that is no help, because f did not exist in 1950. (2) *also* tells us that, today, something now has the same shape type as f just in case it is now similar to f. But that is also no help, because e does not exist today. So we're stuck: (2) is simply silent about this case.

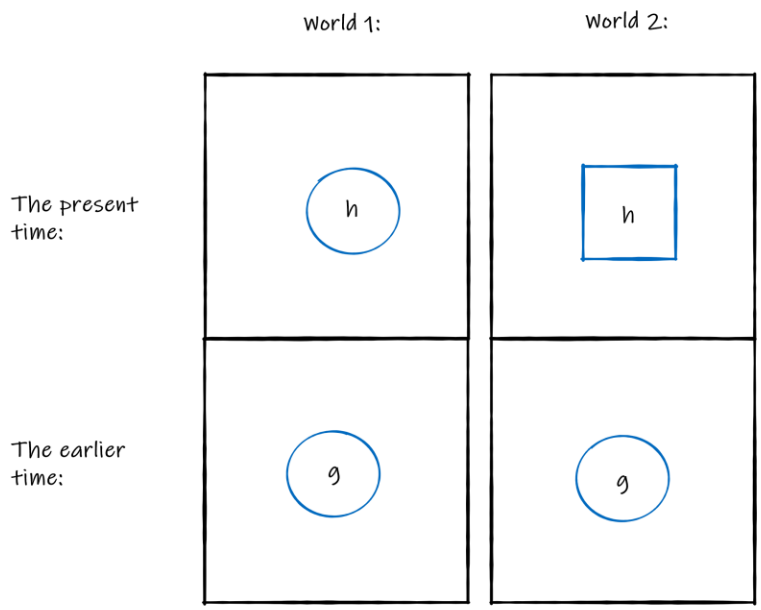
We could put the point like this. (2) enables us to understand claims about the identity and distinctness of shape types *at one time,* but it does not enable us to understand claims about the identity and distinctness of shape types *across times.* (2) is even compatible with the hypothesis that every time has its own class of shape types, so that no two things at different times can share a shape type!

So, as I said, (2) is far too weak.

What we need is a stronger abstraction principle, which enables us to understand claims about the identity and distinctness of shape type across time.

[7] I say that e and f are made from entirely different materials in order to convince you that e and f are numerically distinct. If you are not convinced, you may find your own case in which an object now has the same shape type that a numerically distinct object which no longer exists had in the past.

You may wish to experiment with different formulations; you may find it harder than you expect to find an adequate temporal abstraction principle! Indeed, I think that we cannot succeed in formulating an adequate principle, so long as we confine ourselves to standard tense logic. In standard tense logic, relations are assumed to be *synchronic*. But cross-temporal facts about sameness or distinctness of shape (such as the fact that f today has the shape that e had in 1950) do not supervene on synchronic similarity and non-similarity relations. To see this, consider these two simple possible worlds:



In both worlds, there was a round object g. This object then ceased to exist, and was replaced by a new object, h. In World 1, h is round; in World 2, h is square. Now consider the following sentence:

> h has the shape that g once had.

This sentence is true at World 1 but false at World 2. Now the abstractionist would like to give an "assertibility condition" for this sentence in terms of similarity. I claim that this cannot be done using the resources of standard tense logic. The problem is that, as I said, in standard tense logic, it is assumed that all relations are *synchronic*. Since the synchronic relations of similarity and non-similarity are the same in the two worlds, any sentence of standard tense logic whose only non-logical predicate is "similar" will have the same truth value at both worlds. So no such sentence can be an assertibility condition for "h has the shape that g once had".

My conclusion is that if we are to provide an adequate abstraction principle for the shape types of physical objects, we must go beyond standard tense logic. I know of two ways to do this.

First, a "block universe theorist"[8] may say that, strictly speaking, it is instantaneous objects (objects wholly located at one time) which have shapes. We should interpret the quantifiers in our abstraction principle as ranging over *all* such instantaneous objects — past, present, and future. So the abstraction principle becomes:

(3) $\forall x \, \forall y \, (\mathrm{Shape}(x) = \mathrm{Shape}(y) \leftrightarrow x$ is similar to $y)$
(The variables range over all instantaneous physical objects, including those in the present, those in the past, and those in the future.)

When we say that e in 1950 had the same shape that f now has, what we mean is that f has an instantaneous part, located now, which is similar to an instantaneous part of e, located in 1950.

Second, we may use a hybrid logic which allows for cross-temporal relations.[9] Our abstraction principle could be formulated as follows:

(4) For any physical object x_1 at time t_1, and any physical object x_2 at time t_2, the shape of x_1 at t_1 is identical to the shape of x_2 at t_2 just in case x_1 at t_1 is similar to x_2 at t_2.

The two abstraction principles are similar in an important respect: they both assume cross-temporal similarity relations. And this seems to be inevitable. Recall that e is the house that burnt down and f is its replacement. Then statement (a) is clearly true,[10] and so the abstractionist will wish to give an assertibility condition for (a) in terms of similarity. The only plausible condition is (b), which apparently asserts a cross-temporal similarity relation:

(a) f has the shape that e had.
(b) e is similar to f.

This reveals an apparent incompatibility between abstractionism and a view in the philosophy of time called "serious presentism". The serious presentist holds the following two views. First, they deny that "merely past" or "merely future" objects exist. For example, they insist that Neanderthals do not exist.[11] Second, they

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8 See for example Sider 2001.

9 Kai Wehmeier (2012) describes a modal hybrid logic which allows cross-world predication. What I have in mind is an equivalent hybrid tense logic.

10 I am going to assume here for the sake of simplicity that e didn't change its shape over time, and that f hasn't and won't change shape.

11 They will agree, of course, that Neanderthals *did* exist — but this is an entirely different matter …

claim that things that don't exist can't instantiate properties, or stand in relations. For example, one might be tempted to say that some Neanderthals stand in the relation *ancestor* to some people today. The serious presentist will deny this, saying that because Neanderthals don't exist, they can't stand in relations.[12]

Now consider the story about the burning house, from a serious presentist point of view. The serious presentist will deny that there is a similarity relation between the house today and the house from 1950, insisting that the latter does not exist and so cannot stand in relations. The abstractionist, by contrast, will have to insist that the two houses are similar. And so abstractionism appears to be inconsistent with serious presentism.

Critics of serious presentism may protest at this point that it is obviously absurd to deny that e bears the relation of similarity to f — just look at the picture! This is an instance of the "problem of cross-time relations" — a venerable and much discussed problem in the philosophy of time.[13] The standard serious presentist response to cases like this goes as follows. We first observe that the truth of (a), "f has the shape that e had", poses no threat to serious presentism. In "loglish", this sentence can be paraphrased:

$$\exists S(S \text{ is a shape} \wedge S(f) \wedge \text{PAST: } S(e))$$

This makes it clear that (a) does not imply that e exists, or that e instantiates a property or stands in a relation. (It does assert that e *did* instantiate a property, but that is perfectly compatible with serious presentism). Having argued that the truth of (a) is compatible with serious presentism, the serious presentist may go on to say that (b) is true, so long as (b) is regarded merely as a *paraphrase* of (a). Thus the serious presentist *can* accommodate the truth of (b) after all, as long as it is understood in a particular way.[14]

This is, it seems to me, an entirely adequate response to the challenge to serious presentism posed by sentence (b). It is, however, inconsistent with abstractionism. The abstractionist thinks that our understanding of claims about shape types is derived from a *prior* understanding of claims about the similarity and non-similarity of non-physical objects. This claim is incoherent if the latter are understood as mere paraphrases or reformulations of the former.[15]

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[12] For a defence of serious presentism, see for example Bourne 2006.

[13] John Bigelow (1996) tells us that the problem was discussed by the Stoics and the Epicureans; there is also a discussion in Arthur Prior's 1967 book *Past, Present and Future* (Pg. 170).

[14] This strategy appeared long ago in Prior 1967 (Pg. 170). For a more recent example, see De Clercq 2006.

[15] It is worth mentioning that the problem of cross-time relations appears in other cases. Consider, for example, the allegation that *causal* relations obtain between merely past events and events now — e.g. yesterday's drinking causes one's current hangover.

3. Abstraction and modality

In this section, I will consider abstraction and modality. Before doing so, however, I should explain why this is a particularly pressing problem for the abstractionist. Consider the following familiar theorem from geometry:

> (a) There are, among the shape types, exactly five Platonic solids. (That is, there are, among the shape types, exactly five convex polyhedra, whose faces are congruent regular polygons, with the same number of faces meeting at each vertex.)

What is the assertibility condition of this statement? If we apply Linnebo's method for producing assertibility conditions, we get:

> (b) There are (up to similarity) exactly five physical objects which are polyhedral, convex, with faces that are congruent regular polygons, with the same number of faces meeting at each vertex.

This is not entirely satisfactory. I assume that (a) is true, but it may well be that that there are no dodecahedral physical objects, in which case (b) is simply false. And even if (b) is true, this is only a lucky accident.

Recognizing this problem (in a different case), Linnebo responds by appealing to modality.[16] We may argue that there *could have been* dodecahedral objects, even if in fact there are none, and the *possible existence* of a token suffices for the *actual existence* of the corresponding type.

This is, I think, a convincing response to the problem. But it illustrates that for the abstractionist, modality is no mere side issue. An adequate form of abstractionism must be modal, and an adequate abstraction principle must be modal.

In section 2, I tried to persuade you that an adequate temporal abstraction principle must assume cross-temporal similarity and non-similarity relations. In this section, my claim is that an adequate modal abstraction principle must assume cross-world similarity and non-similarity relations. Since the two cases are closely parallel, I can be brief.

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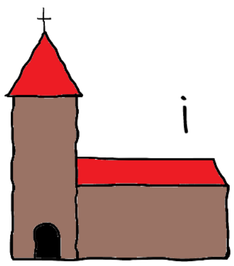
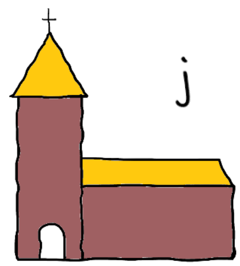
16 For Linnebo, the issue arises in the discussion of numeral types: we wish to say that there are infinitely many numeral types in the Arabic system, even though there are only finitely many concrete numerals. See Linnebo 2018, 187-190.

Consider a story. A church was to be built. Two architects, Ivy and Jeff, submitted designs. It was understood that only one of the two designs would be used. Ivy's design was chosen, and a church was built according to their plans. Now that the building had been completed, Jeff looks at the building, and says, "Huh! What an extraordinary coincidence! If my design had been chosen, the church would have been built from totally different materials,[17] but it would have had exactly the shape type as the actual church."



the actual church



the church that would have been built,
had Jeff's design been chosen

Now consider this sentence:

(a) Ivy's church has the same shape type that Jeff's church would have had.

We may suppose that this sentence is true. But what is its assertibility condition? The abstractionist's answer will have to be some variant of this:

(b) Ivy's church is similar to the church that Jeff would have built.

And (b) appears to posit a similarity relation between the actual church and a merely possible church — a cross-world similarity relation.

I can see two ways of formulating an adequate modal abstraction principle, paralleling the two temporal abstraction principles considered at the end of the last section. First, a Lewisian modal realist (Lewis 1986) might formulate the abstraction principle for the shapes of physical objects thus:

(5) $\forall x \, \forall y \, (\text{Shape}(x) = \text{Shape}(y) \leftrightarrow x \text{ is similar to } y)$
(The variables range over physical objects in all parts of the pluriverse.)

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[17] I say that Jeff's church would have been made from entirely different materials to the actual church in order to convince you that, had Jeff's design been used, the actual church would not have existed. If you are not convinced, you should modify the case accordingly.

From a Lewisian point of view, Ivy's Church is a physical object, and Jeff's church is a physical object, and it makes perfect sense to say that they are similar even though they are in different possible worlds. We might alternatively use a hybrid logic that allows for cross-world relations:

> (6) For any physical object x_1 at world w_1, and any physical object x_2 at world w_2, the shape of x_1 at w_1 is identical to the shape of x_2 at w_2 just in case x_1 at w_1 is similar to x_2 at w_2.

Kai Wehmeier has introduced a formalism, subjunctive logic, which allows us formalize this abstraction principle:[18]

$$\Box_1 \forall_1 x_1 \Box_2 \forall_2 x_2 (Shape^{s_1}(x_1) = Shape^{s_2}(x_2) \leftrightarrow Similar^{s_1, s_2}(x_1, x_2))$$

Wehmeier's formalism is intended to help us understand the logic of *mood* in natural language, and so we might attempt to reformulate (6) using mood instead of explicit quantification over worlds. This is not easy, but here is my best attempt:

> The following is necessarily true. Given any object, it is necessary than an object *would* have the same shape that the former object *does* have if and only if the latter object, as it would be, is similar to the former object, as it is.

If this line of thought is on the right track, it reveals an incompatibility between abstractionism and a view in the philosophy of modality called "serious actualism". The serious actualist maintains first that there are no "merely possible things". For example, while it is true that there could have been dragons, there are no such things as *possible dragons.* The serious actualist adds that things that don't exist can't instantiate properties or stand in relations. For example, one might be tempted to say that certain dinosaurs stand in the relation of resemblance to merely possible dragons; the serious actualist would deny this.[19] Now I have argued that the abstractionist will have to assume cross-world similarity relations. The serious actualist will reject this manoeuvre and with it, abstractionism.

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18 See Wehmeier 2012. Wehmeier doesn't include functions (such as our shape function) in his formalism, but I see no obstacle to adding them. For further discussion of hybrid modal logic see Kocurek 2016.

19 I was converted to serious actualism when I read Quine 1948. For more recent defences, see Forbes 1989 (ch. 3), or Stephanou 2007.

4. The Metaphysics of Abstraction

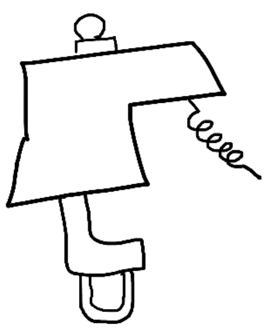
I have now argued that abstractionism is incompatible with both serious actualism and serious presentism. In this section, I will make my way to this conclusion by a slightly different route, focusing on the abstractionist's claims about metaphysical ground.[20, 21]

Recall that, according to Linnebo, facts about the identity and distinctness of shape types are grounded by facts about the similarity and non-similarity of physical objects. For example, it might be that the fact that $\mathrm{Shape}(a) = \mathrm{Shape}(b)$ is grounded by the fact that a and b are similar, and the fact that $\mathrm{Shape}(a) \neq \mathrm{Shape}(c)$ is grounded by the fact that a and c are not similar.

Now consider again the story about the house, e, which burnt down to be replaced by another, f, with the same shape. What grounds the fact that e had the shape that f now has? The abstractionist, it seems, will have to say that this fact is grounded by the fact that e is similar to f — which apparently violates serious presentism.

And consider the story about the churches. What grounds the fact that Ivy's church has the shape that Jeff's would have had? The abstractionist it seems will have to say that this fact is grounded by the fact that i is similar to j — which apparently violates serious actualism.

To reinforce these points, consider a slightly different example. Suppose I walk through a junkyard, and come across an oddly shaped piece of broken machinery, k.



k

20 Recall that Linnebo stresses that it is a particular *species* of ground that is at issue (2018, pg. 18). As far as I can see, differences between different species of ground do not matter to the current argument, so I will continue to speak of grounding *simpliciter*.

21 In earlier work (2016), I presented a different challenge to the metaphysical component of abstractionism. I now believe that it is a strength of Linnebo's "dynamic" abstractionism is that offers a solution to the challenge — see Donaldson forthcoming for details.

Let's stipulate that 'S' refers rigidly to k's shape type. Now what grounds the fact that k has shape type S?

This is a difficult question for the abstractionist to answer. Presumably, the abstractionist will wish to say that this fact is grounded by facts about similarity, but which?

Surely, the fact that k has S is not grounded by the similarity of k with something else. For there may be no other thing with this shape. And even if, by some strange coincidence, there is in some distant place another thing with this exact shape, this other thing is clearly irrelevant to the fact that k has it.

Perhaps the abstractionist will say that the fact that k has S is grounded by the fact that k is similar to itself. But this cannot be right. I assume that if a fact A grounds a fact B, then A necessitates B.[22] The fact that k is similar to itself does not necessitate the fact that k has shape type S, because k would have been similar to itself even if k had had a different shape. Therefore, the fact that k is similar to itself does not ground the fact that k has shape type S.

So, again, what grounds the fact that k has shape type S? It seems to me that the abstractionist will have to reply in the following way. Consider the property *being similar to k as it actually is.* This is a property that an object x at world w has just in case x at w is similar to k at the actual world. Clearly, it is necessary that an object has this property only if it has shape type S. So the abstractionist may claim that the fact that k has shape type S is grounded by the fact that k is similar to k as it actually is.

Now it seems to me that this is not a terribly intuitive view. It seems to me that *having shape S* is a non-modal and intrinsic property of S, but on the proposed account it is modal and relational.[23] But I would not want to put too much emphasis on such muddy intuitions. The more important point is that there is again an apparent conflict between abstractionism and serious actualism.

5. Numbers

It will perhaps be replied that these considerations are not relevant to the neo-Fregean abstractionist, whose primary concern is mathematics: mathematical objects are necessary, and the mathematical facts don't vary from time to time or from world to world, and so these modal and temporal considerations can be put aside.

I think that this reply is mistaken, for three reasons.

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22 There may be some exceptions to this general principle (Skiles 2015). I hope that *this application* of grounding necessitarianism is uncontentious, however.

23 Similar points are made in Fine 2016. See also Whittle 2015.

First, abstractionism is attractive in large part because it offers us a *general* theory of types and tokens. If the abstractionist abandons this aspiration, restricting their account to the unchanging and the necessary, their account must be regarded as unacceptably *ad hoc.*

Second, and more importantly, it is not legitimate for the abstractionist to simply *assert* that the mathematical facts are necessary and unchanging — this is a thesis that must be justified, and this is not straightforward. Let's take the cardinal numbers as an example. Our abstraction principle in this case (usually called "Hume's Principle") is as follows:

Given any pluralities xx and yy:

$$\text{Cardinal}(xx) = \text{Cardinal}(yy) \leftrightarrow \exists R(R \text{ is a one-to-one correspondence between } xx \text{ and } yy)$$

Now I assume that abstractionists will claim that the cardinal numbers are necessary beings.[24] However, Hume's Principle fails to imply this, even when it is preceded by a box. The reason is simple: a boxed version of this abstraction principle tells us only that *within each world* concepts are assigned the same cardinal just in case they are equinumerous. But it is compatible with this that each world has its own cardinals, so that no cardinal number appears at more than one world.[25]

The abstractionist may respond to this point by adducing some *other* justification for the claim that the numbers are necessary beings — perhaps appealing to considerations of parsimony.[26] But even if some such justification is found, a third problem remains: it is difficult to give a temporal or modal version of Hume's Principle without running into difficulties paralleling those discussed above. Let's briefly consider the temporal case:

The number of Jesus' disciples is identical to the number of provinces of the Netherlands.

This is a true statement, but what is its assertibility condition? Presumably that there is a one-to-one correspondence between Jesus' disciples and the provinces of the Netherlands. But to posit this one-to-one correspondence conflicts with serious presentism.

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24 See Linnebo 2019, section 11.1; Hale 2013*;* and Tennant 1997.

25 This is related to an argument made against the "Frege-Russell Definition of Number" by Robert Hambourger (1977). I would like to thank an anonymous reviewer for the reference.

26 I would like to thank two anonymous reviewers for making this point, and for drawing my attention to a relevant interchange on this issue between Hartry Field (on the one hand) and Bob Hale and Crispin Wright (on the other): Hale and Wright 1992, Field 1993, Hale and Wright 1994. Much of the exchange is devoted to Field's fictionalism, which is not my current concern, but it also helpfully clarifies Hale and Wright's views on the modal status of the numbers.

6. Options for the abstractionist

How are we to respond?

It is of course open to the abstractionist to conclude simply that serious presentism and serious actualism are false. One might even say that the above discussion constitutes a persuasive argument against the two views. (Indeed, in conversation I have heard this reply several times.) This is not the occasion on which to thoroughly review the arguments for and against the two views, but perhaps it will not be out of place for me to explain why I personally am unwilling to settle on this conclusion. I tend to think that to reject serious presentism would be no great cost, because the case against serious presentism is in any case rather strong.[27] Serious actualism is different: pending some very forceful argument to the contrary, I think we should endorse serious actualism both on grounds of common sense (of courses there are no such things as possible dragons!) and on grounds of ontological parsimony.[28]

So I would like to be able to provide a reconciliation of abstractionism and serious actualism. I have not myself been able to effect such a reconciliation, but I will finish by outlining what may be the first few steps in the right direction.[29]

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27 On this, see Sider 2001.

28 It is sometimes said that we should reject serious actualism for the following reason. Consider a world at which Socrates doesn't exist. At that world, Socrates instantiates the property of non-existence. And so it is possible for non-existent things to instantiate properties after all! It seems to me that the example is credible only if we fail to distinguish "Socrates exemplifies the property of non-existence" with "not: Socrates exemplifies the property of existence". The former is false at the world in question; the latter is true. On this point see Hanson 2018.

29 I would like to mention, in passing, some other approaches that might be considered.

(a) One might argue that the actualist should allow that there *can* be cross-world relations, in the special case where the relation in question is *internal.* I can't play tennis with my merely possible older sister, but I can be the same height as her. See Brogaard's 2006 for relevant discussion.

(b) One might try to get around the problem by appealing to ersatzism. That is, one could avoid positing similarity relations between merely possible objects by positing instead similarity relations between ersatz stand-ins. This would involve explaining what it means to say that two ersatz stand-ins are "similar". I don't know whether this could be done in a satisfactory way. More importantly, the stand-ins that ersatzers posit are usually abstract objects (e.g. sentences in idealized languages). This is problematic for the abstractionist: the abstractionist wants to explain our knowledge of such abstract things, and so cannot assume such knowledge from the start. For more on ersatzism and serious actualism, see for example Adams 1974, Plantinga 1974, Stalnaker 1976, or, more recently, Wang 2015.

(c) A reviewer briefly described an intriguing alternative approach. Perhaps we should say that, within the actual world, facts about shape types are grounded by facts about similarity and non-similarity. Once the existence of these shape types is secured, they can then be used to make cross-world geometrical comparisons — in particular, they can be used to explain cross-world similarity and non-similarity claims. In summary:

Actual similarity facts ⇒ Facts about shape types ⇒ Cross-world similarity facts

Perhaps I will be able to return to this idea in later work.

I want to suggest that, with some effort, the serious actualist might consistently endorse a version of Hume's Principle in a hybrid modal logic. At a first pass:

> The following is necessarily true. Given any plurality, it is necessary than a plurality would have the same number that the former plurality does have if and only if the latter plurality is equinumerous with the former plurality.

The most important challenge for this approach is to explain (in a manner consistent with serious actualism) what it means to say that two pluralities are similar, when they exist at different worlds. Usually, one interprets the claim that xx_1 and xx_2 are equinumerous as asserting that there is a certain sort of relation, a one-to-one correspondence, between the two pluralities. But this interpretation is unavailable, given the serious actualist's attitude to cross-world relations. So what is to be done?

I propose the following interpretation. To say that xx_1 and xx_2 are equinumerous is to assert the following:

$$\Box(\text{Exists}(xx_1) \wedge \text{Exists}(xx_2) \rightarrow \exists \text{R}(\text{R is a one-to-one correspondence between } xx_1 \text{ and } xx_2))$$

An example may clarify the idea. Suppose I say, "I could have had some daughters, equinumerous with my actual sons." As serious actualists, we cannot interpret the equinumerosity claim here as asserting that there is a one-to-one correspondence between my actual sons and the merely possible daughters — because the merely possible daughters don't exist and so can't stand in one-to-one correspondence. However, we *can* interpret the claim in this way: I could have had daughters, such that, had the daughters co-existed with my actual sons, there would have been a one-to-one correspondence between the daughters and the sons. In this way, we can make cross-world equinumerosity claims without positing cross-world relations.

The strategy has its shortcomings. For one thing, it crucially relies on the assumption that a plurality cannot have different members at different worlds — so a plurality has the same cardinality at every world at which it exists. This means that the approach does not generalize to, for example, the case of the shapes of physical objects: physical objects *can* vary in shape from world to world. At best, we have a "trick" that works in one special case. Moreover, our analysis of equinumerosity will fail if there are pluralities which cannot co-exist. Suppose, for example, it is maintained that the Norse gods could have existed, and the Greek gods could have existed, but there is no world which contains both the Norse and the Greek gods. On this view, our analysis implies — incorrectly, I suppose — that the Norse and Greek gods are equinumerous.

So this proposal is at best a partial solution to the puzzle, but perhaps it is enough to persuade the reader that there may be some hope for serious actualist abstractionism.[30] My conclusion, then, is that there serious presentism and serious actualism apparently conflict with abstractionism. It will not be easy to effect a reconciliation, but perhaps there is some hope.

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[30] I would like briefly to mention that some of the issues discussed here echo discussions in the metaphysics of science about quantities, such as mass and charge. For a recent survey, see Sider 2020, ch. 4.

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