**Hobbes’s Model of Refraction and Derivation of the Sine Law**

**Abstract**: This paper aims both to tackle the technical issue of deciphering Hobbes’s derivation of the sine law of refraction, and to throw some light to the broader issue of Hobbes’s mechanical philosophy. I start by recapitulating the polemics between Hobbes and Descartes concerning Descartes’ optics. I argue that, first, Hobbes’s criticisms do expose certain shortcomings of Descartes’ optics which presupposes a twofold distinction between real motion and inclination to motion, and between motion itself and determination of motion; second, Hobbes’s optical theory presented in *Tractatus Opticus* *I* constitutes a more economical alternative, which eliminates the twofold distinction and only admits actual local motion, and Hobbes’s derivation of the sine law presented therein, which I call “the early model” and which was retained in *Tractatus Opticus II* and *First Draught*, is mathematically consistent and physically meaningful. These two points give Hobbes’s early optics some theoretical advantage over that of Descartes. However, an issue that has baffled commentators is that, in *De Corpore* Hobbes’s derivation of the sine law seems to be completely different from that presented in his earlier works, furthermore, it does not make any intuitive sense. I argue that the derivation of the sine law in *De Corpore* does make sense mathematically if we read it as a simplification of the early model, and Hobbes has already hinted toward it in the last proposition of *Tractatus Opticus I*. But now the question becomes, why does Hobbes take himself to be entitled to present this simplified, seemingly question-begging form without having presented all the previous results? My conjecture is that the switch from the early model to the late model is symptomatic of Hobbes’s changing views on the relation between physics and mathematics.

**Keywords**: Hobbes; optics; Descartes; mixed mathematics.

**1. Introduction**

Hobbes’s optical theory is perhaps his most original and important work as a natural philosopher, and this paper’s main objective is to provide support for this claim by revealing the novelty and consistency of Hobbes’s treatment of refraction. But before we embark on this task, let me briefly outline the chronological order and contents of Hobbes’s optical texts, as the reader might not be familiar with this less known aspect of the thought of the Englishman.

 Hobbes’s optical texts can be separated into two groups, the published and the unpublished[[1]](#footnote-1):

|  |  |
| --- | --- |
| Published Works | Unpublished Works |
| 1644 (1640-41) | *Tractatus Opticus I* (TO I) | 1640 | *Human Nature* (Chap. 2) |
| 1655 | *De Corpore* (Chap. 19, 22, 24, 27) | 1642 | *Tractatus Opticus II* (TO II) |
| 1658 | *De Homine* (Chap. 2-9) | 1642-43 | *Anti-White* (Chap. 9-10) |
|  |  | 1645-46 | *A Minute or First Draught of the Optiques* (FD) |

There are two points worth bringing out about chronology. First, TO I was published as part of the seventh book of Mersenne’s *Universae geometriae mixtaeque mathematicae synopsis* (1644), however, as has been firmly established by Brandt, it was originally part of Hobbes’s critique of Descartes’s *Dioptrique* which was sent to Descartes via Mersenne and which resulted in the unpleasant exchange of letters between the two in 1641.[[2]](#footnote-2) Second, although the exact date of TO II has long been disputed,[[3]](#footnote-3) it has more recently been established on the grounds of the scribal hand of the original manuscript and other evidences that TO II was written after Hobbes arrived in Paris, most likely around 1642.[[4]](#footnote-4), [[5]](#footnote-5)

 Chronological issues aside, I think the chart clearly demonstrates how much a reader would be missing if she only studies Hobbes’s optics through his published works or only the *Elementa*. Not only would the reader be unable to see the connections between his optics and others aspects of his system,[[6]](#footnote-6) she would also underestimate the extent to which Hobbes was confronting Descartes and how much he actually succeeded in coming up with a version of optics that is at least on a par with that of Descartes.

 In what follows, I shall focus on a specific topic that is especially relevant to the polemic between Hobbes and Descartes, i.e. refraction. The explanation of refraction and the derivation of the law of refraction, the so-called sine law, occupies most of TO I; and as the subsequent letters between Hobbes and Descartes show, Hobbes’s main objection (along with other objections) against Cartesian optics is that when trying to explain refraction and derive the sine law, Descartes relied on a distinction between the motion itself and the determination of motion, which distinction is groundless according to Hobbes’s own ontology. I shall read the objection into the treatment of refraction in TO I, and thereby reveal that Hobbes’s explanation of refraction indeed has conspicuous theoretical advantages over that of Descartes. In order to do so, in the next section (section 2) I will present Descartes’ treatment of refraction and its underlying metaphysical grounds, including the aforementioned distinction between motion and determination of motion. In section 3 I shall present Hobbes’s objections to the metaphysical grounds of the Cartesian model and the disadvantages of the Cartesian model revealed by these objections, then, in section 4 I will reconstruct Hobbes’s own model of refraction and illustrate its theoretical advantages over the Cartesian.

 A reader of *De Corpore* might find the above unsatisfactory, since not only the model of refraction in *De Corpore* appears to be completely different from the earlier one, it also does not seem to make any sense at first glance. In section 5 I will show that such an impression is false, because the treatment of refraction in *De Corpore* actually makes much sense if we read it as a mathematical simplification based on the early model. What does not make much sense, however, is the way in which Hobbes presents this model without any preliminaries and as a self-standing model that has physical significance in its own right. In the last section of this paper I will try to explain why Hobbes switched to the late model in *De Corpore*.

**2. Descartes’ Model of Refraction**

 Like Hobbes, Descartes has held a keen interest in optics since the beginning of his career.[[7]](#footnote-7) With regard to the issue of refraction, around 1629 Descartes has proposed to Beeckman that the incident and refracted rays could be treated as two arms of a balance.[[8]](#footnote-8) However, it is clear that Descartes soon abandoned this approach definitively, because Descartes has chosen to tackle the problem of refraction with an altogether different model in the Second Discourse of the *Dioptrique*, which was published in 1637 along with the other *Essais*, but already completed around 1630.[[9]](#footnote-9) There the explanation of refraction and the derivation of the sine law are considered with an analogy to the case in which a tennis ball hits upon a cloth and breaks through it:



First let us suppose that a ball impelled from A towards B meets, at point B, […] a cloth CBE, which is so weak and loosely woven that this ball has the force to rupture it and to pass completely through it, while losing only a part of its speed [*vitesse*]—namely, for example, a half. Now given this, to know what path it must follow let us consider once more that its motion [*mouvement*] differs entirely from its determination to move [*determination à se mouvoir*] in one direction rather than another, from which follows the quantity of these [i.e. motion *and* determination] must be examined separately. And let us also consider that, of the two parts of which we can imagine this *determination* to be composed, only the one that was causing the ball to tend from high to low can be changed in any manner through the encounter with the cloth; and that the one that was causing it to tend toward the right hand must always remain the same as it was, because in no way does this cloth oppose its going in this direction. Then, having described from the center B the circle AFD, and drawn at right angles to CBE the three straight lines AC, HB, FE in such a way that there is twice as much distance between FE and HB as between HB and AC, we will see that this ball must tend toward the point I. For, since it loses half of its speed by going through the cloth CBE, it must take twice as much time to pass below, from B to a certain point of the circumference of the circle AFD, as it took above to come from A to B. And since it loses nothing at all of the *determination* that it had to advance toward the right side, in twice as much time as it took to pass from the line AC to HB, it must make twice as much headway toward this same side, and as a result arrive at a certain point of the straight line FE at the same instant that it arrives at a certain point of the circumference of the circle AFD. This would be impossible were it not going toward I, inasmuch as that is the only point below the cloth CBE where the circle AFD and the straight line FE intersect.

Figure 1, AT VI, pp. 97-98

After considering the ball, Descartes declares that the case of the “action of light” “follows the same laws as the motion of this ball”,[[10]](#footnote-10) except that contrary to the ball, light travels more easily in a denser or harder medium because it is not actual motion but, as the second analogy in the First Discourse shows, an “action or inclination to be moved.”[[11]](#footnote-11) This difference does not matter much as to the calculation, because when considering the case in which the light enters a harder medium, we just need to refer it to the case in which the ball somehow gets accelerated when breaking through the cloth.[[12]](#footnote-12) Also, the sine law, that is, the ratio between the sine of the angle of incidence and that of the angle of refraction is constant, follows easily from above, since the ratio in question, sinABH / sinGBI, is equal to AH / HF, which is determined only by the speeds of the ball above and below the cloth, and is therefore irrelevant to the angle of incidence.

 Although as shown by the reconstruction of Sabra, the mathematics of the above model is clear and consistent,[[13]](#footnote-13) its underlying physical and metaphysical grounds are very complicated. Two problems are most prominent among others, namely, first, why is Descartes entitled to compare the action of light, which is not actual motion but an inclination or tendency to motion, to the actual motion of the tennis ball and thereby claim they “follow the same laws”? Second, what exactly is the all-important distinction between motion and its determination that enables us to treat them differently during refraction? Before we try to answer these questions, it is crucial to note that inclination or tendency [*inclination*, *conatus*][[14]](#footnote-14) is not identical to determination [*determination*, *determinatio*]. Indeed, as we shall see, inclination or tendency to motion, which light is, is motion in potentiality, while determination is one of the two modes of motion.

 Now let me get to the first question. According to Descartes’ official theory of light laid out in *Principia* III. 55-81, the nature of light does not consist in the actual local motion of particles, but rather the centrifugal tendency, endeavor, or inclination of the globules of the second element which constitute the celestial realm originating from the subtle matter of the first element that makes up the stars.[[15]](#footnote-15) Later Hobbes would relentlessly reject Descartes’ conception of tendency to motion which is nonetheless different from actual local motion as unintelligible, which I will discuss in the next section. For now, I would like to first clarify the way in which Descartes himself conceives the notion. Descartes himself obviously thinks that the tennis ball analogy can be accommodated into his continuum theory of light because “the action or the inclination to motion, which I have said must be understood as light, must in this case follow the same laws as motion.”[[16]](#footnote-16) In other words, Descartes thinks that the actual motion of the tennis ball is but a more straightforward way for the less sophisticated minds, the intended audience of *Dioptrique*, to think about the transmission of light in a plenum; more importantly, such a way of thinking about light is harmless because the actual motion of the ball and the transmission of light “follow the same laws”, hence we can still reach the same results by going either way.

However, one can question whether there is ground for Descartes’ assertion that actual motion and the tendency to motion follow the same laws. Almost immediately following the publication of *Diotrique*, Fermat has voiced this very objection, doubting “if inclination to motion should follow the same laws as motion, since there is as much difference between one and another, as there is between power and act [*de la puissance à l’acte*],”[[17]](#footnote-17) to which Descartes replies:

In fact we cannot justifiably doubt that the laws followed by motion [*mouvement*], which as he himself said is an act, are not also observed by the inclination to motion [*l’inclination à se mouvoir*], which is the power of this act: because even though it might not be always true that what was once in power should be in act, it is nevertheless totally impossible that there should be something in act that was not once in power.[[18]](#footnote-18)

Here Descartes happily accepts Fermat’s categorizing inclination and motion as power or potentiality and act or actualized potentiality, and it is clear that for him light, as an inclination to motion, is potential, unactualized motion which nevertheless is like motion in almost every aspect, most importantly, as we shall soon see, in that both motion and the tendency to motion have two modes, speed and determination.

 This leads us to the consideration of the second question. Although Costabel has complained that despite the importance of the distinction between motion and determination, Descartes has not given a precise definition of determination,[[19]](#footnote-19) in fact we do have something very close: in two letters addressed to Mersenne and Clerselier Descartes has described determination as one of the two “modes” or “variations [*varietez*]” of motion [*mouvement*]:

In motion [*mouvement*] two diverse modes must be considered: one is motion alone or speed [*la motion seule ou la vitesse*], and the other is the *determination* of this motion toward a certain direction [*determination de cette motion vers certain coté*], which two modes change with the same extent of difficulty one as another.[[20]](#footnote-20)

As Gabbey notes, here Descartes deliberately distinguishes between two different senses of motion: one is motion, *mouvement*, in a more general sense as the subject of modes; the other is *motion*, a rare French word, meaning motion in a narrower sense as one of the two modes of *mouvement* ormotion as subject.[[21]](#footnote-21) If we read these remarks into the cited text from *Dioptrique*, we can see that the “motion” that Descartes is distinguishing from determination is not motion as subject, but motion as mode or speed [*vitesse*]. Therefore, the underlying ontology behind Descartes’ explanation of refraction is that first, there is motion of the ball as a whole or as subject, then, this motion has two different modes, i.e. a speed and the determination of this speed toward a certain direction. It is worth noting that, unlike what many commentators such as Fermat and Roberval have interpreted,[[22]](#footnote-22) determination is not the mere direction of speed; rather, it is the “directional mode of the motive force” or more simply “the projection of its velocity (or momentum) on that direction”,[[23]](#footnote-23) because otherwise the unchanged determination in the parallel direction cannot be expressed as a distance.

 Now we’ve answered the two questions, we can see that despite its simple appearance, the underlying metaphysical picture of Descartes’ model of refraction is in fact very complicated. We can summarize it with a twofold distinction: first, there is the implicit distinction between the propagation of light as potential motion and the movement of the ball as actual motion, both of which can nevertheless be treated in the same way; second, there is within both potential motion and actual motion a distinction between their two modes, i.e. their speed and determination. In the *Diotrique* passage cited above, when a tennis ball or a light ray is being refracted, the speed of its actual or potential motion is affected as a whole, while the determination of that motion is only affected in the perpendicular direction.

**3. Hobbes’s Critique of the Cartesian Model**

 In TO I, TO II and his correspondence with Descartes Hobbes has launched a systematic attack on the metaphysical grounds of the Cartesian model, especially concerning the two aforementioned distinctions. First, for Hobbes the distinction between tendency or inclination to motion and actual motion is ill-founded. This is because the distinction admits a kind of potential motion which is not local motion yet still an action having causal power, while for Hobbes “every action is local motion [*motus localis*] in the agent, just as every passion is local motion in the patient,”[[24]](#footnote-24) that is, for Hobbes causal action is reduced to local motion. In TO II he further explicates that the reason behind such radical reduction is that “action without motion” is not “imaginable [*imaginabile*]”[[25]](#footnote-25) or “completely unthinkable [*omnino incogitabilis*]”[[26]](#footnote-26):

If someone should strive to conceive it [Cartesian inclination], he would find it inconceivable [*inconceptibile*], because if by *inclination* [*inclinationem*] he understands the power to move simpliciter [*potentiam ad motum simpliciter*], I admit that inclination is not motion but nor would it be endeavor [*conatum*] or beginning of motion, therefore nor would it be action. If he understands [inclination as] the power to move in this or that way [*hac vel illa via*], that is, determined like the power to move downwards in the wine vat,[[27]](#footnote-27) then the *inclination* will indeed be endeavor and action [*conatus quidem et actio*], but truly and properly speaking it is motion, albeit exiguous motion, because the beginning or the first part of motion, as well as every part of motion, is motion.[[28]](#footnote-28)

In this argument one can get a taste of what Bernhardt calls the “radically actualist mechanism” of Hobbes[[29]](#footnote-29): On the one hand, if the Cartesian inclination were pure potentiality, it would not be able to give rise to any determinate subsequent motion, therefore could not be a causal action; on the other hand, if it were determinate enough to produce certain motion, it would indeed be a causal action, but in that case it would already involve the beginning of that motion hence actual motion. Therefore, for Hobbes the Cartesian notion of inclination which is both a causal action and potential motion is inherently paradoxical and unconceivable, and only actual local motion has the causal power to produce certain effects or motions.

 Although Hobbes’s underlying assumption that all causal action only consists in local motion might seem too strong, I think his objection reveals some real weakness in Descartes’ theory of light. As we have seen, Descartes relies on the tennis ball analogy when trying to derive the sine law, and now Hobbes points out that this is because Descartes cannot help but think of the action of light in terms of actual local motion,[[30]](#footnote-30) which shows that local motion is the only candidate for causal action. Admittedly, this is an uncharitable reading of Descartes, nevertheless, if we realize that for *both* Hobbes and Descartes the physical realm is entirely made up of extended matter devoid of substantial forms, and physical objects are but geometrical objects made real, then Hobbes’s objection can in fact be taken to mean that such inclination to motion or *conatus* cannot be fitted into the mechanical conception of nature which he and Descartes share. This objection is further corroborated by the fact that *conatus* in Descartes is not like the positive, active striving that each individual has in the system of Spinoza or Leibniz, but consists more in the counterfactual possibility that *were the body freed from any external impediment, it would stay in its current state*, which does not imply that the *conatus* or inclination is any real property possessed by the body.[[31]](#footnote-31) Thus not only is Descartes’ distinction between actual motion and inclination to motion or *conatus* complicated, it is also questionable whether *conatus* can be accommodated into his own mechanical conception of nature as something that really exists.

 As for the second distinction between motion, whether potential or actual, and determination, Hobbes and Descartes wrangled extensively yet unfruitfully in their correspondence, and Hobbes’s main issue with the notion of determination is also about its ontological status. Both Hobbes and Descartes grant that motion is not a substance, but an accident or mode of body or material substance.[[32]](#footnote-32) Therefore, were determination to be a real mode of motion, it would be a mode in a mode, which Hobbes thinks is absurd:

But what does he mean by the determination being “in motion”? He surely does not mean as in a subject? That is absurd, since motion is an accident; just as it would be absurd to say that whiteness is in a colour, whereas whiteness is a determination of colour in the same way that being moved to the right or the left is a determination of motion.[[33]](#footnote-33)

To this objection Descartes simply sees no reason why a mode or accident cannot be in another mode.[[34]](#footnote-34) The underlying basis of Hobbes’s rejection is a traditional understanding of *determinatio* as the actualization or individualization of what is potential, universal, or indeterminate,[[35]](#footnote-35) combined with an extremely nominalist view:

Firstly, one must know that although the name “man” is a common name [*vox communis*] (one, in fact, of the five names that Porphyry expounds in his *Isagogue*), every man is either Peter or Socrates or some other individual; and that in the same way, therefore, every motion is either this, or that motion, in other words, [a motion] determined by the limits of its *terminus a quo* and [*ad*] *quem*. So just as Socrates and man are not two men, not two things, but one man described by two names (since it is the same thing which is names “Socrates” and named “man”), *in the same way* “motion” and “determined motion” are one motion, and one thing under two names.[[36]](#footnote-36)

What really exist are individual things, and every individual thing is individualized by its proper determinations. However, Hobbes thinks that there is no sense in talking about the “determination” of an individual thing as if it is something different from the individual itself since in reality there are only individuals. Therefore, determination of motion only denotes “determined motion”, a way to consider the individual motion insofar as it is determined.

 Now, if motion and determination are in reality one and the same determined motion, then Descartes’ way of considering the quantities of determination and of motion or speed separately during refraction would be unjustified, since “determination cannot be diminished, unless by determination he [Descartes] understands motion”;[[37]](#footnote-37) furthermore, it would be a self-contradiction to say that on the one hand, “that loss [of motion] must be reckoned in the whole motion [*in motu toto*]”, while on the other hand “only the perpendicular determination not the lateral [determination] is diminished”.[[38]](#footnote-38) Therefore, under this view Descartes’ whole tactic in trying to determine the path of the refracted ray collapses.

 Hobbes’s critique serves to expose certain shortcomings of the Cartesian model: First of all, Hobbes’s ontology, which only admits extended bodies and their actual motions, embodies a central tenet of the mechanistic natural philosophy, that is, everything must be explained in terms of size, shape and motion.[[39]](#footnote-39) And since Descartes shares that ontology with respect to the physical realm, it is not easy to see how the complexities that Descartes wants to introduce into this picture, especially the twofold distinction between inclination/motion and determination/motion, could be fully accommodated. Second, even if we charitably concede its underlying metaphysics, the specific ways in which the “perpendicular” and “lateral” components of the determination are affected during refraction do not seem to follow from it, that is, we still do not understand why is it that only the perpendicular determination is affected while the lateral or parallel determination is not. The second point, already hinted by Hobbes, was explicitly proposed by Fermat,[[40]](#footnote-40) and it has been more recently elaborated by Schuster, who argues that the fact that the velocity of light in a medium is path-independent implies that for Descartes any homogeneous medium is isotropic (i.e. the property of the medium is the same in every direction), while the fact that when entering the new medium only the perpendicular determination is affected implies the opposite.[[41]](#footnote-41)

**4. Hobbes’s Early Model of Refraction**

 In TO I, TO II and FD Hobbes has propounded a model of refraction vastly different from the Cartesian model, in which Hobbes considers the propagation of light as the actual local motion of a “line of light [*linea lucis*],” the “wave front” of the successive motion of the medium propelled by the local motion of the lucid body, and the light ray [*radius/radiatio*] as the path [*via*] traversed by the light ray.[[42]](#footnote-42) The only premises that Hobbes makes use of in his model of refraction are certain rules regulating the motion of line of light through medium.[[43]](#footnote-43) *First, the line of light always moves in its perpendicular direction.* In terms of geometrical figure, the two sides of the light ray formed by the motion of the two end points of the line of light which Hobbes calls the “sides of light [*latera lucis*]” are perpendicular to the line of light and thus parallel to each other. Hobbes’s explanation for this is that the lucid body (star, fire, etc) must act “with all the forces [*totis viribus*]”[[44]](#footnote-44) or “as much as it can [*quantum potest*]”,[[45]](#footnote-45) and only ray propagated perpendicular to the line of light originating from the lucid body realizes the greatest force of the lucid body.[[46]](#footnote-46) *Second, the width of the ray or length of the line of light always remains constant throughout the entire process of propagation.* Hobbes has summoned various explanations for this in different texts: in TO I the explanation seems to be that the line of light is “in the thinnest matter [*materia tenuissima*]” hence cannot be “pulled into quantity greater than itself”;[[47]](#footnote-47) in TO II, Hobbes compares the line of light to a certain “hardest stick [*baculum durissimum*]”[[48]](#footnote-48) which cannot be stretched; in FD, based on the analogy between the propagation of the line of light and the rolling of a cylinder or a cone, the line of light cannot change its shape since “the body [cylinder or cone] is solid, and the parts hold their scituation.”[[49]](#footnote-49) *Third, the velocity of the motion of line of light is inversely proportional to the density of medium.* Hobbes’s explanation for this is that a denser medium is “less movable [*minus mobile*]”, and in which there’s more “obstacle” to the impinging motion, hence making the motion “weaker [*debilior*]” or slower.[[50]](#footnote-50)

 With these rules governing the motion of the line of light being proposed, the inquiry is now reduced to a Euclidean problem: To find out the path of the line of light during refraction such that it satisfies all the above rules.[[51]](#footnote-51) As we shall see, Hobbes has come up with a pretty satisfactory solution to this problem, and the sine law of refraction is but a property of this constructed geometrical figure.[[52]](#footnote-52)

 Suppose (Figure 2) there are two different media separated by the surface CD, above which the medium is rarer, below denser. Next suppose a line of light AB moves through the upper medium, forming the ray ABFE. When the line of light is propagated to FE, it impinges on CD obliquely, striking on CD at E. Now if the media above and below CD were homogeneous, there would be no refraction hence the line of light would keep moving in its original direction and be propagated to GH, thereby entering the medium below CD completely.[[53]](#footnote-53) However, since the medium below CD is denser than above, according to the third rule, the velocity of the line of light at the end of B would be slower than the other end, therefore when A is propagated from F to G, during the same time B would not be propagated from E to H, but somewhere in between. Suppose it is propagated to I.[[54]](#footnote-54) That is to say, the ratio between the velocity of light in upper medium to that in the lower medium is FG / EI, or, by the third rule, the ratio between the densities of the upper and lower medium is EI / FG, which is a constant since it only depends on the nature of the two media. Now, if the line of light were really propagated to GI, it would have become longer than the original line of light, which is AB, EF and GH. And this violates the second rule, that is, the length of the line of light must remain the same. This preliminary attempt shows that the solution is not that easy to find.

Figure 2

 Nevertheless, Hobbes has come up with an ingenious geometrical procedure to find out the path (Figure 3). First, connect GI and extend it until it crosses the extended FE at point O. Next, with O as center and OF as radius, draw a circle which cuts CD at K; connect KO. Then, again with O as center and OE with radius, draw a circle which cuts KO at L.[[55]](#footnote-55) Now if the line of light were to move along the annular sector FEKL, or *frustum coni* as Hobbes calls it,[[56]](#footnote-56) its motion would satisfy all of the three rules: 1) in any instant of the motion the line of light is moving toward its perpendicular direction, along the tangent line of arc FK and arc EL; 2) the length of the line of light is the same throughout the process, that is, the difference between the radii of the two concentric circles; 3) the ratio between the velocity of E and the velocity of F is equal to FK / EL, which is in turn equal to FO / EO thus FG / EI, hence the motion of E and F along FK and EL satisfies the third rule.

Figure 3

So with the help of the three rules we have found out the path of line of light when it is crossing the interface between two media, and now it is propagated to KL, rather than GI; and, since at KL the line of light is completely immersed in the lower medium, and there’s no longer any difference between the velocities of the two ends (rule 3), the light would then be propagated to the direction perpendicular to KL (rule 1), forming the ray with the two lines tangent to the two concentric circles at K and L respectively as its sides.

Now we’ve constructed the figure, let’s turn to the derivation of the sine law. In this model, how do we compare the sines of the angle of incidence and angle of refraction[[57]](#footnote-57)? First, since the two sides of the ray are parallel, then it doesn’t matter which side we’re operating on. So, let’s now draw the normals of the surface CD at points G and K respectively (Figure 4), and angle PGA would be the angle of incidence, and MKQ the angle of refraction. Now since PGD and GFE are all right angles, hence PGA is equal to FEC; also, since MKL and QKD are right angles, MKQ would be equal to LKD. Next draw FR perpendicular to CD at R, LT perpendicular to CD at T, sinFEC would be FR / FE, and sinLKD would be LT / LK; and since FE and LK are lines of light which are equal in length, therefore sinFEC / sinLKD = FR / LT. Then draw IS perpendicular to CD at S, triangles FGR and IES are similar triangles, therefore FG / EI = FR / IS. Now if IS = LT (this is the most important step), the sine law would have been proved, since the ratio between the sines of the angle of incidence and refraction would be equal to FG / EI, which is the constant ratio between densities of the lower medium and the upper medium. This is not hard to prove: connect IL, and IL would be parallel to CD, since the triangles FGK and EIL are similar triangles. Therefore, any line perpendicular to CD and IL would be equal in length, hence IS = LT.[[58]](#footnote-58)

Figure 4

Now, although the geometrical apparatus employed in the model and demonstration might appear formidable, it is actually rather straightforward and involves nothing above most basic acquaintance with Euclidean geometry. What’s more marvelous about the demonstration is how thin the underlying metaphysics is as compared to that of Descartes: it does away with all the complications about inclination and determination, and what’s left in this minimalist picture is but local motion, the actual change of place of *linea lucis*. And the model is also more consistent as it only relies on three basic rules about the properties of the line of light, and these rules also have immediate physical interpretations that have great intuitive appeal. These rules work pretty well throughout the whole demonstration, especially in finding out the path of line of light during refraction; in comparison, Descartes had to supply the ad hoc rule that the parallel determination of light is unaffected by refraction when it comes to delineating the refracted ray.

 Therefore, I think it is fair to say that Hobbes’s early model of refraction is a serious competitor to that of Descartes. While both of them have successfully obtained the same mathematical result, Hobbes’s model assumes a far less demanding metaphysics and is more consistent throughout.

**5. Hobbes’s Model of Refraction in *De Corpore***

 Although Hobbes’s explanation of refraction and the derivation of the sine law based thereupon have been a relative success, Hobbes seems to have abandoned his early results and altered the model and the underlying physical grounds completely in his magnum opus *De Corpore*. Most conspicuously, the line of light, around which the whole early model was built, has disappeared in the model in *De Corpore*. What is more, to the bafflement of commentators, the model in *De Corpore* does not really make much sense. Shapiro has attributed this to a “decline in Hobbes’s scientific abilities”; more recently Horstmann propounded that Hobbes might have abandoned the early model due to Descartes’ objection that the ray should not have a breadth,[[59]](#footnote-59) which is not very convincing as Descartes’ objection was proposed around 1641 but Hobbes upheld the early model at least until the late 1640s. In what follows, I suggest that the model was already hinted at by the early texts as a mathematically simplified form. After this, I will try to explain why Hobbes switched to the late model in *De Corpore*.

***5.1 The model in De Corpore as a mathematical simplification***

 Now, let us first recapitulate the model itself and briefly reveal what makes it so unintelligible. In *De Corpore* XXIV, Hobbes begins the explanation of model by assuming that an endeavour, or “motion made in less space and time than can be given,”[[60]](#footnote-60) which light presumably is, is propagated from its source A and traverses through the boundary Bb between two different kinds of media with different densities (Figure 5), then:

Let the strait line FG be drawn parallel to the straight line AB, meeting with the straight line bB produced in G. Therefore since AF and BG are also parallels, they will be equal; and consequently, during the time in which the endeavour [*conatus*] is propagated through AF, during the same time the endeavour would be propagated through BG, if the medium were of the same density.[[61]](#footnote-61) But because the medium is denser through BG, that is, it resists the endeavour more than the medium in which AF is, the endeavour will be propagated less in BG than in AF, according to the proportion which the density of the medium, in which AF is, has to the density of the medium in which BG is. Let therefore the density of the medium, in which BG is, be to the density of the medium, in which AF is, as BG is to BH; and let the measure of time [*temporis mensura*] be the radius of the circle. Let HI be drawn parallel to BD, meeting with the circumference in I; and from the point I let IK be drawn perpendicular to BD; which being done, BH and IK will be equal; […] Therefore as within time AB, which is the radius of the circle, the endeavour is propagated in the rarer medium through AF, within the same time, that is time BI, it will be propagated in the denser medium from K to I. Therefore BI will be the refracted [line] of the straight [line] AB.[[62]](#footnote-62)

Figure 5

Excerpted from figure 3 of chap. 24, *De Corpore*

The derivation of the sine law based thereupon is extremely straightforward, since the ratio between the sine of the angle of incidence and that of the angle of refraction (or angle refracted) is but AF / KI, which by stipulation is the constant ratio between the densities of the two media. But how are we to understand the model itself? Under one most obvious interpretation Hobbes has retaken a quasi-Cartesian method of decomposing motion, in which only the parallel component of the velocity is altered (represented by AF and BH in the model). But under this interpretation the overall velocity (AB and BI) composed by its two components would remain the same before and after refraction, and consequently the perpendicular component would have been *increased*, which is hardly plausible. Also, Hobbes himself seems to have resisted this interpretation by defining the radius of the circle as the “measure of time,” hence the equality of AB and BI does not denote the equality of velocity, but equality of time.

 However, this only serves to obscure the picture even more – how can time be represented as part of the motion itself and thus have a decisive role in determining its direction? The explanation for this can be partly attributed to the fact that in *De Corpore* Hobbes freely represents time by “any straight line [*recta quaelibet*]” that could be most conveniently found in his diagram, often a line that also represents a moving body itself or the path traversed by it.[[63]](#footnote-63) The justification for this could be that since “time is the phantasm of motion, insofar as we imagine the before and after, or succession, in motion,” therefore “we measure time by motion,”[[64]](#footnote-64) or to put it more succinctly, “motion is the measure of time [*motum esse mensuram temporis*]”[[65]](#footnote-65); hence, time is not something existing independently of the bodies in motion, on the contrary, “we cannot reckon time which is the phantasm of motion except by exposed motion [*per expositum motum*]”.[[66]](#footnote-66) However, this only justifies representing time with the same kind of geometrical figure as motion, but not representing time with *the same* figure which *also* represents the motion in question, since even in Hobbes’s own examples we need another motion independent from the motion in question in order to measure the latter.[[67]](#footnote-67) Therefore, although including time as part of the diagram is justifiable, making AB and BI, which are also the motions to be determined, the measure of time is a gross categorical mistake.

 Therefore, either we interpret the model in a Cartesian way or in a way that seems to be suggested by Hobbes himself, the model has insurmountable difficulties. What, then, are we to make of this model? I think the model is in fact based on the early model but has been extremely simplified mathematically.

 First, it is worth noting that although the demonstration of the sine law can be carried through (as we have done) based solely on the original form of the early model, Hobbes himself actually conducted the proof on the basis of a slightly simplified form.[[68]](#footnote-68) Hobbes could justifiably simplify the original form of the model because the goal of the demonstration is only to determine the *direction* of the refracted line of light and its relation with the incident line of light, hence the exact *location* of the refracted line of light does not matter as long as the same result about its direction can be obtained. Therefore, although in the original form of the model (Figure 3 and 4) the sector FEKL is the actual path of line of light during refraction, its function is only to find out the direction of the refracted ray KL; and since we have proved in the original form that IS = LT, the direction of the line of light after refraction can be determined solely on the basis of the location of I and its distance to the surface because 1) the two endpoints of the line of light after refraction must be respectively located on CD and its parallel line crossing I, 2) the length of line of light remains constant. Any line of light determined by these conditions are parallel to each other (excluding the obviously impossible case in which the direction is turned to the other side of the normal of the surface), therefore their direction would be the same. That is to say, FEKL can be omitted from the model, thus making the model simpler or less “verbose”.[[69]](#footnote-69) In this simplified form (Figure 6), the location of line of light after refraction can be arbitrary: TO I and FD locate it at GL and TO II locates it at KI, using either G or I as anchor point.[[70]](#footnote-70) As GL and KI are parallel, the same result about their direction, i.e. the sine law, can be proved similarly for both since the ratio between the sine of angle of incidence and that of angle of refraction can be proved to be equal to that between FJ and IS or LT, which is in turn equal to FG / EI.

Figure 6

Figure 6

 Now, if we keep in mind that all that matters is the *direction* of the refracted ray, then there is possibility to even further simplify this already simplified form of the early model because the direction of the ray is fixed by only one of its sides. And Hobbes has already sketched a way to do it in TO I.

 In Proposition 12 of TO I, Hobbes does not start the demonstration with a line of light moving through two media; rather, (Figure 7) he takes line AC and CK taken by themselves as the incident ray (*radius inclinatus*) and the refracted ray (*radius refractus*),[[71]](#footnote-71) and all the rest of the apparatus only serves to determine the relationship between them. First, A is an arbitrary point on the incident ray AC; then draw AB which is perpendicular to AC and meets the surface at B, AB would be the temporary line of light at A.[[72]](#footnote-72) Then, draw parallelogram CABD, and a circle with C and CD as its center and radius. The refracted line of light CE must be a radius of the circle, since it is equal to CD and AB. Also, E’s perpendicular distance to the surface, ER, must be in a constant ratio to AP (A’s perpendicular distance to the surface) as has been proved above. E is uniquely determined by these two conditions, and the direction of CK is thereby also determined because CK is perpendicular to CE. Now since the angle of incidence, ACO, is equal to angle ABP, and the angle of refraction KCL equals ECR, the rest of the demonstration follows easily. This demonstration is nothing other than the very same demonstration sketched in Figure 6 focused exclusively on the side AG.

Figure 7, TO I, Prop. 12, p. 239

 This more simplified version of the model is only one step away from that in *De Corpore*. Now suppose that (Figure 8) the incident ray meets the circle at F, and that the refracted ray meets the circle at K. Draw FG perpendicular to CO at G, and KH perpendicular to CL at H. It can be easily proved that triangle CFG and BAP are congruent triangles (having sides and angles that are equal to those of the other), as well as triangle KCH and ECR; hence the sine law can be proved by simply pointing out that the ratio between the sine of angle of incidence FCG and that of angle of refraction KCH is now equal to FG / KH which is in turn equal to AP / ER, which is constant. And now that the fact that FG / KH is constant has been proved, the rest of the figure can be omitted as mere auxiliary apparatus; what’s left of the figure, as we can see, is but the essence of the model in *De Corpore*.

Figure 8

 Therefore, the model in *De Corpore* is actually a legitimate mathematically simplified form based on the original form of the early model. What is problematic, however, is the way in which Hobbes presents it without proving all the previous results, that is, as a self-standing model that has physical significance in its own right. In the next section I will try to explain why Hobbes switched to presenting this simplified model in a seemingly unjustifiable way in *De Corpore*. My answer mainly has to do with the status of geometry or mathematics (for Hobbes mathematics *is* geometry) in the Hobbesian system.

***5.2 Geometry, mixed mathematics and optics***

 Optics, along with astronomy, mechanics, and music etc. are numbered among the so-called mixed mathematical sciences by Aristotle and in the Aristotelian tradition. These mixed mathematical sciences roughly speaking apply mathematical principles to the study of natural phenomena, so they are “subalternated” to the pure mathematical sciences, i.e. geometry and arithmetic.[[73]](#footnote-73) However, this does not imply that for Aristotle or any bona fideAristotelian the mixed mathematical sciences could reveal the nature and the true causes of natural phenomena; on the contrary, the mixed mathematicians only concentrate on the quantitative aspects of natural phenomena and abstract away from their real underlying physical causes because the study of the latter pertains only to the purview of physicists or natural philosophers rather than mathematicians.[[74]](#footnote-74) And I think Galileo, who was one of the major influences on Hobbes,[[75]](#footnote-75) is in fact closer to an Aristotelian mixed mathematician who does not think that his mathematical studies could reveal the real underlying causes of natural phenomena, despite his famous grandiose statement in *Il Saggiatore* that the universe is “written in mathematical language”.[[76]](#footnote-76) This is implied by many statements dispersed through *Dialogo* and *Discorsi* which express the view that mathematics and physics are distinct sciences, and that the former is irrelevant to the real causes of natural phenomena but only studies their quantitative properties, which Galileo has sometimes called their “symptoms” (*sintomi*).[[77]](#footnote-77)

 Now it is my conjecture that the change in Hobbes’s demonstrations of the sine law reflects his increasingly higher appraisal of the role that mathematics or geometry is supposed to play in the study of natural phenomena: in his earlier stages he was more like a mixed mathematician in the Aristotelian sense, while in his later stages he became more and more convinced in the explanatory power of geometry in physics itself such that physics eventually becomes assimilated into geometry. One point that might present some obstacle to my conjecture is the fact that, as has been pointed by Giudice, optics has been numbered among mixed mathematics by Hobbes consistently throughout his career;[[78]](#footnote-78) nevertheless, we may still ask what is the sense in which Hobbes understands mixed mathematics. The following two texts, one taken from *Anti-White*, the other taken from *De Homine*, I think constitute the *termini a quo* and *ad quem* of Hobbes’s thinking about this issue:

Philosophy is the science of general Theorems […] The first part of it, which is the foundation of other [parts], is the science in which Theorems about the attributes of being in general are demonstrated, which is called first Philosophy. […] Another part is concerned with beings by name and one from another separately, where the ways in which natural effects [are generated] through single natural bodies are demonstrated, and it is for this reason called physics, or natural Philosophy […] [Hobbes then talks about Ethics and Politics] Another part of philosophy considers the relations of space to space, time to time, figure to figure, number to number, and this part constitutes Geometry and Arithmetic, which two items are usually comprehended by the single name of Mathematics […] There are also other parts of Philosophy, such as Astronomy, Mechanics, Optics, Music and other parts hitherto untouched which, as they consider quantity and number not abstractly but in the motion of heavy things, or in the action of lucid things and others which produce sound, must be numbered among the mathematics. (*Anti-White*, I. 1, pp. 105-106)

Therefore many theorems are [*a priori*] demonstrable about quantity, the knowledge thereof is called Geometry, because the causes of the Properties that each figure has are in those lines that we ourselves draw, and the generations of figures depend on our will, and nothing more is required for knowing any particular property of a figure than considering everything that follows from the construction which we ourselves produce in delineating the figure. [Hobbes goes on to say that since we ourselves are not the cause of natural things, the science of natural things, i.e. physics only allows *a posteriori* demonstration which proceeds from observable effect to possible causes]. And because in natural things, which are produced by motion, we cannot proceed from the posterior to the prior by ratiocination without the cognition of those things which follow every kind of motion, nor [can we know] the consequences of motion without the cognition of Quantity that is Geometry, it is impossible that nothing should be demonstrated by a priori demonstration by the Physicist. Therefore Physics, (I mean) *true Physics* which is supported by Geometry, should be listed among *mixed Mathematics*, because those sciences which are not learned by use and experience are usually called Mathematics. (*De Homine*, X. 5, OL II, p. 93)

In the first passage from *Anti-White* we do not see the assimilation of physics into mathematics or mixed mathematics, which are presented separately as independent “parts” of philosophy; indeed, physics even occupies a higher place in the scheme of sciences, and there is scarcely a hint at Hobbes’s later view that physical causes can only be revealed or hypothesized through *a priori* geometry. To be more accurate, Hobbes does hold at this early stage that physics depends on hypotheses,[[79]](#footnote-79) but he does not seem to hold that these hypotheses are *geometrical* in nature. And if an Aristotelian should read this passage, although she would definitely find objectionable many specific views expressed therein, yet she would not find the general scheme of sciences problematic.

 By contrast, in the *De Homine* passage physics itself is identified with mixed mathematics, and the Aristotelian distinction between physics on the one hand, and mathematics and mixed mathematics on the other, is thereby abolished. Geometry, according to Hobbes at this stage, is not a study of abstract figures, but a study of bodies in motion.[[80]](#footnote-80) Therefore, since motion is the only kind of causal action in nature, we could only rely on geometry in order to understand the underlying causal mechanisms of natural phenomena. Of course, the transition from the earlier Hobbes to the mature Hobbes is perhaps not as dramatic as it might appear from what I have said,[[81]](#footnote-81) because Hobbes has held throughout his career that local motion is the only kind of causal action, and it seems that local motion can be mathematized in a very straightforward way. However, I find it curious and significative that it is only as late as in the published version of Chapter VI of *De Corpore* that Hobbes’s well-known view that *a priori* geometry is the study of bodies in motion and thus geometry constitutes the basis for all subsequent parts of philosophy, including physics, finds its first explicit expression.[[82]](#footnote-82) In *Anti-White* and even in an early draft of *De Corpore*,[[83]](#footnote-83) Hobbes does not seem to be willing to take the step and assert that physics is entirely subalternated to geometry.

 What this means for our current discussion is that, perhaps the early model of refraction is proposed as a piece of mixed mathematical inquiry *in the more traditional sense*. That is to say, although the delineation of the figure and the calculation of the quantitative properties of the figure come from geometrical principles, the reason why we choose this particular figure as representing what actually happens during refraction is based on our understanding of the physical process. At the beginning of Section 4 I have explicated the underlying physics of the early model with three rules that the motion of the light ray has to satisfy, and I have pointed out that Hobbes freely draws on analogies with observable physical bodies in order to justify these rules. This shows, I think, the early model depends heavily on our physical intuitions about how motion is supposed to behave in nature. Such physical intuitions impose limitations on how the model should be constructed, as well as endow the model with an intuitive physical meaning at the same time.

 In *De Corpore*, however, since physics is reduced to mixed mathematics and thereby subalternated to *a priori* geometry, our physical intuitions no longer constitute the grounds for the construction of the model. The only way in which physics differs from pure geometry now is that in pure geometry our arbitrary construction of figures and models does not aim to meet any goals, while in physics the constructed model needs to be in accord with observed phenomena. But even in *a posteriori* physics, the construction of geometrical model itself is still *a priori*, that is, it is arbitrary insofar as it only depends on our will.[[84]](#footnote-84) This is why, according to my conjecture, the model of refraction which has previously been presented as part of physics is reassigned to the geometrical Part III of *De Corpore*.[[85]](#footnote-85) Although optics has always been a mixed mathematical science according to Hobbes, in his early writings mixed mathematics is still conceived in a more traditional sense. In this more traditional sense, mixed mathematics only studies the quantitative aspects of natural phenomena, and the real underlying mechanisms are grasped only through physics. Therefore, the early model has to come after the physical hypotheses on the nature and origin of light and how it is transmitted, and the construction of the model itself is also based on certain physical intuitions. Presumably the early model also relies on geometrical principles, but these principles are separate from its physical foundations, and they only tell us how to fill out the specifics of the model and calculate its quantitative properties based on our preconceived understanding of the underlying physical process.[[86]](#footnote-86) By contrast, since in *De Corpore* physics is completely subalternated to geometry, the physical foundations of the late model are also thereby assimilated with its *a priori* geometrical principles. This might have led Hobbes to think that it was justifiable to further develop the *a priori* geometrico-physical foundations of refraction as part of geometry in a way that is more in keeping with his mature view on the relation between physics and mathematics, rather than study refraction in the way of a traditional mixed mathematician. The late model, I think, is given exactly in this context, and the reason why it is counterintuitive and devoid of any straightforward physical interpretation is that it does not rely on our commonsense physical intuitions like the early model, but rather itself constitutes the *a priori* geometrical foundations for subsequent *a posteriori* physics. And as a piece of *a priori* geometry, the late model only depends on the will of maker, that is to say, as long as the maker can draw the diagram, it is *ipso facto* valid and could correspond to some possible natural process.

 Before I conclude, I would like to make two interrelated remarks. First, although I have described the change in Hobbes’s conception of the relation between physics and geometry as the assimilation of physics into geometry, this change could also have reshaped Hobbes’s conception of geometry itself as well. It is been pointed out that Hobbesian geometry is concerned with bodies in motion rather than abstract figures and quantities, however, it is in *De Corpore* that we find perhaps the first explicit statement of this idiosyncratic view of geometry.[[87]](#footnote-87) According to an earlier statement in the beginning section of *Anti-White* cited above, geometry and arithmetic study “the relations of space to space, time to time, figure to figure, number to number”, which interestingly does not mention either body or motion. Of course, perhaps it is advisable to not overestimate the extent of such a change in Hobbes’s thought, which is my second point. Already in *Anti-White* we see that Hobbes tends to obfuscate the boundary between geometry as the study of abstract figures and geometry as the study of concrete bodies, which is most clearly testified by his discussions on the angle of contact in Chapter XXIII, where Hobbes even proposes that the two touching lines be considered as having widths.[[88]](#footnote-88) That Hobbes would tend to view geometry as the study of concrete bodies should be no surprise for us, as Hobbes has held a very strictly mechanical conception of the world, including the human mind itself, right from the earliest stage of his career. Nevertheless, I think Hobbes only started to fully embrace the implications of his thought quite late in his career, possibly when he was working on the published version of *De Corpore* after he published the English *Leviathan*.

 Therefore, if my conjecture regarding the change in Hobbes’s view on the relation between physics and mathematics is roughly correct, then the early model of refraction is the product of Hobbes as a more traditional mixed mathematician. And after Hobbes has fully subalternated physics to geometry, the late model is then proposed as the *a priori* geometrical foundation for subsequent *a posteriori* physics or mixed mathematics in the new Hobbesian sense. This explains the reason why the late model does away with the appeal to our physical intuitions. Thus, Hobbes’s treatments of refraction in his different stages not only constitute one of his major scientific achievements, they also provide us with a clue with regard to the development of his thought.

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1. In what follows, I will cite the titles of the texts as indicated in this chart. A detailed bibliography is provided at the end of this article. [↑](#footnote-ref-1)
2. Although Hobbes’s original letter has been lost, Descartes’ quotations in his reply correspond very closely to the published TO I (Brandt 1927, pp. 94-96). [↑](#footnote-ref-2)
3. For a summary of views, see Giudice 1999, pp. 12-13, note 45. [↑](#footnote-ref-3)
4. Raylor 2005 and Malcolm 2005. Thanks to the reviewer for pointing me to these articles. [↑](#footnote-ref-4)
5. I think there is also another reason to date TO II before *Anti-White* which no commentators have hitherto noticed, that is, in TO II Hobbes falsely posited that under the diastole-systole model of the generation of light, the velocity of the propagation of light should decrease with the distance to the center of the lucid body in a relation based on square roots rather than cubic roots, a glitch that was later fixed in *Anti-White*. Cf. TO II, I. 6-7, p. 149; *Anti-White*, IX. 4, pp. 162-163. [↑](#footnote-ref-5)
6. See Giudice 1999 for a discussion on the connection between optics and physics/physiology, and Malet 2001 for a discussion on the place of vision in Hobbes’s philosophical system. [↑](#footnote-ref-6)
7. For an interpretation of some of the early texts, see Sabra 1981 and Schuster 2000. [↑](#footnote-ref-7)
8. AT X, p. 336. Here Descartes has not explained exactly how this analogy with the balance can help to determine the relation between the incident angle and the refracted angle. An earlier text that deals with refraction was written around 1619-1621, but there Descartes only gives the rough estimate that when the light travels from a rarer to a denser medium it would be refracted towards the normal and *vice versa* (AT X, pp. 242-243). [↑](#footnote-ref-8)
9. “Introduction,” *Oeuvres completes III* (Descartes 2009)*,* pp. 18-19. [↑](#footnote-ref-9)
10. AT VI, p. 100. [↑](#footnote-ref-10)
11. AT VI, pp. 87-88; also see chap. 13 of *Le monde* for a detailed description of light as the centrifugal tendency of the second element. (AT XI, pp. 84-97; esp. p. 86 for the explanation of the centrifugal tendency). [↑](#footnote-ref-11)
12. AT VI, pp. 99-100. [↑](#footnote-ref-12)
13. Sabra 1981, pp. 109-111. [↑](#footnote-ref-13)
14. “Tendency” corresponds to the Latin verb “*tendere*” (in French, *tendre*), which is equivalent to *conatus* (cf. *Principia*, III. 57, AT VIII, pp. 108-109), or “endeavor”. *Conatus* was also translated into French as *effort*, or *inclination* (*Principia* III. 56-57, AT IX B, p. 131; cf. *Le monde* XIII, AT XI, p. 84). [↑](#footnote-ref-14)
15. *Principia* III. 55, AT VIII, p. 108. For a lucid exposition of Descartes’ theory of light in comparison to the modern motion of pressure, see Shapiro 1974. Descartes’ theory of light is perhaps proposed to accommodate light, whose transmission is instantaneous and in straight lines, in a plenum universe where actual motions usually occur in circles. Thanks to the reviewer for pointing out the importance of this issue and its relevance to Hobbes’s polemics with Descartes. [↑](#footnote-ref-15)
16. AT VI, p. 89. [↑](#footnote-ref-16)
17. AT I, p. 450. This is Descartes’ quotation of Fermat’s original letter. [↑](#footnote-ref-17)
18. AT I, p. 451. [↑](#footnote-ref-18)
19. Costabel 1975, p. 237. [↑](#footnote-ref-19)
20. Descartes to Clerselier, February 17, 1645 (AT IV, p. 185). Also see Descartes to Mersenne, April 26, 1643 (AT III, p. 650). [↑](#footnote-ref-20)
21. On the use of French word *motion*, see Gabbey 1980, pp. 311-312, note. 125. [↑](#footnote-ref-21)
22. On the objection of Fermat, see Sabra 1981, pp. 116-136; on the objection of Roberval, see Garber 1992, p. 186. [↑](#footnote-ref-22)
23. Gabbey 1980, p. 258; Knudsen and Pedersen 1968, p. 185. Also see Garber 1992, pp. 188-193. [↑](#footnote-ref-23)
24. TO I, Hypothesis I, OL V, p. 217. [↑](#footnote-ref-24)
25. TO II, I. 8, p. 150. [↑](#footnote-ref-25)
26. TO II, I. 10, p. 152. On Hobbes’s critique of Descartes in TO II, especially concerning the rejection of inclination, see Bernhardt 1979, pp. 435-437. [↑](#footnote-ref-26)
27. This is the second analogy in the First Discourse of *Dioptrique*. [↑](#footnote-ref-27)
28. TO II, IV. 13, p. 207. [↑](#footnote-ref-28)
29. Bernhardt 1977, p. 16, note. 34. [↑](#footnote-ref-29)
30. The following passage from TO II is interesting: “Descartes himself, in order to explain the action of light, always employs words signifying actual motion and he talks about (in refraction) the very [here I read *easque* rather than *eamque*] laws of the motion of the ball.” TO II, I. 10, p. 152. Thanks to the reviewer for directing me to relevant passages. [↑](#footnote-ref-30)
31. For a discussion on the Cartesian *conatus* as merely counterfactual, see Garber 1992, pp. 219-223 and Garber 1994. [↑](#footnote-ref-31)
32. For Hobbes, see *De Corpore*, XV. 1, OL I, p. 175; for Descartes, see Garber’s discussion in Garber 1992, pp. 182-185. [↑](#footnote-ref-32)
33. *Correspondence*, Letter 34, p. 103; translation of Noel Malcolm, pp. 108-109. [↑](#footnote-ref-33)
34. *Correspondence,* Letter 36, p. 117. [↑](#footnote-ref-34)
35. Gabbey 1980, p. 259. [↑](#footnote-ref-35)
36. *Correspondence*, Letter 34, p. 103; translation of Noel Malcolm, p. 108. [↑](#footnote-ref-36)
37. *Correspondence*, Letter 30, p. 69. [↑](#footnote-ref-37)
38. Ibid. [↑](#footnote-ref-38)
39. For general backgrounds of Hobbes’s natural philosophy, see Garber 2008. [↑](#footnote-ref-39)
40. Sabra 1981, p. 117. [↑](#footnote-ref-40)
41. Schuster 2000, p. 267. Schuster later in the same article argues that Descartes in a way reverse-engineered the physical explanation from an already obtain mathematical result. [↑](#footnote-ref-41)
42. TO I, OL V, pp. 221-223; TO II, II. 2, p. 160; FD, pp. 122-124. The description of line of light as a wave front is due to Shapiro 1973, in which he argues that Hobbes’s theory is a forerunner of the subsequent wave theory of light. [↑](#footnote-ref-42)
43. Hobbes himself did not set out these rules in a systematic way. I summarized them from his scattered remarks as well as principles at work during his actual derivation. [↑](#footnote-ref-43)
44. TO I, Postulatum, OL V, p. 226. [↑](#footnote-ref-44)
45. TO II, II. 6, p. 163. [↑](#footnote-ref-45)
46. TO I, ibid. This explanation can be traced back to Alhazen, who thinks that the light perpendicular to the surface has the greatest power hence could not be refracted, and adduces several analogies to explain this, e.g. a sword striking an armor perpendicularly can often break through the armor, but when striking obliquely often fails to do so. Cf. Lindberg 1968, pp. 26-27. [↑](#footnote-ref-46)
47. TO I, OL V, p. 223. [↑](#footnote-ref-47)
48. TO II, II. 3 & II. 7, pp. 161, 163. [↑](#footnote-ref-48)
49. FD, pt. 1, chap. 4, p. 124. [↑](#footnote-ref-49)
50. TO II, II. 10, pp. 165-166; TO I, Prop. VI, OL V, p. 227. This is in fact a more traditional view compared to that of Descartes, cf. Lindberg 1968 and Sabra 1981, pp. 98-99. [↑](#footnote-ref-50)
51. In the *Elements* there are two kinds of propositions, *theorem* and *problem*. The former takes the form of an indicative statement, while the latter is expressed in infinitive with imperative meaning. The goal of *problem* is to construct a figure that satisfies certain conditions. Cf. Beere and Morison, “A Mathematical Form of Knowing How: The Nature of Problems in Euclid’s Geometry”. [↑](#footnote-ref-51)
52. My reconstruction is largely indebted to Shapiro’s (Shapiro 1973, pp. 258-263), though he did not explicitly point out the three rules assumed by Hobbes’s model and Hobbes’s justifications for them. [↑](#footnote-ref-52)
53. TO I, Prop VI, OL V, p. 227; TO II, II. 10, p. 166. [↑](#footnote-ref-53)
54. Ibid. [↑](#footnote-ref-54)
55. TO II, II. 10, p. 166; FD, p. 126a. [↑](#footnote-ref-55)
56. Cf. TO I, OL V, p. 223. [↑](#footnote-ref-56)
57. Hobbes calls “angle refracted [*angulus refractus*]” what we usually understand to be the angle of refraction, i.e. the angle formed by the refracted light and the normal of the surface (cf. *De Corpore*, XXIV, Def. V, OL I, pp. 305-306). But this is simply a terminological issue, and I shall stick to what is more in keeping with our usage. [↑](#footnote-ref-57)
58. Hobbes only used this form of the model to demonstrate the much weaker claim that the angle of incidence or refraction is smaller on the side of denser medium (TO I, OL V, p. 224; FD, pp. 125-126a), but did not demonstrate the sine law based on it. However, as we have seen, it is not hard to prove the sine law under this form of the model, and the form of the model that Hobbes actually used to prove to sine law was only a simplified form based on the face that LT = IS. Cf. TO II, II. 10, pp. 166-167 and Shapiro’s reconstruction of it in Shapiro 1973, p. 260, note 410. [↑](#footnote-ref-58)
59. Shapiro 1973, p. 172; Horstmann 2000, pp. 437-439. [↑](#footnote-ref-59)
60. *De Corpore*, XV. 2, OL I, p. 177. [↑](#footnote-ref-60)
61. Hobbes stipulates that the surface itself belong to the lower medium. [↑](#footnote-ref-61)
62. *De Corpore*, XXIV. 4, OL I, pp. 309-310. Translation based on *Concerning Body*, EW I, pp. 379-380. [↑](#footnote-ref-62)
63. *De Corpore*, XVI. 1, OL I, p. 185. Examples can be found throughout Part III, but one can get a taste of it by taking a look at the following article, XVI. 2, where Hobbes discusses uniform motion. [↑](#footnote-ref-63)
64. *De Corpore*, VII. 3, OL I, p. 85. [↑](#footnote-ref-64)
65. *De Corpore*, XV. 1, OL I, p. 176. [↑](#footnote-ref-65)
66. *De Corpore*, VIII. 16, OL I, p. 101. [↑](#footnote-ref-66)
67. Such as the motion of a horologe, ibid. [↑](#footnote-ref-67)
68. See note 58. [↑](#footnote-ref-68)
69. TO I, OL V, p. 225. [↑](#footnote-ref-69)
70. TO I, OL V, p. 225 and FD, pp. 126j-126m; TO II, II. 10, p. 166. [↑](#footnote-ref-70)
71. TO I, OL V, p. 239, last line and p. 240, l. 2, 14, etc. [↑](#footnote-ref-71)
72. TO I, OL V, p. 240, l. 21. It should be noted that here the length of the line of light is not fixed, since if A were at another place, AB would have been different. Therefore, here the line of light is only a temporary device to determine the direction of CK and is not an actual part of the incident and refracted rays. Shapiro has taken the line of light as a given, and hence has not noticed that here only AC and CK are the rays proper (Shapiro 1973, p. 262). [↑](#footnote-ref-72)
73. For some statements about mixed mathematics in Aristotle, see *Posterior Analytics*, I.7, 75b13-20; I.9, 76a10-15; I.13, 78b34-79a16. [↑](#footnote-ref-73)
74. For a clear exposition of the difference between physics and the mathematical sciences in Aristotle, see Lennox 1986 and Biener 2008, chap. 2. [↑](#footnote-ref-74)
75. For an overview on Galileo’s influence on Hobbes, see Giudice 2016 and Baldin 2020. [↑](#footnote-ref-75)
76. *Opere* VI, p. 232. [↑](#footnote-ref-76)
77. See, e.g., Salviati’s claim that the cause of gravity cannot be known for certain in the Second Day of *Dialogo*, *Opere* VII, pp. 260-261, and Salviati’s similar claim in the Third Day of *Discorsi* that the causes of accelerated motion are largely irrelevant to the mathematical investigations on the properties (*passiones*) of such motion, *Opere* VIII, p. 202. Galileo also said in two letters that he only wanted to investigate the *sintomi* of motion (*Opere* X, pp. 351-352; XVIII, p. 12), and *sintomi* here roughly denote the characteristics and properties of motion in contrast with its generation and true nature. [↑](#footnote-ref-77)
78. Giudice 2016, sec. 4. [↑](#footnote-ref-78)
79. See the opening paragraph of TO II, p. 147. There Hobbes contrasts *a posteriori* physics with *a priori* mathematics, but he does not express the view that the hypotheses that physics must make are made from mathematics. [↑](#footnote-ref-79)
80. On this point see Jesseph 1999, chap. 3. [↑](#footnote-ref-80)
81. Thanks to the reviewer for making me reconsider the issue. [↑](#footnote-ref-81)
82. *De Corpore*, VI. 6, OL I, pp. 62-63. Besides, the chapters on the classification of the sciences in the English *Leviathan* (1651) and its Latin translation (1668) also differ with respect to the status of geometry. The Latin *Leviathan* is more in accord with the view in *De Corpore*, while in the English *Leviathan* we do not see a definitive statement of the prominent status of geometry in Hobbes’s system (cf. Curley’s edition, pp. 47-50). [↑](#footnote-ref-82)
83. Notes on an early draft of *De Corpore* is published by the editors of *Anti-White* as Appendix III of the volume. The early version of the methodological Chapter VI can be found in pp. 472-473, which curiously does not mention geometry or mathematics. As the editors of the volume remark, these notes do not seem to have been written continuously, but rather roughly span the period between 1645 and 1649 (pp. 83-84). [↑](#footnote-ref-83)
84. We could say that Hobbesian geometry (or geometrical physics) is *arbitrary* because it does not need to rely on any physical premise about how bodies are supposed to behave but is only concerned with whether our geometrical construction could be carried through. Arbitrariness in this sense is different from the arbitrariness of conventional definitions, which is also a much-discussed topic in Hobbesian studies. As for a refutation of the view that the Hobbesian science is not arbitrary in the latter sense, see Jesseph 2010. [↑](#footnote-ref-84)
85. Giudice has observed this fact in Giudice 2016, p. 102. [↑](#footnote-ref-85)
86. Hobbes’s derivation of the quantitative relation between the quickness of light and the distance from its lucid origin on the basis of his early model of the generation of light also constitutes an example for this more traditional kind of mixed mathematics: one first assumes a specific physical process – in this case the diastole and systole of the star, and then delineates and derives its quantitative properties (for texts in TO II and *Anti-White*, see note 5). Galileo’s studies of various natural phenomena, e.g. his study of uniformly accelerated motion in the Third Day of *Discorsi*, all roughly follow this approach. As mentioned above, Galileo deliberately avoids entering into discussions of the real causes of these phenomena, but rests content with attaining quantitative results that best fit with our experiments. [↑](#footnote-ref-86)
87. *De Corpore*, VI. 6. And in the earlier draft of *De Corpore* cited in note 83 we do not find such a statement. [↑](#footnote-ref-87)
88. *Anti-White*, XXIII. 18, pp. 283-284. [↑](#footnote-ref-88)